Scaling FFAG accelerators

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Transverse motion

Transverse motion in particle accelerators

Linearized equations of motion:

$$\frac{\partial^2 a}{\partial a^2} + K_a(s)a = 0$$

a: horizontal (x)
or vertical (y)

Periodic case: Hill's equations

→ General solution: $a = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\nu\phi(s) + \phi_0)$

Betatron oscillations: pseudo-harmonic oscillation of frequency ν (tune) and varying amplitude $\sqrt{\beta(s)}$.

Betatron resonances

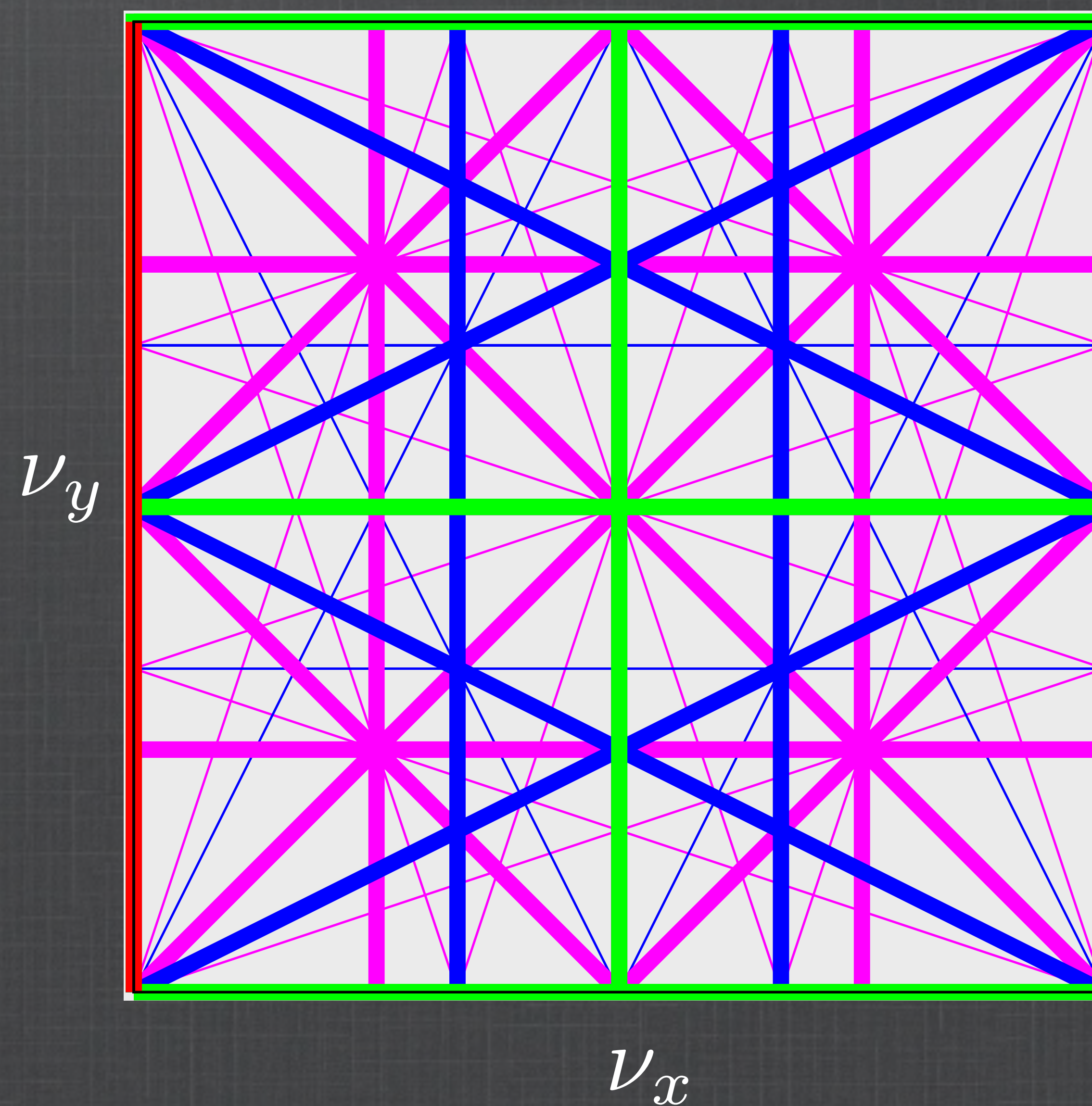
Non-linear components are considered as perturbations of the linear equations of motion.

