Introduction to Accelerator Physics

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Outline of the talk

1. Why do we need accelerators?
2. Principles.
3. Electromagnetic fields, their properties and how can you use them?
4. Equation of motion.
5. Linear optics.
6. Examples.
7. Beam cooling.

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Why do we need accelerators?

Accelerators are used:

- to produce beams of charge particles.
- in order to probe matter at fundamental level ($\lambda = h/p$).
- for production of neutrons and synchrotron radiation needed in life science and technology.
- in particle beam therapy for cancer treatment.
- for production of radioactive isotopes for medical applications, etc.

How do they work?

Physical principles of accelerators are very simple ☺:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

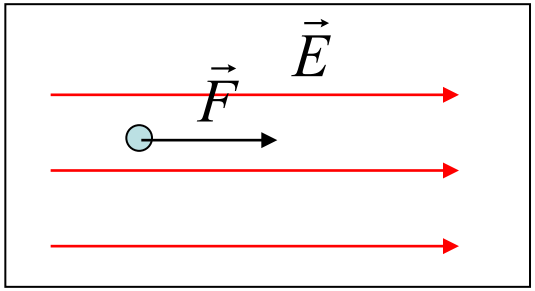
q – electric charge of particle,

F – force

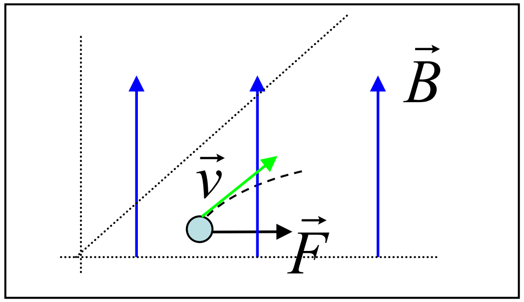
E – electric field

B – magnetic field

v – speed < c



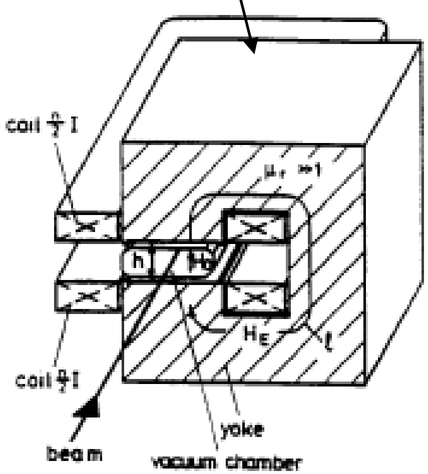
Effect of electric field

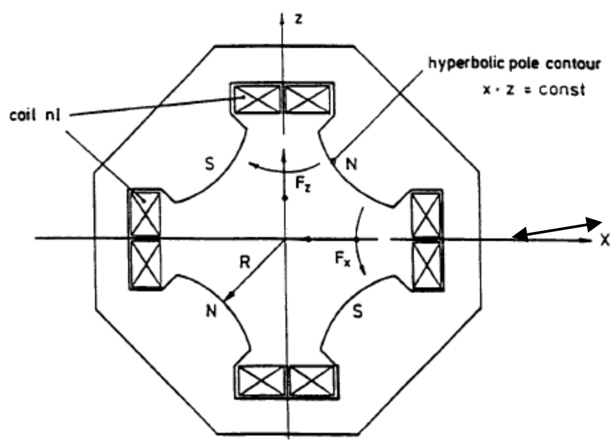


Effect of magnetic field

Beam steering and focusing

- Magnetic field B is used to bend particle orbits in dipole magnets.
- Focusing is performed in quadrupole magnets (magnetic lenses for charged particles)





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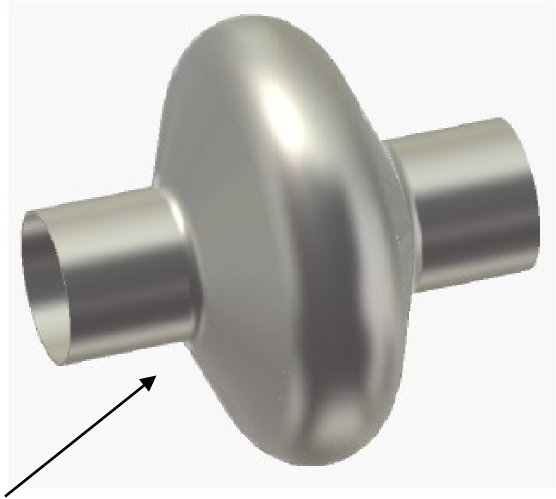
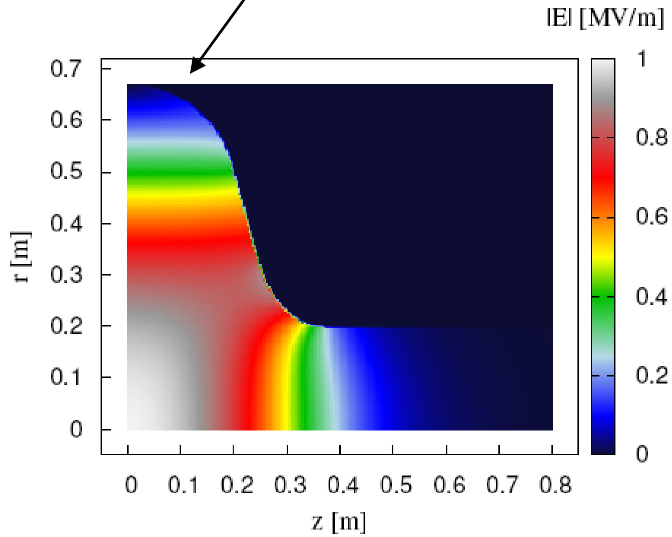
Acceleration

- Particle acceleration can be done only by using the electric field E!
- Constant voltage can be applied only at low energy (breakdown).
- At high energy Radio Frequency (RF) cavities are used.



RF cavity



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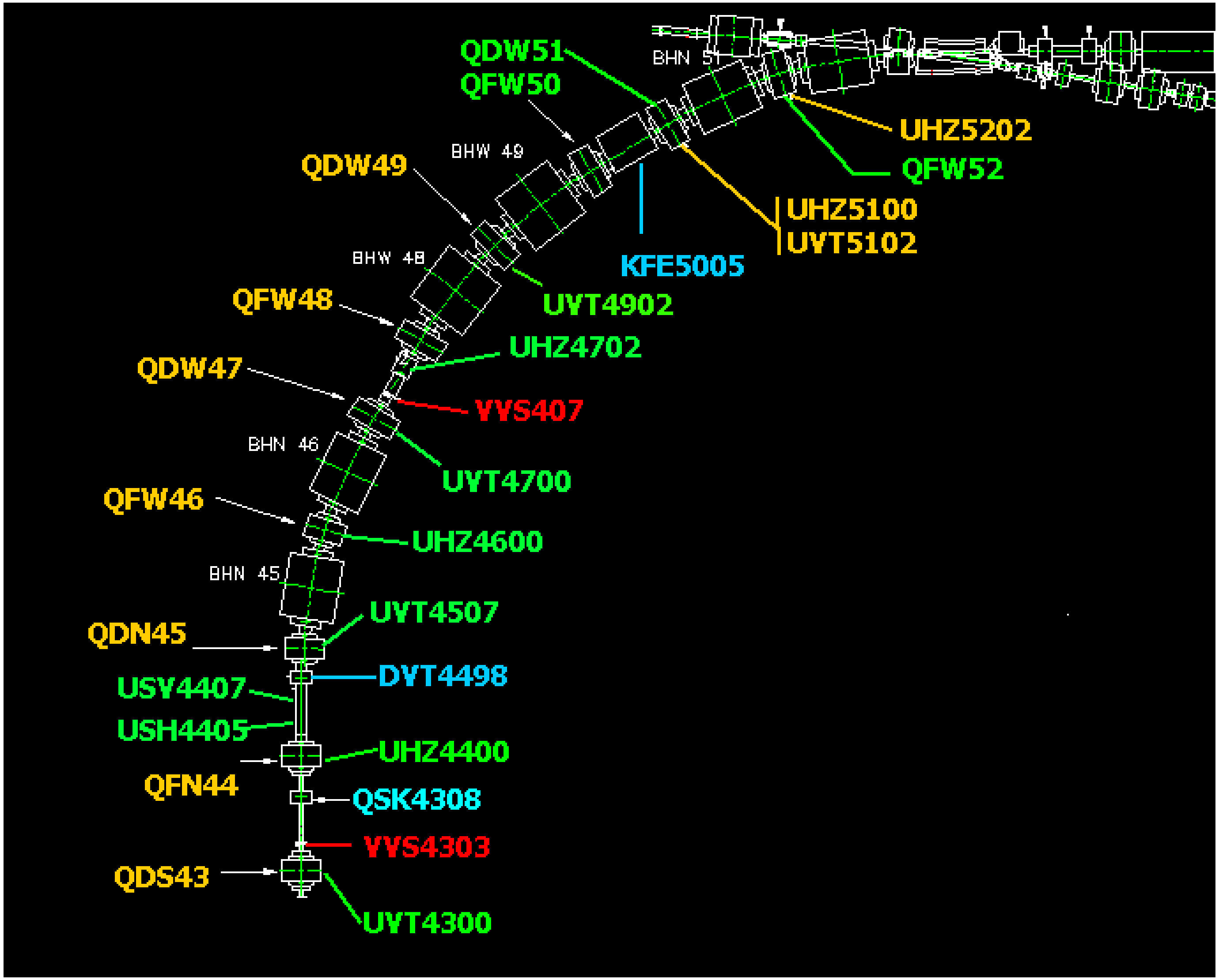
Electric field in the RF cavity

