

CKM from semi-leptonic B decays

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Overview

- $b \rightarrow cl\nu \Rightarrow |V_{cb}|$
 - exclusive $B \rightarrow D/D^* l\nu$
 - inclusive $B \rightarrow X_c l\nu$
- $b \rightarrow ul\nu \Rightarrow |V_{ub}|$
 - exclusive $B \rightarrow \pi/\rho/\eta/\eta'/\omega l\nu$
 - inclusive $B \rightarrow X_u l\nu$
- precise knowledge of $|V_{cb}|$ & $|V_{ub}|$ is important
 - fundamental parameters of the Standard Model
 - * radius of the so-called Unitarity Clock in the unitarity triangle proportional to the ratio $|V_{ub}/V_{cb}|$
 - dealing with B semileptonic decays relatively easy
 - * HQET and better controlled effects of strong interactions (compared to non-leptonic)
 - assumption that tree-level processes not significantly affected by new physics at current level of achievable precision

Exclusive $B \rightarrow D/D^* l\nu$

- main problem given by the non perturbative evaluation of the form factor of matrix elements $\langle D|V^\mu(A^\mu)|B \rangle$.

- form factors depend on meson masses, and on $w = v_B \cdot v_D \geq 1$ $w_{max} \simeq 1.5$

$$\frac{d\Gamma(\bar{B} \rightarrow D l \bar{\nu})}{dw} \propto (w^2 - 1)^{3/2} |V_{cb}|^2 |\mathcal{G}(w)| \quad \frac{d\Gamma(\bar{B} \rightarrow D^* l \bar{\nu})}{dw} \propto (w^2 - 1)^{1/2} |V_{cb}|^2 |\mathcal{F}(w)|$$

- $|V_{cb}| |\mathcal{F}/\mathcal{G}(w)|$ are fitted by data at $w \neq 1$ (due to vanishing phase space at $w = 1$)
 - data are extrapolated to the non-recoil point ($w = 1$, or $v_B = v_D$, with leptons back to back and mesons at rest) where constraints from heavy quark symmetries
 - results for $|\mathcal{F}/\mathcal{G}(1)|$ from theory (f.i.lattice) may be used to obtain values for V_{cb}
- both decays are measured simultaneously \Rightarrow the comparison of their form factors can validate the QCD predictions: f.i. factor ratio at zero recoil $\mathcal{G}(1)/\mathcal{F}(1) = 1.23 \pm 0.09$ confirms the lattice QCD prediction

Exclusive $B \rightarrow D/D^* l\nu$: lattice results

- it is possible to determine the form factors much more precisely than in most leptonic and semileptonic decays
 - V_{cb} connected with the semileptonic form factors via quantities in which the uncertainties cancel almost completely ($D l\bar{\nu}$) or to a high accuracy ($D^* l\bar{\nu}$) in the heavy-quark symmetry limit

– f.i. for $\bar{B} \rightarrow D$

$$|h(1)_+|^2 = \frac{\langle D|\bar{c}\gamma_0 b|\bar{B}\rangle \langle \bar{B}|\bar{b}\gamma_0 c|D\rangle}{\langle D|\bar{c}\gamma_0 c|D\rangle \langle \bar{B}|\bar{b}\gamma_0 b|\bar{B}\rangle} \quad \langle D|V^\mu|B\rangle \propto (v_B+v_D)^\mu h_+ + (v_B-v_D)^\mu h_-$$

Statistical errors in the numerator and denominator are highly correlated and largely cancel as well as most of the normalization uncertainty

- most recent results

– unquenched lattice calculation (Fermilab/MILC) in 2+1 flavor

$$\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016$$

– Using $|V_{cb}|\mathcal{G}(1) = (43.0 \pm 1.9 \pm 1.4) \times 10^{-3}$ from Babar (04/2009) this produces

$$|V_{cb}| = (39.8 \pm 1.8 \pm 1.3 \pm 0.9_{FF}) \times 10^{-3} \quad (1)$$

where the last error is due to the theoretical uncertainty in $\mathcal{G}(1)$

- HFAG average (winter 09); includes untagged $B \rightarrow DXl\nu$

$$|V_{cb}|\mathcal{G}(1) = 42.3 \pm 0.7 \pm 1.3$$

(not yet updated with recent Babar)

