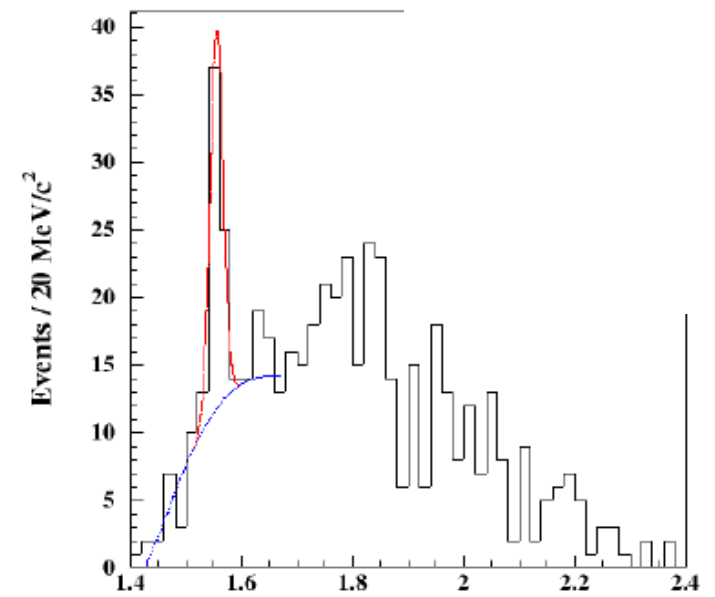
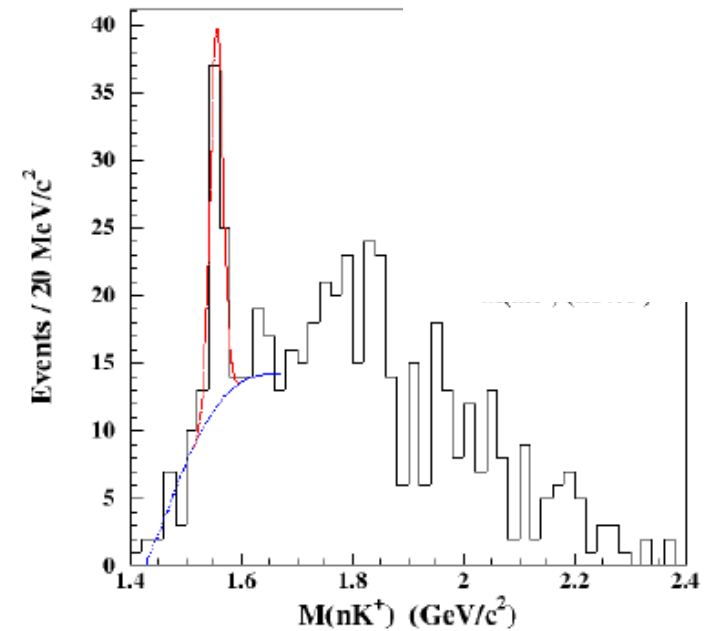


Is there evidence for a peak in this data?



Is there evidence for a peak in this data?



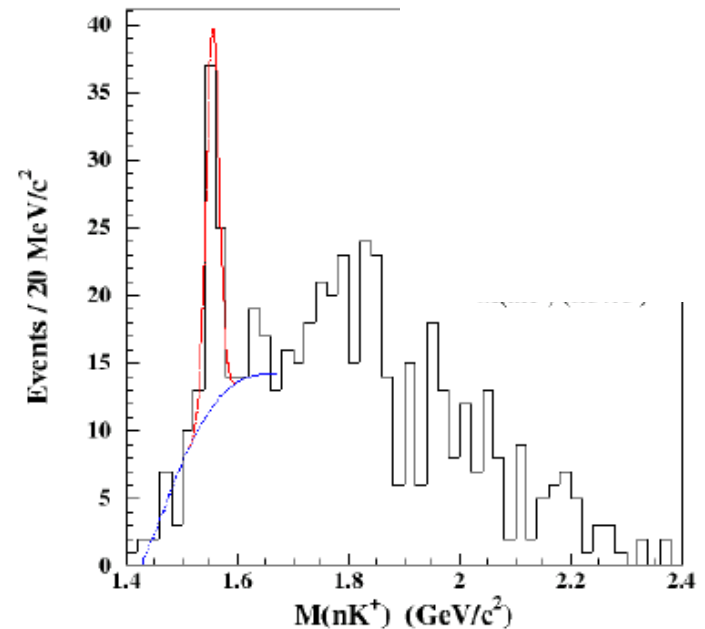
“Observation of an Exotic $S=+1$

Baryon in Exclusive Photoproduction from the Deuteron”

S. Stepanyan et al, CLAS Collab, Phys.Rev.Lett. 91 (2003) 252001

“The statistical significance of the peak is $5.2 \pm 0.6 \sigma$ ”

Is there evidence for a peak in this data?



“Observation of an Exotic $S=+1$

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“The statistical significance of the peak is $5.2 \pm 0.6 \sigma$ ”

“A Bayesian analysis of pentaquark signals from CLAS data”

D. G. Ireland et al, CLAS Collab, Phys. Rev. Lett. 100, 052001 (2008)

“The $\ln(\text{RE})$ value for g2a (-0.408) indicates weak evidence in favour of the data model without a peak in the spectrum.”

Comment on “Bayesian Analysis of Pentaquark Signals from CLAS Data”
Bob Cousins, <http://arxiv.org/abs/0807.1330>

Statistical Issues in Searches for New Physics

Louis Lyons and Lorenzo Moneta
Imperial College, London & Oxford
CERN

Theme: Using data to make judgements about H1 (New Physics) versus H0 (S.M. with nothing new)

Why?

Experiments are expensive and time-consuming
so

Worth investing effort in statistical analysis
→ better information from data

Topics:

- p-values

 - What they mean

 - Combining p-values

- Significance

- Blind Analysis

- LEE = Look Elsewhere Effect

- Why 5σ for discovery?

- Wilks' Theorem

- Background Systematics

- p_0 v p_1 plots

- Higgs search: Discovery, mass and spin

Conclusions

Examples of Hypotheses

1) Event selector (Event = particle interaction)

Events produced at CERN LHC at enormous rate

Online 'trigger' to select events for recording (~ 1 kiloHertz)

e.g. events with many particles

Offline selection based on required features

e.g. H0: Event contains top H1: No top

Possible outcomes: Events assigned as H0 or H1

2) Result of experiment

e.g. H0 = nothing new

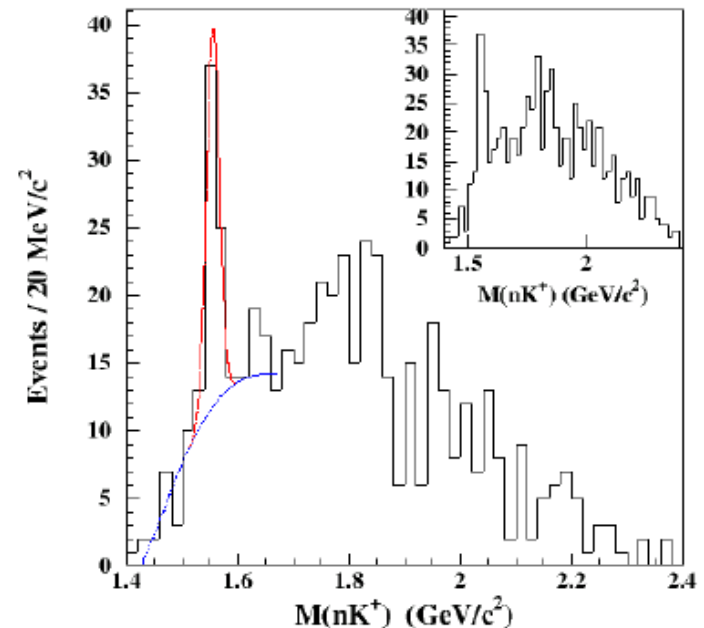
H1 = new particle produced as well
(Higgs, SUSY, 4th neutrino,.....)

Possible outcomes	H0	H1	
	✓	X	Exclude H1
	X	✓	Discovery
	✓	✓	No decision
	X	X	?

WRONG DECISIONS

E1: Reject H0 when H0 true (Loss of effic in 1))

E2: Fail to reject H0 when H1 true (Contamination)



H0 or H0 versus H1 ?

H0 = null hypothesis

e.g. Standard Model, with nothing new

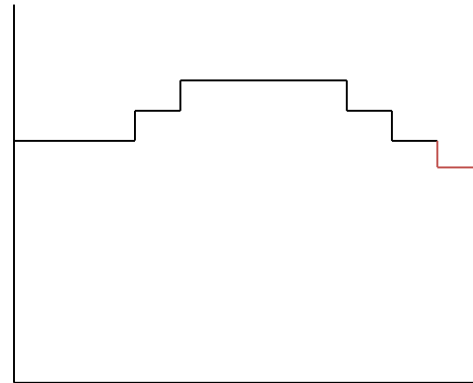
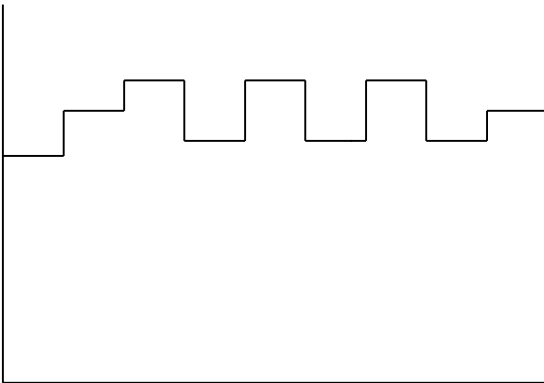
H1 = specific New Physics e.g. Higgs with $M_H = 125$ GeV

H0: “Goodness of Fit” e.g. χ^2 , p-values

H0 v H1: “Hypothesis Testing” e.g. \mathcal{L} -ratio

Measures how much data favours one hypothesis wrt other

H0 v H1 likely to be more sensitive for H1



Choosing between 2 hypotheses

Possible methods:

$\Delta\chi^2$

p-value of statistic →

$\ln\mathcal{L}$ -ratio

Bayesian:

Posterior odds

Bayes factor

Bayes information criterion (BIC)

Akaike (AIC)

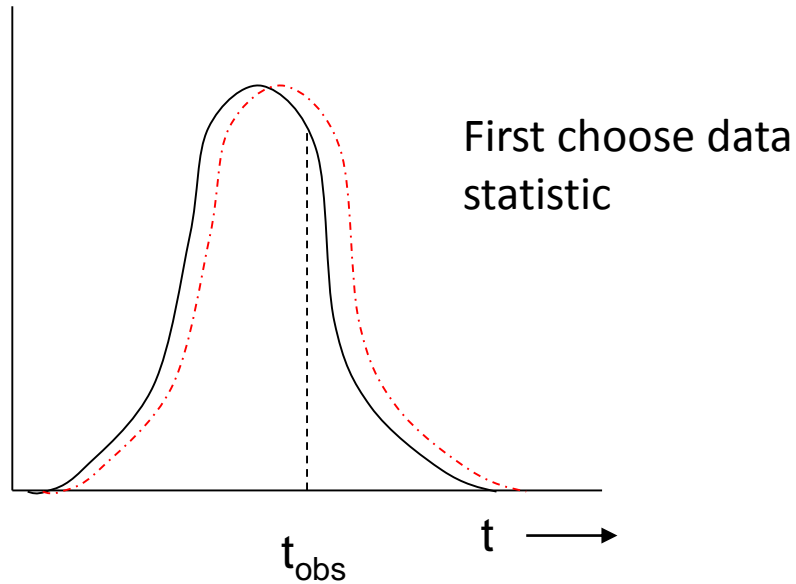
Minimise “cost”

See ‘Comparing two hypotheses’

