KAON DECAYS IN CHIRAL PERTURBATION THEORY

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Contents

- **Introduction**
- **Semileptonic decays**

  K_{l3} **and CKM**

  $P_{e2}/P_{\mu 2}$

  LD contributions for $K \to \pi\nu\overline{\nu}$

- **Nonleptonic decays**

  $K \to 2\pi, 3\pi$

  Rare decays

  $K_S \to \gamma\gamma, K_L \to \pi^0\gamma\gamma, K_S \to \pi^0\pi^0\gamma\gamma$

  Other rare decays $\longrightarrow$ **Giancarlo D'Ambrosio**

- **Conclusions**

Introduction

Chiral Perturbation Theory (CHPT)

effective field theory of the Standard Model

in the hadron sector at low energies ($E \ll 1$ GeV)

powerful tool for hadron phenomenology for more than 25 years

Weinberg, Gasser, Leutwyler, . . .

main goals

- **reliable results in the confinement regime**
- **search for traces of new physics**

long way from first steps beyond current algebra till today

$\longrightarrow$ **manifested by effective Lagrangian (meson sector)**

Effective chiral Lagrangian (meson sector)

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}^{\text{odd}}_{p^4}(0) + \mathcal{L}^{\Delta S=1}_{G_F p^2}(2) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1)$ $+ \mathcal{L}^{\text{em}}_{e^2 p^0}(1) + \mathcal{L}^{\text{leptons}}_{\text{kin}}(0)$	$L = 0$
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}^{\text{odd}}_{p^6}(23) + \mathcal{L}^{\Delta S=1}_{G_8 p^4}(22) + \mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28)$ $+ \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14) + \mathcal{L}^{\text{em}}_{e^2 p^2}(13) + \mathcal{L}^{\text{leptons}}_{e^2 p^2}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

LECs $\equiv$ **low energy constants**

- **relevant for semileptonic decays**

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- relevant for semileptonic decays
- including radiative corrections

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- **nonleptonic decays (lowest order)**

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- nonleptonic decays (lowest order)
- higher orders and radiative corrections

Semileptonic decays

rich field for CHPT: $K_{l2}, K_{l3}, K_{l4}, \ldots$

K_{l3}

fully inclusive decay rate $\Gamma(K_{\ell 3[\gamma]})$

$$\Gamma = \frac{G_F^2\, M_K^5\, C_K^2}{192\,\pi^3} S_{ew}\, |V_{us}\, f_+^{K^0\pi^-}(0)|^2\, I_{K\ell}^{(0)}(\lambda_i)\, \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)$$

state-of-the-art radiative corrections for all K_{l3} decay modes
available since last summer Cirigliano, Gianotti, Neufeld

both for EM correction factors $\delta_{EM}^{K\ell}$ and for decay distributions

$\delta_{SU(2)}^{K\pi}$: isospin breaking corrections for K^+ (relative to K^0)
depends on quark mass ratio $R = (m_s - \widehat{m})/(m_d - m_u)$

NEW: theor. and exp. determinations of $\delta^{K\pi}_{SU(2)}$ now agree

source	$\delta^{K\pi}_{SU(2)}$ (in %)
$\eta \to 3\pi$ decays	$4.72(44)$
kaon mass splitting	$5.8(8)$
FlaviaNet Kaon WG	$5.4(8)$

N.B.: agreement is evidence for large deviation from Dashen's limit

$$\Delta M_\pi^2|_{\rm EM} = \Delta M_K^2|_{\rm EM}$$

Kastner, Neufeld

Ananthanarayan, Moussallam

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K_{l3}: best source for CKM matrix element V_{us} at present

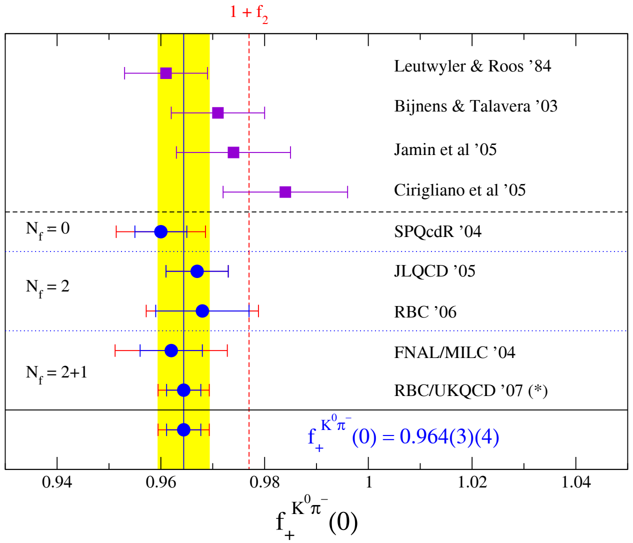
collaboration theory + exp. $\longrightarrow$ $|V_{us}|f_+(0) = 0.21660(47)$