Accelerator LEGO Blocks



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Electromagnetic fields

- On the fundamental level the electromagnetic fields and their properties originate from existence of photon and electric charge.
- On classical level they are described by Maxwell equations (in vacuum):

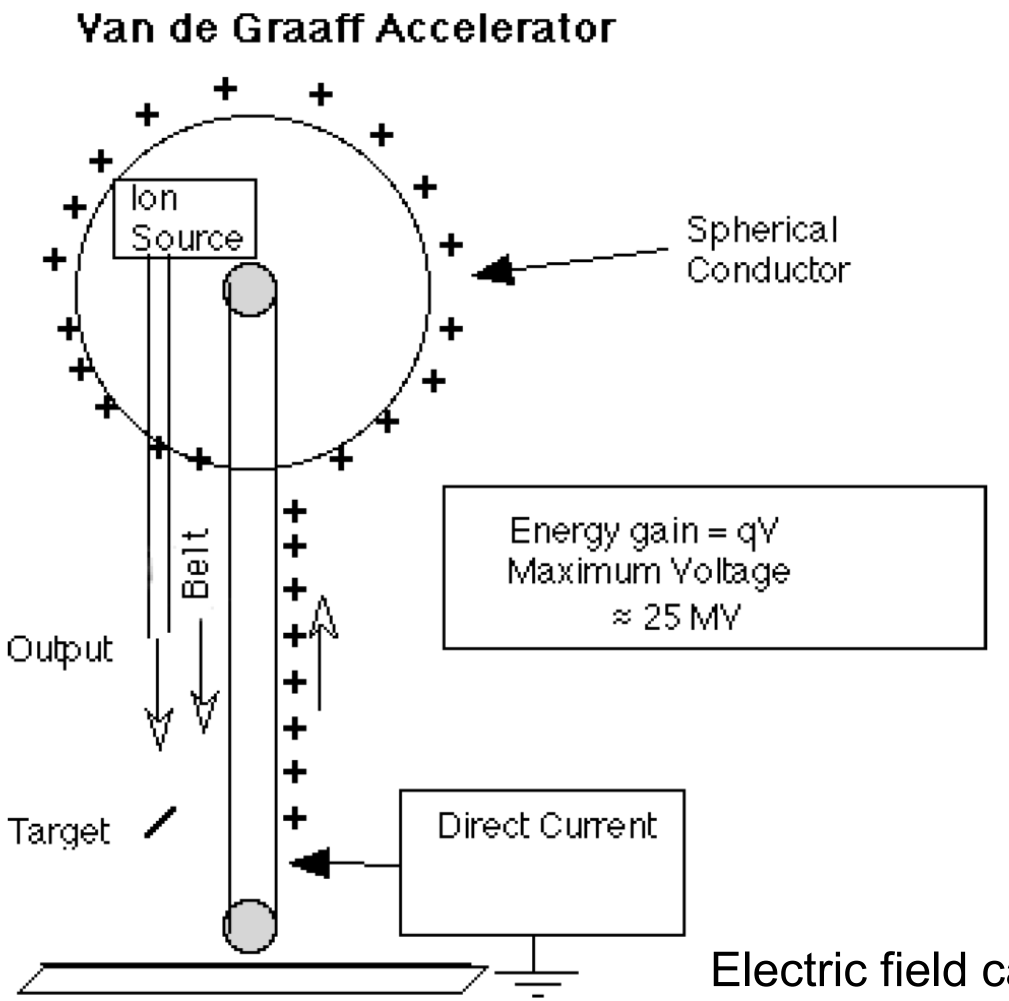
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 J$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Electrostatic Acceleration



ig. 11-2. A Van de Graaff accelerator.

Electric field can be generated by storing as large charge as possible (limited by breakdown).

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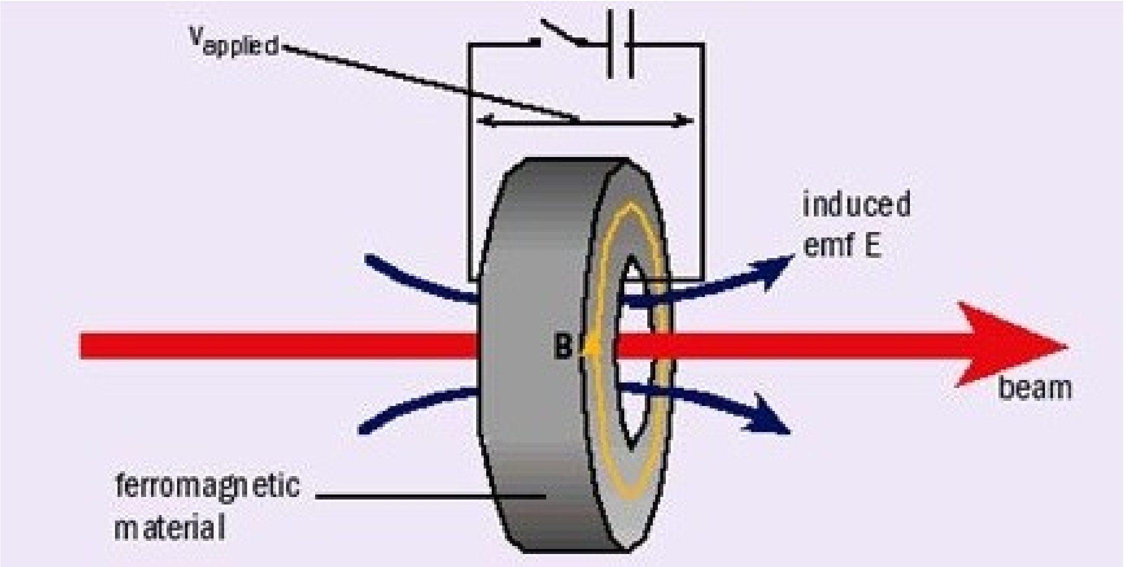
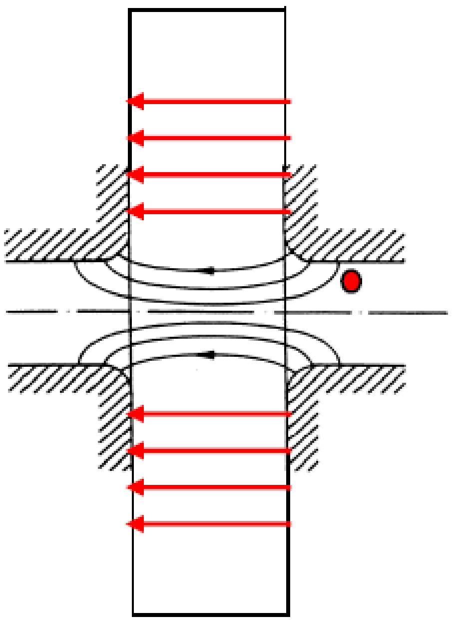
Induction acceleration

Magnetic field changing in time will generate electric field, which may be used for acceleration (induction acceleration). May accept large current, but limited in energy gain.



RF acceleration



•Wave nature of electromagnetic fields allows for Radio Frequency (RF) acceleration, where the sinusoidal variation of electric field can be produced inside a metallic cavity.

• By adjusting the cavity geometry the oscillations of the wave can be synchronised with a particle beam (smaller with the speed of light).

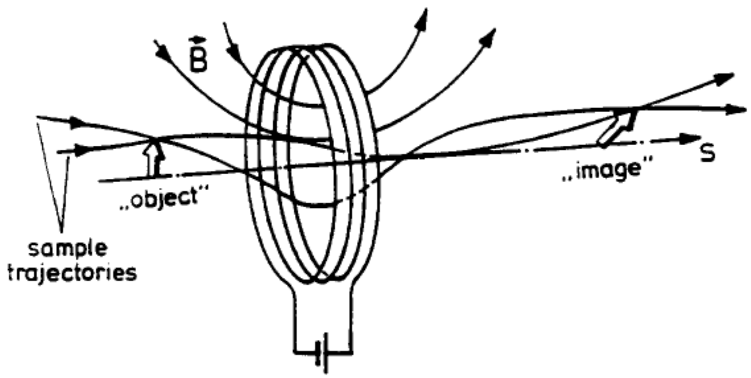
•In the cavities used for acceleration inside ring accelerators the frequency can be varied (with the cost of obtaining smaller gradient).

• The most common method of acceleration.