Resonance conditions:

$$m_x \nu_x + m_y \nu_y = q$$

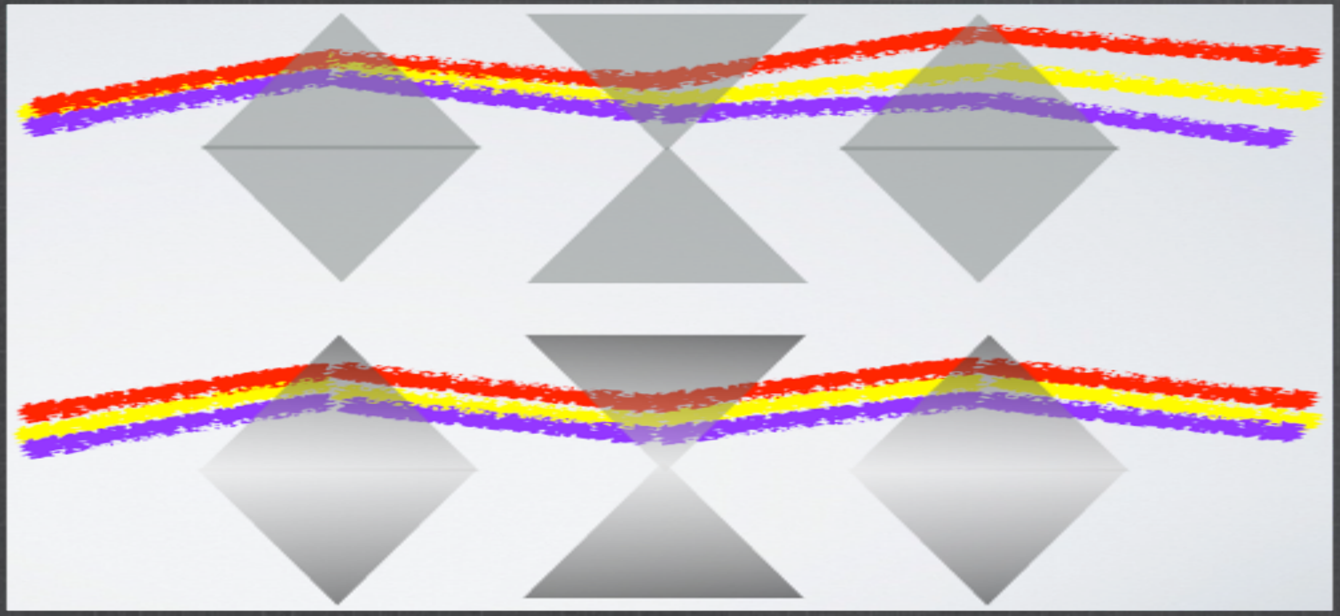
(m_x, m_y, q) integer numbers

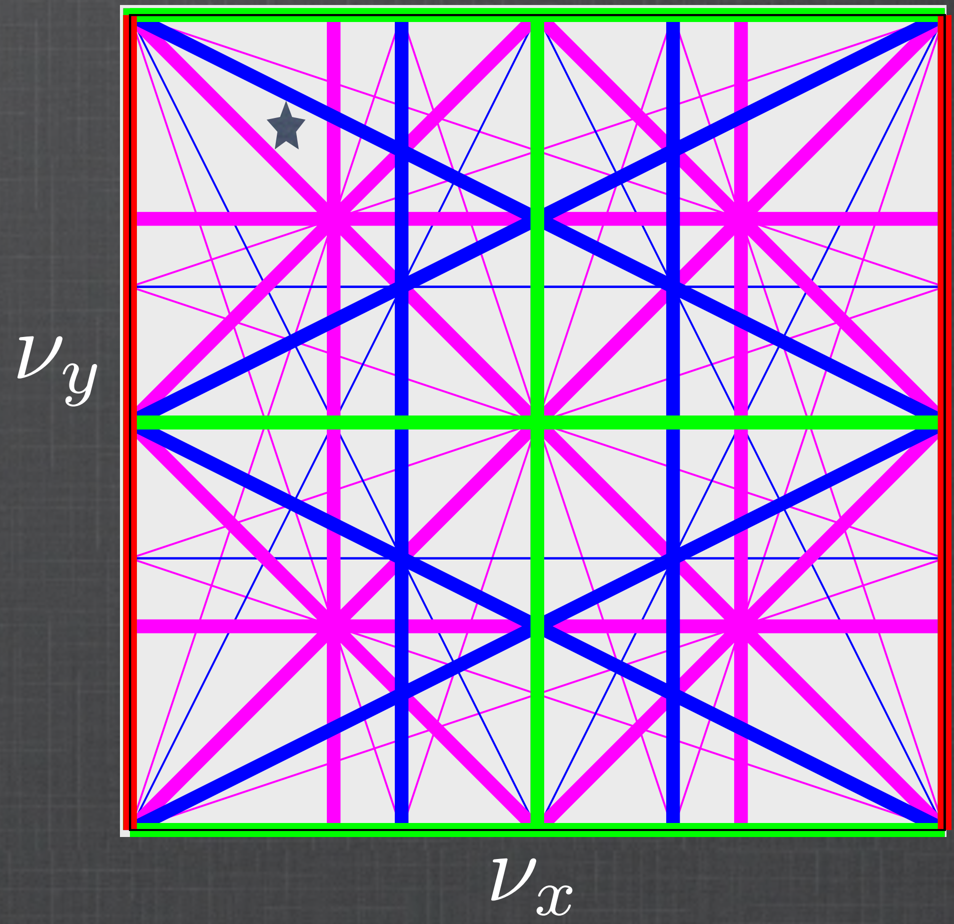
Working point (ν_x, ν_y) positioned in the tune diagram.



Chromaticity

Variation of tune with respect to particle energy.





$$\xi = \frac{\frac{\Delta\nu}{\nu}}{\frac{\Delta p}{p}} = -\frac{q}{4\pi\nu p}\int \beta(s)\frac{\partial B_y}{\partial x}(s)ds$$

FFAG classification

2 categories:

➡ Achromatic machines

SCALING FFAG

- large momentum acceptance
- large transverse acceptance
- no beam loss during acceleration
- fast acceleration if desirable
- high repetition rate (up to 1kHz or even CW)
- possibly big machines

➡ Chromatic machines

➡ Isochronous cyclotrons

CYCLOTRONS

- efficient acceleration
- CW (Continuous Wave) acceleration
- big machines
- need to treat resonances one by one
- limited energy

➡ Linear non-scaling FFAG

NON-SCALING FFAG

- very strong focusing
- CW (Continuous Wave) acceleration
- rapid acceleration necessary

Scaling FFAG

keep independent of momentum the transverse linearized equations of motion.

➡Analytical solution.

➡achromatic system for any momentum range.

There are also numerical ways to keep the tune constant over a certain momentum range (“non-linear non-scaling” FFAG).

Circular scaling FFAG

Linearized equations of motion for a momentum p:

$$\begin{cases} \dfrac{d^2x}{d\Theta^2} + \dfrac{R^2}{\rho^2}(1-n)x = 0, \\ \dfrac{d^2y}{d\Theta^2} - \dfrac{R^2}{\rho^2}ny = 0. \end{cases}$$

(x, s, y): curvilinear coordinates.

New system of coordinates (x, Θ, y)

$\Theta = s/R$ with $R = \dfrac{1}{2\pi}\oint ds$

n: field index

ρ : curvature radius

Independent of momentum p:

$$\begin{cases} \left(\dfrac{\partial(R/\rho)}{\partial p}\right)_\Theta = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_\Theta = 0. \end{cases}$$

→ Similarity of the reference trajectories.