FLAVIANet Kaon WG

some spread in predictions for $f_+(0)$

dominated by lattice results (agreeing with 1984 prediction of Leutwyler, Roos)

Lellouch (Lattice 08)



$$f_+(0) = 0.9644(49)$$ (**UKQCD/RBC**)

yields

$$|V_{us}| = 0.2246(12)$$

perfect agreement with CKM unitarity
(with V_{ud} from nuclear β decays)

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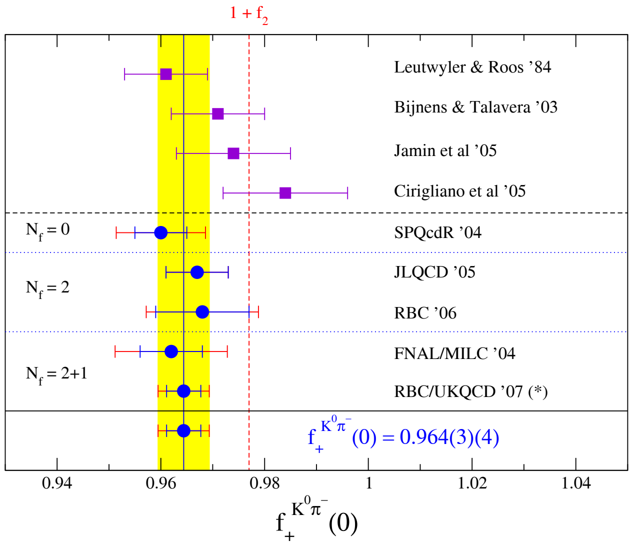

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Comments

- recent CHPT estimates for $f_+(0)$ all rely on same 2-loop calculation
  Bijnens, Talavera
- lattice results involve extrapolation to small quark masses: under control?
- V_{us} from hadronic τ decays
  $|\Delta S| = 0$, $|\Delta S| = 1$ τ decay widths and (small) $SU(3)$ breaking effects
  $$|V_{us}| = 0.2159(30)_{\rm exp}(5)_{\rm theor}$$ Gamiz et al.

N.B.: **very small theoretical uncertainty !**

similar analysis of Maltman et al.

$\longrightarrow$ **wait for final data from BELLE and BaBar**

☛ **how well do we know V_{ud}?**

from nuclear β decay (Towner, Hardy)

$$|V_{ud}| = 0.97424(22)$$

on the other hand: from recent precise measurement of neutron lifetime (Serebrov et al. 2005, incompatible with PDG world average)

$$\longrightarrow \quad |V_{ud}| = 0.9786(4)_{\tau_n}(18)_{g_A}(2)_{\rm RC} \qquad \textbf{PDG 2008}$$

for the devil's pleasure:

CKM unitarity + V_{ud} (Serebrov et al.) $\longrightarrow$

$$|V_{us}| = 0.2058 \pm$$

even smaller than current value from τ decays

$\Gamma(P \to e\nu_e)/\Gamma(P \to \mu\nu_\mu) \quad (P = \pi, K)$

$V - A$ structure of charged currents $\longrightarrow$

$$R_{e/\mu}^{(P)} = \Gamma(P \to e\nu_e[\gamma])/\Gamma(P \to \mu\nu_\mu[\gamma])$$ **helicity suppressed**

$\longrightarrow$ **sensitive probe for new physics**

(charged Higgs exchange, violation of lepton universality, ...)

	PDG 2008	Marciano, Sirlin 1993	Finkemeier 1996
$R_{e/\mu}^{(\pi)} \cdot 10^4$	1.230 ± 0.004	1.2352 ± 0.0005	1.2354 ± 0.0002
$R_{e/\mu}^{(K)} \cdot 10^5$	2.45 ± 0.11		2.472 ± 0.001

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$P \to \ell\nu_\ell$ to $O(p^2)$

$$T_\ell^{p^2} = -2\,i\,G_F\,V_{ud}\,F\,m_\ell\,\bar{u}_L(p_\nu)\,v(p_\ell) \qquad (P = \pi)$$

$$O(p^{2n}), e = 0: \quad F \to F_P^{(2n)} \quad \longrightarrow \quad R_{e/\mu}^P = \frac{m_e^2}{m_\mu^2}\left(\frac{M_P^2 - m_e^2}{M_P^2 - m_\mu^2}\right)^2$$

$\longrightarrow$ **nontrivial corrections (hadronic structure) only for $e \neq 0$**

amplitudes of $O(e^2p^2)$

1-loop diagrams $\longrightarrow$ $T_\ell^{e^2p^2}$ **corresponds to point-like approximation**