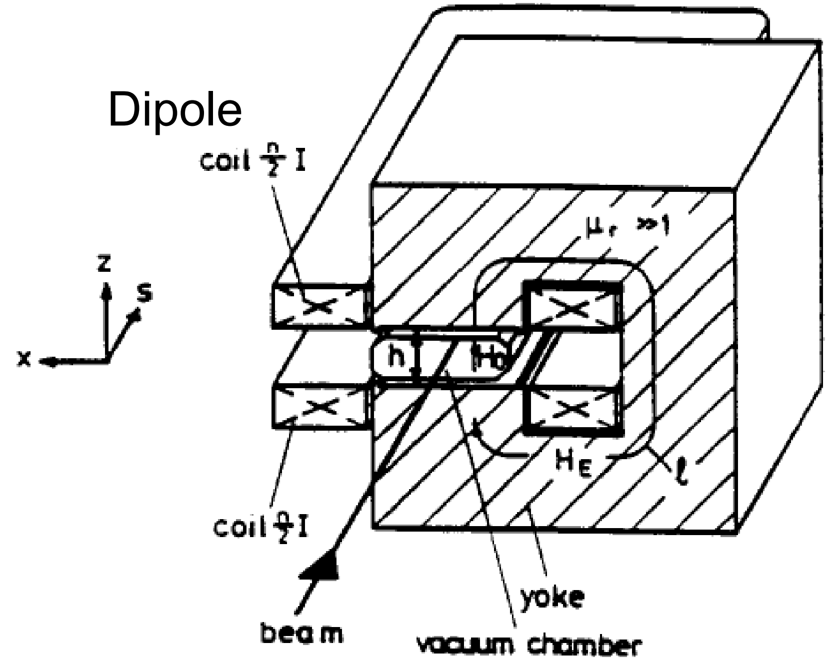
$$E_z = E_z(t) = E_0 \cos(\omega t + \phi)$$

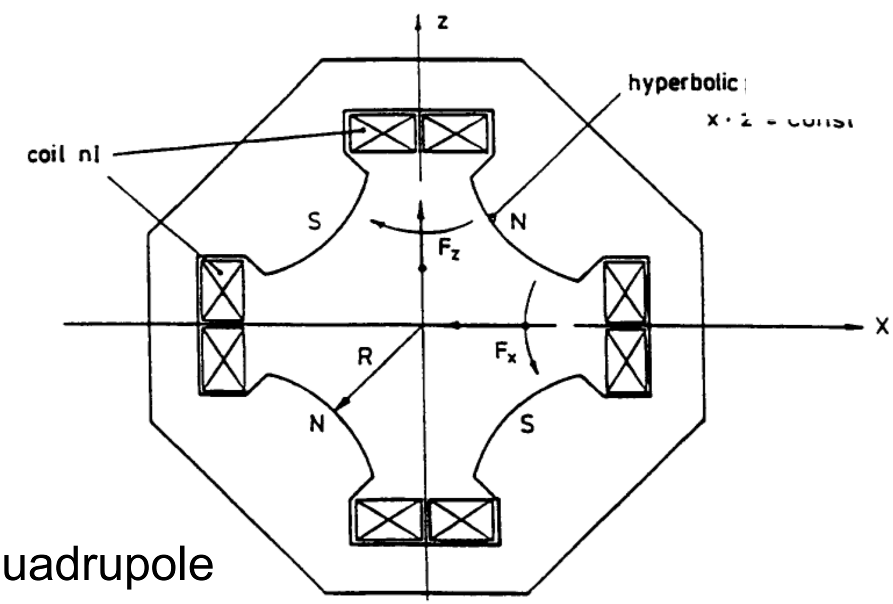
Magnets

Solenoid



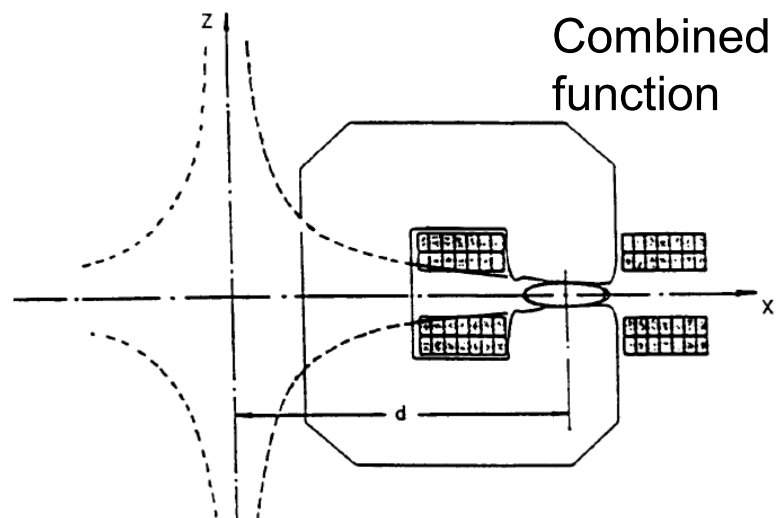
Dipole





Quadrupole

Combined function



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Hamiltonian in the magnetic field

The full Hamiltonian describing the dynamics of charge particle in the B field:

$$H_{CS} = -\frac{e\vec{A}_s}{cp}(1+\frac{x}{\rho}) - (1+\frac{x}{\rho})\sqrt{1-(\vec{P}_x - \frac{e\vec{A}_x}{cp})^2 - (\vec{P}_y - \frac{e\vec{A}_y}{cp})^2}$$

(the Courant-Snyder Hamiltonian), where the vector potential:

$$\frac{e}{cp}A_s = -\frac{1}{2}(1+\frac{x}{\rho}) + \frac{1}{2}k(y^2 - x^2) + \frac{1}{6}m(x^3 - 3xy^2) + O(4)$$

corresponds to the magnetic field, which can be expressed as a combination of multipoles:

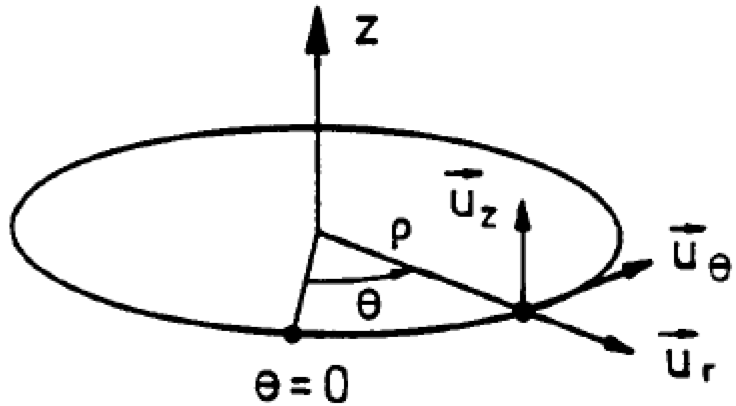
$$\frac{e}{cp}B_x = ky + mxy + O(3)$$

quidence (dipole) — Focusing (quadrupole) — Chromaticity correction (sextupole, nonlinear term)

$$\frac{e}{cp}B_y = \frac{1}{\rho} + kx + \frac{1}{2}m(x^2 - y^2) + O(3)$$

There are also nonlinear terms coming from the fields at the magnet ends- the so called fringe fields.

Simplified derivation of equations of motion



Coordinate system

Particle position:

$$\mathbf{R} = \mathbf{R}_o + r\mathbf{u}_r + z\mathbf{u}_z \qquad \mathbf{R}_o = const. \qquad d\mathbf{u}_r = d\Theta\mathbf{u}_\Theta,\ d\mathbf{u}_\Theta = -d\Theta\mathbf{u}_r,\ d\mathbf{u}_z = 0$$

Newton equation:

$$m\ddot{\mathbf{R}} = -e\mathbf{v}\times\mathbf{B} = -e\left[\left(r\dot{\Theta}B_z - \dot{z}B_\Theta\right)\mathbf{u}_r + \left(\dot{z}B_r - \dot{r}B_z\right)\mathbf{u}_\Theta + \left(\dot{r}B_\Theta - r\dot{\Theta}B_r\right)\mathbf{u}_z\right]$$

Assuming no solenoidal field:

$$m\left(\ddot{r} - r\dot{\Theta}^2\right) = -er\dot{\Theta}B_z(r,z,\Theta)$$

$$m\ddot{z} = er\dot{\Theta}B_r(r,z,\Theta)$$

Simplified derivation of equations of motion (2)

Assuming: $r = \rho + x$ with $\rho = const$

$$m\left(\ddot{x} - r\dot{\Theta}^2\right) = -er\dot{\Theta}\left(B_\circ - gx\right)$$
$$m\ddot{z} = -er\dot{\Theta}gz$$

Changing an independent variable from time into longitudinal position:

$$s = vt, \quad \ddot{x} = v^2 x'', \quad x'' = \frac{d^2x}{ds^2}$$

Now using:

$$mv = p = p_\circ\left(1 + \frac{\Delta p}{p_\circ}\right) \qquad \frac{1}{r} \approx \frac{1}{\rho}\left(1 - \frac{x}{\rho}\right)$$

$$p_\circ = eB_\circ, \quad \frac{1}{\rho} = \frac{eB_\circ}{p_\circ}, \quad k = \frac{eg}{p_\circ}$$

Magnetic rigidity:

$$B\rho[Tm] = \frac{p[GeV/c]}{0.2998}$$

Review of optics in the circular machine (1)

The equations for horizontal and vertical motion:

$$x'' + (k + \frac{1}{\rho^2})x = \frac{\Delta p}{p_0}\frac{1}{\rho} \qquad z'' - kz = 0$$

In circular machine:

$$k(s) = k(s+L)$$

$$\rho(s) = \rho(s+L)$$

where L is length of machine period or circumference

The solution of equation of motion can be expressed in matrix formalism:

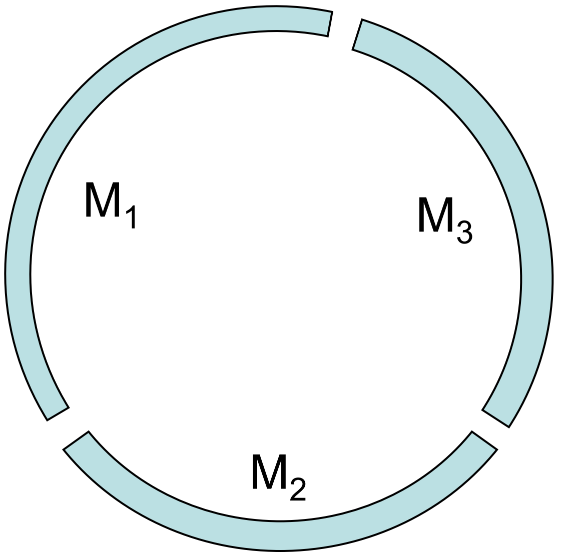
$$\begin{bmatrix} x(s) \\ x'(s) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x(0) \\ x'(0) \end{bmatrix} = M\begin{bmatrix} x(0) \\ x'(0) \end{bmatrix}$$

The Courant-Snyder invariant (emittance) is given by:

$$\frac{1}{\beta}(x^2 + (\alpha x + \beta x')^2) = \gamma x^2 + 2\alpha\gamma x' + \beta x'^2 = \varepsilon$$

Review of optics in the circular machine (2)

Matrix multiplication rule: $M_{turn} = M_1 M_2 M_3$.