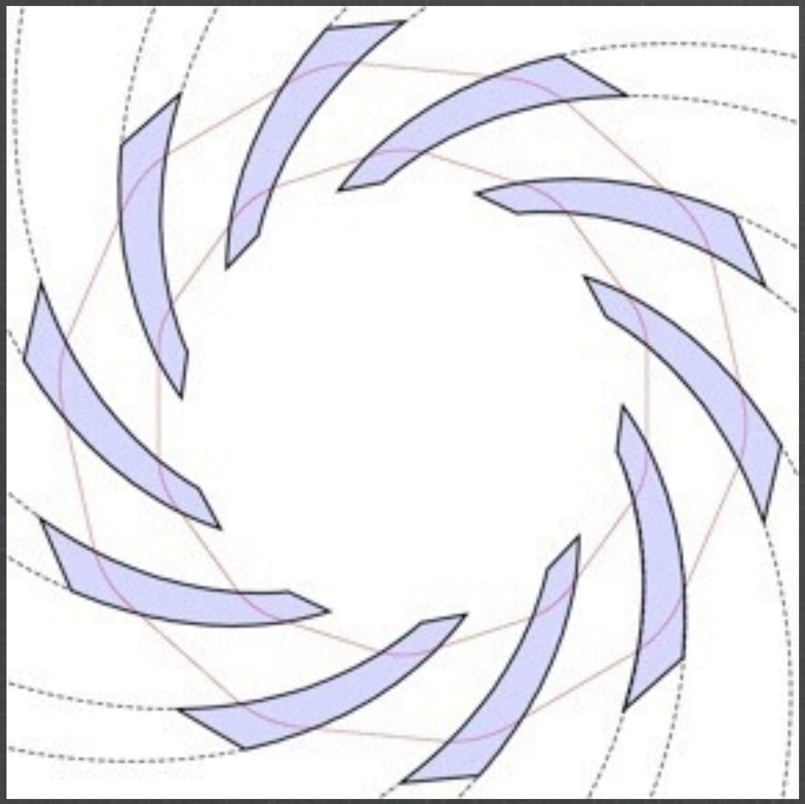
→ Invariance of the focusing strength.

Circular scaling FFAG

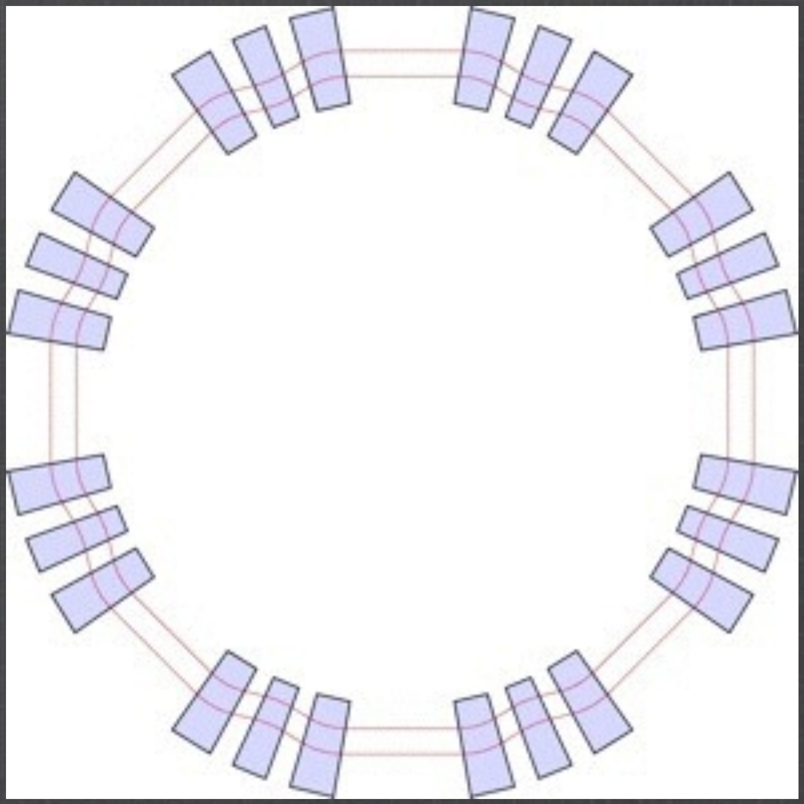
Invariance of the betatron oscillations ⟷ similarity of the closed orbits <u>and</u> invariance of the field index