Kinoshita; Marciano, Sirlin

Knecht, Neufeld, Rupertsberger, Talavera

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$O(e^2p^4)$ **Cirigliano, Rosell**

(up to) 2-loop diagrams $\longrightarrow$ **involve meson structure (form factors)**

loop integration divergent $\longrightarrow$ $T_\ell^{e^2p^4}$ **contains counterterm**

Cirigliano, Rosell: **counterterm fixed by**

matching form factors with large-N_c QCD

- **inclusion of real photon corrections +**
  summation of leading logs $\alpha^n \log^n(m_\mu/m_e)$ (Marciano, Sirlin)

Final results

	Cirigliano, Rosell	Marciano, Sirlin	Finkemeier
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☛ **discrepancy with previous calculation of $R_{e/\mu}^{(K)}$**

[asymptotic behaviour of form factors incompatible with QCD]

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most recent experimental results (KAON09)

	KLOE	NA62 prel.
$R_{e/\mu}^{(K)} \cdot 10^5$	$2.493 \pm 0.025 \pm 0.019$	$2.500 \pm 0.012 \pm 0.011$

LD contributions for $K \to \pi\nu\bar{\nu}$, $K_L \to \pi^0\ell^+\ell^-$

hadronic matrix elements

$$\langle\pi^+|\bar{s}\gamma^\mu d|K^+\rangle$$
$$\langle\pi^0|\bar{s}\gamma^\mu d|K^0\rangle$$

related to K_{l3} form factors by isospin

recent improvements:

isospin breaking effects of $O[(m_u - m_d)p^4]$, radiative corrections of $O[e^2p^2]$

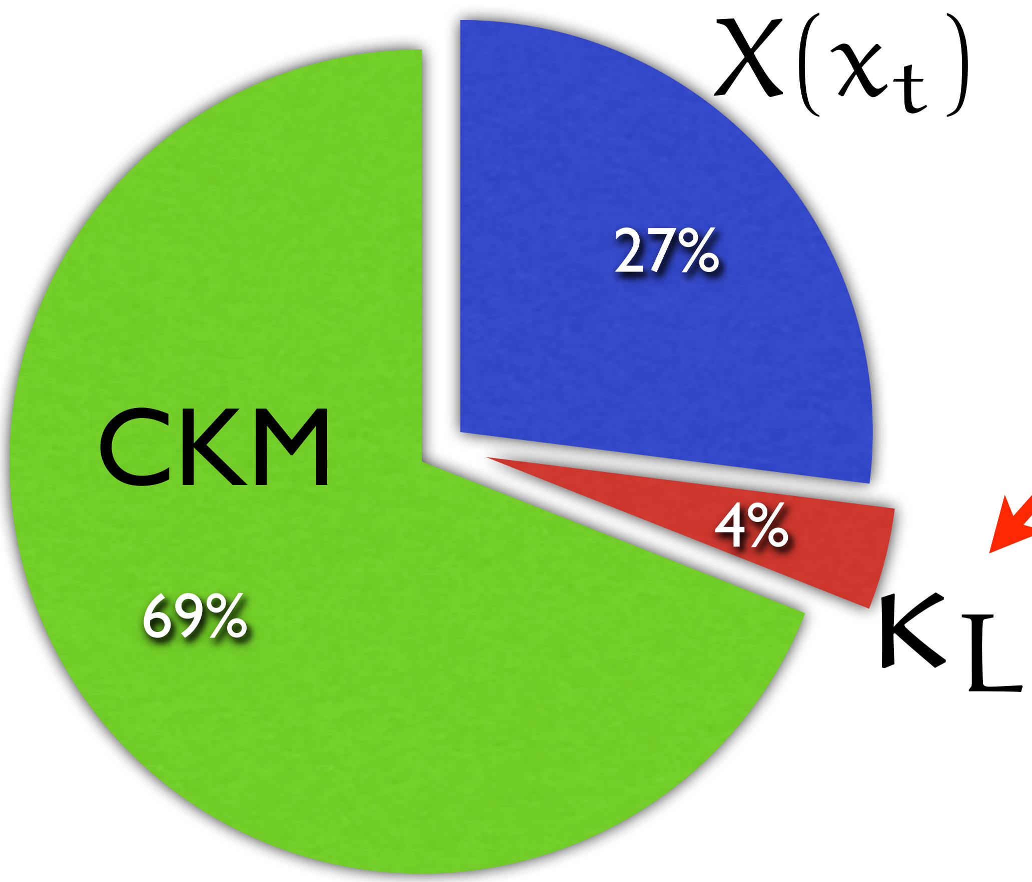
Mescia, Smith 2007

isospin breaking effects of $O[(m_u - m_d)p^6]$ Bijnens, Ghorbani 2007

important consequence:

theoretical uncertainties for $B(K^+ \to \pi^+\nu\bar{\nu})$, $B(K_L \to \pi^0\nu\bar{\nu})$
now completely dominated by short-distance part $\longrightarrow$ Gorbahn (KAON09)
improvement also for direct CP-violating contribution to $K_L \to \pi^0\ell^+\ell^-$

