


# Theory Predictions for Neutrino Oscillations



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# Neutrino Mass beyond the SM

- SM: effective low energy theory with non-renormalizable terms
- new physics effects suppressed by powers of small parameter  $\frac{M_W}{M}$
- neutrino masses generated by dim-5 operators

$$\frac{\lambda_{ij}}{M} H H L_i L_j \Rightarrow m_\nu = \lambda_{ij} \frac{v^2}{M}$$

$\lambda_{ij}$  are dimensionless couplings;  $M$  is some high scale

- $m_\nu$  small: non-renormalizable terms ( $M$  is high)

**lowest higher dimensional operator that probes high scale physics**

- total lepton number and family lepton numbers broken
  - ➔ lepton mixing and CP violation expected
  - ➔  $\mu \rightarrow e \gamma$  ;  $\tau \rightarrow \mu \gamma$  ;  $\tau \rightarrow e \gamma$  decays ;  $\mu$  - e conversion



# Tri-bimaximal Neutrino Mixing

Schwetz, Tortola, Valle (Aug 2008)

- Neutrino Oscillation Parameters ( $2\sigma$ )

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

$$\sin^2 \theta_{23} = 0.5_{-0.12}^{+0.14}, \quad \sin^2 \theta_{12} = 0.304_{-0.032}^{+0.044}$$

- indication for non-zero  $\theta_{13}$ :

Bari group, June 2008

$$\sin^2 \theta_{13} = 0.01_{-0.011}^{+0.016} (1\sigma) \quad \text{consistent with } \theta_{13} = 0$$

- Tri-bimaximal neutrino mixing: Harrison, Perkins, Scott, 1999

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2 \theta_{\text{atm, TBM}} = 1/2$$

$$\sin \theta_{13, \text{TBM}} = 0.$$

$$\sin^2 \theta_{\odot, \text{TBM}} = 1/3$$

$$\tan^2 \theta_{\odot, \text{TBM}} = 1/2$$

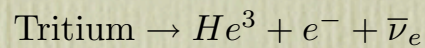
$$\tan^2 \theta_{\odot, \text{exp}} = 0.429$$

new KamLAND result:  $\tan^2 \theta_{\odot, \text{exp}} = 0.47_{-0.05}^{+0.06}$



# Neutrino Mass Spectrum

- search for absolute mass scale:
- end point kinematic of tritium beta decays:

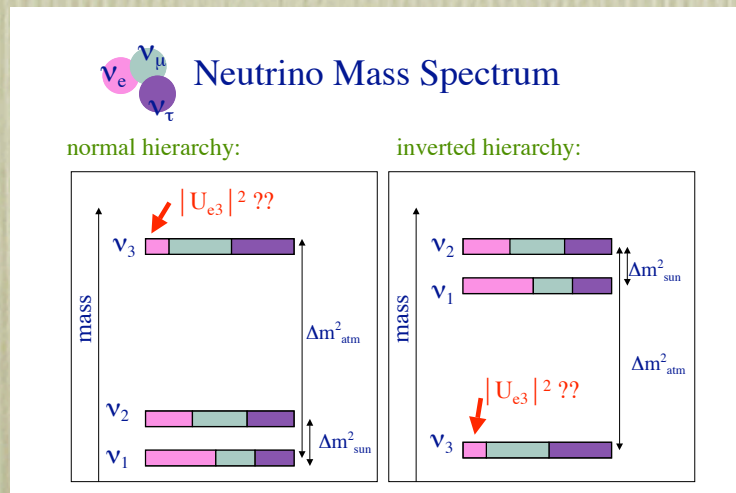


Mainz:  $m_\nu < 2.2$  eV

KATRIN: increase sensitivity  $\sim 0.2$  eV

- WMAP + 2dFRGS + Ly $\alpha$ :  $\sum(m_{\nu_i}) < (0.7-1.2)$  eV
- neutrinoless double beta decay

current bound:  $|\langle m \rangle| < (0.19 - 0.68)$  eV (CUORICINO, Feb 2008)



## The known unknowns:

- How small is  $\theta_{13}$ ?
- $\theta_{23} > \pi/4$ ,  $\theta_{23} < \pi/4$ ,  $\theta_{23} = \pi/4$ ?
- Neutrino mass hierarchy ( $\Delta m_{13}^2$ ) ?
- CP violation in neutrino oscillations?



# Need for Precision Measurements

- current data post two challenges:
  - why  $m_\nu \ll m_{u,d,l}$
  - why lepton mixing large while quark mixing small
- To answer the first question  $\Rightarrow$  Seesaw mechanism: most appealing scenario
- Seesaw: not sufficient to explain the whole mass matrix with mass hierarchy and two large and one small mixing angles
  - \* flavor symmetry: there is a structure
    - ▶ Possible symmetries show up only in the lepton sector
    - ▶ Connection between quark and lepton sectors (GUT symmetry)
- These scenarios have drastically different predictions
- To tell these models apart: Precision measurements important



possible textures:

Texture	Hierarchy	$ U_{e3} $	$ \cos 2\theta_{23} $ (n.s.)	$ \cos 2\theta_{23} $	Solar Angle
$\frac{\sqrt{\Delta m_{13}^2}}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$	Normal	$\sqrt{\frac{\Delta m_{12}^2}{\Delta m_{13}^2}}$	O(1)	$\sqrt{\frac{\Delta m_{12}^2}{\Delta m_{13}^2}}$	O(1)
$\sqrt{\Delta m_{13}^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$	Inverted	$\frac{\Delta m_{12}^2}{ \Delta m_{13}^2 }$	-	$\frac{\Delta m_{12}^2}{ \Delta m_{13}^2 }$	O(1)
$\frac{\sqrt{\Delta m_{13}^2}}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	Inverted	$\frac{\Delta m_{12}^2}{ \Delta m_{13}^2 }$	O(1)	$\frac{\Delta m_{12}^2}{ \Delta m_{13}^2 }$	$ \cos 2\theta_{12}  \sim \frac{\Delta m_{12}^2}{ \Delta m_{13}^2 }$
$\sqrt{\Delta m_{13}^2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	Normal <sup>a</sup>	> 0.1	O(1)	-	O(1)