- for $\bar{B} \rightarrow D^* l \bar{\nu}$ theoretical ratios with less complete error cancellation, better experimental determination
- first lattice calculation with with three flavors of sea quarks (Fermilab/MILC)

$$\mathcal{F}(1) = 0.921(13)_{stat}(20)_{syst}$$

- Using $|V_{cb}|\mathcal{F}(1) = (35.9 \pm 0.8) \times 10^{-3}$ from PDG

$$|V_{cb}| = (38.7(0.9)_{exp}(1.0)_{theo}) \times 10^{-3}$$

results from $B \rightarrow D/D^*$ not straightforward to combine, correlations between the two sets of measurements and two calculations not been analyzed

- Assuming a correlation of 50% for both, average value

$$|V_{cb}| = (38.6 \pm 1.1) \times 10^{-3}$$

where experimental and lattice-QCD errors have been added in quadrature (CKM08; not includes latest Babar results)

- extrapolations to $w = 1$ introduce a systematic error into the extraction of $|V_{cb}| \Rightarrow$ can be eliminated by calculating the form factors at non zero recoil
 - A first step on this route has been done by applying the step scaling method to calculate, in the quenched approximation, $\mathcal{G}(w)$ and $\mathcal{F}(w)$ for values of w where experimental data are directly available
 - * pro: info on the systematics on $|V_{cb}|$ coming from the extrapolation of the experimental decay rates at zero recoil
 - * contra: quenching uncertainty
 - substantial agreement at zero recoil with the full QCD results \Rightarrow the unestimated quenching error seems to be comparable to the present statistical error.
- $B \rightarrow D/D^*\tau\nu$ studies with non perturbative accuracy opens the possibility to perform lepton-flavor universality checks on the extraction of $|V_{cb}|$ from this channel (Babar 2009)

$$B(B \rightarrow D\tau\nu) = (0.86 \pm 0.24 \pm 0.11 \pm 0.06)\% \quad B(B \rightarrow D^*\tau\nu) = (1.62 \pm 0.31 \pm 0.10 \pm 0.05)\%$$

where the additional third uncertainty is from the normalization mode.

Higher mass states in SL decays

- Studies of $B \rightarrow D/D^*/D^{**}l\nu$ are quite relevant
 - D^{**} : short name for $D(*)n\pi(n > 0)$ final states
 - D^{**} : includes Narrow states (D_1, D_2^*): well established; Broad States (D_0^*, D_1') ; Non resonant ($D^*n\pi$ and $Dn\pi$)
 - Historical puzzle: sum of known exclusive states misses inclusive semileptonic BR
 - Need detailed understanding of $B \rightarrow D/D^*/D^{**}l\nu$ spectra to fix background for $|V_{ub}|$ measurements and $B \rightarrow D/D^*\tau\nu$ studies
- In the past assume single pion saturates D^{**} decays: but $D(\star)\pi\pi$ are possible and start to be measured (could solve the problem with the missing decay modes)

Inclusive $B \rightarrow X_q l \bar{\nu}$

Effective Fermi weak hamiltonian (gluons with virtualities between m_W and m_b)

$$H_W = \frac{4G_F}{\sqrt{2}} V_{qb} J^\mu J_\mu^l = \frac{4G_F}{\sqrt{2}} V_{qb} \bar{q} \gamma^\mu P_L b \bar{l} \gamma_\mu P_L \nu_l$$

neglecting em correction, the decay rate factors into a product of a leptonic tensor and a hadronic tensor $W_{\alpha\beta}$. By optical theorem

$$W_{\alpha\beta} = -\frac{1}{\pi} \text{Im } T_{\alpha\beta}$$

where

$$T_{\alpha\beta} = -i \int d^4x e^{-iq \cdot x} \frac{\langle \bar{B} | T [J_\alpha^\dagger(x) J_\beta(0)] | \bar{B} \rangle}{2m_B}$$