$K_L \to \pi^0 \bar{\nu} \nu$: Theoretical Status



Matrix element extracted from K_{l3} decays. $N^{\frac{3}{2}}LO\ \chi PT$

[Mescia, Smith ´07; Bijnens, Ghorbani ´07]

No further long distance uncertainty

$$\mathcal{B}r_{K_L} = (2.6 \pm 0.4) \times 10^{-11}$$

$X(x_t)$: NLO QCD calculation: ±1% error

[Misiak, Urban ´99; Buchalla, Buras ´99]

$X(x_t)$: Electroweak (EW) corrections: ±2% error

[Buchalla, Buras ´99]

Reduce error with 2 loop electroweak calculation

Nonleptonic K decays

Dominant decays: $K \longrightarrow 2\pi,\ 3\pi$

only nonleptonic decays occurring to lowest chiral order (except Bremsstrahlung)

NLO — Kambor, Missimer, Wyler

NLO + isospin violation + rad. corrs. — Cirigliano, E., Neufeld, Pich; Bijnens, Borg

most recent analysis (Bijnens, Borg)

- chiral corrections sizable
- isospin violating corrections small in most cases
- good fits for rates and slope parameters (in terms of LECs)

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main current interest:

- $K \longrightarrow \pi\pi$ — **$\pi\pi$ s-wave phase shift difference**
- $K \longrightarrow 3\pi$ — **$\pi\pi$ s-wave scattering lengths**

$\pi\pi$ phase shifts from $K \to 2\pi$

general parametrization

$$
\begin{aligned}
A(K^0 \to \pi^+\pi^-) &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2} \\
A(K^0 \to \pi^0\pi^0) &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} \\
A(K^+ \to \pi^+\pi^0) &= \frac{3}{2} A_2^+\, e^{i\chi_2^+}
\end{aligned}
$$

isospin limit

$$
A_2 = A_2^+\,, \qquad \chi_I = \delta_I(M_K)
$$

$\delta_I(M_K)$: **s-wave $\pi\pi$ phase shifts (isospin I)**

$\pi\pi$ phase shifts from $K \to 2\pi$

general parametrization

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isospin limit

$$A_2 = A_2^+ \,, \qquad \chi_I = \delta_I(M_K)$$

$\delta_I(M_K)$: s-wave $\pi\pi$ phase shifts (isospin I)

ancient problem:

- up to 2002 (**KLOE**: $\Gamma(K_S \to \pi^+\pi^-(\gamma))/\Gamma(K_S \to \pi^0\pi^0)$):

$$\chi_0 - \chi_2 \simeq \delta_0(M_K) - \delta_2(M_K) + 10^\circ$$

- isospin violating corrections for A_2, χ_2 enhanced by $\Delta I = 1/2$ rule ($A_0/A_2 \sim 22$)

theoretical problem:

- ☛ $\chi_I = 0$ at LO CHPT
- ☛ NLO analysis $\longrightarrow$ large uncertainties for phases

more robust method Cirigliano, E., Pich

only theory input: isospin violating amplitudes at NLO

fit $\delta_I(M_K)$, $\overline{A}_I$ (isospin limit) directly from the rates

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final result

$\delta_0(M_K) - \delta_2(M_K)$		
$(52.5 \pm 0.8_{\rm exp} \pm 2.8_{\rm theor})^\circ$	Cirigliano, E., Pich	$K \to \pi\pi$
$(47.7 \pm 1.5)^\circ$	Colangelo, Gasser, Leutwyler	$\pi\pi$
$(50.9 \pm 1.2)^\circ$	Kaminski, Pelaez, Yndurain	$\pi\pi$

- ☛ reasonable agreement

Cusp in $K \to 3\pi$ decays

first seen in $M_{\pi^0\pi^0}$ distribution in $K^+ \to \pi^+\pi^0\pi^0$ — NA48/2 2006