Constant geometrical field index: $k = \frac{R}{\bar{B}} \frac{d\bar{B}}{dR}$

$$\boxed{B(r,\theta) = B_0 \left(\frac{r}{r_0}\right)^k \cdot \mathcal{F}(\theta - \tan\zeta \ln \frac{r}{r_0})}$$



Spiral sector: $\zeta = const.$



Radial sector: $\zeta = 0$

Straight scaling FFAG

Linearized equations of motion for a momentum *p*:

$$\begin{cases} \dfrac{d^2x}{ds^2} + \dfrac{(1-n)}{\rho^2}x = 0, \\ \dfrac{d^2y}{ds^2} - \dfrac{n}{\rho^2}y = 0. \end{cases}$$

(x, s, y): curvilinear coordinates

n: field index

ρ : curvature radius

Independent of momentum *p:*

$$\begin{cases} \left(\dfrac{\partial \rho}{\partial p}\right)_s = 0, \\ \left(\dfrac{\partial n}{\partial p}\right)_s = 0. \end{cases}$$

$\left(\dfrac{\partial \rho}{\partial p}\right)_s = 0$ ➡ Similarity of the reference trajectories

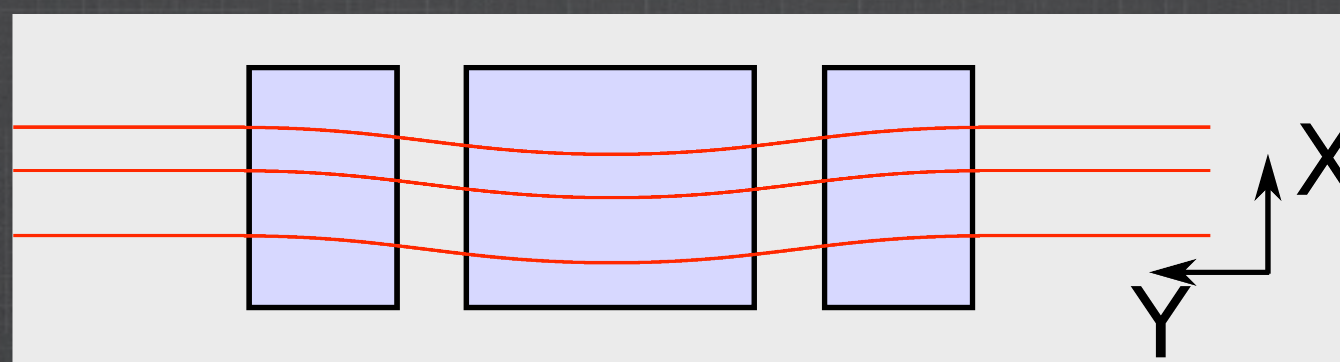
$\left(\dfrac{\partial n}{\partial p}\right)_s = 0$ ➡ Invariance of the focusing strength

Straight scaling FFAG

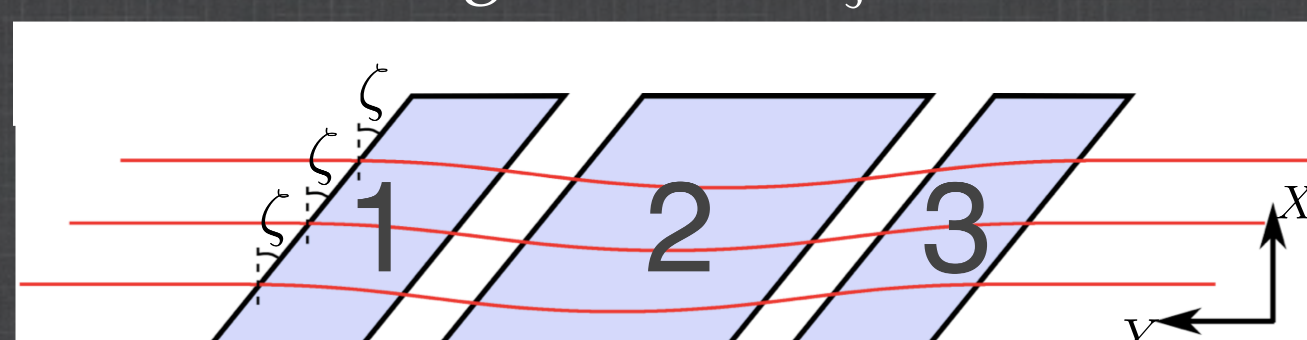
Invariance of the betatron oscillations ⟷ similarity of the closed orbits and invariance of the field index