Altarelli, Feruglio, Masina, 02;  
Hall, Murayama, Weiner;  
Sato, Yanagida; Barbieri et al; ...

leptonic family symmetry:

Symmetry breaking	$\theta_{13}$	$\theta_{23} - \pi/4$
none	0	0
$\mu$ - $\tau$ sector only	$\sim \Delta m_{12}^2 / \Delta m_{31}^2$	$\leq 8^\circ \sim \sqrt{\Delta m_{12}^2 / \Delta m_{31}^2}$
e-sector only	$\sim \sqrt{\Delta m_{12}^2 / \Delta m_{31}^2}$	$\leq 4^\circ \sim \Delta m_{12}^2 / \Delta m_{31}^2$
dynamical	$\sim \sqrt{\Delta m_{12}^2 / \Delta m_{31}^2}$	large


R. N. Mohapatra ('04)



# SO(10) GUT

- RH neutrino accommodated in the model

$$16 = \bar{5} + 10 + \textcircled{1}$$


  
 $\nu_R$

$$16 = (3, 2, 1/6) \sim \begin{bmatrix} u & u & u \\ d & d & d \end{bmatrix}$$

$$+ (3^*, 1, -2/3) \sim (u^c \ u^c \ u^c)$$

$$+ (3^*, 1, 1/3) \sim (d^c \ d^c \ d^c)$$

$$+ (1, 2, -1/2) \sim \begin{bmatrix} \nu \\ e \end{bmatrix}$$

$$+ (1, 1, 1) \sim e^c$$

$$+ (1, 1, 0) \sim \nu^c$$

- Natural for seesaw: offer both ingredients, i.e. RH neutrino & heavy scale neutrino oscillation strongly support SO(10)!!
- Quark & Leptons reside in the same GUT multiplets
- One set of Yukawa coupling for a given GUT multiplet
  - ➔ SO(10) relates quarks and leptons (intra-family relations)
  - ➔ reduce # of parameters in Yukawa sector



# Models Based on SUSY SO(10)

- large neutrino mixing from neutrino sector

$$U_{MNS} = U_{e,L}^+ U_{\nu,L}$$

SO(10) GUT + SU(2) family symmetry

Barbieri, Hall, Raby, Romanino; ...

$$\begin{aligned} \text{SO}(10) &\rightarrow \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R \\ &\rightarrow \text{SU}(3) \times \text{SU}(2)_L \times \text{U}(1)_Y \end{aligned}$$

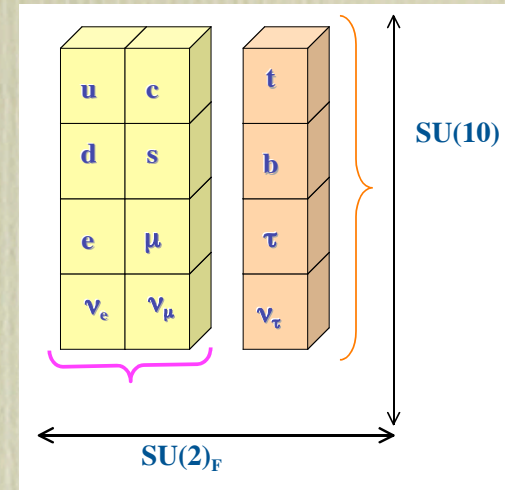
- symmetric mass matrices:

M.-C.C & K.T. Mahanthappa

Up-type quarks  $\Leftrightarrow$  Dirac neutrinos

Down-type quarks  $\Leftrightarrow$  charged leptons

$$\text{seesaw} \Rightarrow M_\nu \sim \begin{pmatrix} 0 & 0 & * \\ 0 & 1 & 1 \\ * & 1 & 1 \end{pmatrix}$$



12 parameters accommodate 22 fermion masses, mixing angles and CP phases in both quark and lepton sectors

- prediction for  $\theta_{13}$ :

$$\sin \theta_{13} \sim \left( \frac{\Delta m_{sun}^2}{\Delta m_{atm}^2} \right)^{1/2} \sim O(0.1) \Rightarrow \text{LMA}$$

continuous family symmetries:  
to get bi-maximal (TBM)  $\Rightarrow$   
specific values for parameters  
(couplings)



# Tri-bimaximal Neutrino Mixing

- Neutrino mass matrices:

$$M = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \longrightarrow \sin^2 2\theta_{23} = 1 \quad \theta_{13} = 0$$

solar mixing angle NOT fixed

- $S_3$  Mohapatra, Nasri, Yu, 2006; ...
  - $D_4$  Grimus, Lavoura, 2003; ...
  - $\mu$ - $\tau$  symmetry Fukuyama, Nishiura, '97; Mohapatra, Nussinov, '99; Ma, Raidal, '01; ...
- if  $A+B = C + D \longrightarrow \tan^2 \theta_{12} = 1/2$  TBM pattern
    - $A_4$  Ma, '04; Altarelli, Feruglio, '06; .....
    - $Z_3 \times Z_7$  Luhn, Nasri, Ramond, 2007

[Other discrete groups: Hagedorn, Lindner, Plentinger; Chen, Frigerio, Ma; and many others...]