The time ordered product of currents is expanded in a series of local operators by OPE, which corresponds to an expansion of the rate in inverse powers of m_b by HQET

$$T_{\alpha\beta} = \sum_i C_{\alpha\beta}^i(v \cdot q, q^2, m_b, m_c) O_i(9)$$

Up to second order, HQET operators

$$\begin{aligned}
 \langle O_3 \rangle &= \frac{1}{2m_B} \langle \bar{B}(p_B) | \bar{b}_v \gamma^\mu v_\mu b_v | \bar{B}(p_B) \rangle = 1 \\
 \langle O_{\text{kin}} \rangle &= \frac{1}{2m_B} \langle \bar{B}(p_B) | \bar{b}_v (iD)^2 b_v | \bar{B}(p_B) \rangle = -\mu_\pi^2 \\
 \langle O_{\text{mag}} \rangle &= \frac{1}{2m_B} \langle \bar{B}(p_B) | \bar{b}_v \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | \bar{B}(p_B) \rangle = \mu_G^2
 \end{aligned}$$

where $b(x) = e^{-im_b vx} b_v(x)$

$1/m_b$ corrections are absent, as there is no independent gauge-invariant operator of dim 4 in OPE (bound state effects strongly suppressed)

- The nonperturbative input is given by the matrix elements of local operators
- Wilson coefficients of the operators are independent of the external states: can be calculated perturbatively using partonic initial and final states

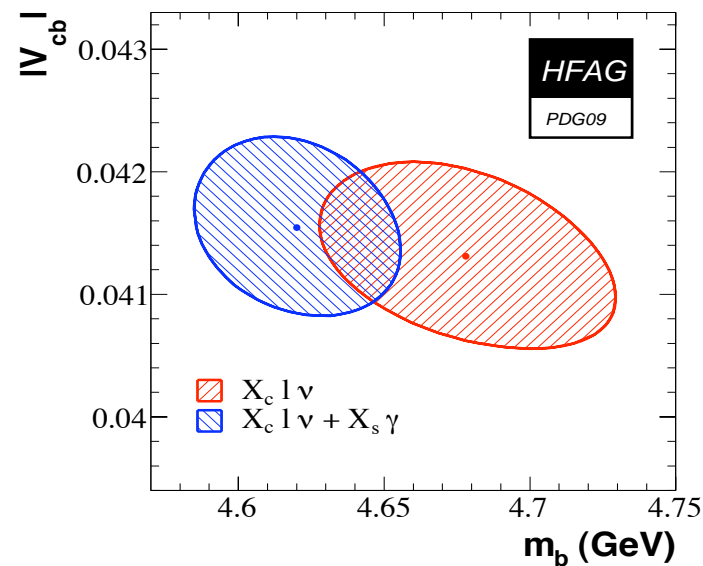
The quantities $|V_{cb}|$, m_b , m_c , and nonperturbative parameters describing effects of the strong interaction can be determined from the measured rates and moments using expansions in $1/m_b$ and the strong coupling constant with reliable uncertainty estimates.

HFAG (winter 09) Global fits to the moments of inclusive distributions in $B \rightarrow X_c l \nu$ (lepton energy and hadron mass) and $B \rightarrow X_s \gamma$ (photon energy) transitions are performed using calculations in the kinetic mass scheme:

second line: $B \rightarrow X_c l \nu$ only

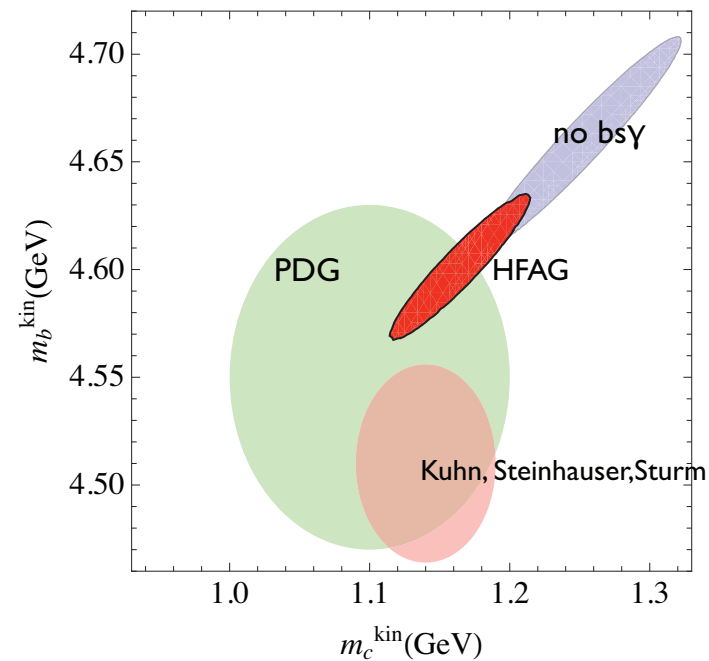
On $|V_{cb}|$ additional uncertainties: The second error corresponds to the uncertainty in the B lifetime, the third error is an additional theoretical uncertainty of 1.4% in the expression of $|V_{cb}|$.

$ V_{cb} (10^{-3})$	m_b^{kin} GeV	$\mu_\pi^2 \text{GeV}^2$	m_c^{kin} GeV
$41.54 \pm 0.43(\text{fit}) \pm 0.08(\tau_B) \pm 0.58(\text{th})$	4.620 ± 0.035	0.424 ± 0.042	1.190 ± 0.052
$41.31 \pm 0.49(\text{fit}) \pm 0.08(\tau_B) \pm 0.58(\text{th})$	4.679 ± 0.051	0.410 ± 0.046	1.276 ± 0.072



Recently $B \rightarrow X_s \gamma$ is argued unreliable, because of presence of uncalculable non perturbative theoretical contributions, starting at order $\alpha_s \Lambda/m_b$ (effect on the branching ratio of roughly $-1.5 \pm 1.5\%$) (Lee, Neubert, Paz, 2007) An analysis of all additional effects not included in the standard calculation gives a factor of about $+1.6\%$ (Misiak 2008)

From P. Gambino talk at CKM08



- recent results from Babar (2009)
 - While lepton-energy moments are known with good accuracy, the precision of the hadronic-mass and photon energy moments is limited by statistics.
 - new measurement of the hadronic-mass moments $\langle m_X^k \rangle$ with $k = 1, \dots, 6$
 - combined fit to the hadronic mass moments, moments of the lepton-energy spectrum, and moments of the photon-energy spectrum in decays $B \rightarrow X_s \gamma$

$$|V_{cb}| = 42.05 \pm 0.45 \pm 0.70 \times 10^{-3}$$

- BaBar $B \rightarrow D$ analyses reach the precise measurements era
 compatibility with both inclusive and $B \rightarrow D^*$
 $|V_{cb}|$ from $B \rightarrow D^*$ lower by about 2.5σ than inclusive determination

Charmless semileptonic decays

- exclusive decays $B \rightarrow \pi/\rho/\eta/\eta'/\omega l\nu$
main theoretical uncertainty due to calculations of the form factors, e.g.

$$\frac{d\Gamma(B \rightarrow \pi l\nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 p_\pi^3 |f_+(q^2)|^2$$

$V_{ub} [10^{-3}]$	FF calculation	
$3.34 \pm 0.12 + 0.55 - 0.37$	Ball-Zwicky (2005)	LC sum rules unquenched lattice, NRQCD unquenched lattice, HQET different q^2 dependence
$3.40 \pm 0.20 + 0.59 - 0.39$	HPQCD (2006-2007)	
$3.62 \pm 0.22 + 0.63 - 0.41$	FNAL (2005)	
3.38 ± 0.36	FNAL (2009)	