It can be shown:

$$M_{turn} = \begin{bmatrix} \cos(\Delta\mu) + \alpha\sin(\Delta\mu) & \beta\sin(\Delta\mu) \\ -\gamma\sin(\Delta\mu) & \cos(\Delta\mu) - \alpha\sin(\Delta\mu) \end{bmatrix}$$

$Det(M_{turn})=1$,

$$\cos(\mu) = \frac{1}{2}Tr(M_{turn}) = \frac{1}{2}(a+d)$$

$$\beta\gamma - \alpha^2 = 1$$

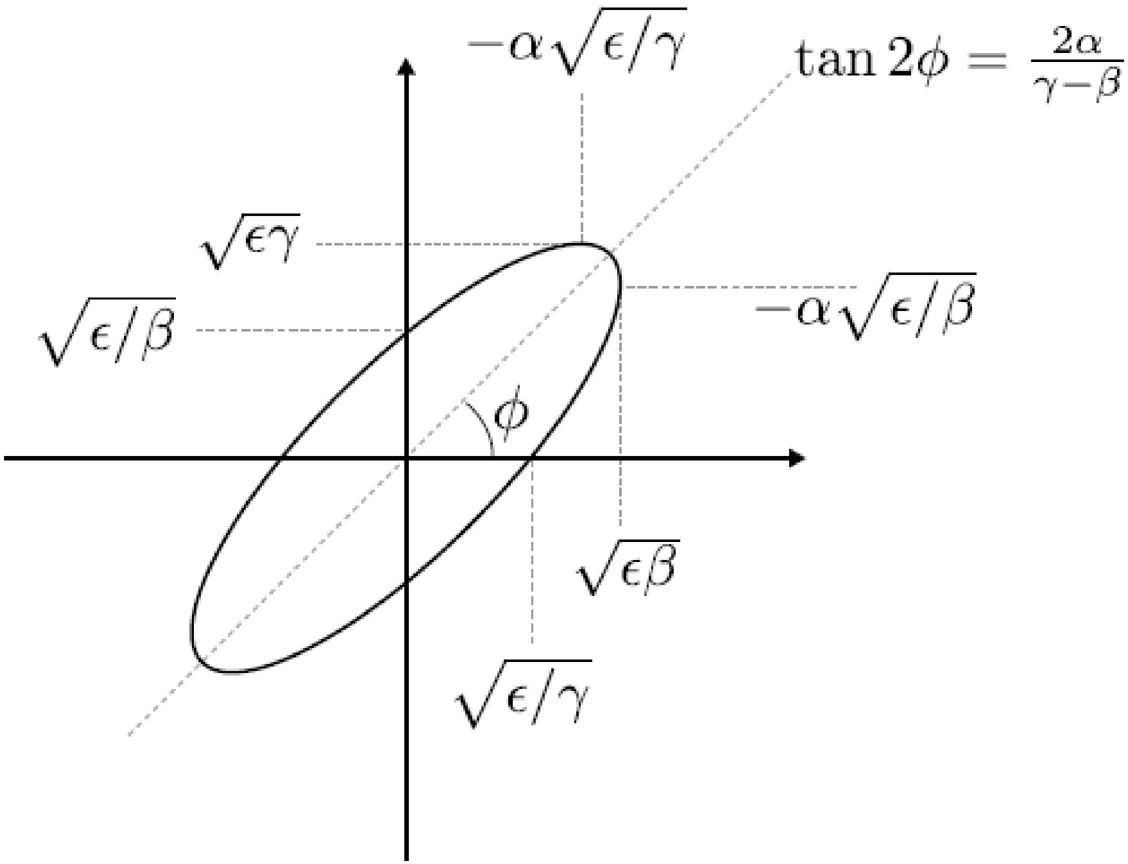
Solution of equation of motion cab be expressed as:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\mu - \mu_0)$$

$$Q = \frac{\Delta\mu}{2\pi} = \frac{1}{2\pi}\int_0^s \frac{ds}{\beta}$$

Machine tune, number of betatron oscillations per turn.

Interpretation of Twiss parameters



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Optics for off-momentum particles

Motion for off-momentum particle:

$$x'' + (k + \frac{1}{\rho^2})x = \frac{\Delta p}{p_0}\frac{1}{\rho}$$

$$x_D = D\frac{\Delta p}{p_0}$$

In circular machines periodic dispersion function is given by:

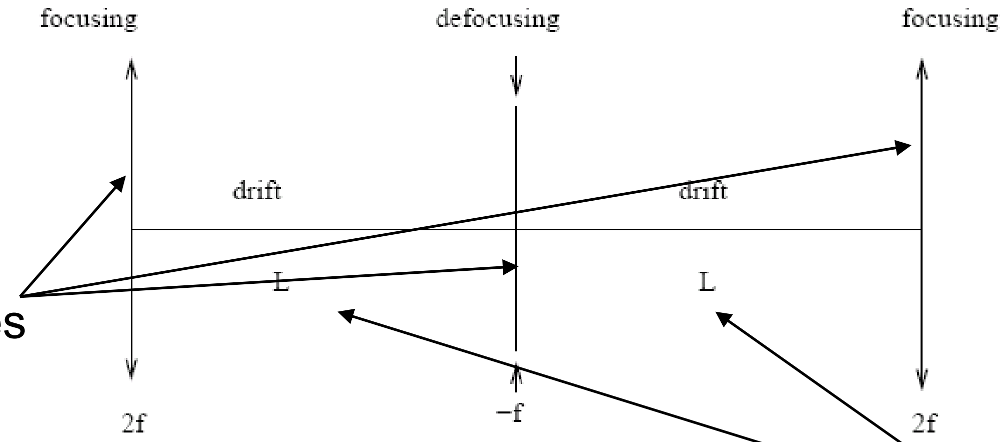
$$D(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)}\oint \frac{\sqrt{\beta(t)}}{\rho(t)}\cos(|\mu(t) - \mu(s) - \pi Q)dt$$

Matrix formalism can be extended to describe dispersion:

$$\begin{bmatrix} D(s) \\ D'(s) \\ \frac{\Delta p}{p} \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D(0) \\ D'(0) \\ \frac{\Delta p}{p} \end{bmatrix}$$

FODO cell

FODO lattice is one of the most common and useful focusing structure:



Quadrupoles

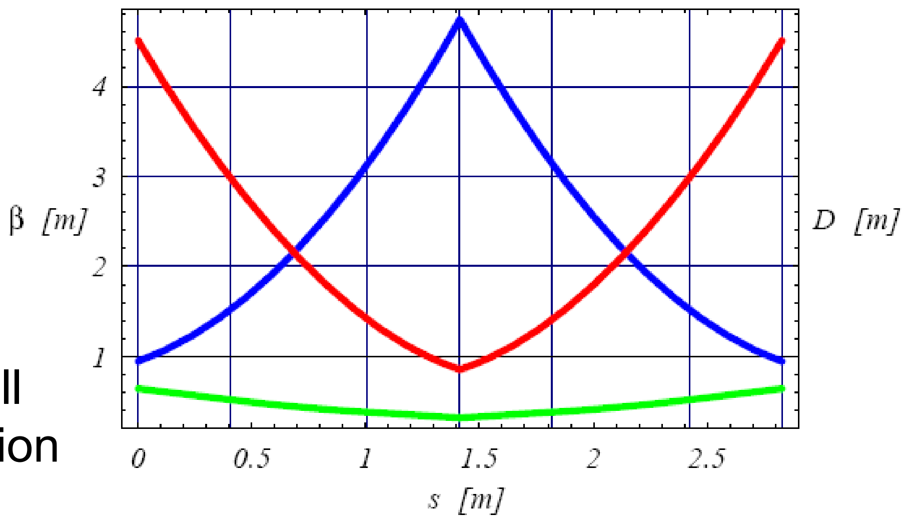
We can put bending dipoles here.

$$M_q = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \quad f = \frac{1}{k l_Q} \qquad M_L = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

Thin lens approximation.

$$\sin\left(\frac{\mu}{2}\right) = \frac{L}{2f}$$

Betatron functions and dispersion in the FODO cell in the thin lens approximation