recent claim:  $S_4$  unique group for TBM [C.S. Lam, 2008]

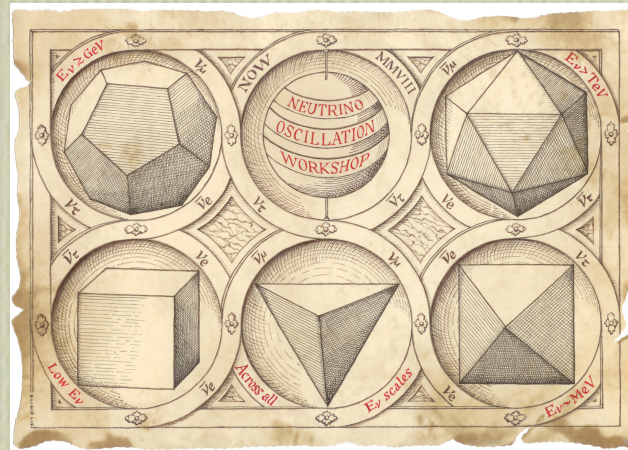


# Perfect Geometric Solids & Family Symmetries

solid	faces	vert.	Plato	Hindu	sym.
tetrahedron	4	4	fire	Agni	$A_4$
octahedron	8	6	air	Vayu	$S_4$
cube	6	8	earth	Prithvi	$S_4$
icosahedron	20	12	water	Jal	$A_5$
dodecahedron	12	20	quintessence	Akasha	$A_5$

From E. Ma, talk at WHEPP-9, Bangalore

$A_5$



$A_5$

आकाश

$S_4$

$S_4$



$A_4$





# Non-abelian Finite Family Symmetry

- TBM mixing matrix: can be realized in finite group family symmetry based on  $A_4$  Ma & Rajasekaran, '01

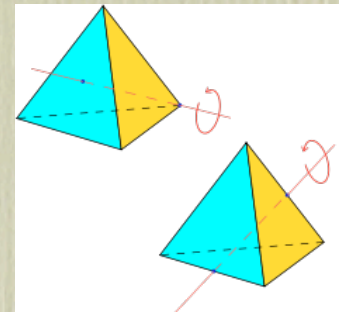
- even permutations of 4 objects

$$S: (1234) \rightarrow (4321)$$

$$T: (1234) \rightarrow (2314)$$

- invariance group of **Tetrahedron**
- orbifold compactification:

$$6D \rightarrow 4D \text{ on } T_2/Z_2$$



Altarelli, Feruglio, '06

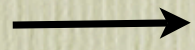
- Deficiencies:
  - does NOT give rise to CKM mixing:  $V_{ckm} = I$
  - does NOT explain mass hierarchy
  - all CG coefficients real



# The Double Tetrahedral $T'$ Symmetry

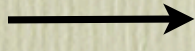
- consider double covering of  $A_4$
- Classified as a candidate family symmetry that can arise from Type-II B String theories  
Frampton, Kaphart, 1995, 2001
- can account for quark sector: Carr, Frampton, '07; Feruglio, Hedgedorn, Lin, Merlo, '07

exist in  $A_4$ :  $1, 1', 1'', 3$



TBM for neutrinos

not in  $A_4$ :  $2, 2', 2''$



2 + 1 assignments for quarks

- Combined with GUT:  $T' \times SU(5)$  GUT  
M.-C.C & K.T. Mahanthappa  
Phys. Lett. B652, 34 (2007)
  - only 9 operators allowed: highly predictive model
  - all 22 masses, mixing angles (CKM & MNS) and CPV measures are “accommodated”
    - lepton mixing
    - CPV in quark and lepton sectors
- } CG coefficients of  $T'$  &  $SU(5)$   
 $\Rightarrow$  pure geometrical in origin!

\* In RS warped extra dimension: prevent tree-level FCNCs in both quark and lepton sectors

M.-C.C, K.T. Mahanthappa, F.Yu, arXiv:0907.3963



# Group Theory of T'

- generators:

$$S^2 = R, T^3 = 1, (ST)^3 = 1, R^2 = 1$$

R=1: 1, 1', 1'', 3 (vector)  
 R=-1: 2, 2', 2'' (spinorial)

- generators: in 3-dim representations, T-diagonal basis

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

★ complex CG coefficients in T'

complexity cannot be avoided by different basis choice

- spinorial x spinorial  $\supset$  vector:

$$2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1$$

$$3 = \begin{pmatrix} (\frac{1-i}{2})(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}$$

J. Q. Chen & P. D. Fan,  
J. Math Phys 39, 5519 (1998)

- spinorial x vector  $\supset$  spinorial:

$$2 \otimes 3 = 2 \oplus 2' \oplus 2''$$

$$2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}$$



# A Novel Origin of CP Violation

M.-C.C., K.T. Mahanthappa, arXiv:0904.1721

- Conventionally:
  - Explicit CP violation: complex Yukawa couplings
  - Spontaneous CP violation: complex Higgs VEVs
- ★ complex CG coefficients in  $T'$   $\Rightarrow$  explicit CP violation
  - real Yukawa couplings, real Higgs VEVs
  - CP violation in both quark and lepton sectors determined by complex CG coefficients
  - no additional parameters needed  $\Rightarrow$  extremely predictive model!!



# Tri-bimaximal Mixing from Family Symmetry

- fermion charge assignments:

$$\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}_L \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'', \quad \tau_R \sim 1' \quad \xi \sim 3, \quad \eta \sim 1 \quad \langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- SM Higgs  $\sim$  singlet under  $T'$

- operator for neutrino masses:  $\frac{HHLL}{M} \left( \frac{\langle \xi \rangle}{\Lambda} + \frac{\langle \eta \rangle}{\Lambda} \right)$

- TBM neutrino mixing from CG coefficients**

Form diagonalizable!