Constant normalized field gradient: $m = \frac{1}{B_y}\frac{dB_y}{dx}$

$$B(X,Y) = B_0 e^{m(X-X_0)} \mathcal{F}\left(Y - (X - X_0)\tan\zeta\right)$$



Rectangular case: $\zeta = 0$



Tilted straight case: $\zeta = const.$

Vertical scaling FFAG

Linearized equations of motion for a momentum p:

$$\begin{cases} \dfrac{d^2x}{d\theta^2} + x - n_y y = 0, \\ \dfrac{d^2y}{d\theta^2} + n_y x = 0. \end{cases}$$

(x, s, y): curvilinear coordinates

ρ : curvature radius

θ: polar coordinate.

Vertical field index $n_y = -\dfrac{\rho}{B_y}\dfrac{\partial B_y}{\partial y}$

Independent of momentum *p:*

$$\left(\frac{\partial \rho}{\partial p}\right)_s = 0,$$

→ Similarity of the reference trajectories

$$\left(\frac{\partial n_y}{\partial p}\right)_s = 0.$$

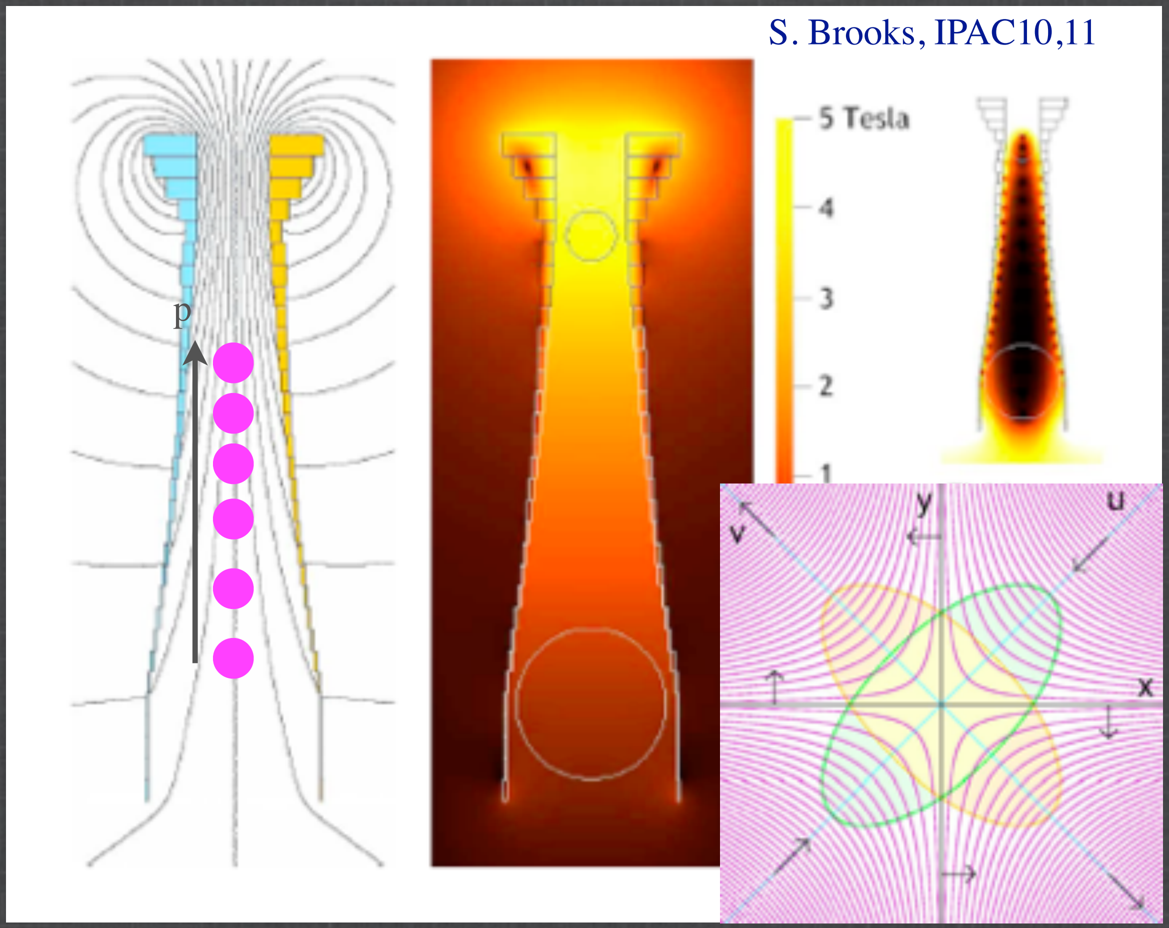
→ Invariance of the focusing strength

Vertical scaling FFAG

Invariance of the betatron oscillations ⟷ similarity of the closed orbits <u>and</u> invariance of the field index

Constant normalized vertical field gradient: $m_y = \frac{1}{B_y}\frac{dB_y}{dy}$