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FODO cell (thick lens approximation)

$$M_F = \begin{pmatrix} \cos(l\sqrt{|K|}) & \frac{1}{\sqrt{|K|}}\sin(l\sqrt{|K|}) \\ -\frac{1}{\sqrt{|K|}}\sin(l\sqrt{|K|}) & \cos(l\sqrt{|K|}) \end{pmatrix}$$

$$M_D = \begin{pmatrix} \cosh(l\sqrt{|K|}) & \frac{1}{\sqrt{|K|}}\sinh(l\sqrt{|K|}) \\ \frac{1}{\sqrt{|K|}}\sinh(l\sqrt{|K|}) & \cosh(l\sqrt{|K|}) \end{pmatrix}$$

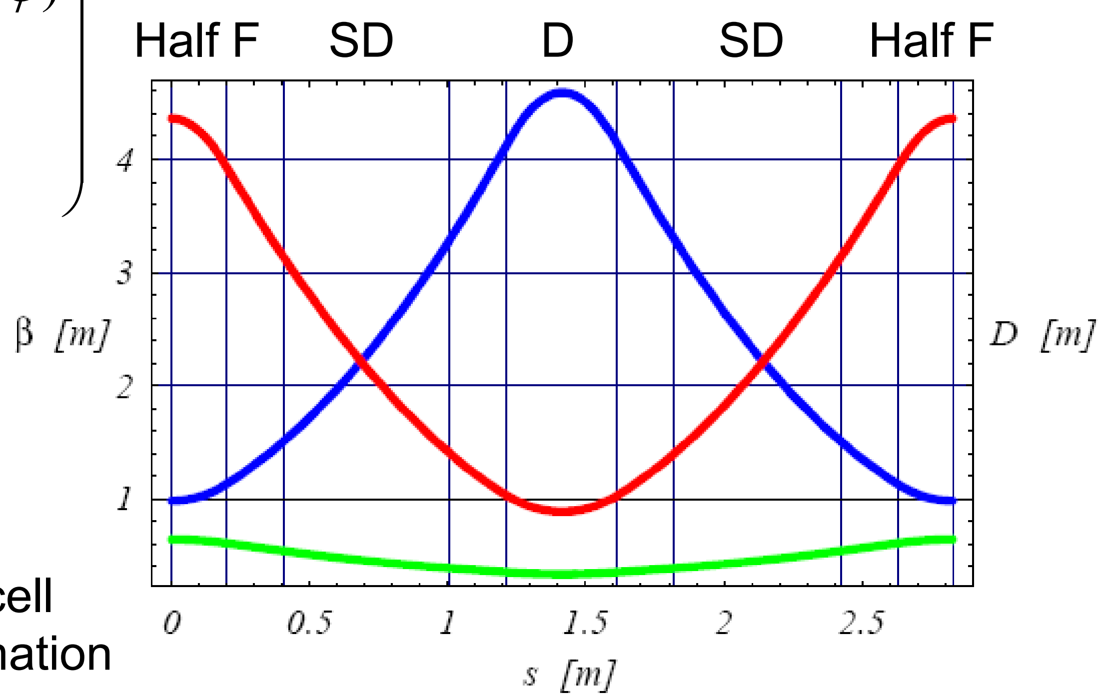
2x2 matrix representation for thick quadrupoles of lengh l.

In 3x3 matrix representation:

$$\begin{pmatrix} M_F & 0 \\ 0 & 1 \end{pmatrix}$$

$$M_{SD} = \begin{pmatrix} \cos\varphi & \rho\sin\varphi & \rho(1-\cos\varphi) \\ -\frac{1}{\rho}\sin\varphi & \cos\varphi & \sin\varphi \\ 0 & 0 & 1 \end{pmatrix}$$

3x3 matrix representation for sector dipole (φρ=l).



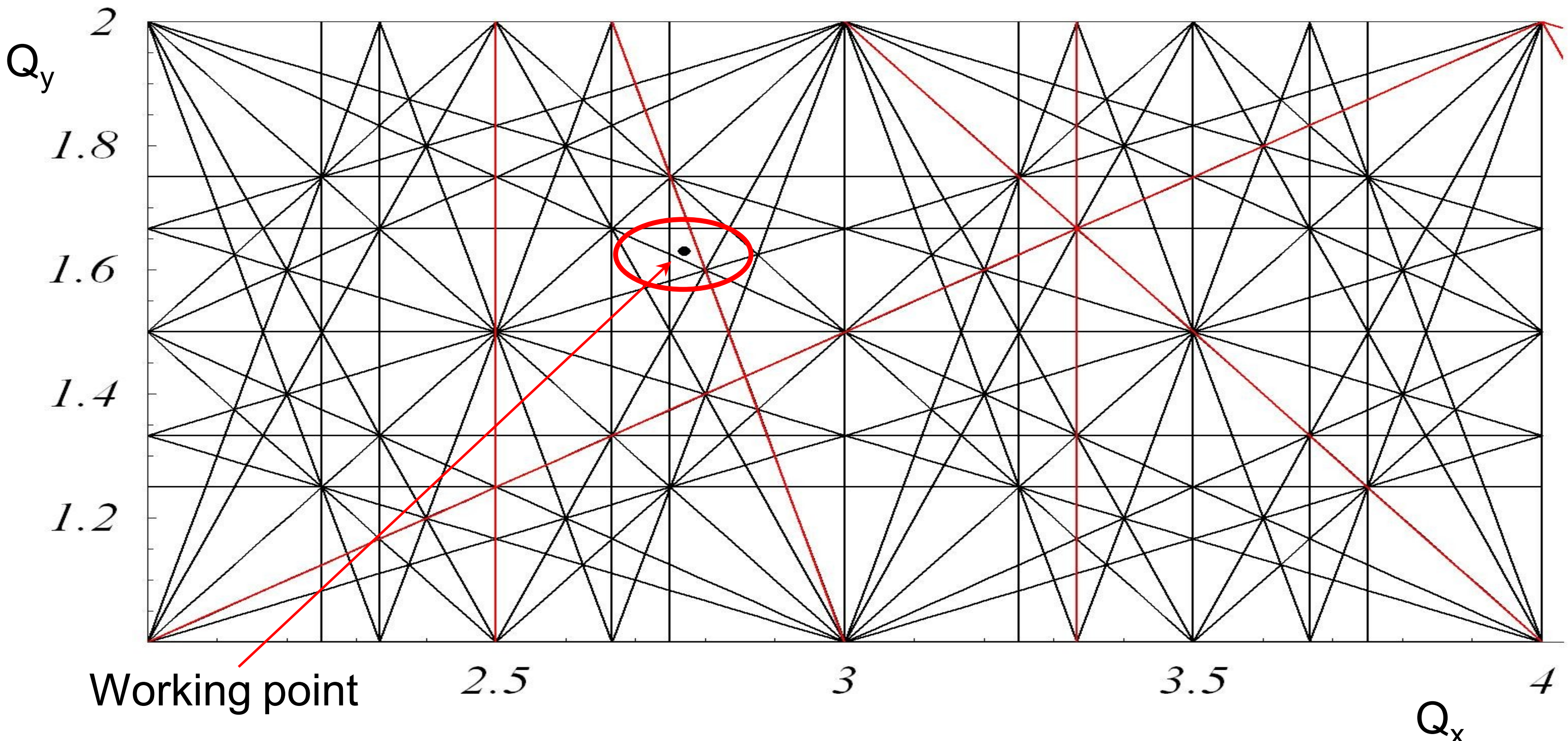
Betatron functions and dispersion in the FODO cell in the thick lens approximation

Resonances

The resonance conditions have to be avoided:

$$mQ_x + nQ_y = l$$

where m, n, l are integers and |m|+|n| gives order of resonance. Resonances of first and second order are driven by linear fields and higher orders are driven by nonlinear ones. They can be a source of an unacceptable beam loss!



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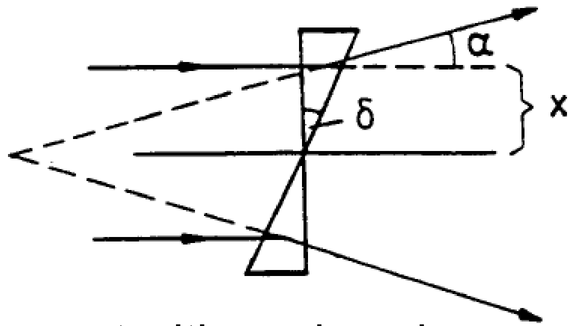
Wedge and fringe field focusing



Piece of magnet with wedge shape provides focusing effect: originating from geometry in horizontal plane and from fringe fields in the vertical one.