- indirect determination of $|V_{ub}|$ by UTfit

$$|V_{ub}| = (3.60 \pm 0.12) \times 10^{-3}$$

- inclusive approach (HFAG)

$V_{ub} [10^{-3}]$	Collab.
$4.06 \pm 0.15 + 0.25 - 0.27$	BLNP
$4.25 \pm 0.15 + 0.21 - 0.17$	DGE
$4.03 \pm 0.15 + 0.20 - 0.25$	GGOU
$3.84 \pm 0.13 + 0.23 - 0.20$	ADFR
$4.87 \pm 0.24 + 0.38 - 0.38$	BLL

Inclusive determination

- OPE factorization of short and long distance dynamics and double series in α_s and Λ/m_b
- dependence on quark masses and HQET expansions parameters (2 parameters at $\mathcal{O}(1/m_b^2)$, 2 more at $\mathcal{O}(1/m_b^3)$...)
- input parameters, included quark masses, determined by a momentum fit with $B \rightarrow X_c l \nu$ and $B \rightarrow X_s \gamma$ in a particular quark mass scheme
- the large $b \rightarrow c$ background implies cuts and use of differential rate
 - no more "enough" inclusive
- available phase-space to QCD partons strongly reduced; regions where OPE fails become relevant

$$m_X \ll E_X$$

- Final gluon radiation strongly inhibited \Rightarrow soft and collinear singularities \Rightarrow perturbative expansion of spectra affected by large logarithms $\approx \alpha_s^n \log^{2n}(2E_X/m_X) \Rightarrow$ logs resummation needed to all orders in α_s
- non-perturbative effect related to a small vibration of the b quark in the B meson (Fermi motion) enhanced at $m_X \approx \sqrt{\Lambda E_X}$

Experimental cuts

- background given by $B \rightarrow X_c l \nu$ suppressed by means of different experimental cuts

$$E_l > (m_B^2 - m_D^2)/2m_B, \quad m_X < m_D, \quad q^2 > (m_B - m_D)^2, \quad p_+ < m_D^2/m_B$$

E_l energy of the charged lepton, m_X hadronic invariant mass of the final state, q^2 invariant mass squared of the lepton-neutrino pair, p_+ plus component of the total momentum of the final- state hadrons. From HFAG

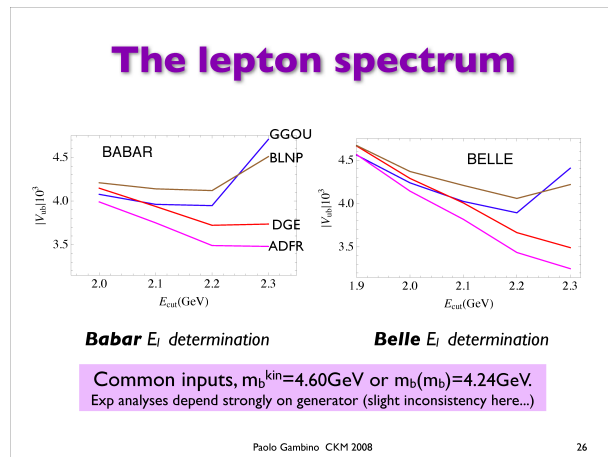
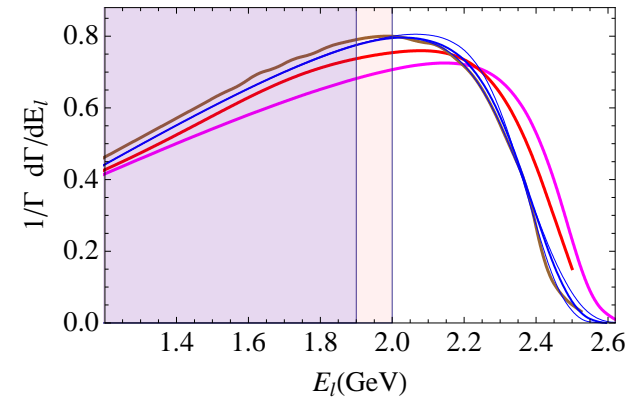
1) <i>BABAR</i> (E_ℓ) 06	2) Belle (E_ℓ) 05	3) CLEO (E_ℓ) 02	4) <i>BABAR</i> (m_X) 08
5) Belle (m_X) 05	6) <i>BABAR</i> ((m_X, q^2)) 08	7) Belle ((m_X, q^2)) 04	8) <i>BABAR</i> (P_+) 08