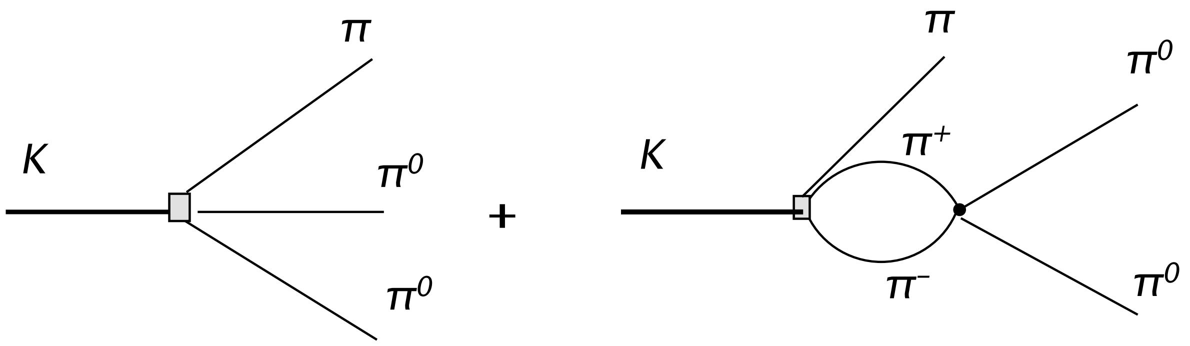
now also seen (effect much smaller) in $K_L \to 3\pi^0$ — KTeV 2008

cusp due to rescattering of final state pions — Cabibbo 2004

(Meißner, Müller, Steininger 1997)

$$K^+ \to \pi^+(\pi^+\pi^-)^* \to \pi^+\pi^0\pi^0$$

basic mechanism: **interference between tree and 1-loop amplitudes**



square root singularity generates cusp at $M^2_{\pi^0\pi^0} = 4M^2_{\pi^+}$

sensitive to $a_0 - a_2 \sim A(\pi^+\pi^- \longrightarrow \pi^0\pi^0)_{\rm thresh}$

- surprisingly accurate method to extract s-wave scattering lengths a_I
  especially for difference $a_0 - a_2$

various approaches:

unitarity + analyticity, expansion in powers of a_I — Cabibbo, Isidori

” + CHPT — Gamiz, Prades, Scimemi

dispersive approach — Kampf, Knecht, Novotný, Zdráhal

nonrelativistic EFT (tailored to near-threshold region) — Bissegger, Colangelo, Fuhrer, Gasser, Kubis, Rusetsky

most advanced treatment (radiative corrections)

unlike standard CHPT: valid to all orders in quark masses

- **cusp effect produces very accurate results**
  especially when combined with K_{e4} decays $\longrightarrow$ Bloch-Devaux

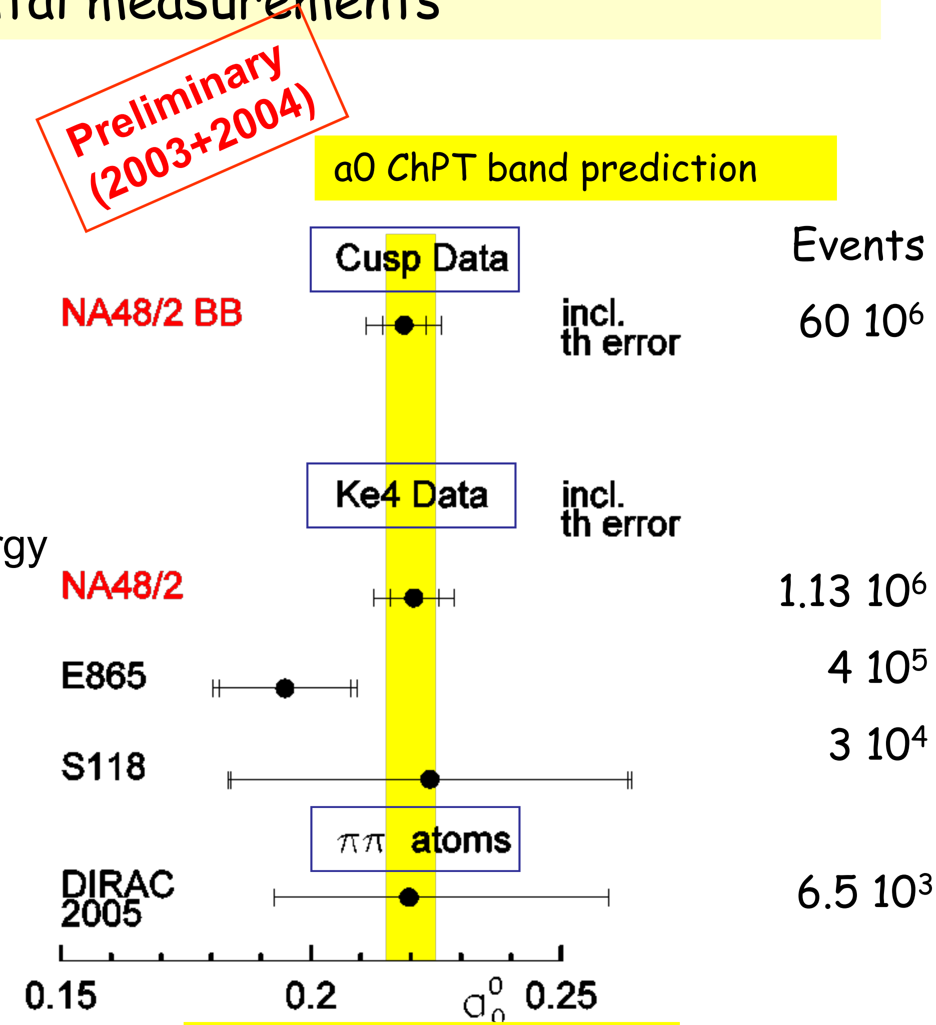
Comparison with other experimental measurements