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}, \quad V_\nu = U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

-- no adjustable parameters  
-- neutrino mixing from CG coefficients!

$$V_\nu^T M_\nu V_\nu = \text{diag}(u + 3\xi_0, u, -u + 3\xi_0) \frac{v_u^2}{M_x}$$

General conditions for Form Diagonalizability  
in seesaw: M.-C. C, S. F. King, 2009

- charged lepton mass matrix in non-GUT model: diagonal

- in  $SU(5)$  model: corrections to TBM due to GUT relations

$$\langle \phi \rangle = \phi_0 \Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- corrections at leading order in terms of  $\theta_c$  and CG only

M.-C.C & K.T. Mahanthappa  
Phys. Lett. B652, 34 (2007); arXiv:0904.1721



# Quark and Lepton Mixing Matrices

M.-C.C & K.T. Mahanthappa  
 Phys. Lett. B652, 34 (2007);  
 arXiv:0904.1721

- CKM mixing matrix:

$$M_u = \begin{pmatrix} i\phi_0^3 & \frac{1-i}{2}\phi_0^3 & 0 \\ \frac{1-i}{2}\phi_0^3 & \phi_0^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u \quad M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi'_0 & 0 \\ -(1-i)\phi_0\psi'_0 & \psi_0 N_0 & 0 \\ \phi_0\psi'_0 & \phi_0\psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

$\xrightarrow{\text{CKM}} V_{cb} \quad \quad \quad \xrightarrow{\text{CKM}} V_{ub}$

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha} \sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0 N_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \quad \longrightarrow \quad \theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3} \sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3} \theta_c$$

**Georgi-Jarlskog relations**  $\Rightarrow V_{d,L} \neq \mathbf{I}$   
 $SU(5) \Rightarrow M_d = (M_e)^T$   
 $\Rightarrow$  corrections to TBM related to  $\theta_c$

$$U_{MNS} = V_{e,L}^\dagger U_{TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2 \theta_\odot \simeq \tan^2 \theta_{\odot, TBM} + \frac{1}{2} \theta_c \cos \delta$$

$$\theta_{13} \simeq \theta_c / 3\sqrt{2}$$

**new QLC relation!**

**leptonic Dirac CP phase  $\Leftarrow$  complex CG**



# Quark-Lepton Complementarity

*lepton mixing*

parameter	Best-fit value	3 $\sigma$ range
$\theta_{12}$	33.2°	28.7° – 38.1°
$\theta_{23}$	45°	35.7° – 55.6°
$\theta_{13}$	2.6°	0 – 12.5°

*quark mixing*

parameter	Best-fit value	3 $\sigma$ range
$\theta_c$	12.88°	12.75° – 13.01°
$\theta_{23}^q$	2.36°	2.25° – 2.48°
$\theta_{13}^q$	0.21°	0.17° – 0.25°

$$\theta_{12} + \theta_c = 45^\circ$$

Raidal, '04; Smirnov & Minakata, '04

quark-lepton complementarity relation

quark-lepton unification?

more generally:

$$\theta_{12} + \theta_C \left( \frac{1}{\sqrt{2}} + \frac{\theta_C}{4} \right) \approx \frac{\pi}{4}$$

Plentinger, Seidl, Winter, 08; Frampton, Matsuzaki, 08; King 05; King Antusch, 05

RG effects:  $\Delta\theta_c \sim \theta_c^4$

MSSM: normal hierarchy  $\Delta\theta_{12} < 0.1^\circ$  Schmidt & Smirnov, '06

Motivate measurements of neutrino mixing angles to at least the accuracy of the measured quark mixing angles



# Numerical Results

M.-C.C & K.T. Mahanthappa  
Phys. Lett. B652, 34 (2007);  
arXiv:0904.1721

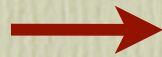
- diagonalization matrix for charged leptons:

$$\begin{pmatrix} 0.997e^{i177^\circ} & 0.0823e^{i131^\circ} & 1.31 \times 10^{-5}e^{-i45^\circ} \\ 0.0823e^{i41.8^\circ} & 0.997e^{i176^\circ} & 0.000149e^{-i3.58^\circ} \\ 1.14 \times 10^{-6} & 0.000149 & 1 \end{pmatrix}$$

- MNS Matrix:

$$|m_1| = 0.0156 \text{ eV}, \quad |m_2| = 0.0179 \text{ eV}, \quad |m_3| = 0.0514 \text{ eV}$$

$$\begin{pmatrix} 0.838 & 0.542 & 0.0583e^{-i227^\circ} \\ -0.385 - 0.0345e^{i227^\circ} & 0.594 - 0.0224e^{i227^\circ} & 0.705 \\ 0.384 - 0.0346e^{i227^\circ} & -0.592 - 0.0224e^{i227^\circ} & 0.707 \end{pmatrix} \rightarrow |U_{MNS}| = \begin{pmatrix} 0.838 & 0.542 & 0.0583 \\ 0.362 & 0.610 & 0.705 \\ 0.408 & 0.577 & 0.707 \end{pmatrix}$$



$$\sin^2 2\theta_{atm} = 1, \quad \tan^2 \theta_\odot = 0.419, \quad |U_{e3}| = 0.0583$$

prediction for Dirac CP phase:  $\delta = 227$  degrees

$$J_\ell = -0.00967$$

Note that these predictions do NOT depend on  $u_0$  and  $\xi_0$

- neutrino masses: using best fit values for  $\Delta m^2$

$$u_0 = -0.0593, \quad \xi_0 = 0.0369, \quad M_X = 10^{14} \text{ GeV}$$

2 parameters in  
neutrino sector

$$|m_1| = 0.0156 \text{ eV}, \quad |m_2| = 0.0179 \text{ eV}, \quad |m_3| = 0.0514 \text{ eV}$$

$$\alpha_{21} = \pi \quad \alpha_{31} = 0.$$

predicting: 3 masses,  
3 mixing angles, 3 CP Phases;  
both  $\theta_{sol}$  &  $\theta_{atm}$  agree with exp