	ADFR	DGE	BLNP	GGOU
1)	$3.46 \pm 0.14^{+0.24}_{-0.23}$	$4.06 \pm 0.27^{+0.27}_{-0.26}$	$4.18 \pm 0.24^{+0.29}_{-0.31}$	$4.05 \pm 0.23^{+0.22}_{-0.32}$
2)	$3.26 \pm 0.17^{+0.22}_{-0.22}$	$4.56 \pm 0.42^{+0.28}_{-0.24}$	$4.64 \pm 0.43^{+0.29}_{-0.31}$	$4.53 \pm 0.42^{+0.22}_{-0.30}$
3)	$3.49 \pm 0.20^{+0.24}_{-0.24}$	$3.58 \pm 0.42^{+0.28}_{-0.25}$	$3.83 \pm 0.45^{+0.32}_{-0.33}$	$3.68 \pm 0.43^{+0.24}_{-0.38}$
4)	$4.04 \pm 0.19^{+0.25}_{-0.26}$	$4.23 \pm 0.20^{+0.21}_{-0.16}$	$4.02 \pm 0.19^{+0.27}_{-0.29}$	$3.98 \pm 0.19^{+0.26}_{-0.28}$
5)	$3.93 \pm 0.26^{+0.24}_{-0.24}$	$4.03 \pm 0.27^{+0.26}_{-0.20}$	$3.90 \pm 0.26^{+0.24}_{-0.26}$	$3.86 \pm 0.26^{+0.18}_{-0.21}$
6)	$4.15 \pm 0.27^{+0.24}_{-0.24}$	$4.26 \pm 0.28^{+0.23}_{-0.19}$	$4.32 \pm 0.28^{+0.29}_{-0.31}$	$4.22 \pm 0.28^{+0.38}_{-0.35}$
7)	$3.97 \pm 0.42^{+0.23}_{-0.23}$	$4.20 \pm 0.44^{+0.23}_{-0.18}$	$4.23 \pm 0.45^{+0.29}_{-0.310}$	$4.14 \pm 0.44^{+0.33}_{-0.34}$
8)	$3.56 \pm 0.23^{+0.23}_{-0.23}$	$3.70 \pm 0.24^{+0.31}_{-0.24}$	$3.65 \pm 0.24^{+0.25}_{-0.27}$	$3.43 \pm 0.22^{+0.28}_{-0.27}$

Four Different approaches

- predictions based on parameterizations of shape function, and OPE constraints
 - [BLNP](#) B.O. Lange, M. Neubert and G. Paz, Phys. Rev. D72:073006 (2005), and references therein.
 - [GGOU](#) P. Gambino, P. Giordano, G. Ossola, N. Uraltsev, JHEP 0710:058,2007
- predictions based on resummed pQCD
 - [DGE](#) Dressed Gluon Exponentiation, J.R. Andersen and E. Gardi, JHEP 0601:097 (2006); [arXiv:0806.4524]
 - [ADFR](#) - U. Aglietti, F. Di Lodovico, G. Ferrera , G. Ricciardi, EPJC, Vol. 59 (2009), U. Aglietti, G. Ferrera and G. Ricciardi, Nucl. Phys. B768, 85 (2007) and references therein.