$$\boxed{B_y(y,\theta) = B_0 e^{m_y(y-y_0)}\mathcal{F}(\theta)}$$



scaling FFAG

Important parameters

	horizontal circular scaling	straight scaling	vertical scaling
Field law	$B_y = B_0 \left(\frac{r}{r_0}\right)^k$	$B_y = B_0 e^{m(x-x_0)}$	$B_y = B_0 e^{m_y(y-y_0)}$
Dispersion	$D_x = p_0 \left(\frac{\partial r_{co}}{\partial p}\right)_{p_0} = \frac{r}{k+1}$	$D = p_0 \left(\frac{\partial x_{co}}{\partial p}\right)_{p_0} = \frac{1}{m}$	$D_y = p_0 \left(\frac{\partial y_{co}}{\partial p}\right)_{p_0} = \frac{1}{m_y}$
momentum compaction factor	$\alpha = \frac{\Delta r_{co}/r_{co}}{\Delta p/p} = \frac{1}{k+1}$	$\alpha = \frac{\Delta x_{co}/x_{co}}{\Delta p/p} = 0$	$\alpha = \frac{\Delta y_{co}/y_{co}}{\Delta p/p} = 0$

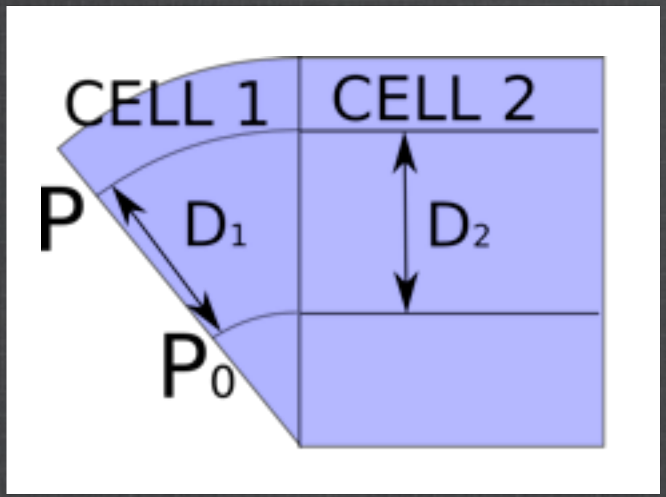
Insertions

Matching of different scaling FFAG cells

1) Matching of the curves s = const.

Radial for the circular case, rectangular in the straight case.

2) Matching of the reference trajectories



a) Matching of a special momentum P_0.