# Neutrino Mass Sum Rule

- sum rule among three neutrino masses:  $m_1 - m_3 = 2m_2$
- including CP violation:

$$\begin{aligned}
 m_1 &= u_0 + 3\xi_0 e^{i\theta} \\
 m_2 &= u_0 \\
 m_3 &= -u_0 + 3\xi_0 e^{i\theta}
 \end{aligned}
 \qquad
 \begin{aligned}
 \Delta m_{atm}^2 &\equiv |m_3|^2 - |m_1|^2 = -12u_0\xi_0 \cos\theta \\
 \Delta m_{\odot}^2 &\equiv |m_2|^2 - |m_1|^2 = -9\xi_0^2 - 6u_0\xi_0 \cos\theta
 \end{aligned}$$

- leads to sum rule

$$\Delta m_{\odot}^2 = -9\xi_0^2 + \frac{1}{2}\Delta m_{atm}^2 \quad \longrightarrow \quad \Delta m_{atm}^2 > 0$$

normal hierarchy  
predicted!!

- constraint on Majorana phases:

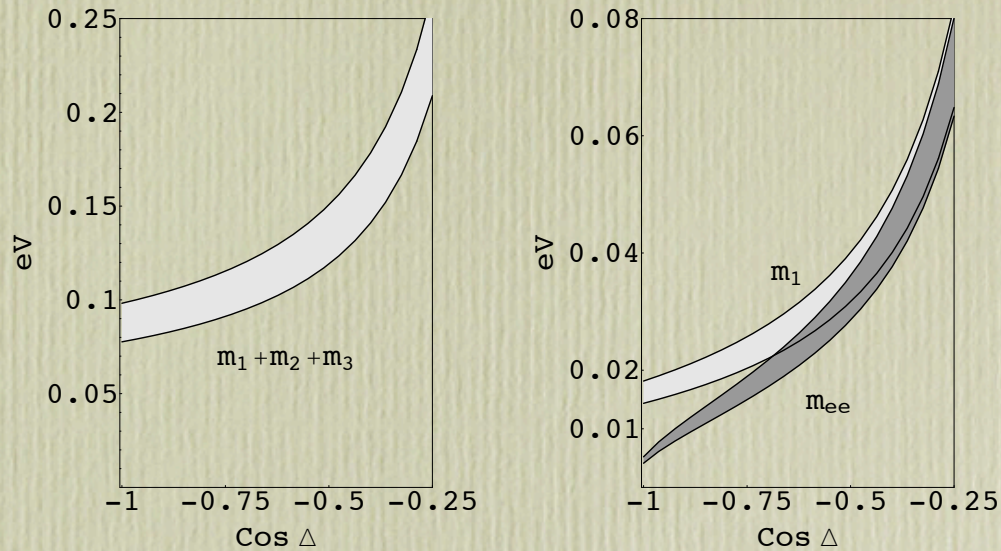
$$0 > \cos\theta > -\frac{3\xi_0}{2u_0}$$

- mass sum rule:

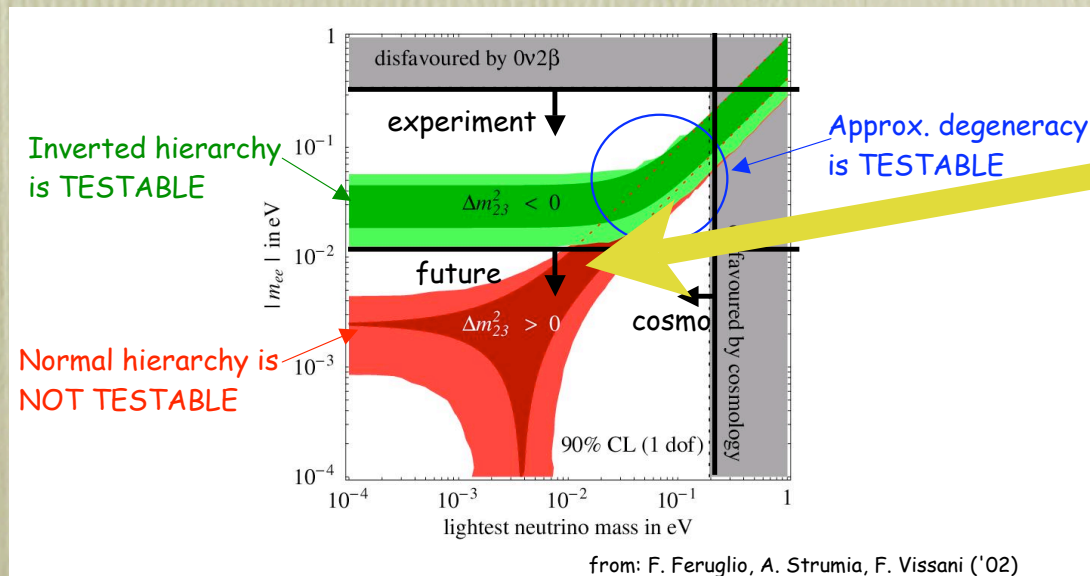
- ➔ large neutrino-less double beta decay matrix element
- ➔ large sum of three absolute masses (cosmology)