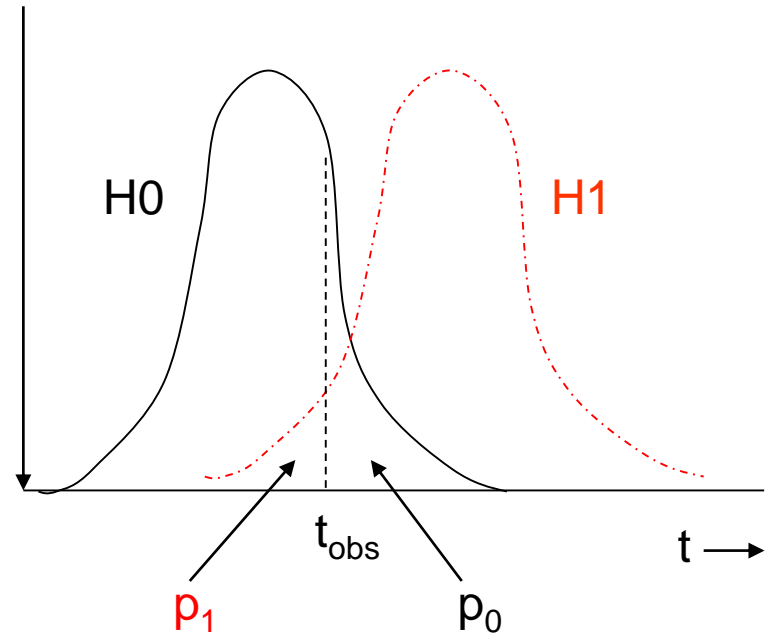
<http://www-cdf.fnal.gov/physics/statistics/notes/H0H1.pdf>

p-values

(a)

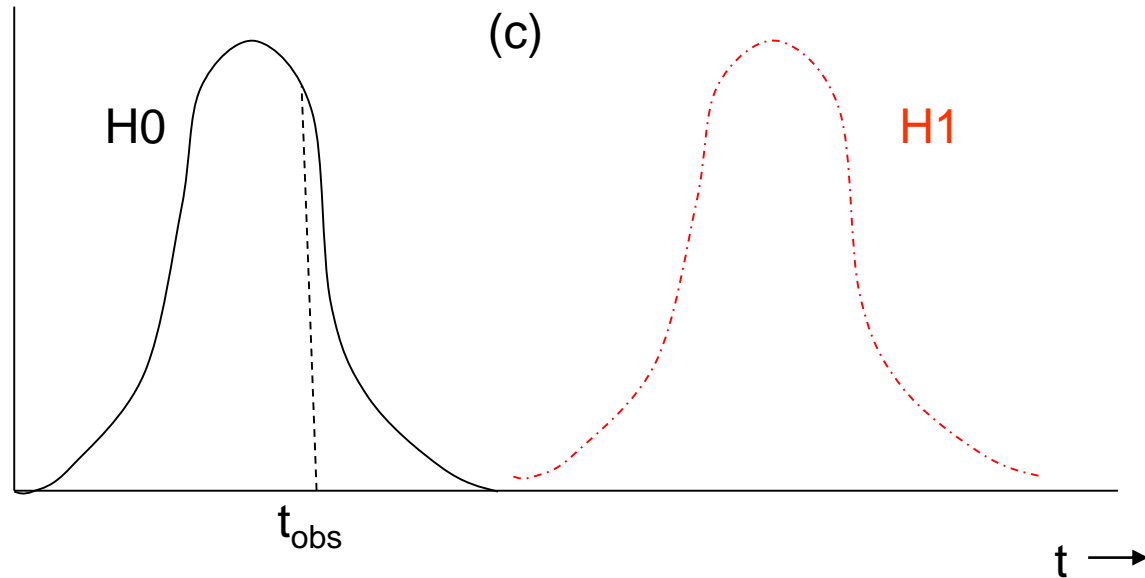


(b)



With 2 hypotheses,
each with own pdf,
p-values are
defined as tail
areas, pointing in
towards each other

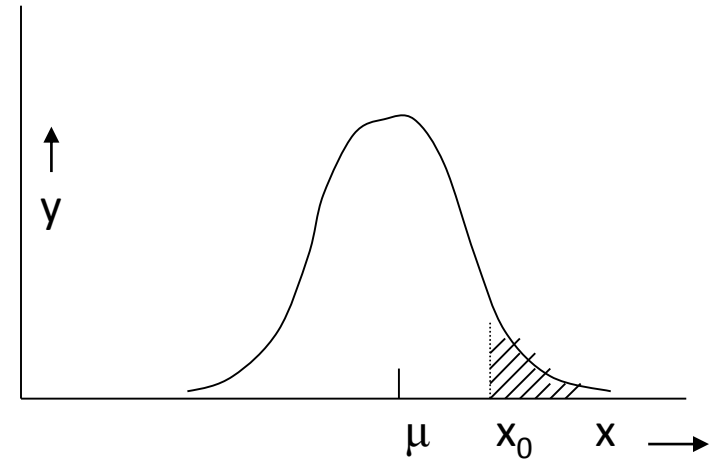
(c)



p-values

Concept of pdf

Example: **Gaussian**



y = probability density for measurement x

$$y = \frac{1}{\sqrt{(2\pi)\sigma}} \exp\{-0.5*(x-\mu)^2/\sigma^2\}$$

p-value: probability that $x \geq x_0$

Gives probability of “extreme” values of data (in interesting direction)

$(x_0-\mu)/\sigma$	1	2	3	4	5
p	16%	2.3%	0.13%	0.003%	$0.3*10^{-6}$

i.e. **Small p = unexpected**

p-values, contd

Assumes:

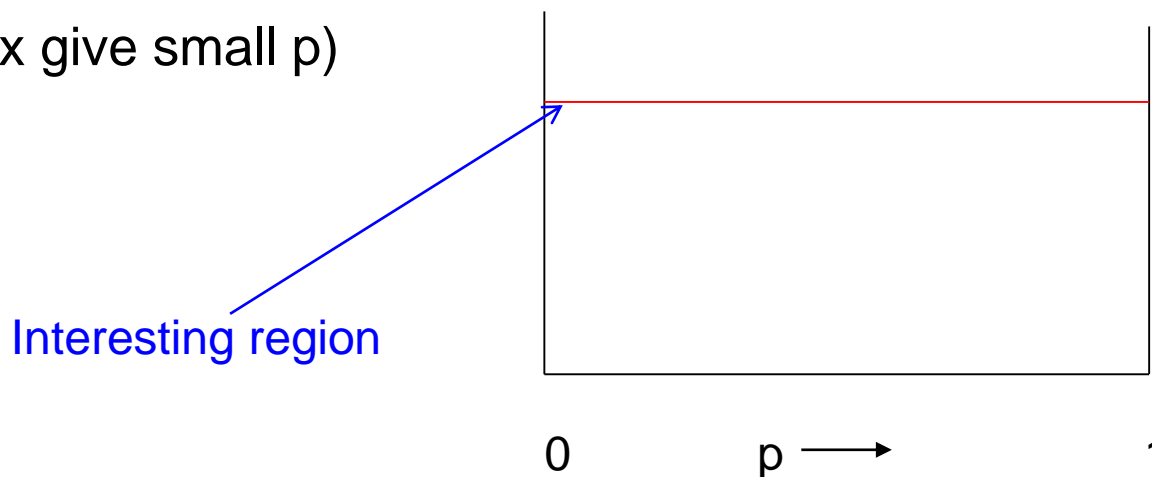
Specific pdf for x (e.g. Gaussian, no long tails)

Data is unbiased

σ is correct

If so, and x is from that pdf \Rightarrow **uniform p-distribution**

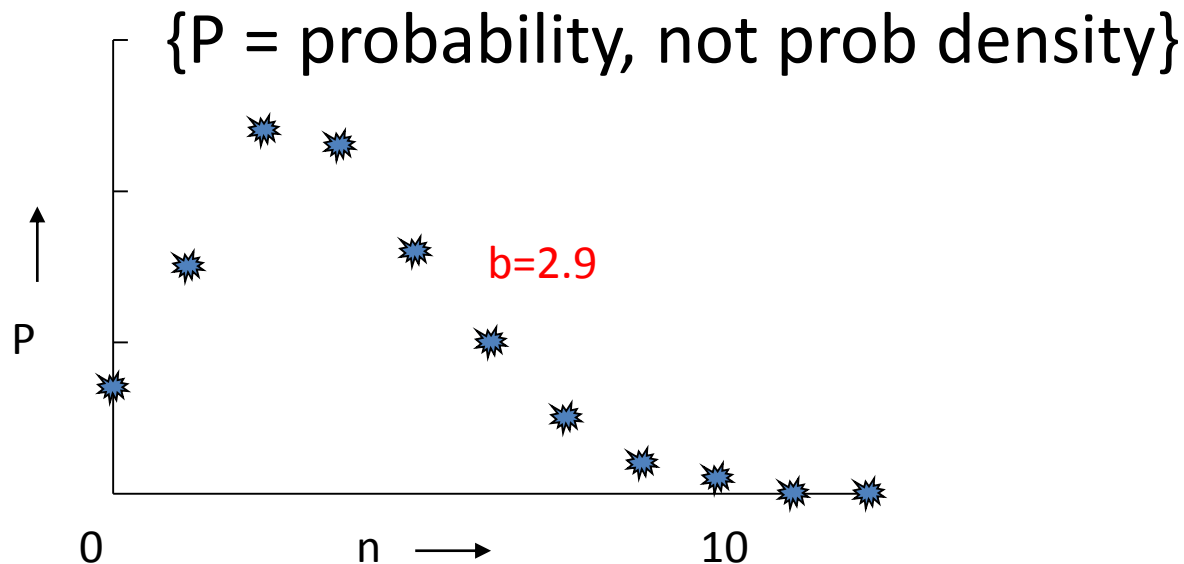
(Events at large x give small p)



p-values for non-Gaussian distributions

e.g. **Poisson** counting experiment, $\text{bgd} = b$

$$P(n) = e^{-b} * b^n / n!$$



For $n=7$, $p = \text{Prob}(\text{at least 7 events}) = P(7) + P(8) + P(9) + \dots = 0.03$

Significance

Significance = S/\sqrt{B} or similar ?

Potential Problems:

- Uncertainty in B
- Non-Gaussian behaviour of Poisson, especially in tail
- Number of bins in histogram, no. of other histograms [LEE]
- Choice of cuts, bins (Blind analyses)

For future experiments:

- Optimising: Could give $S = 0.1$, $B = 10^{-4}$, $S/\sqrt{B} = 10$

CONCLUSION:

Calculate p properly (and allow for LEE if necessary)

p-values and σ

p-values often converted into equivalent Gaussian σ

e.g. 3×10^{-7} is “ 5σ ” (one-sided Gaussian tail)

Does NOT imply that pdf = Gaussian

(Simply easier to remember number of σ , than p-value.)

What p-values are (and are not)

Reject H_0 if $t > t_{\text{crit}}$ ($p < \alpha$)

p-value = prob that $t \geq t_{\text{obs}}$

Small $p \rightarrow$ data and theory have poor compatibility

Small p-value does **NOT** automatically imply that theory is unlikely

Bayes $\text{prob}(\text{Theory}; \text{data})$ related to $\text{prob}(\text{data}; \text{Theory}) = \text{Likelihood}$

by Bayes Th, including Bayesian prior

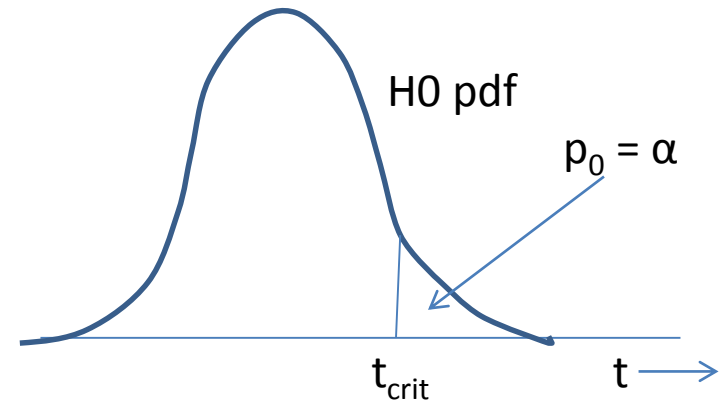
$$P(A;B) \neq P(B;A)$$

p-values are misunderstood. e.g. Anti-HEP jibe:

“Particle Physicists don’t know what they are doing, because half their $p < 0.05$ exclusions turn out to be wrong”

Demonstrates lack of understanding of p-values

[**All** results rejecting energy conservation with $p < \alpha = .05$ cut will turn out to be ‘wrong’]



Criticisms of p-values

(p-values banned by journal [*Basic and Applied Social Psychology*](#))

1) Misunderstood

So ban relativity, matrices.....?