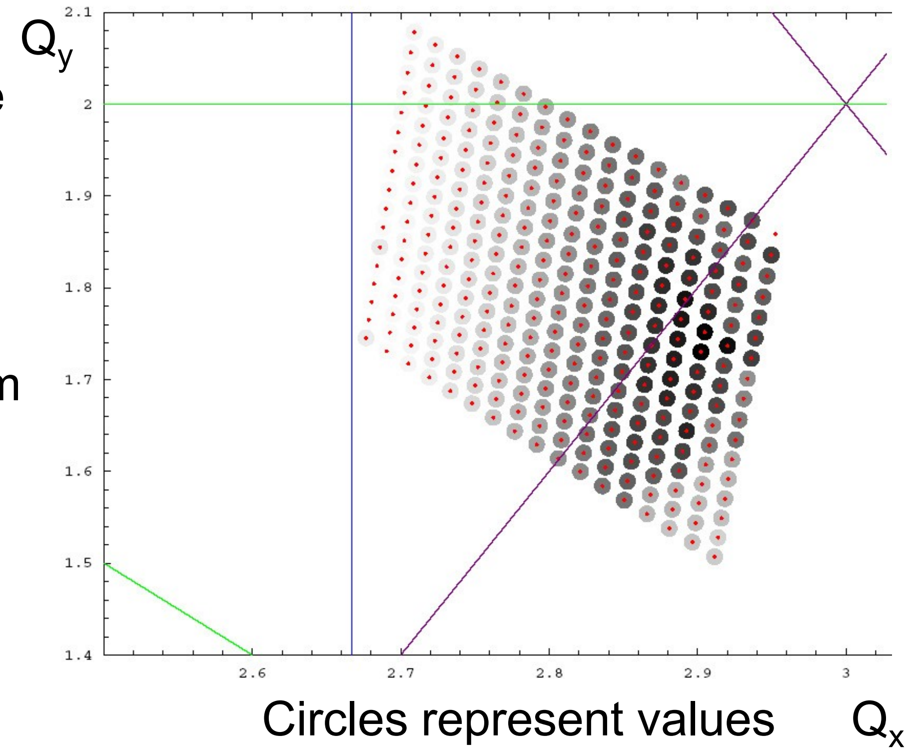
It can be represented by the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{\rho}\tan\delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Nonlinear transverse optics

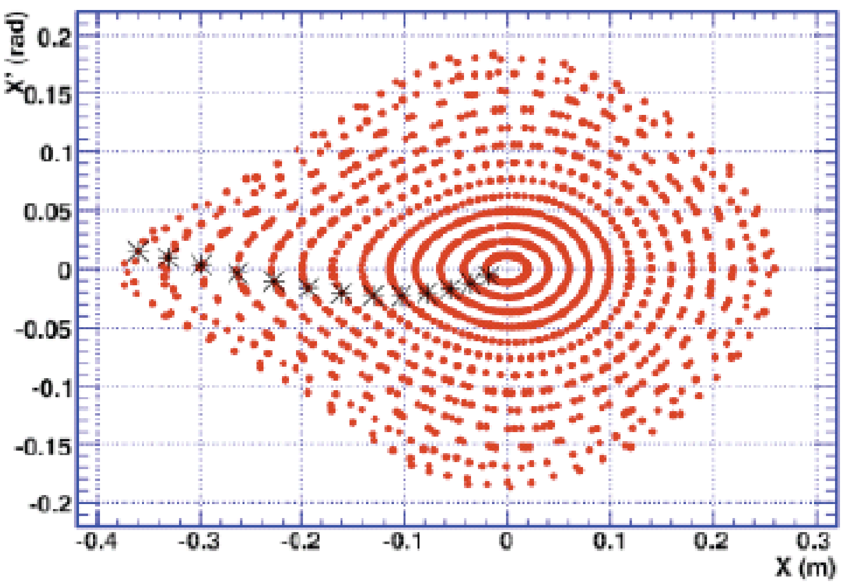
Nonlinear fields limits stable amplitudes. The dynamical apperture is defined by the highest amplitude which can be transported taking into account only beam dynamics limitations.

The amplitude size dictated by the vacuum chamber geometry is called physical aperture.



Circles represent values of DA

The effects of nonlinearities are especially strong close to resonance lines



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Momentum compaction and slippage factors

- Synchronization condition for RF acceleration: $\omega_{RF}=h\,\omega_{Rev}$

•Transition:

$$\frac{\Delta L}{L} \Big/ \frac{\Delta p}{p} = \frac{1}{L}\oint \frac{D}{\rho} ds = \alpha_c = \frac{1}{\gamma_T^2}$$

Momentum compaction factor

L – machine circumference
T - revolution time
β - relativistic factor

$$T = \frac{L}{c\beta}$$

$$\frac{\Delta T}{T} = \left(\frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p} = \eta \frac{\Delta p}{p}$$

Slippage factor

When γ of particle is higher, than γ_T higher energy particles are „slower" in circular machines.
At $\gamma = \gamma_T$ all particles move with the same revolution frequency – transition point.

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ISIS synchrotron at RAL

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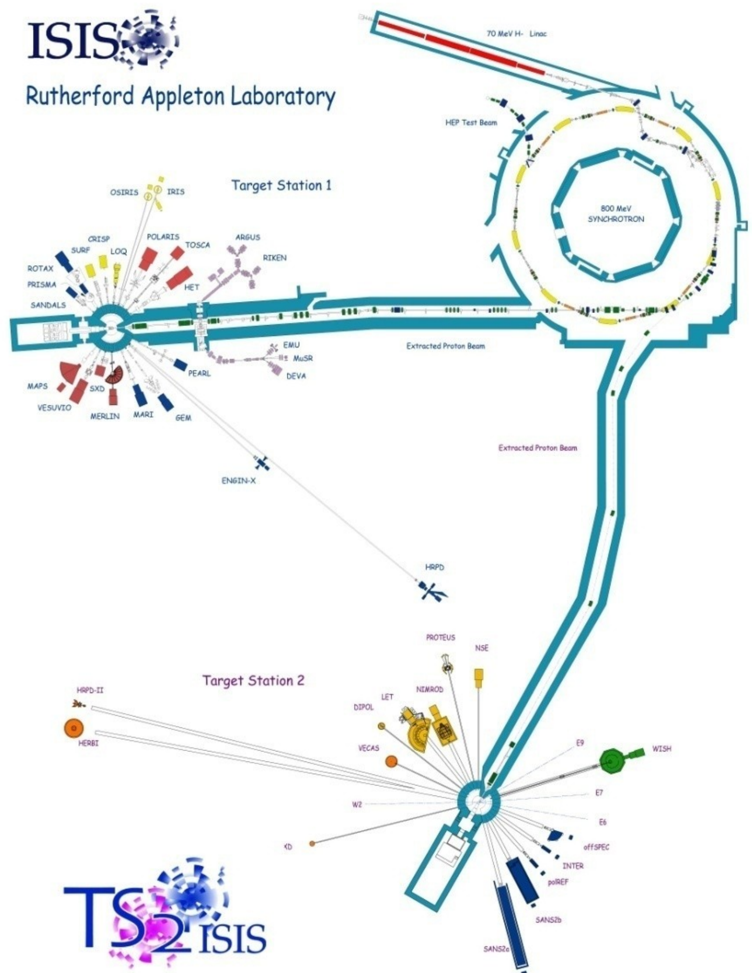
70 MeV H^{-} linac

TS-1



800 MeV proton synchrotron

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ISIS synchrotron

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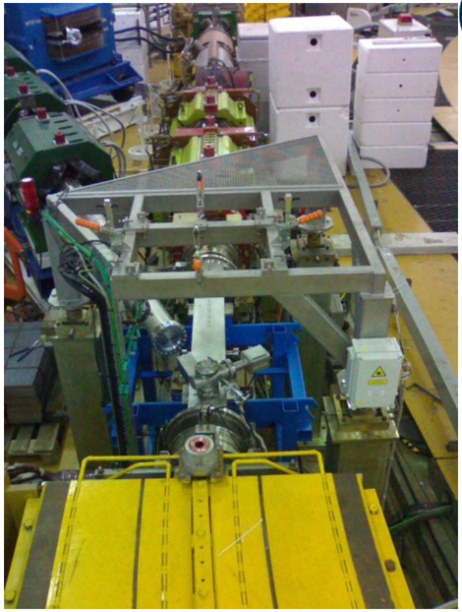
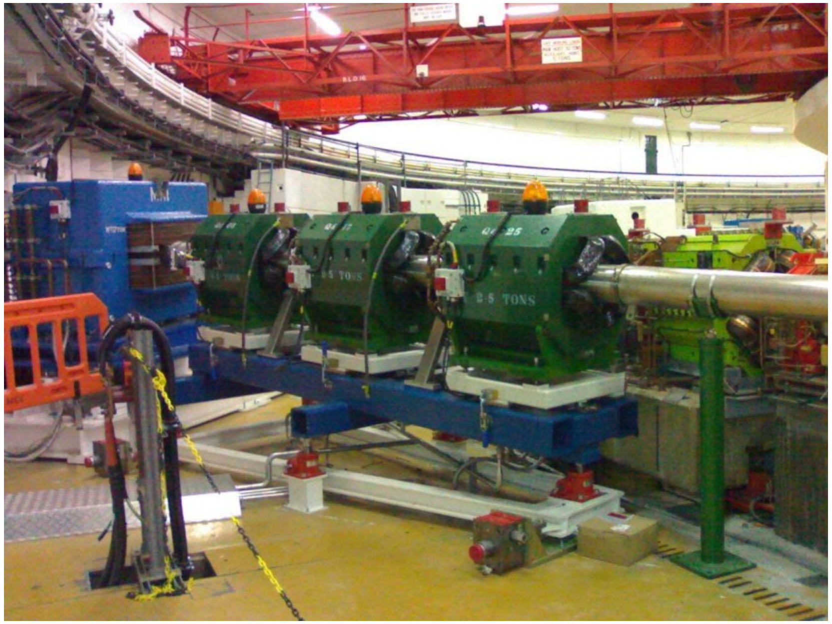
Injection