From Gambino talk at CKM08



recent results from Belle $p_l^{*B} > 1.0 \text{ GeV}/c$
(arXiv:0907.0379):

$$\begin{aligned}
 V_{ub} 10^3 &= 4.37 \text{ (BLNP)} \\
 &= 4.46 \text{ (DGE)} \\
 &= 4.41 \text{ (GGOU)}
 \end{aligned}$$

compatible with other models
overall uncertainty $\sim 12\%$

A Shape function primer

in the threshold region OPE and the heavy quark expansions fail (dynamics is influenced by soft and collinear gluons)

$$m_X^2 \sim E_X \Lambda_{QCD}$$

an inclusive description is still possible, with the introduction of a non perturbative distribution function (shape function)

- Shape function takes care of singular terms in the theoretical spectrum
it has the role of a momentum distribution function of the b quark in the B meson.
- at leading order is universal
It can be measured in the radiative decay $\bar{B} \rightarrow X_s \gamma$ and the results applied to the calculation of the $\bar{B} \rightarrow X_u l \bar{\nu}_l$ (or independent relation between observable) (f.i Mannel, Recksiegel, 1999)
- Subleading shape functions are difficult to constrain and are not process independent
only first two moments of the leading shape function reasonably well constrained, (related to m_b and μ_π respectively)
- the cuts give rise to a large dependence on m_b

ADFR idea: to introduce nonperturbative effects by introducing an effective, infrared-safe, low energy QCD coupling constant, which mimics, in this specific threshold framework, non perturbative Fermi motion effects

the proper coupling requirements

- starts from universality of perturbative threshold resummation
- non-perturbative effect (Fermi motion) relegated into an effective QCD coupling, which is inserted in the standard soft-gluon resummation formulas
- resummation formulas with the effective coupling are automatically regulated, no need for a prescription
- the coupling is universal (radiative decay processes as well as B fragmentation processes) and it can be constructed on the basis of analyticity arguments
- no non perturbative contribution added through shape function
- no free parameters
- the whole fragmentation process is described in a perturbative framework, no double counting

physical picture: assume that B fragmentation into the b -quark and the spectator quark can be described as a radiation process off the b with a proper coupling