2) Incorrect statements

3) p-values smaller than \mathcal{L} -ratios

Measure different quantities

p is only for one hypothesis

\mathcal{L} -ratio compares two hypotheses

(Is length or mass 'better' for comparing mouse and elephant?)

Combining different p-values

Several results quote independent p-values for same effect:

p_1, p_2, p_3, \dots e.g. 0.9, 0.001, 0.3

What is combined significance? Not just $p_1 * p_2 * p_3, \dots$

If 10 expts each have $p \sim 0.5$, product ~ 0.001 and is clearly **NOT** correct
combined p

$$S = z * \sum_{j=0}^{n-1} (-\ln z)^j / j! , \quad z = p_1 p_2 p_3 \dots$$

(e.g. For 2 measurements, $S = z * (1 - \ln z) \geq z$)

Problems:

1) Recipe is not unique (Uniform dist in n-D hypercube \rightarrow uniform in 1-D)

2) Formula is not associative

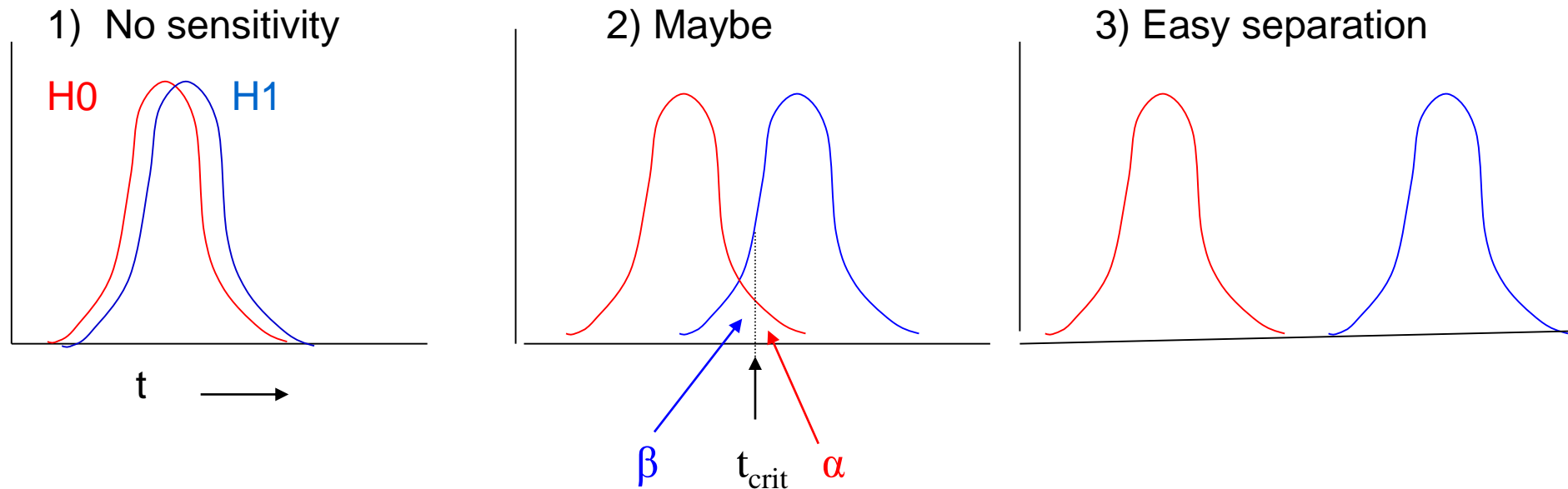
Combining $\{p_1 \text{ and } p_2\}$, and then p_3 gives different answer
from $\{p_3 \text{ and } p_2\}$, and then p_1 , or all together

Due to different options for “more extreme than x_1, x_2, x_3 ”.

3) Small p's due to different discrepancies

***** Better to combine data *****

Procedure for choosing between 2 hypotheses



Procedure: Obtain expected distributions for data statistic (e.g. \mathcal{L} -ratio) for H_0 and H_1

Choose α (e.g. 95%, 3σ , 5σ ?) and CL for p_1 (e.g. 95%)

Given b , α determines t_{crit}

$b+s$ defines β . For $s > s_{\text{min}}$, separation of curves \rightarrow discovery or excln

$1-\beta$ = Power of test

Now data: If $t_{\text{obs}} \geq t_{\text{crit}}$ (i.e. $p_0 \leq \alpha$), discovery at level α

If $t_{\text{obs}} < t_{\text{crit}}$, no discovery. If $p_1 < 1 - \text{CL}$, exclude H_1 (or $\text{CLs} = p_1/(1-p_0)$)

For event selector, $1-\alpha$ = efficiency for signal events; β = mis-ID prob from other events

BLIND ANALYSES

Why blind analysis? Data statistic, selections, corrections, method

Methods of blinding

- Add random number to result *
- Study procedure with simulation only
- Look at only first fraction of data
- Keep the signal box closed
- Keep MC parameters hidden
- Keep unknown fraction visible for each bin

Disadvantages

- Takes longer time
- Usually not available for searches for unknown

After analysis is unblinded, don't change anything unless

* Luis Alvarez suggestion re “discovery” of free quarks

Look Elsewhere Effect (LEE)

Prob of bgd fluctuation at that place = local p-value

Prob of bgd fluctuation '**anywhere**' = global p-value

Global $p >$ Local p

Where is '**anywhere**'?

- a) Any location in this histogram in sensible range
 - b) Any location in this histogram
 - c) Also in histogram produced with different cuts, binning, etc.
 - d) Also in other plausible histograms for this analysis
 - e) Also in other searches in this PHYSICS group (e.g. SUSY at CMS)
 - f) In any search in this experiment (e.g. CMS)
 - g) In all CERN expts (e.g. LHC expts + NA62 + OPERA + ASACUSA +)
 - h) In all HEP expts
- etc.

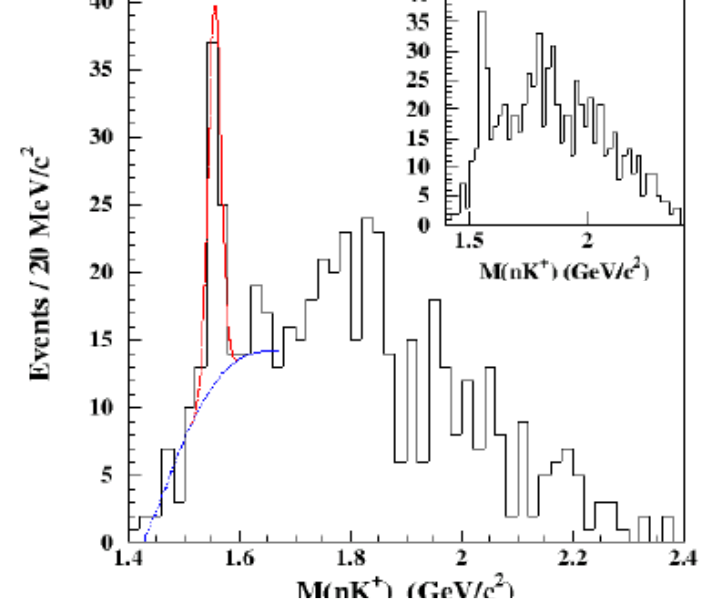
d) relevant for graduate student doing analysis

f) relevant for experiment's Spokesperson

INFORMAL CONSENSUS:

Quote local p , and global p according to a) above.

Explain which global p



Example of LEE: Stonehenge



Are alignments significant?

- Atkinson replied with his article "Moonshine on Stonehenge" in [Antiquity](#) in 1966, pointing out that some of the pits which had used for his sight lines were more likely to have been natural depressions, and that he had allowed a margin of error of up to 2 degrees in his alignments. Atkinson found that the probability of so many alignments being visible from 165 points to be close to 0.5 rather than the "one in a million" possibility which had claimed.
- had been examining stone circles since the 1950s in search of astronomical alignments and the [megalithic yard](#). It was not until 1973 that he turned his attention to Stonehenge. He chose to ignore alignments between features within the monument, considering them to be too close together to be reliable. He looked for landscape features that could have marked lunar and solar events. However, one of 's key sites, Peter's Mound, turned out to be a twentieth-century rubbish dump.

Why 5σ for Discovery?

Statisticians ridicule our belief in extreme tails (esp. for systematics)

Our reasons:

1) Past history (Many 3σ and 4σ effects have gone away)

2) LEE

3) Worries about underestimated systematics

4) Subconscious Bayes calculation

$$\frac{p(H_1|x)}{p(H_0|x)} = \frac{p(x|H_1)}{p(x|H_0)} * \frac{\pi(H_1)}{\pi(H_0)}$$

Posterior Likelihood Priors
prob ratio

“Extraordinary claims require extraordinary evidence”

N.B. Points 2), 3) and 4) are experiment-dependent

Alternative suggestion:

L.L. “Discovering the significance of 5σ ”

<http://arxiv.org/abs/1310.1284>

How many σ 's for discovery?

SEARCH	SURPRISE	IMPACT	LEE	SYSTEMATICS	No. σ
Higgs search	Medium	Very high	M	Medium	5
Single top	No	Low	No	No	3
SUSY	Yes	Very high	Very large	Yes	7
B_s oscillations	Medium/Low	Medium	Δm	No	4
Neutrino osc	Medium	High	$\sin^2 2\theta, \Delta m^2$	No	4
$B_s \rightarrow \mu \mu$	No	Low/Medium	No	Medium	3
Pentaquark	Yes	High/V. high	M, decay mode	Medium	7
$(g-2)_\mu$ anom	Yes	High	No	Yes	4
H spin $\neq 0$	Yes	High	No	Medium	5
4 th gen q, l, ν	Yes	High	M, mode	No	6
Dark energy	Yes	Very high	Strength	Yes	5
Grav Waves	No	High	Enormous	Yes	8



Suggestions to provoke discussion, rather than 'carved in stone' on Mt. Sinai'

Bob Cousins: "2 independent expts each with 3.5σ better than one expt with 5σ "

Wilks' Theorem

Data = some distribution e.g. mass histogram

For H_0 and H_1 , calculate best fit weighted sum of squares S_0 and S_1

Examples: 1) H_0 = polynomial of degree 3

H_1 = polynomial of degree 5

2) H_0 = background only

H_1 = bgd+peak with free M_0 and cross-section

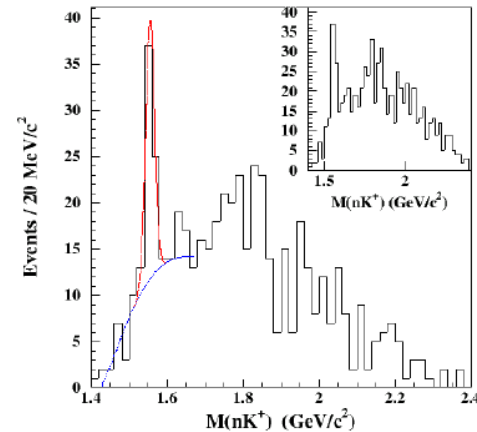
3) H_0 = normal neutrino hierarchy

H_1 = inverted hierarchy

If H_0 true, S_0 distributed as χ^2 with $\text{nfd} = \nu_0$

If H_1 true, S_1 distributed as χ^2 with $\text{nfd} = \nu_1$

If H_0 true, what is distribution of $\Delta S = S_0 - S_1$? Expect not large. Is it χ^2 ?