MICE pion line



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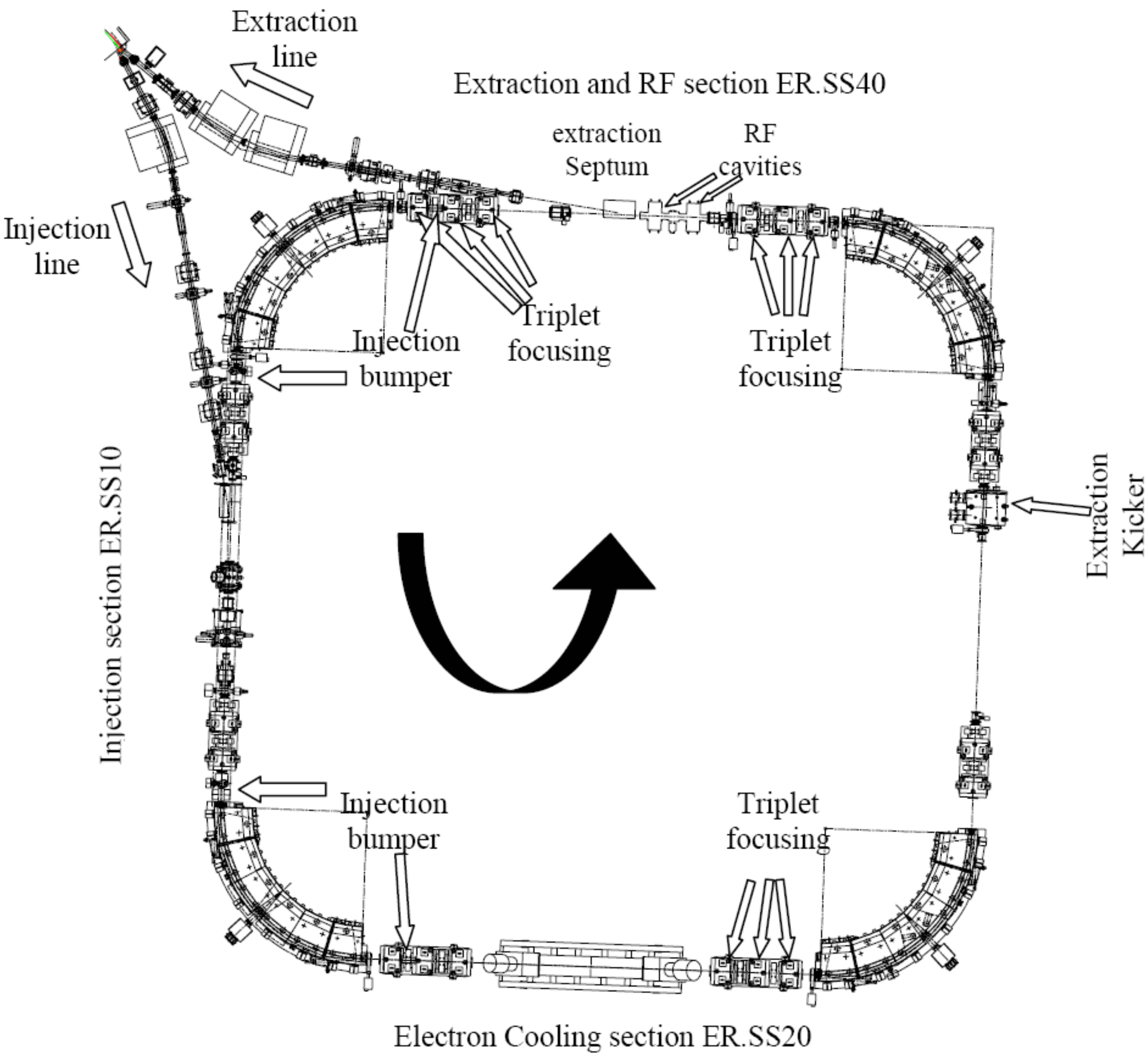
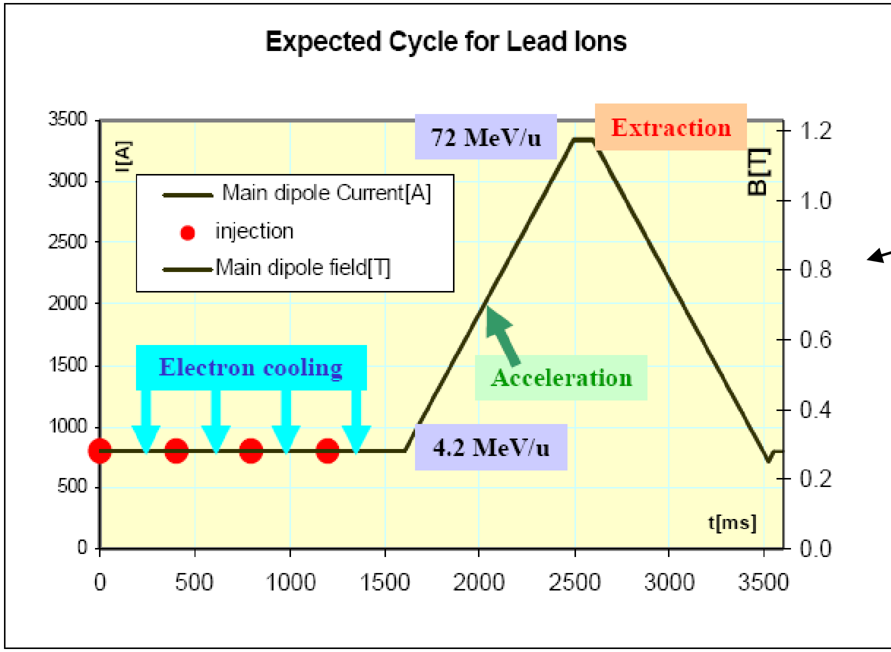
Example synchrotron - LEIR



The **L**ow **E**nergy **I**on **R**ing (Ion accumulator for LHC):

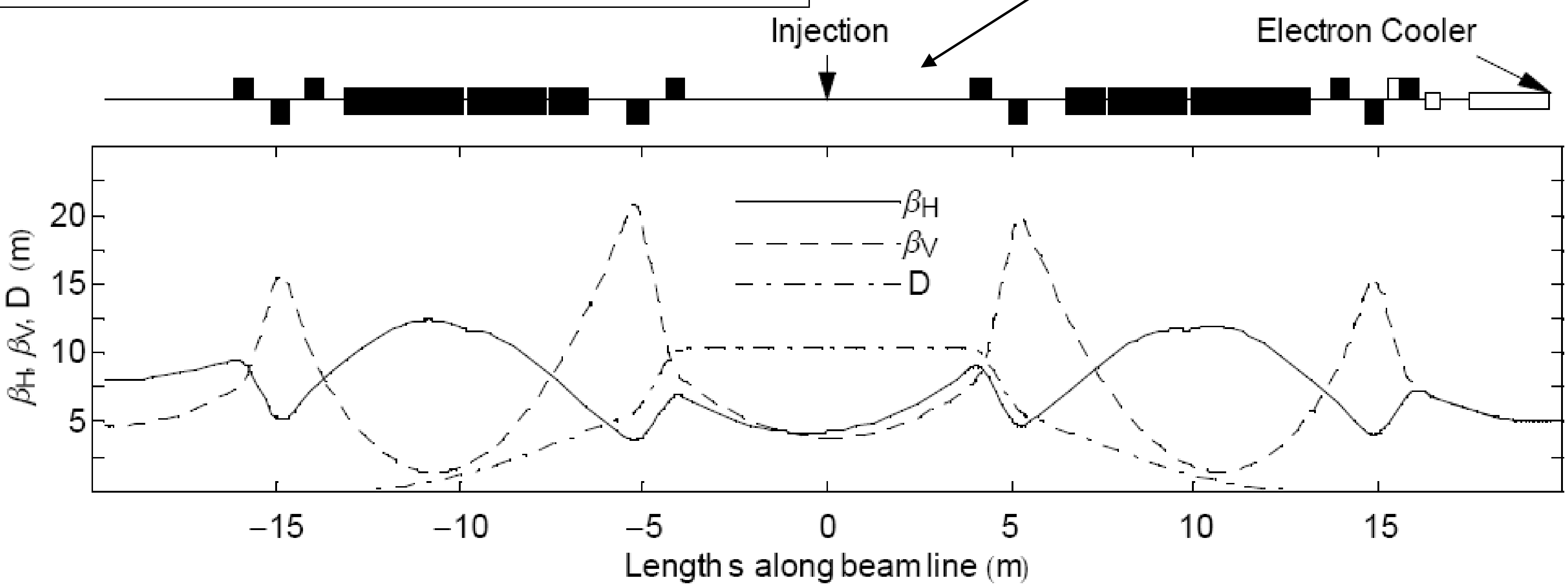
- Multiturn injection
- Fast electron cooling
- Ultra High Vacuum
- Bunching
- Aceleration and transfer into the CERN PS

Example synchrotron – LEIR (2)