# Models with Tri-bimaximal Neutrino Mixing



For A4: Altarelli et al, 2006



prediction in  
A<sub>4</sub> and T' models



# TBM $\longleftrightarrow$ Leptogenesis

- TBM from broken discrete symmetries through type-I seesaw

➔ exact TBM mixing

E. Jenkins, A. Manohar, 2008

$$\sin \theta_{13} = 0 \Rightarrow J_{CP}^{lep} \propto \sin \theta_{13} = 0$$

CP violation through Majorana phases:  $\alpha_{21}, \alpha_{31}$

➔ no leptogenesis as  $Im(y_D y_D^\dagger) = 0$

➔ true even when flavor effects included

- corrections to TBM pattern due to high dim operators

small symmetry breaking parameter  $\eta \ll 1$  :

$$\sin \theta_{13} \sim \eta \sim 10^{-2}, \epsilon \sim 10^{-6} \text{ can be generated}$$

- SU(5) x T' model: corrections to TBM from charged lepton sector

- without flavor effects  $\Rightarrow \epsilon = 0$

M.-C.C., K.T. Mahanthappa, under preparation

- with flavor effects  $\Rightarrow \epsilon \sim 10^{-6}$  right amount for leptogenesis

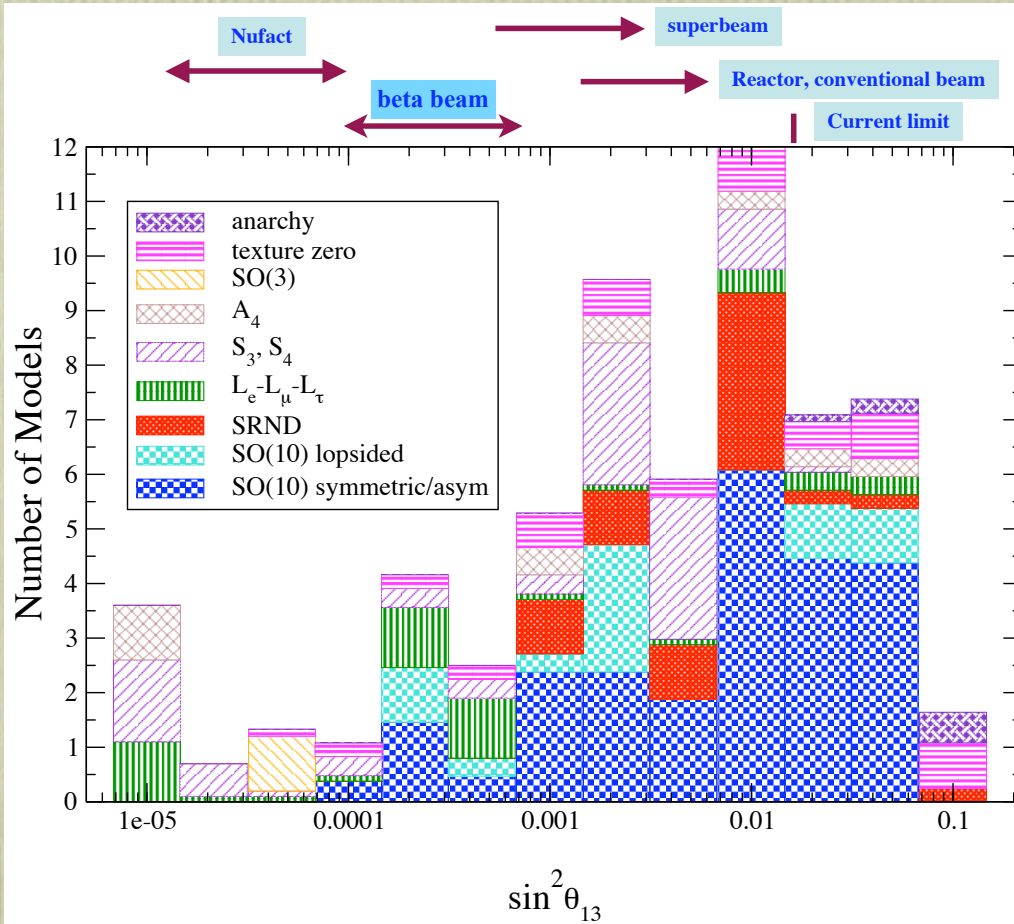
- Dirac phase the only non-vanishing leptonic CPV phase