Wilks' Theorem: ΔS distributed as χ^2 with $\text{nfd} = \nu_0 - \nu_1$ provided:

- a) H_0 is true
- b) H_0 and H_1 are nested
- c) Params for $H_1 \rightarrow H_0$ are well defined, and not on boundary
- d) Data is asymptotic

Wilks' Theorem, contd

Examples: Does Wilks' Th apply?

1) H_0 = polynomial of degree 3

H_1 = polynomial of degree 5

YES: ΔS distributed as χ^2 with ndf = $(d-4) - (d-6) = 2$

2) H_0 = background only

H_1 = bgd + peak with free M_0 and cross-section

NO: H_0 and H_1 nested, but M_0 undefined when $H_1 \rightarrow H_0$. $\Delta S \neq \chi^2$
(but not too serious for fixed M)

3) H_0 = normal neutrino hierarchy

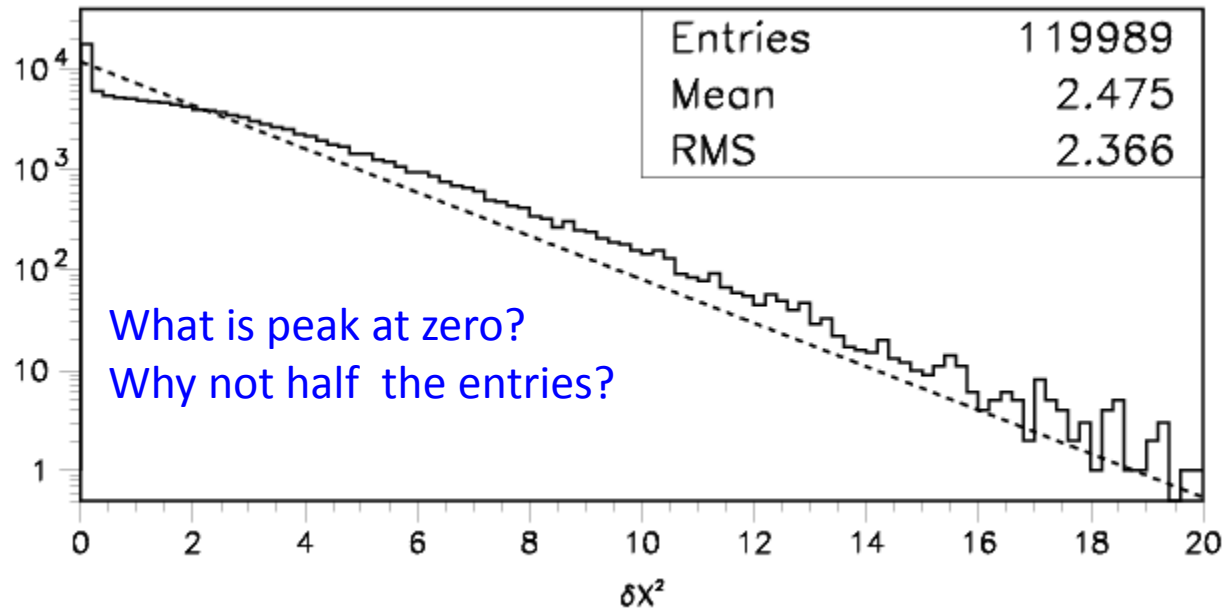
H_1 = inverted hierarchy

NO: Not nested. $\Delta S \neq \chi^2$ (e.g. can have ΔS negative)

N.B. 1: Even when **W. Th.** does not apply, it does not mean that ΔS is irrelevant, but you cannot use **W. Th.** for its expected distribution.

N.B. 2: For large ndf, better to use ΔS , rather than S_1 and S_0 separately

Is difference in S distributed as χ^2 ?

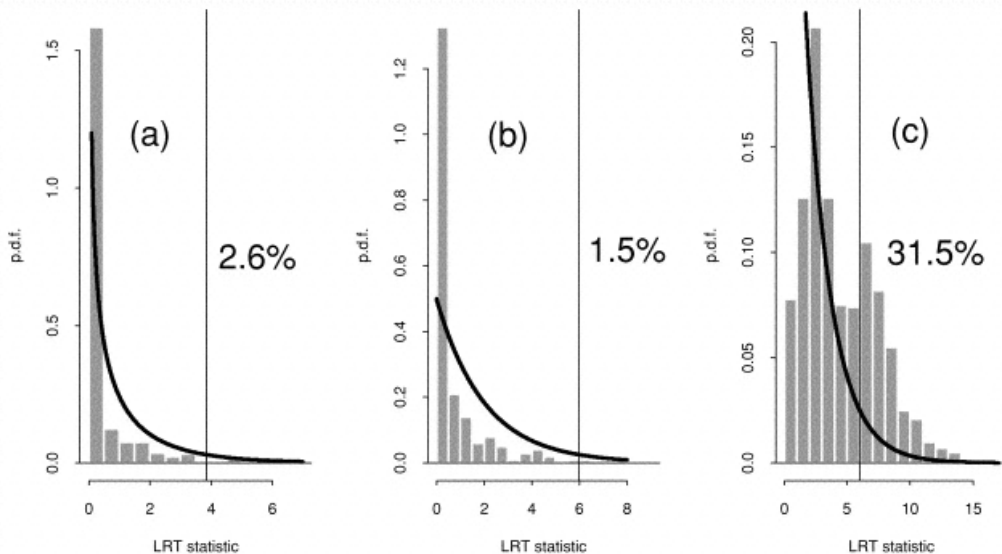


Demortier:

H0 = quadratic bgd

H1 = +

Gaussian of fixed width,
variable location & ampl



Protassov, van Dyk, Connors,

H0 = continuum

(a) H1 = narrow emission line

(b) H1 = wider emission line

(c) H1 = absorption line

Nominal significance level = 5%

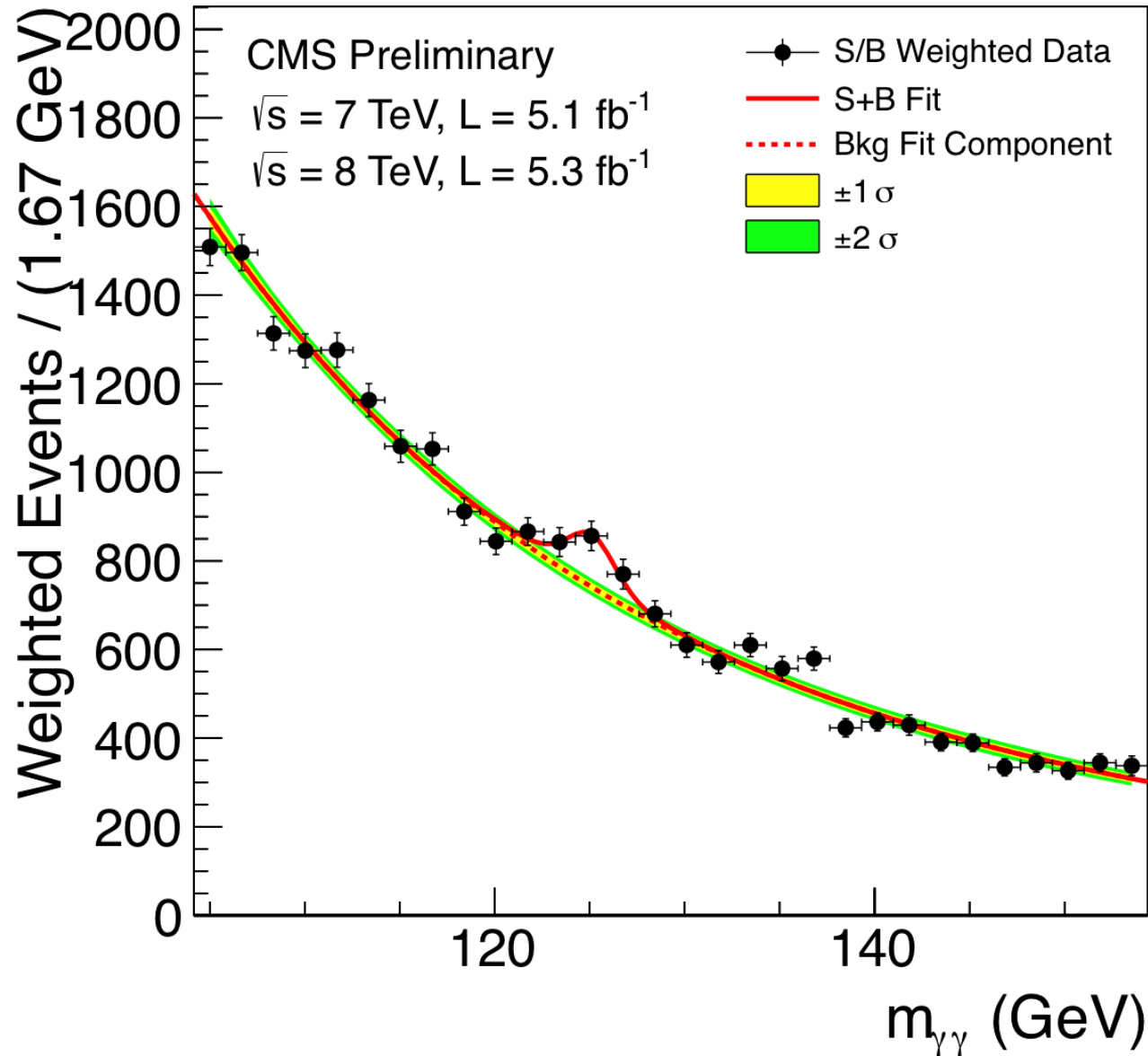
Is difference in S distributed as χ^2 ?, contd.

So need to determine the ΔS distribution by Monte Carlo

N.B.

- 1) For mass spectrum, determining ΔS for hypothesis H_1 when data is generated according to H_0 is not trivial, because there will be lots of local minima
- 2) If we are interested in 5σ significance level, needs lots of MC simulations (or intelligent MC generation)
- 3) Asymptotic formulae may be useful (see K. Cranmer, G. Cowan, E. Gross and O. Vitells, 'Asymptotic formulae for likelihood-based tests of new physics', <http://link.springer.com/article/10.1140%2Fepjc%2Fs10052-011-1554-0>)

Background systematics



Background systematics, contd

Signif from comparing χ^2 's for H0 (bgd only) and for H1 (bgd + signal)

Typically, bgd = functional form f_a with free params

e.g. 4th order polynomial

Uncertainties in params included in signif calculation

But what if functional form is different ? e.g. f_b

Typical approach:

If f_b best fit is bad, not relevant for systematics

If f_b best fit is ~comparable to f_a fit, include contribution to systematics

But what is 'comparable'?