Cusp : (a_0-a_2) ChPT fit with 2 models, BB is the most complete in terms of radiative corrections

Ke4 : apply isospin corrections to published phase points of all experiments and perform a_0 ChPT fit

Note : E865 result dominated by highest energy data point, otherwise compatible

ππ atoms DIRAC: |a_0-a_2| errors from PLB619 (2005), use ChPT constraint (only 40% Data analyzed)



Yellow band: best ChPT prediction

a_0 (ChPT) = 0.220 ± 0.005

NA48/2 experimental precision and most precise theory prediction now at the same level!

final results from NA48/2 — Bloch-Devaux (KAON09)

	NA48/2 (combined)	Colangelo, Gasser, Leutwyler
$a_0 - a_2$	$0.2640 \pm 0.0026_{\rm exp} \pm 0.0035_{\rm th}$	0.264 ± 0.004
a_0	$0.2196 \pm 0.0034_{\rm exp} \pm 0.0048_{\rm th}$	0.220 ± 0.005

N.B.: experimental errors for combined results
now smaller than theoretical uncertainties **!**

Rare nonleptonic decays

recall: **except for $K \to 2\pi, 3\pi$ (including Bremsstrahlung)**

nonleptonic amplitudes start at $O(G_F p^4)$ only

Rare nonleptonic decays

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$$K_S \to \gamma\gamma, K_L \to \pi^0\gamma\gamma, K_S \to \pi^0\pi^0\gamma\gamma$$

theoretical status at $O(G_F p^4)$

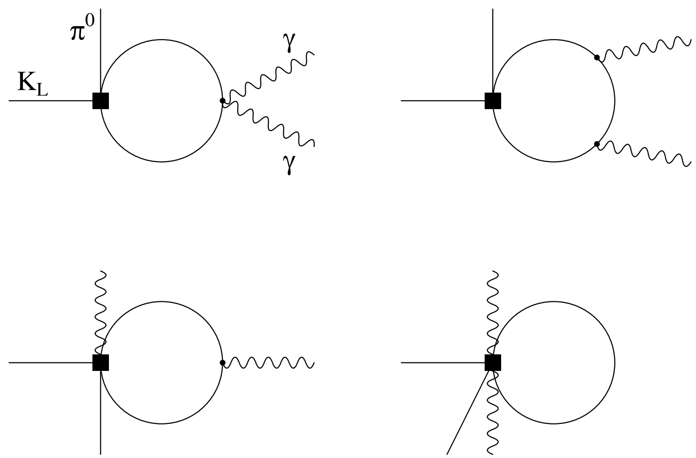
$K_S \to \gamma\gamma$	D'Ambrosio, Espriu; Goity
$K_L \to \pi^0\gamma\gamma$	E., Pich, de Rafael; Cappiello, D'Ambrosio
$K_S \to \pi^0\pi^0\gamma\gamma$	Funck, Kambor

$O(G_F p^2)$ no contribution (neutral mesons)

$O(G_F p^4)$ LECs do not contribute $\longrightarrow$ finite loop amplitude

$O(G_F p^6)$ rescattering corrections

resonance exchange $\to$ LECs



estimates of higher-order corrections (for $K_S \to \gamma\gamma$ and $K_L \to \pi^0\gamma\gamma$)

☛ **rescattering (unitarity) corrections largely model independent**

Cappiello, D'Ambrosio, Miragliuolo; Cohen, E., Pich; Kambor, Holstein

$K_S \to \gamma\gamma$	"trivial" in terms of $K \to 2\pi$ rate
$K_L \to \pi^0\gamma\gamma$	more involved but straightforward