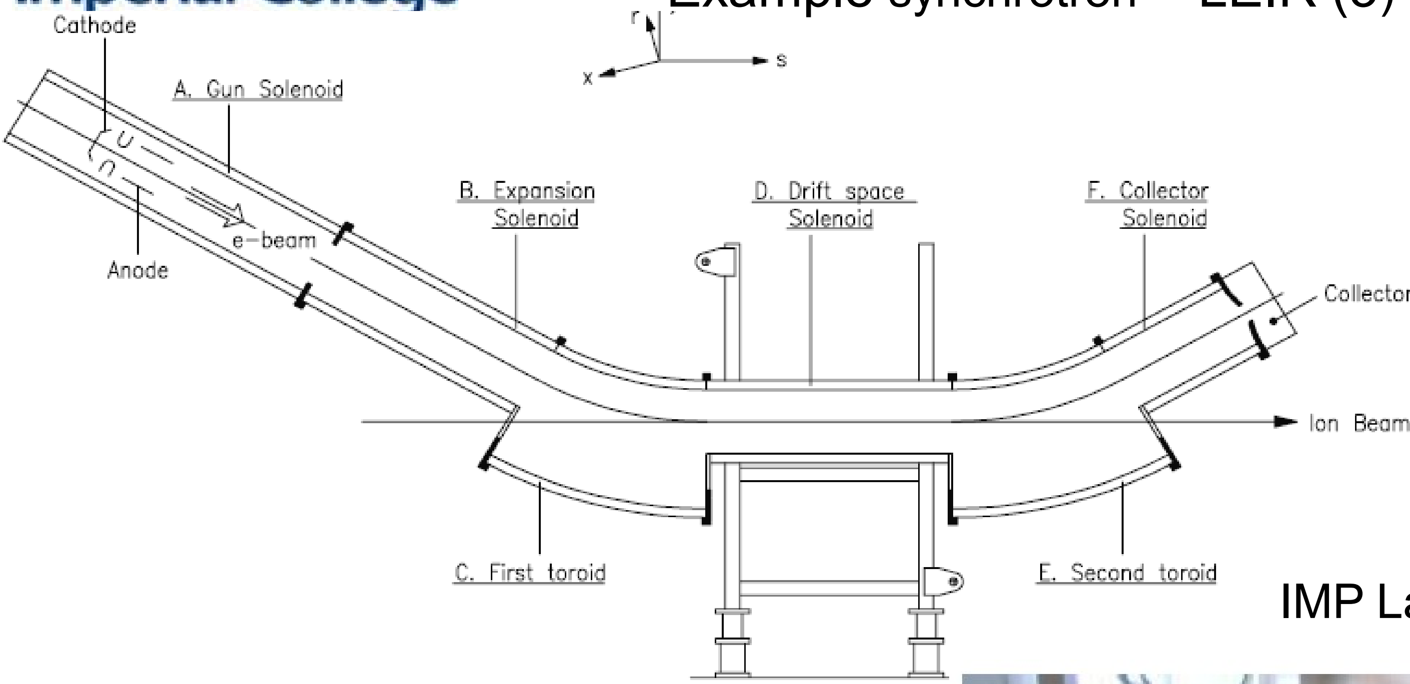


LEIR synchrotron cycle

Transverse optical function in the machine period



Example synchrotron – LEIR (3)



General layout of the electron cooler

IMP Lanzhou electron cooler

Beam cooling methodes in accelerator physics:

- electron cooling
- radiation cooling
- stochastic cooling
- laser cooling
- ionization cooling

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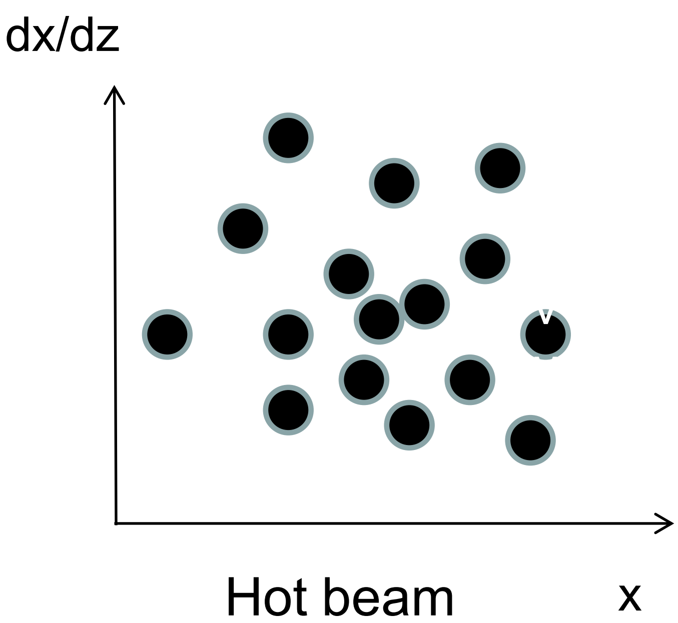
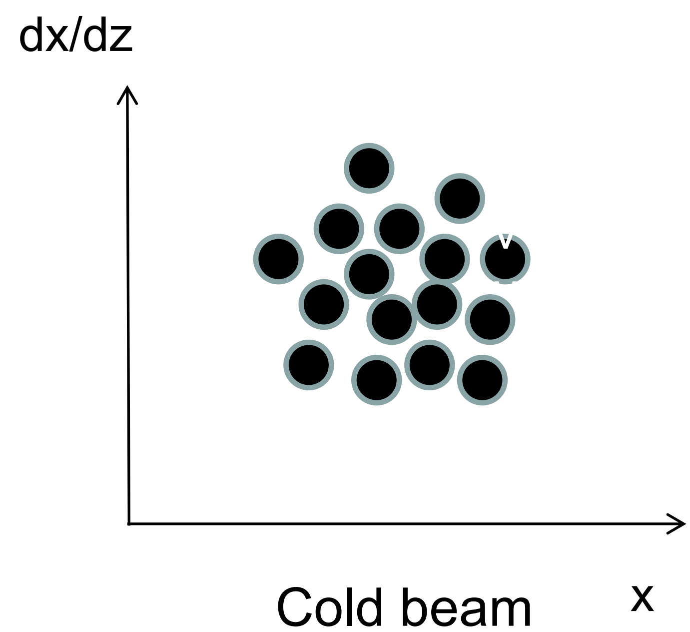
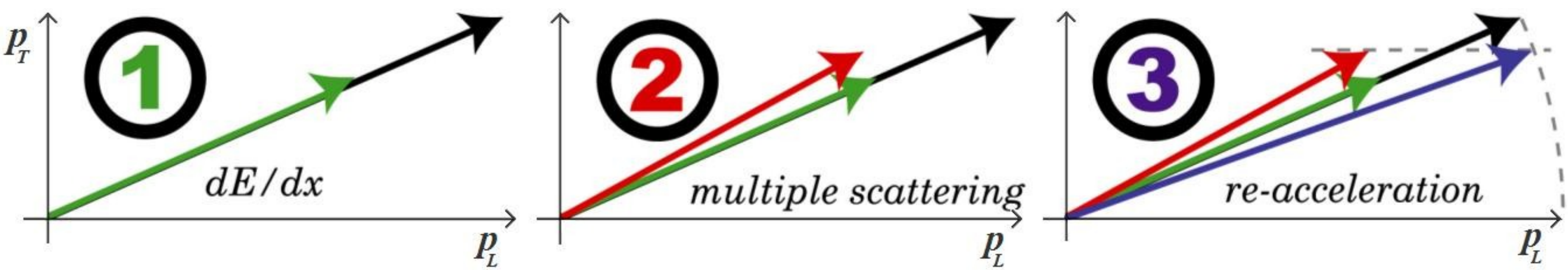
What is cooling?





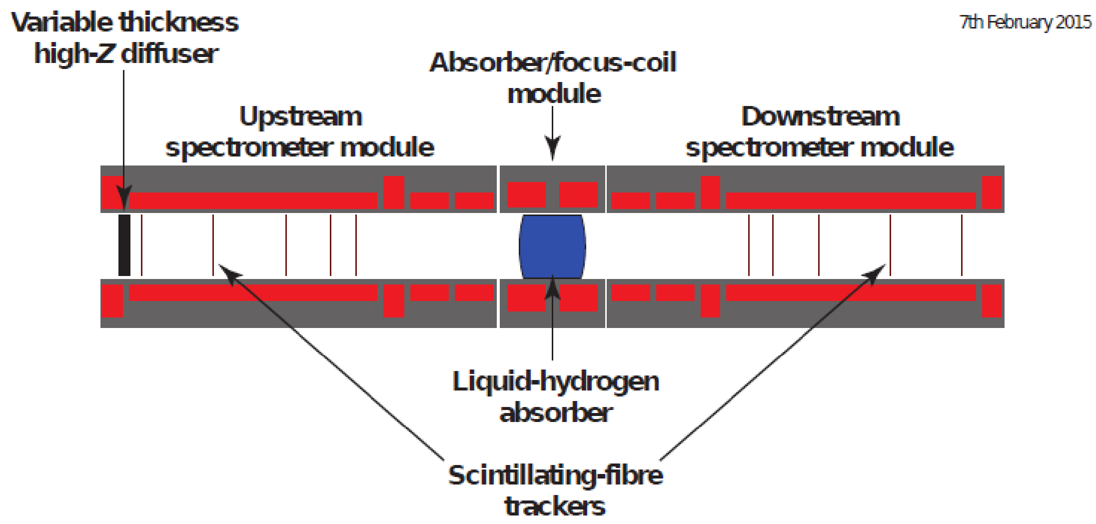
Quantity, which is a measure of beam phase space volume is called emittance [mm.mrad]. Cold beam has small emittance.

Basics of ionization cooling



- Muons pass trough absorber (liquid hydrogen) and acelerating cavity (RF).
- As a net effect transverse momentum is reduced.
- Strong focusing (using solenoids), low Z material as absorber and high RF gradient are necessary.

MICE:
Muon Ionization Cooling Experiment



- **MICE Goals:**
  - **Design, build, commission, and operate a realistic section of cooling channel**
  - **Measure its performance in a variety of modes of operation and beam conditions**

…results will be used to optimize Neutrino Factory, Muon Collider and future high brightness muon beam designs.

- Thank you and let’s turn into the main part of the school:

FFAGs!