Other approaches:

Profile likelihood over different bgd parametric forms

<http://arxiv.org/pdf/1408.6865v1.pdf?>

Background subtraction

sPlots

Non-parametric background

Bayes

etc

No common consensus yet among experiments on best approach

{Spectra with multiple peaks are more difficult}

“Handling uncertainties in background shapes: the discrete profiling method”

Dauncey, Kenzie, Wardle and Davies (Imperial College, CMS)

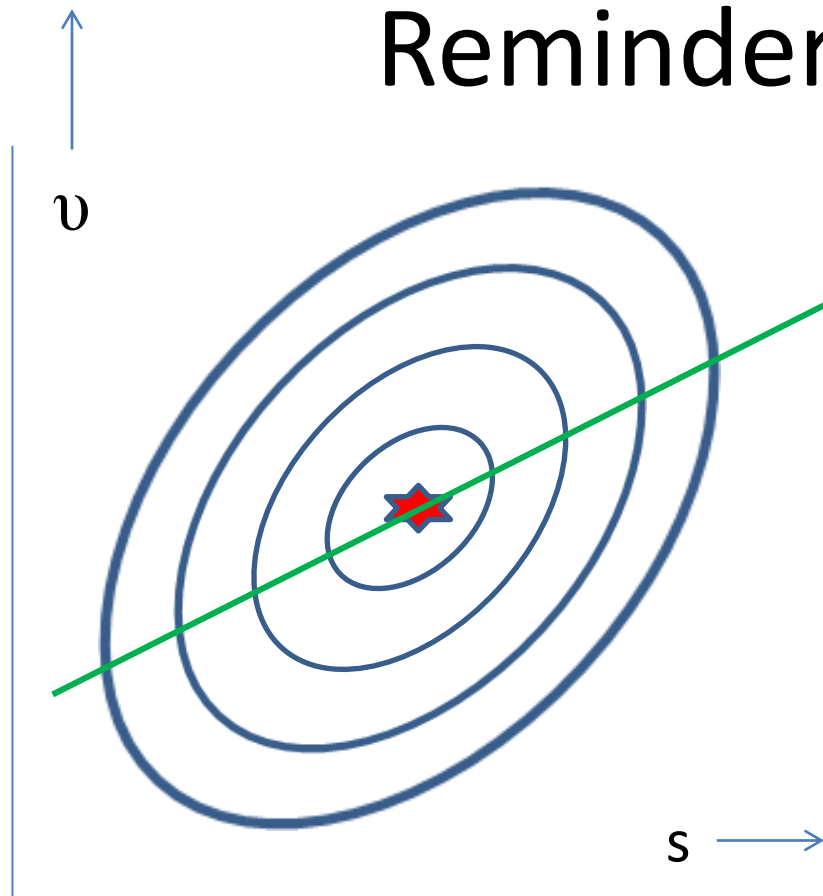
[arXiv:1408.6865v1](https://arxiv.org/abs/1408.6865v1) [physics.data-an]

Has been used in CMS analysis of $H \rightarrow \gamma\gamma$

Problem with ‘Typical approach’: Alternative functional forms do or don’t contribute to systematics by hard cut, so systematics can change discontinuously wrt $\Delta\chi^2$

Method is like profile \mathcal{L} for continuous nuisance params
Here ‘profile’ over discrete functional forms

Reminder of Profile \mathcal{L}

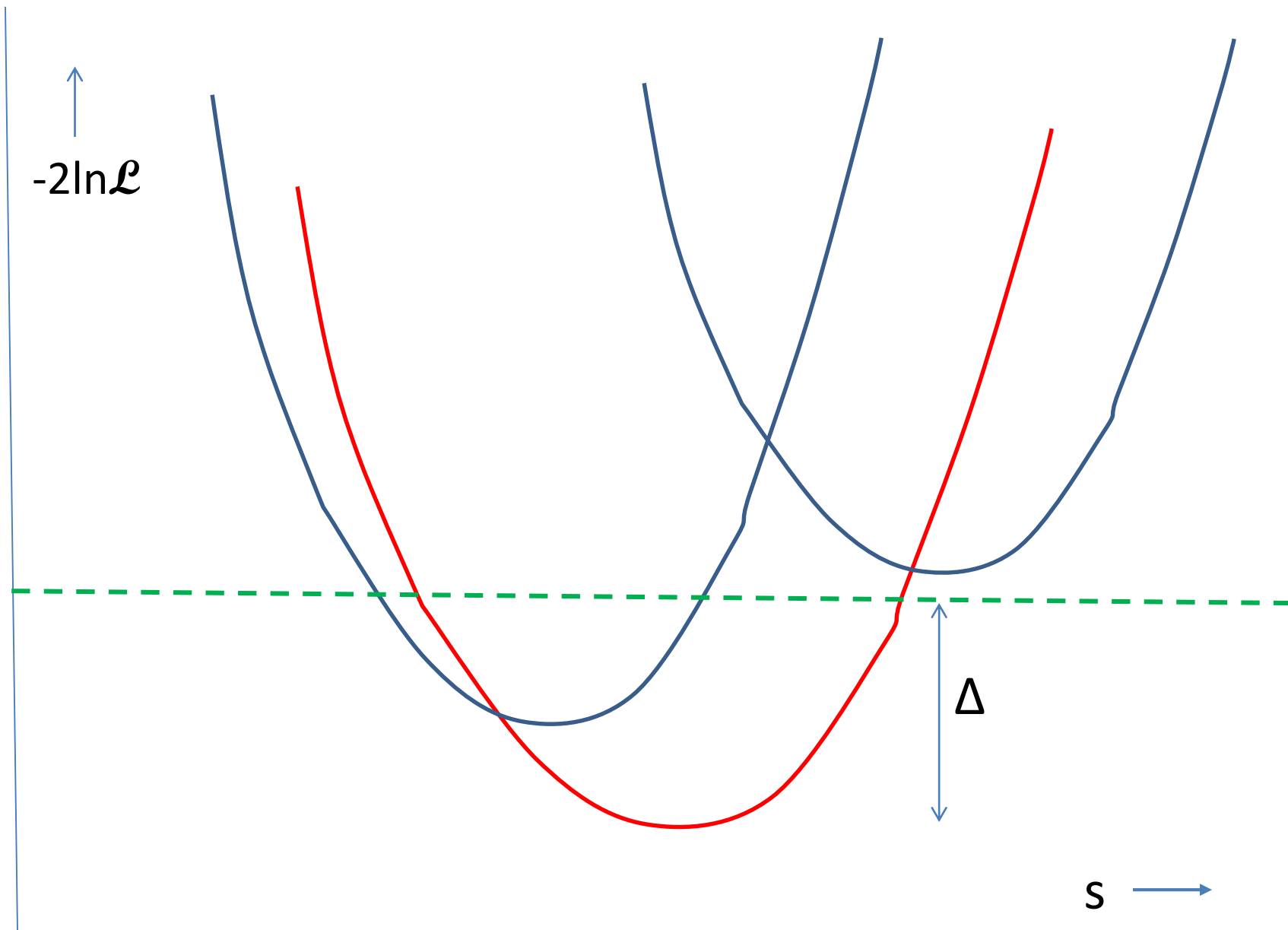


Stat uncertainty on s from width of \mathcal{L} fixed at v_{best}

Total uncertainty on s from width of $\mathcal{L}(s, v_{\text{prof}(s)}) = \mathcal{L}_{\text{prof}}$
 $v_{\text{prof}(s)}$ is best value of v at that s
 $v_{\text{prof}(s)}$ as fn of s lies on green line

Contours of $\ln \mathcal{L}(s, v)$
 s = physics param
 v = nuisance param

Total uncert \geq stat uncertainty



Red curve: Best value of nuisance param ν

Blue curves: Other values of ν

Horizontal line: Intersection with red curve \rightarrow
statistical uncertainty

‘Typical approach’: Decide which blue curves have small enough Δ
Systematic is largest change in minima wrt red curves’.

Profile L: Envelope of lots of blue curves

Wider than red curve, because of systematics (ν)

For $\mathcal{L} = \text{multi-D Gaussian}$, agrees with ‘Typical approach’

Dauncey et al use envelope of finite number of functional forms

Point of controversy!

Two types of 'other functions':

a) Different function types e.g.

$$\sum a_i x_i \text{ versus } \sum a_i / x_i$$

b) Given fn form but different number of terms

DDKW deal with b) by $-2\ln L \rightarrow -2\ln L + kn$

n = number of extra free params wrt best

$k = 1$, as in AIC (= Akaike Information Criterion)

Opposition claim choice $k=1$ is arbitrary.

DDKW agree but have studied different values, and say $k=1$ is optimal for them.

Also, any parametric method needs to make such a choice

p_0 v p_1 plots

Preprint by Luc Demortier and LL,
“Testing Hypotheses in Particle Physics:
Plots of p_0 versus p_1 ”
<http://arxiv.org/abs/1408.6123>

For hypotheses H_0 and H_1 , p_0 and p_1
are the tail probabilities for data
statistic t

Provide insights on:

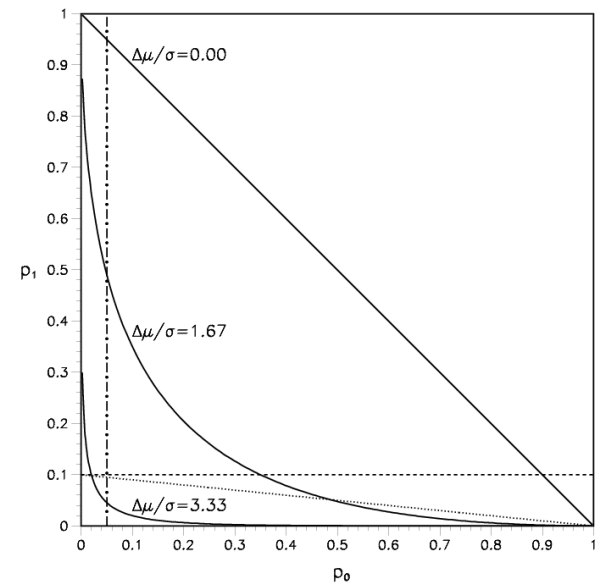
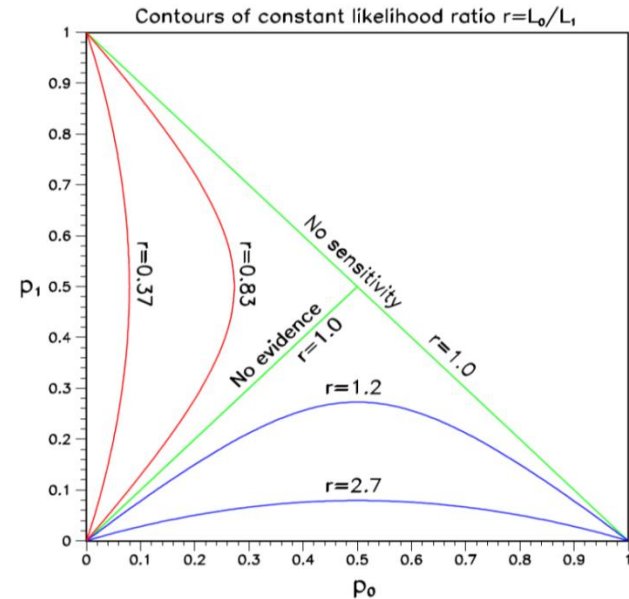
CLs for exclusion