☛ **resonance contributions**

Cohen, E., Pich; D'Ambrosio, Portolés; Buchalla, D'Ambrosio, Isidori

$K_S \to \gamma\gamma$	small (vector mesons cannot contribute)
$K_L \to \pi^0\gamma\gamma$	vector meson contribution model dependent (weak transition!)
	good approximation: single parameter a_V

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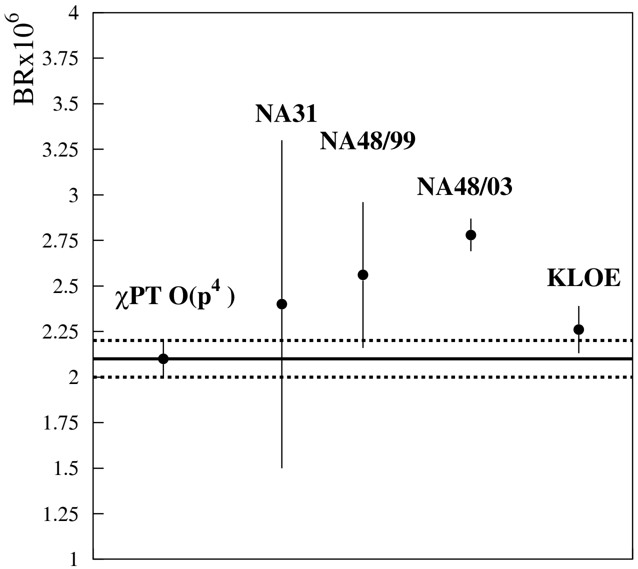
$K_S \to \gamma\gamma$

puzzling result of NA48 (2003):
rate much bigger than $O(p^4)$ result

new KLOE measurement (2008)

$B(K_S \to \gamma\gamma) = 2.26(12)(06) \times 10^{-6}$

$\longrightarrow$ **perfect agreement**



$K_L \to \pi^0\gamma\gamma$

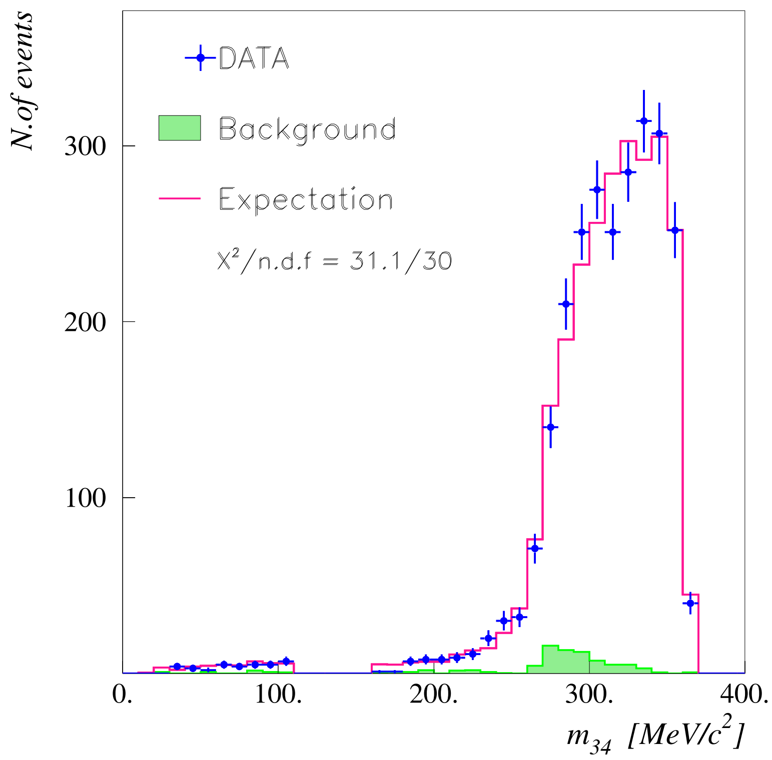
for reasonable values of a_V:

pion loop dominates 2γ-spectrum

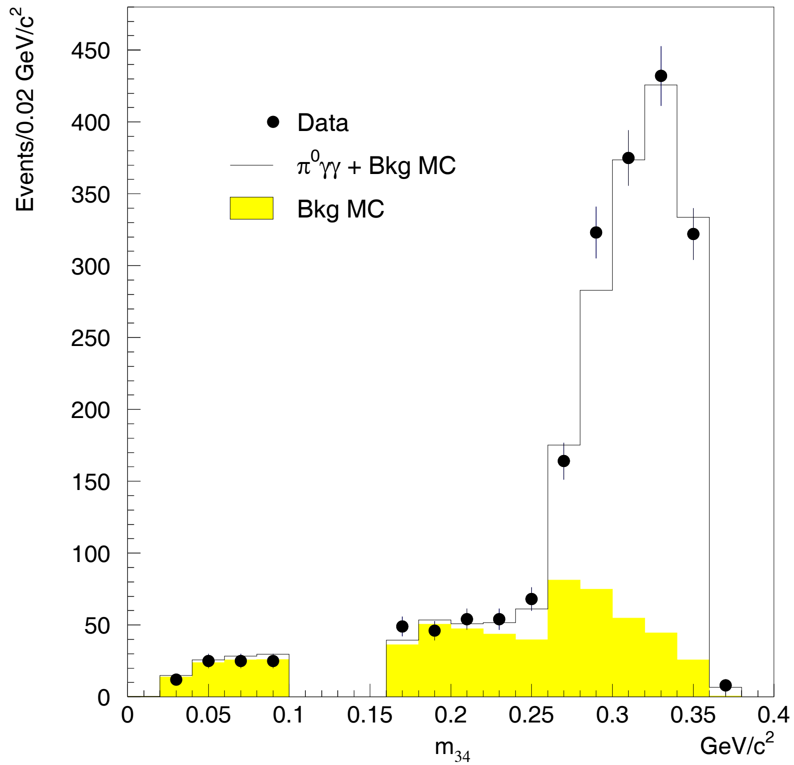
rate more affected both by rescattering corrections and by a_V

experimental situation only settled last year

$K_L \to \pi^0\gamma\gamma$ decay distribution in $m_{34} = M_{\gamma\gamma}$



NA48 (2002)



KTeV (2008)

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for reasonable values of a_V:

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☛ **excellent agreement between theory and experiment for both rate and spectrum**

$$B(K_L \to \pi^0\gamma\gamma)\cdot 10^6 = \begin{cases} 1.27 \pm 0.04 \pm 0.01 & \text{NA48 (2002)} \\ 1.28 \pm 0.06 \pm 0.01 & \text{KTeV (2008)} \\ 1.273 \pm 0.034 & \text{PDG (2008)} \end{cases}$$

$$a_V = \begin{cases} -0.46 \pm 0.03 \pm 0.04 & \text{NA48 (2002)} \\ -0.31 \pm 0.05 \pm 0.07 & \text{KTeV (2008)} \\ -0.43 \pm 0.06 & \text{PDG (2008)} \end{cases}$$

important consequence

CP conserving contribution to $K_L \to \pi^0 e^+e^-$ via $K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+e^-$ negligible in comparison with CP violating amplitudes

$K_S \to \pi^0\pi^0\gamma\gamma$

at $O(p^4)$: **also completely determined by loop amplitude** **Funck, Kambor**

$B(K_S \to \pi^0\pi^0\gamma\gamma)$ **with cut on 2-photon mass** $M_{\gamma\gamma}$

$$|M_{\gamma\gamma} - M_\pi| \geq \Delta m$$

Δm (MeV)	$B(K_S \to \pi^0\pi^0\gamma\gamma) \cdot 10^9$
1	5.9
10	5.3
30	4.0

might be within reach of KLOE

for comparison: $B(K_S \to 3\pi^0)_{\rm SM} \simeq 2 \cdot 10^{-9}$

Other rare decays

depend on LECs of $O(G_F p^4)$

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$
$\pi^0\gamma^*\ (S)$	$\pi^0\pi^0\gamma^*\ (L)$		$2N_{14}^r + N_{15}^r$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma\ (S)$		"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma\ (S)$	$\pi^+\pi^0\pi^0\gamma$	"
		$\pi^+\pi^-\pi^0\gamma\ (L)$	"
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17}^r)$
	$\pi^+\pi^-\gamma^*\ (L)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma^*\ (S)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma\ (L)$	$\pi^+\pi^-\pi^0\gamma\ (S)$	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma\ (L)$	$6N_{28} + 3N_{29} - 5N_{30}$

rare K decay final states: local amplitudes of $O(G_8 p^4)$

Other rare decays

depend on LECs of $O(G_F p^4)$

Comments

- all electric couplings $N_{14}, \ldots, N_{18}$ can in principle be extracted from experiment
- only 3 combinations of the 4 magnetic couplings $N_{28}, \ldots, N_{31}$ occur in measurable amplitudes
- several relations between different amplitudes (low-energy theorems)
- however: higher-order effects may be sizable

present status $\longrightarrow$ Giancarlo D'Ambrosio

Conclusions

- **CHPT provides comprehensive framework for all K decays**
- **only reliable method for isospin violating and electromagnetic corrections**
- **testing the SM: patience often needed**

  examples: $K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$
- **kaon side of the flavour problem:**

  still no evidence for new physics ! ?

Conclusions

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  still no evidence for new physics ! ?
- nonetheless

Kaon physics will continue to play a role in the LHC era