Prescription for the effective coupling

minimal possible "shifting" from standard QCD coupling

$$\alpha_S^{lo}(Q^2) = \frac{1}{\beta_0 \log Q^2 / \Lambda_{QCD}^2},$$

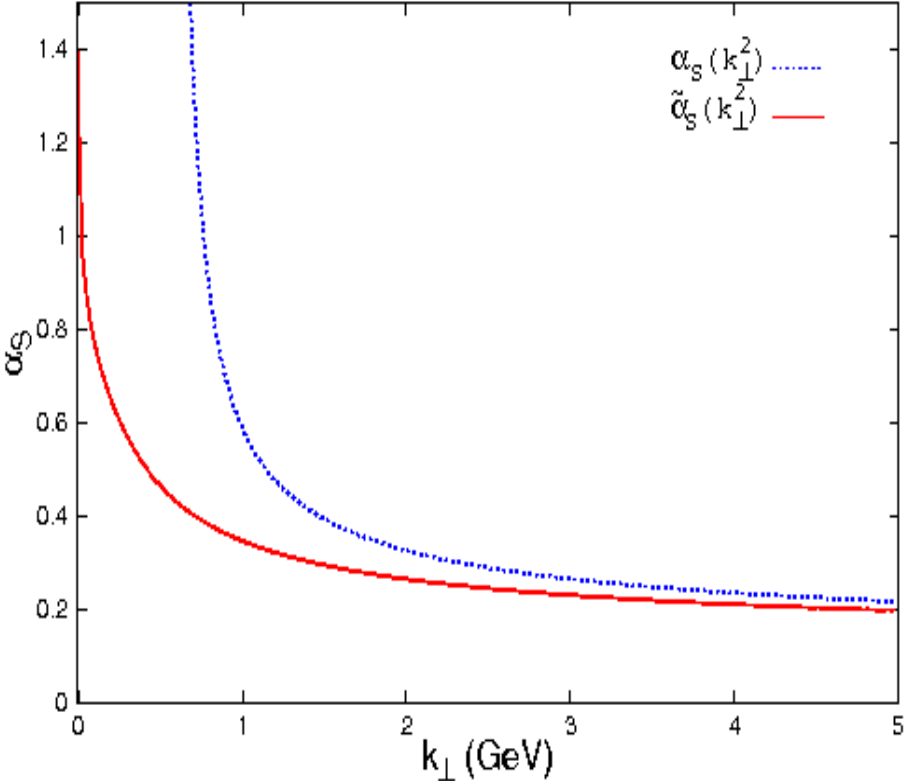
the effective coupling

1. has the same physical discontinuity as α_S along the cut $Q^2 < 0$ (related to the decay of a time-like gluon into secondary parton);
2. is analytic elsewhere in the complex plane (thus removing the unphysical simple pole for $Q^2 = \Lambda^2$ —"Landau ghost")
3. includes secondary emissions off the radiated gluons, with the absorptive parts of the gluon polarization function (the " $-i\pi$ " terms)

$$\tilde{\alpha}_S(k_\perp^2) = \frac{i}{2\pi} \int_0^{k_\perp^2} ds \text{Disc}_s \frac{\bar{\alpha}_S(-s)}{s}$$

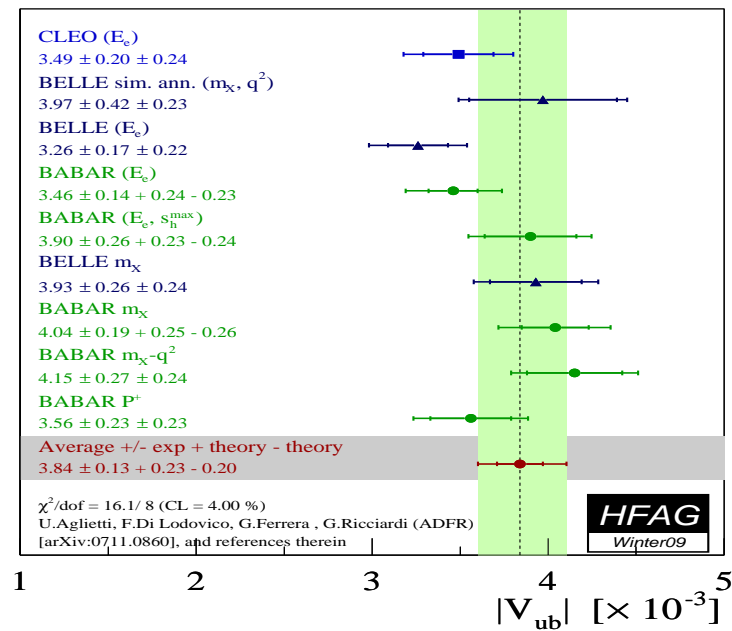
where $\bar{\alpha}_S$ is the ghost-less coupling built according to the preceding prescriptions.

QCD couplings in NNLO (blu, standard, red, effective)



Extracted values of $|V_{ub}|$ for all the uncorrelated analyses and their corresponding average

errors are experimental and theoretical, respectively. The experimental error includes both the statistical and systematic errors



Conclusions

- new unquenched lattice results for form factors by Fermilab/MILC; comparison with other groups awaited , as well as progress beyond the zero recoil point
- impressive recent progress on the experimental side for both $|V_{cb}|$ (Babar) and V_{ub} (Belle)
- still tension in both cases between exclusive (lower) and inclusive determination
- sum of known exclusive states still misses inclusive semileptonic BR, further investigation of SL decays are required and multipion D^{**} final states are the possible missing part: more data are needed, also to improve V_{ub} background
- A preliminary result from BELLE uses $E_l > 1.0$ GeV, and quotes an experimental uncertainty of 6% on $|V_{ub}|$
inclusive approaches need improvement (WA diagrams impact, high q^2 singularity in the shape function, further constraints on SL shape function and value of m_b)