Punzi definition of sensitivity

Relation of p-values and Likelihoods

Probability of misleading evidence

Jeffreys-Lindley paradox

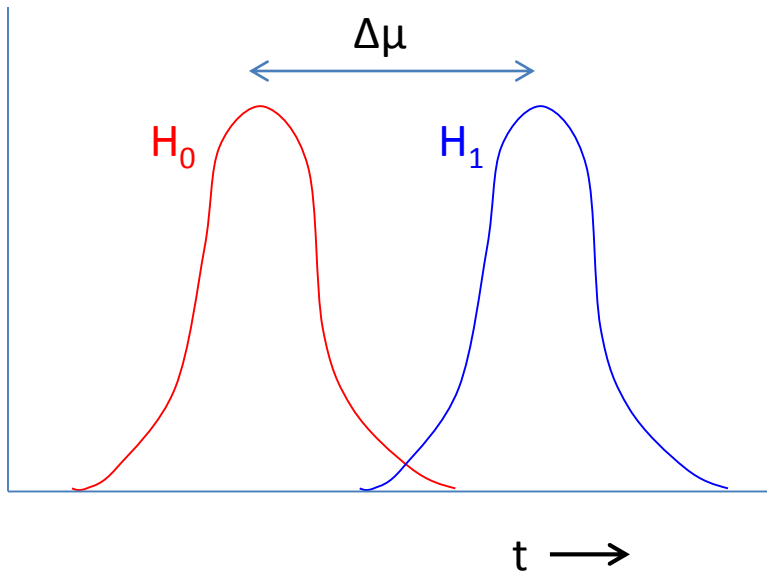


CLs = $p_1/(1-p_0)$ \rightarrow diagonal line

Provides protection against excluding H_1 when little or no sensitivity

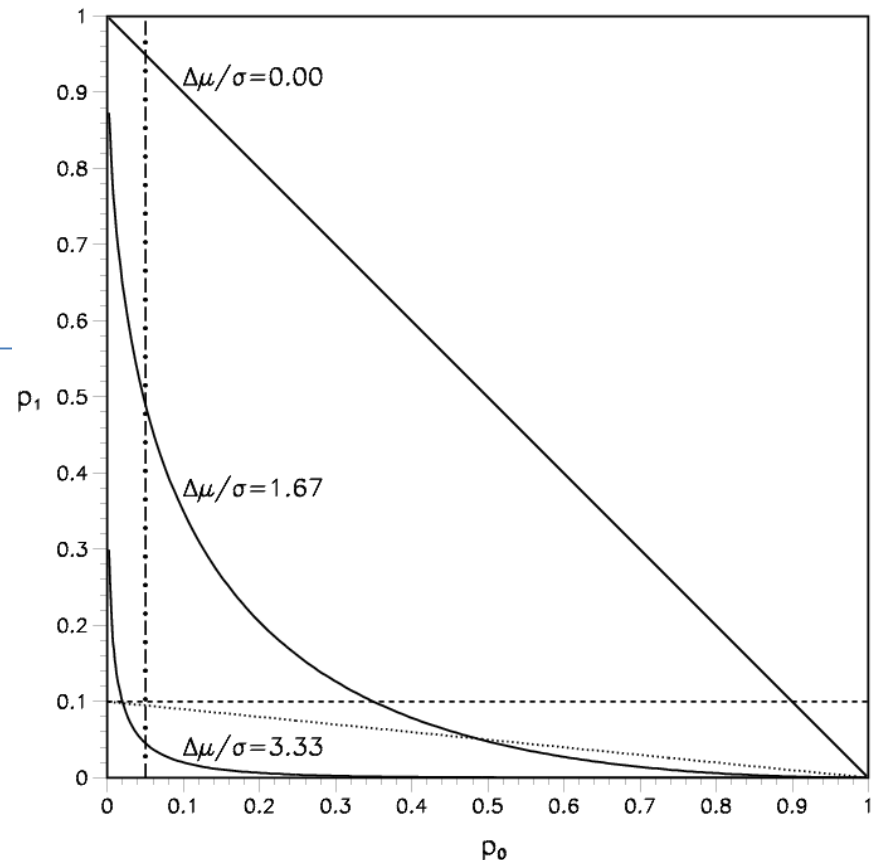
Punzi definition of sensitivity:

Enough separation of pdf's for no chance of ambiguity



Can read off power of test
e.g. If H_0 is true, what is
prob of rejecting H_1 ?

N.B. p_0 = tail towards H_1
 p_1 = tail towards H_0



α , β , Errors of 1st and 2nd Kind, etc.

e.g. H_0 = event with top H_1 = no top

α = prob of rejecting H_0 when H_0 true = E1

$p_0 < \alpha$, reject as top event

$p_0 > \alpha$, accept as top event

Effic for H_0 = $1 - \alpha$

β = value of p_1 when $p_0 = \alpha$

= prob of not rejecting H_0 when H_1 true = E2

= mis-ID of 'no top' events

Power = prob of rejecting H_0 when H_1 true = $1 - \beta$

Contamination in signal sample depends on β , and relative frequencies for H_0 and H_1 events.

ROC curves plot '1- Bgd Mis-ID' versus 'Signal Efficiency'

= '1- p_1 ' versus '1- p_0 ' (Cf p_1 v p_0 plots)

Why $p \neq$ Likelihood ratio

Measure different things:

p_0 refers just to H_0 ; \mathcal{L}_{01} compares H_0 and H_1

Depends on amount of data:

e.g. Poisson counting expt little data:

For H_0 , $\mu_0 = 1.0$. For H_1 , $\mu_1 = 10.0$

Observe $n = 10$ $p_0 \sim 10^{-7}$ $\mathcal{L}_{01} \sim 10^{-5}$

Now with 100 times as much data, $\mu_0 = 100.0$ $\mu_1 = 1000.0$

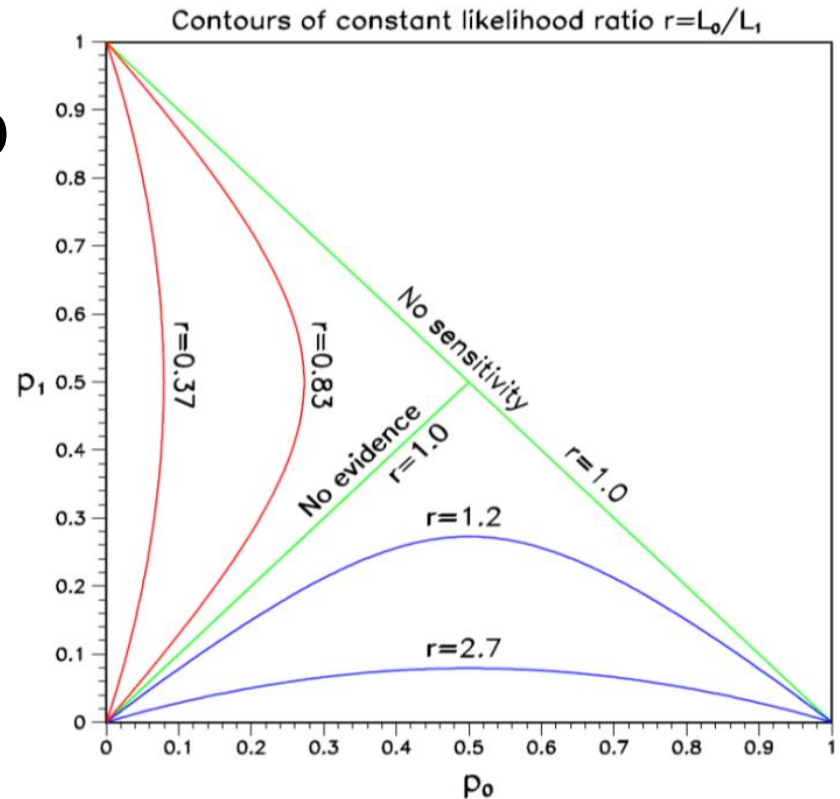
Observe $n = 160$ $p_0 \sim 10^{-7}$ $\mathcal{L}_{01} \sim 10^{+14}$

N.B. In HEP, data statistic is typically \mathcal{L}_{01}

Can think of method as:

p-value, where data statistic just happens to be \mathcal{L}_{01} ; or

\mathcal{L}_{01} method where p-values are just used for calibration.



Jeffreys-Lindley Paradox

H_0 = simple, H_1 has μ free

p_0 can favour H_1 , while B_{01} can favour H_0

$$B_{01} = L_0 / \int L_1(s) \pi(s) ds$$

Likelihood ratio depends on signal :

e.g. Poisson counting expt small signal s :

For H_0 , $\mu_0 = 1.0$. For H_1 , $\mu_1 = 10.0$

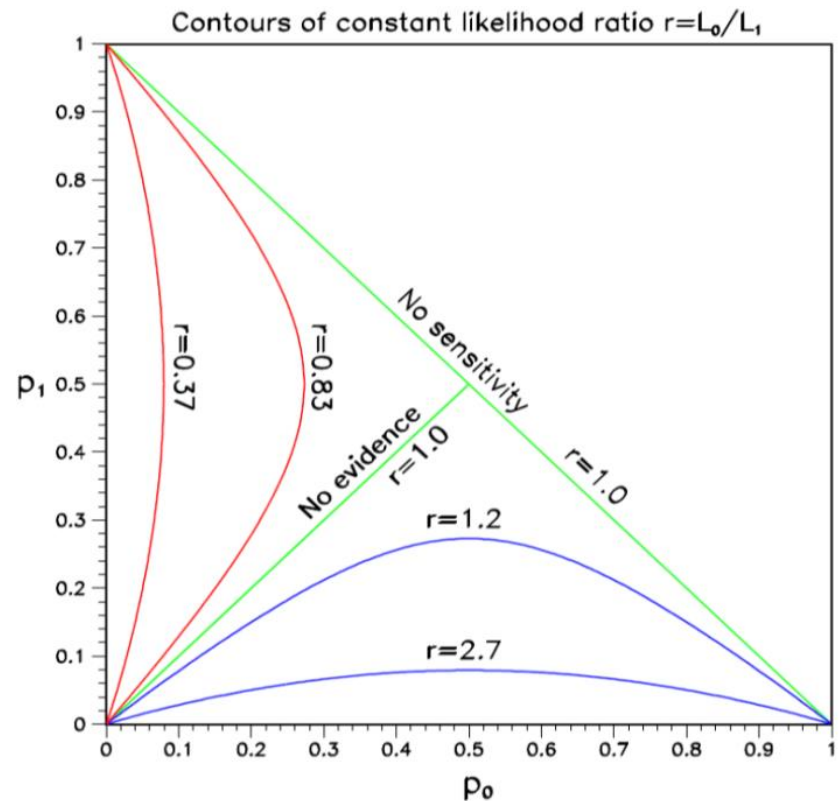
Observe $n = 10$ $p_0 \sim 10^{-7}$ $L_{01} \sim 10^{-5}$ and favours H_1

Now with 100 times as much signal s , $\mu_0 = 100.0$ $\mu_1 = 1000.0$

Observe $n = 160$ $p_0 \sim 10^{-7}$ $L_{01} \sim 10^{+14}$ and favours H_0

B_{01} involves integration over s in denominator, so a wide enough range will result in favouring H_0

However, for B_{01} to favour H_0 when p_0 is equivalent to 5σ , integration range for s has to be $O(10^6)$ times Gaussian widths



WHY LIMITS?

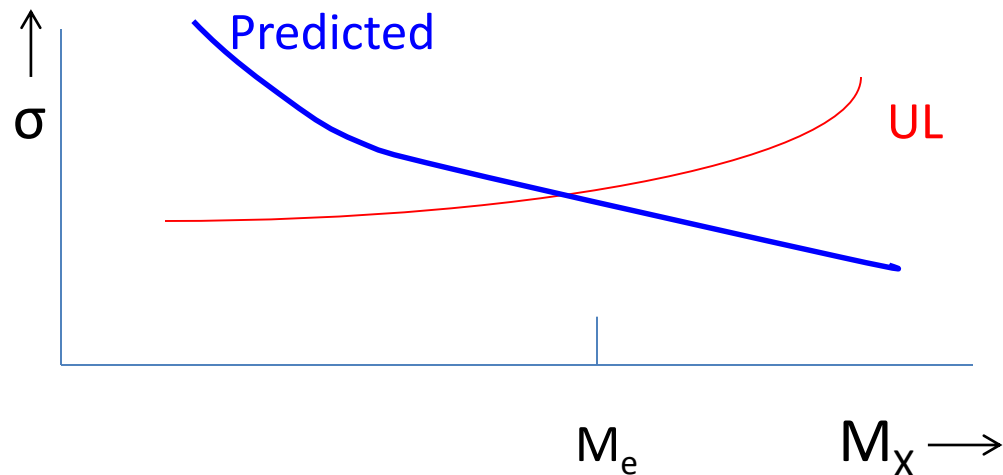
Michelson-Morley experiment → death of aether

HEP experiments:

If UL on expected rate for new particle < expected, exclude particle

Do as function of M_X → excluded mass range below M_e

Compare with expected
 M_e → expt's sensitivity



CERN CLW (Jan 2000)

FNAL CLW (March 2000)

Heinrich, PHYSTAT-LHC, "Review of Banff Challenge"

Methods (no systematics)

Bayes (needs priors e.g. const, $1/\mu$, $1/\sqrt{\mu}$, μ ,

Frequentist (needs ordering rule,
possible empty intervals, F-C)

CLs

Likelihood (DON'T integrate your \mathcal{L})