$\Rightarrow$  connection between leptogenesis & low energy CPV



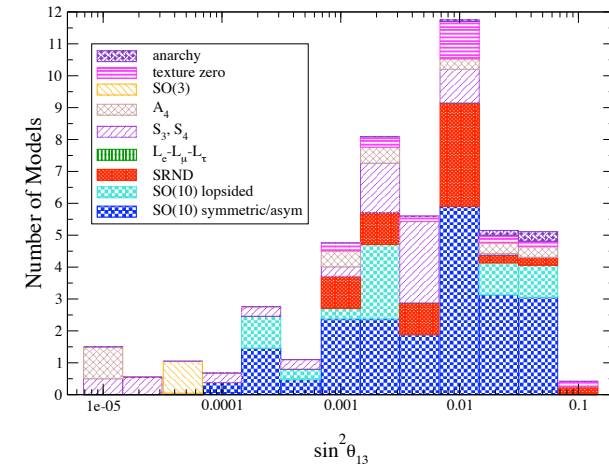
# Distinguishing Models

C. Albright & M.-C.C, 2006

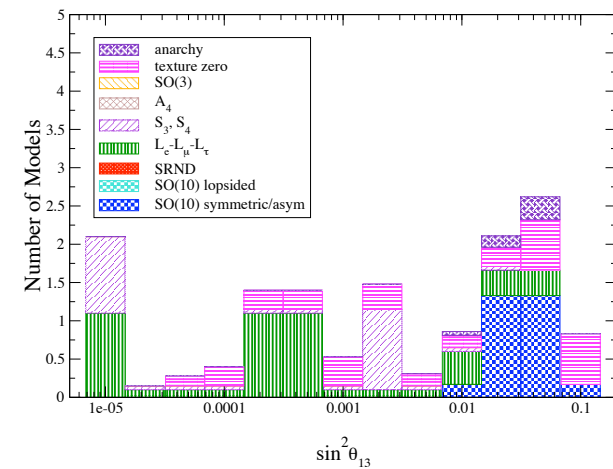


	$\sin^2 2\theta_{13}$	$\sin \theta_{13}$
current limit	$10^{-1}$	0.16
reactor	$10^{-2}$	0.05
Conventional beam	$10^{-2}$	0.05
superbeam	$3 \times 10^{-3}$	$2.7 \times 10^{-2}$
Neutrino factory	$(5-50) \times 10^{-5}$	$(3.5-11) \times 10^{-3}$

Models with Normal Hierarchy



Models with Inverted Hierarchy



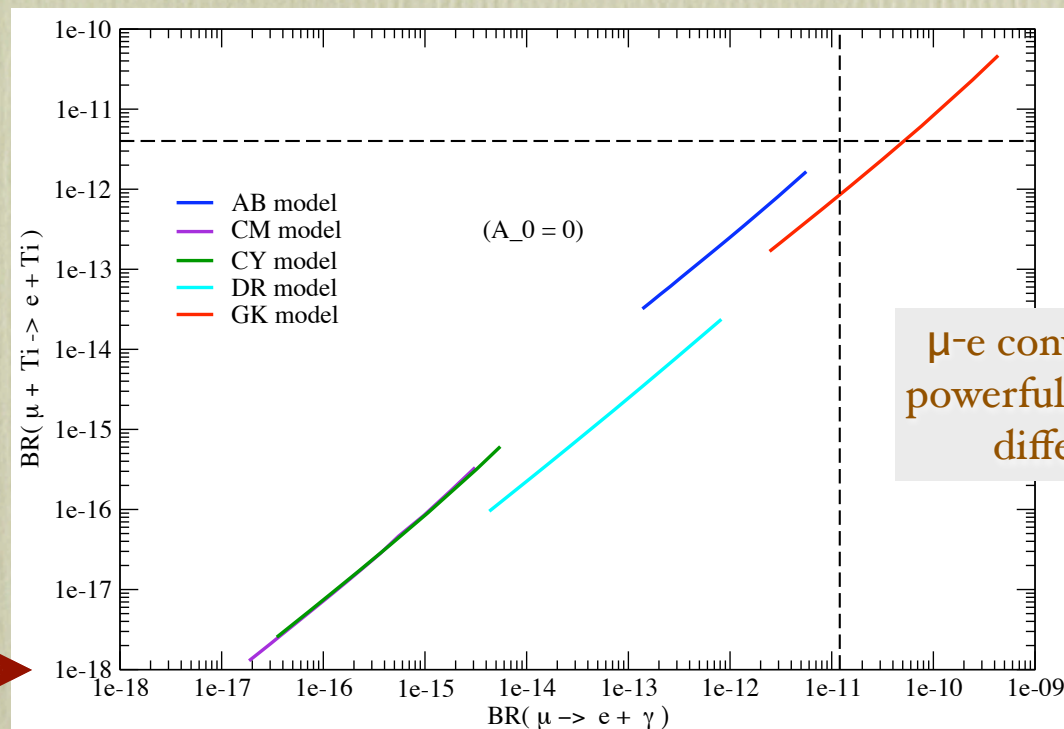


# LFV Rare Processes

C. Albright & M.-C.C, 2008

predictions for LFV processes in five viable SUSY SO(10) models:

- assuming MSUGRA boundary conditions
- including Dark Matter constraints from WMAP (lower bound on predictions)



sensitivity of proposed  
MECO-type exp

reach at MEG



# TeV Scale Seesaw

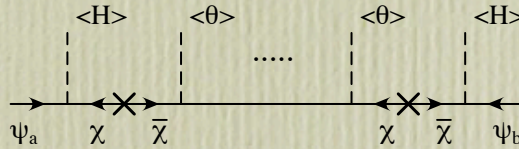
M.-C.C, A. de Gouvea, B. Dobrescu, 2006

- SM x  $U(1)_{NA} + 3 \nu_R$ : charged under  $U(1)_{NA}$  symmetry, broken by  $\langle \phi \rangle$
- $U(1)_{NA}$  forbids usual dim-4 Dirac operator and dim-5 Majorana operator

$$m_{LL} \sim \frac{HHLL}{M} \rightarrow M \sim 10^{14} \text{ GeV}$$

- neutrino masses generated by very high dimensional operators

$$m_{LL} \sim \left( \frac{\langle \phi \rangle}{M} \right)^p \frac{HHLL}{M} \rightarrow M \sim \text{TeV}, \quad \text{for large } p \quad \frac{\langle \phi \rangle}{M} \sim \text{not too small}$$