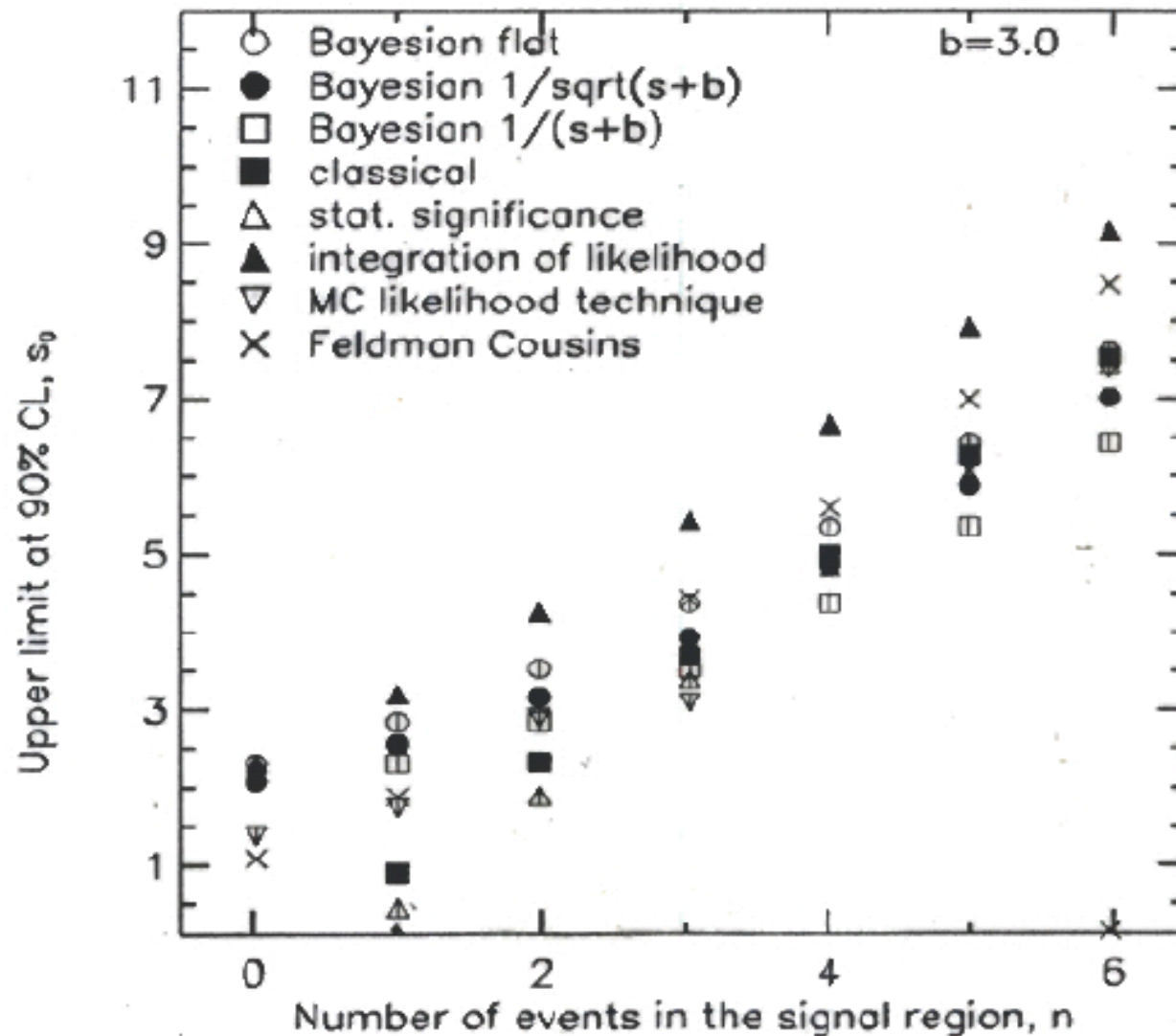
$$\chi^2 (\sigma^2 = \mu)$$

$$\chi^2 (\sigma^2 = n)$$

Recommendation 7 from CERN CLW: “Show your \mathcal{L} ”

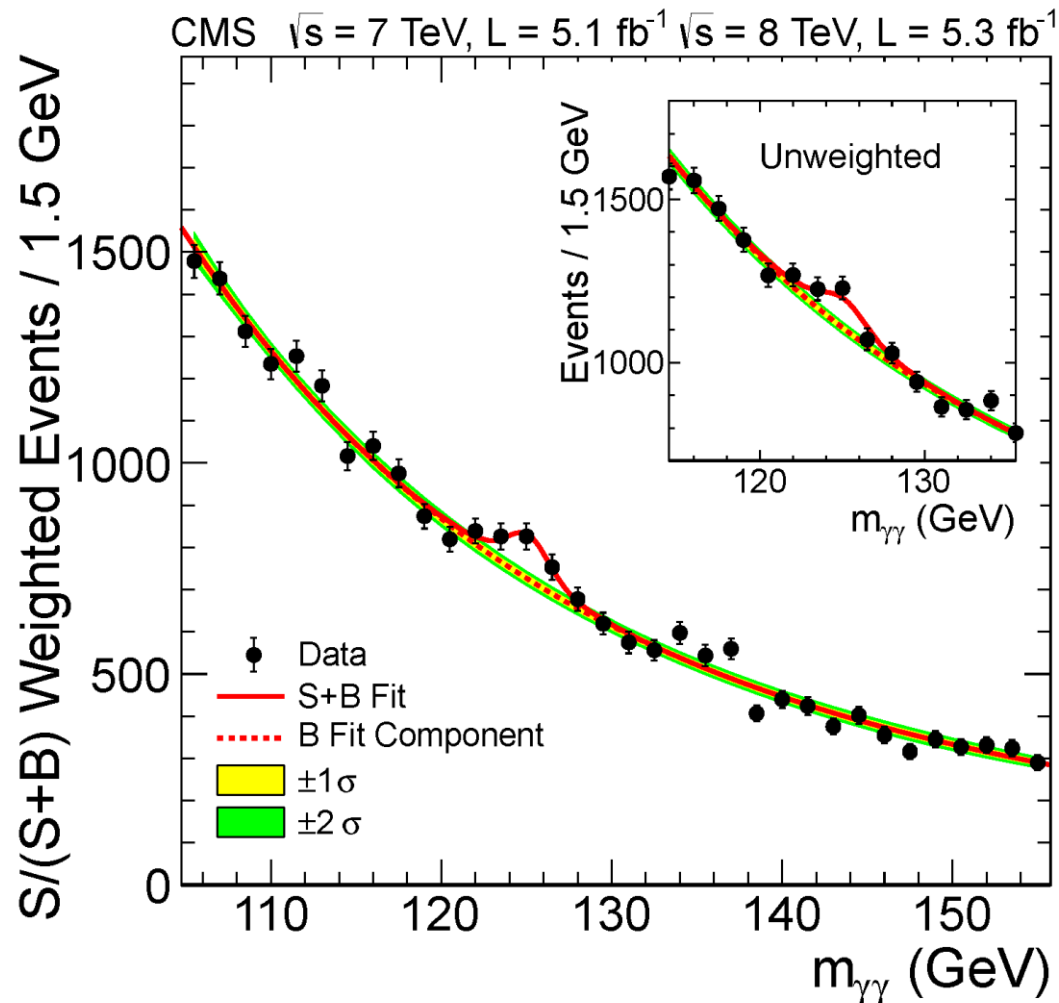
1) Not always practical

2) Not sufficient for frequentist methods

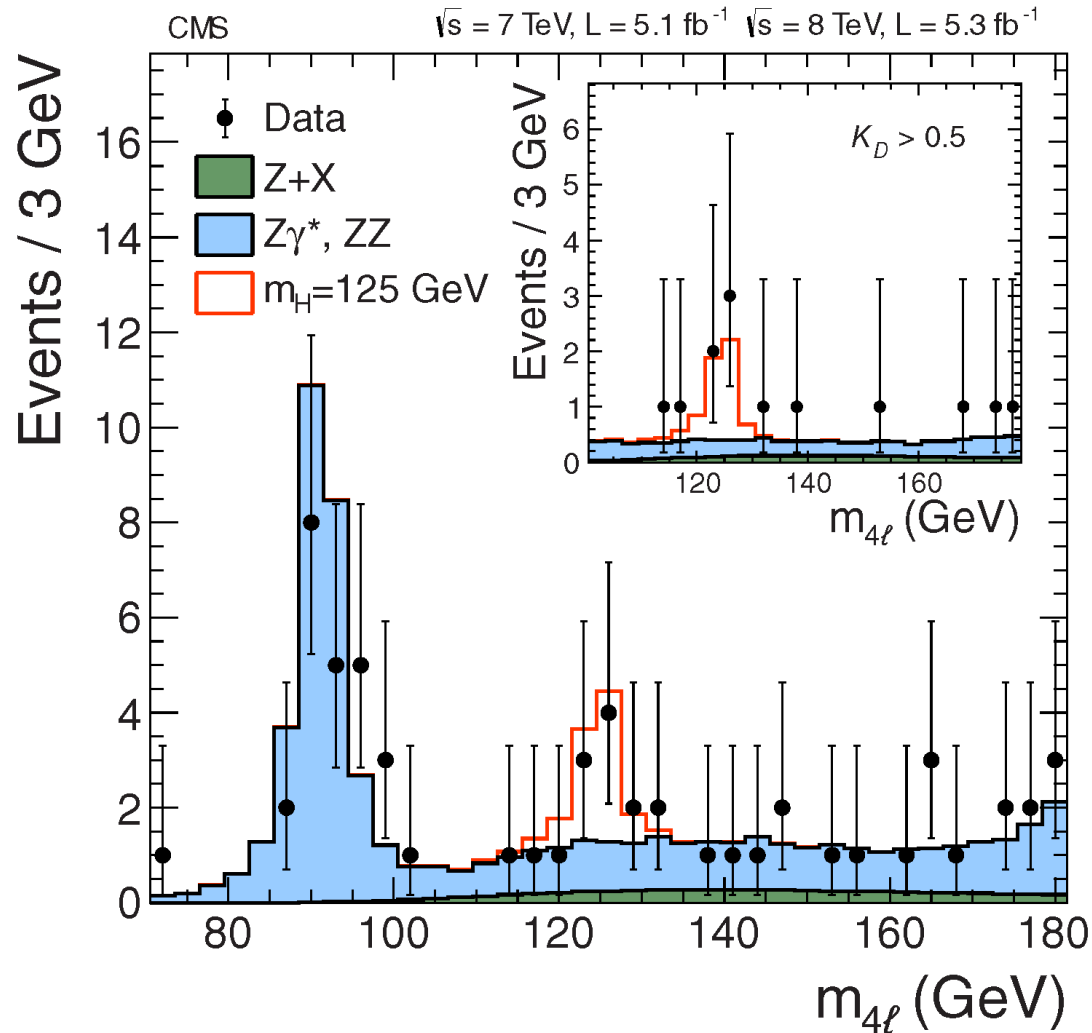


Search for Higgs:

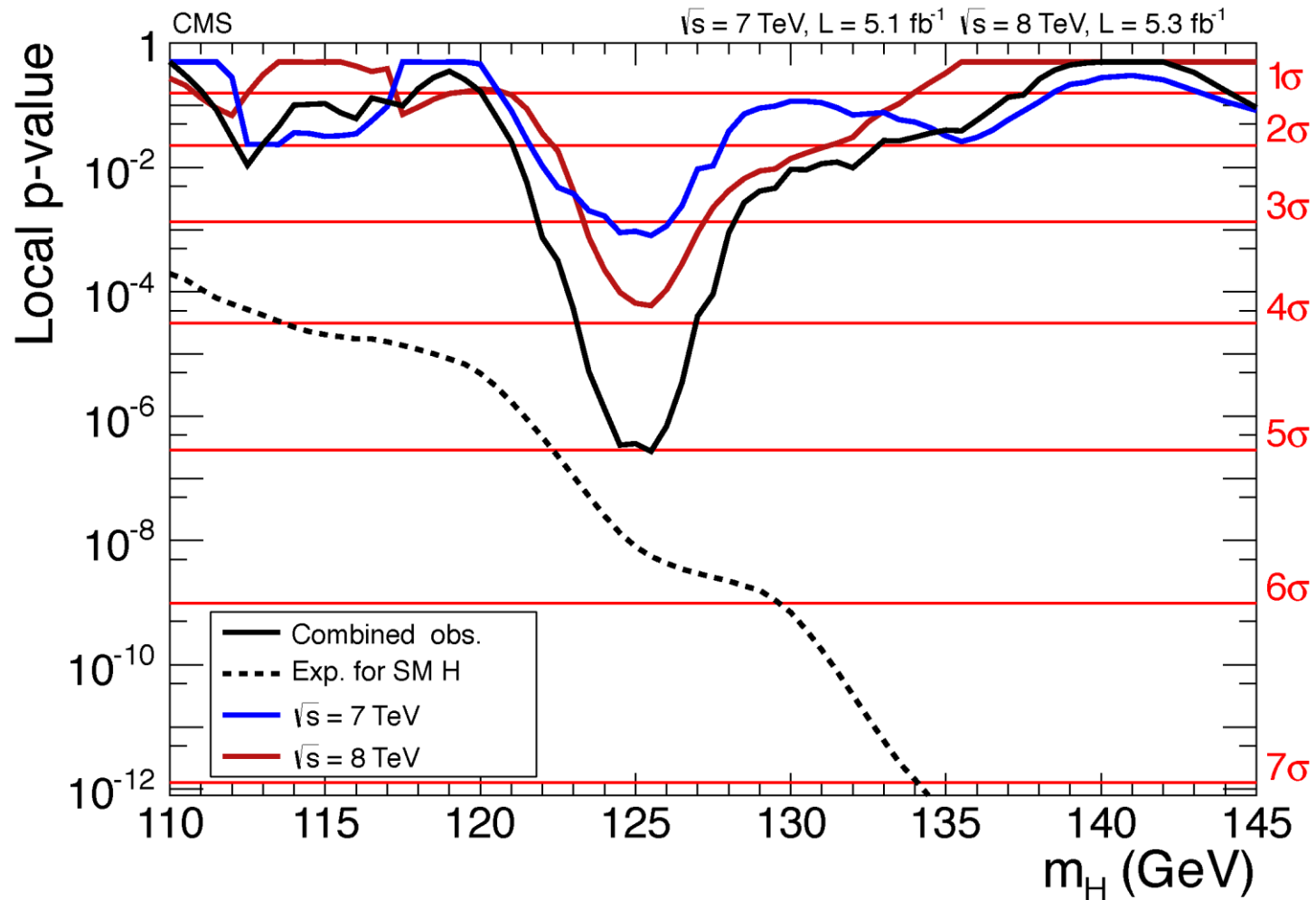
$H \rightarrow \gamma \gamma$: low S/B, high statistics

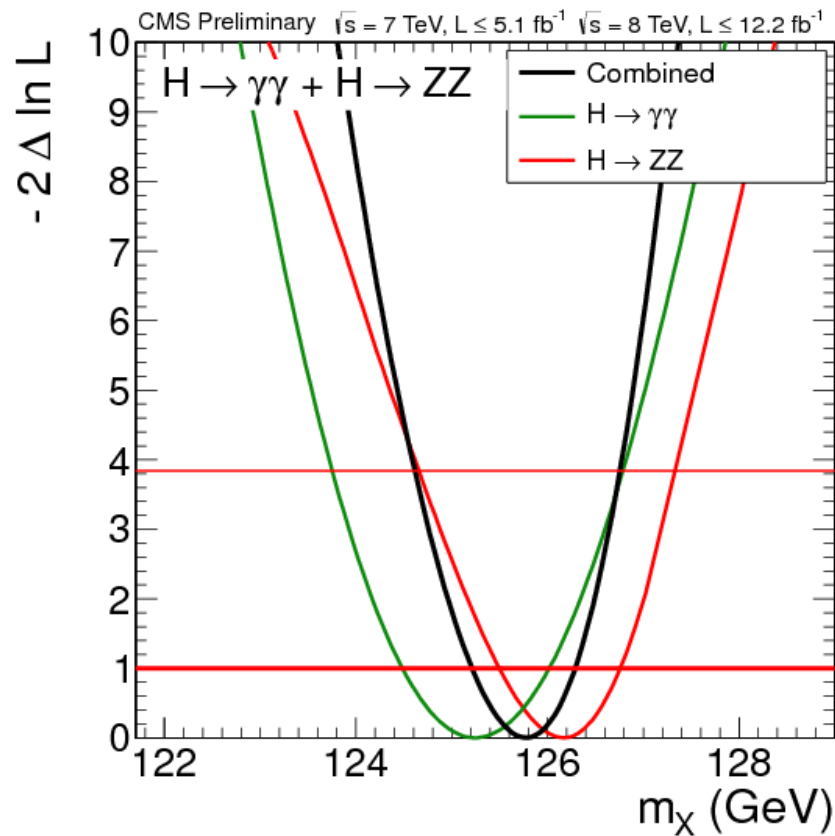


$H \rightarrow Z Z \rightarrow 4 \ell$: high S/B, low statistics

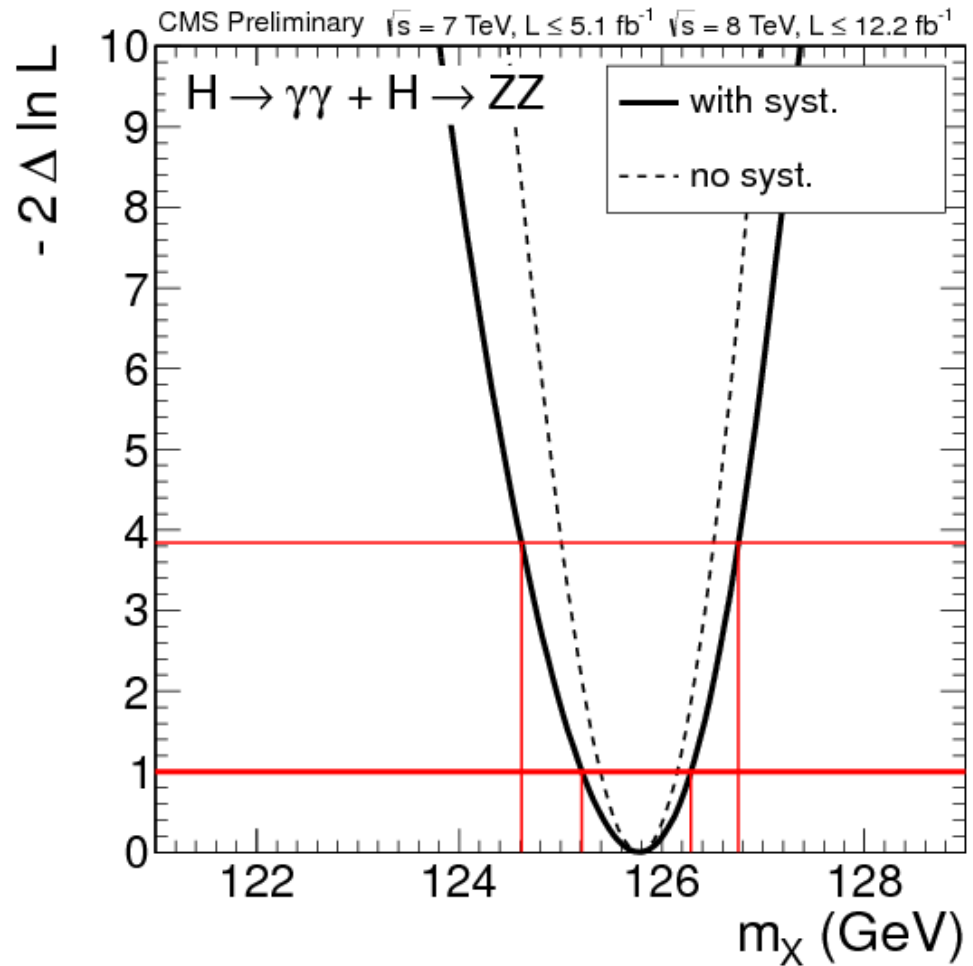


p-value for 'No Higgs' versus m_H

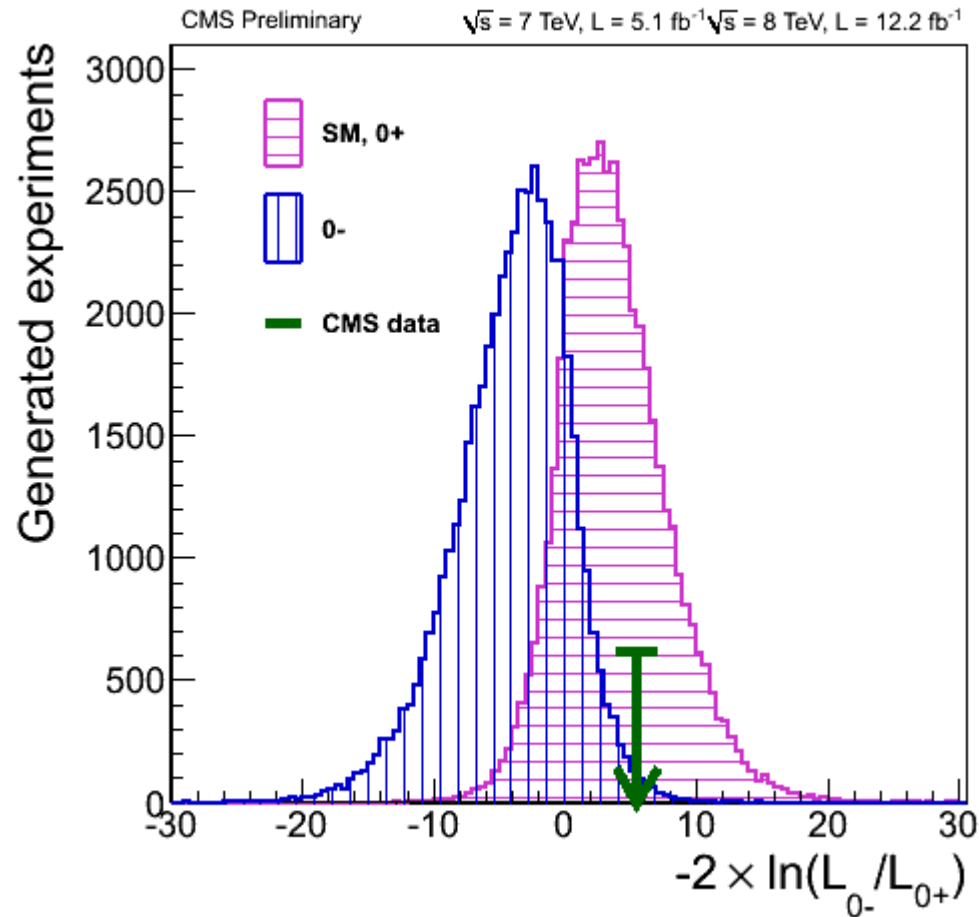




Mass of Higgs: Likelihood versus mass



Comparing 0^+ versus 0^- for Higgs (like Neutrino Mass Hierarchy)



<http://cms.web.cern.ch/news/highlights-cms-results-presented-hcp>

Conclusions

Resources:

Software exists: e.g. RooStats

Books exist: Barlow, Cowan, James, Lista, Lyons, Roe,.....

New: `Data Analysis in HEP: A Practical Guide to Statistical Methods', Behnke et al.

PDG sections on Prob, Statistics, Monte Carlo

CMS and ATLAS have Statistics Committees (and BaBar and CDF earlier) – see their websites

Before re-inventing the wheel, try to see if Statisticians have already found a solution to your statistics analysis problem.

Don't use a square wheel if a circular one already exists.

“Good luck”