low seesaw scale achieved  
with all couplings  $\sim O(1)$

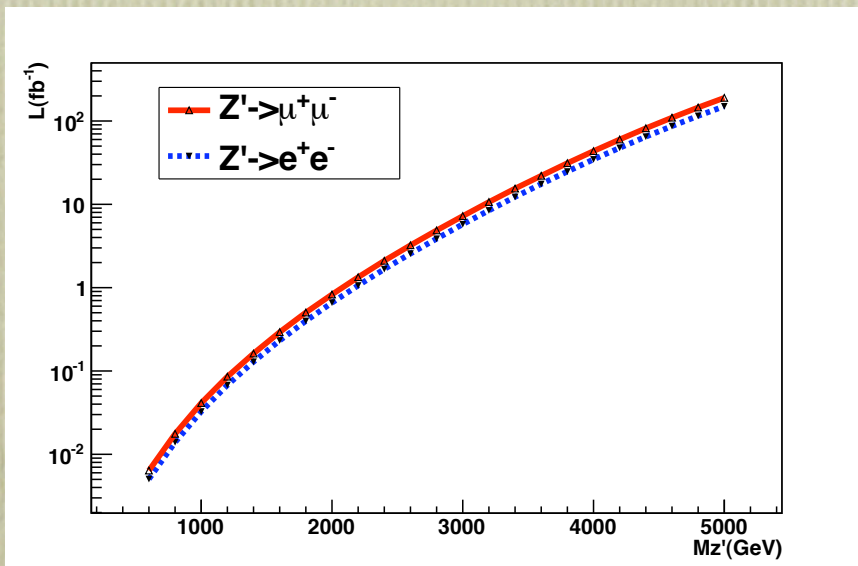
- anomaly cancellations: charges of different families of fermions related  
=> predict flavor mixing
- Through couplings to  $Z'$ : can probe neutrino sector at colliders



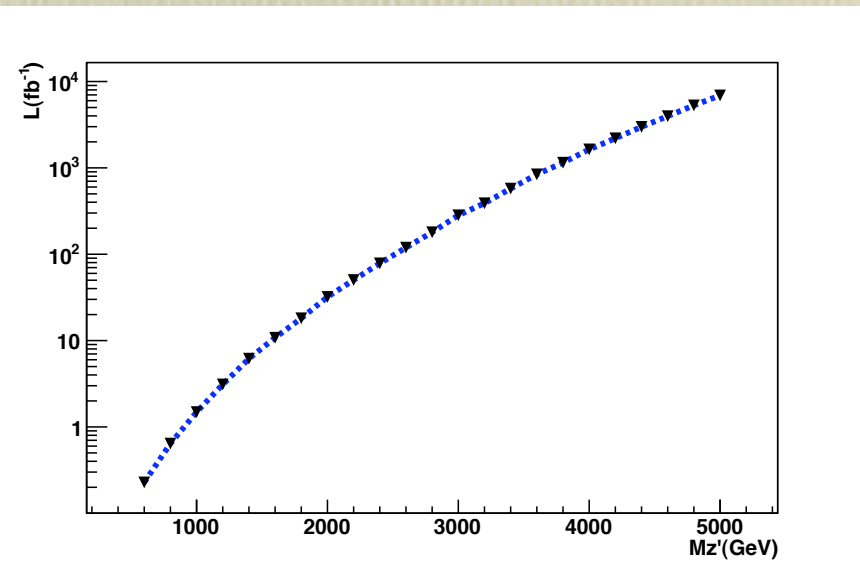
# LHC Potential

M.-C.C, J.R. Huang, under preparation

lepton charges:  $q_e = -55/8$  ,  $q_\mu = q_\tau = 49/8$



Integrated Luminosity  
needed for  $5\sigma$  discovery

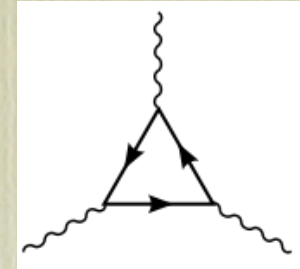


Integrated Luminosity  
needed for  $5\sigma$   
distinction



# Non-anomalous v.s. Anomalous U(1)

- anomaly cancellations: relating charges of different fermions
  - $[U(1)]^3$  condition generally difficult to solve
- most models utilized anomalous U(1):



- mixed anomaly: cancelled by Green-Schwarz mechanism
- $[U(1)]^3$  anomaly: cancelled by exotic fields besides RH neutrinos
- U(1) broken at fundamental string scale
- earlier claim that U(1) has to be anomalous to be compatible with  $SU(5)$  while giving rise to realistic fermion mass and mixing patterns

constraints not  
as stringent

L.E. Ibanez, G.G. Ross 1994

- non-anomalous U(1) can be compatible with SUSY  $SU(5)$  while giving rise to realistic fermion mass and mixing patterns
  - no exotics other than 3 RH neutrinos M.-C.C, D.R.T.Jones, A. Rajaraman, H.B.Yu, 2008
  - U(1) also forbids Higgs-mediated proton decay



# Conclusion

- finite group family symmetry: group theoretical origin for mixing and CP violation
- Predictions of existing models for  $\theta_{13}$ : 0 - current bound
- Precision measurements for the  $\theta_{13}$  and mass hierarchy can tell different scenarios apart:
  - leptonic family symmetry vs GUT
  - inverted hierarchy, small 1-3 mixing => lepton symmetry
  - large 1-3 mixing => inconclusive
- deviation from maximal  $\theta_{23}$  may tell how symmetry is broken
- May probe other interesting relations: e.g.
  - quark-lepton complementarity:  $\theta_{12} + \theta_c = 45^\circ$
  - new quark-lepton complementarity:  $\tan^2 \theta_\odot \simeq \tan^2 \theta_{\odot, TBM} + \frac{1}{2} \theta_c \cos \delta$
- LFV rare processes can be a robust test

Precision Measurements Indispensable!!