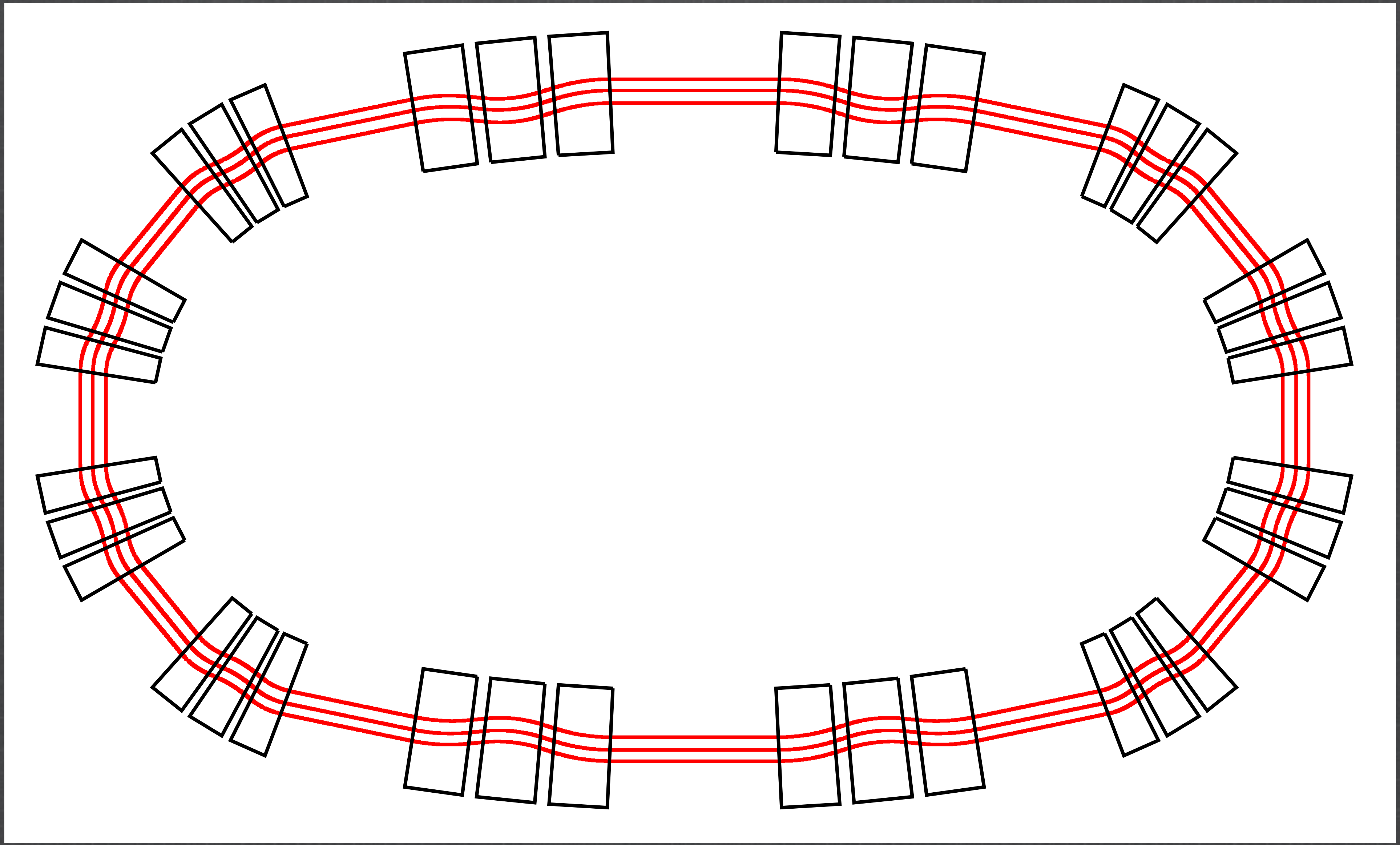
b) Matching to the first order in $\Delta R/R_0$ by matching of the cell dispersion of the different parts.

3) Matching of the periodic linear parameters

Often difficult → π-phase advance for one of the parts

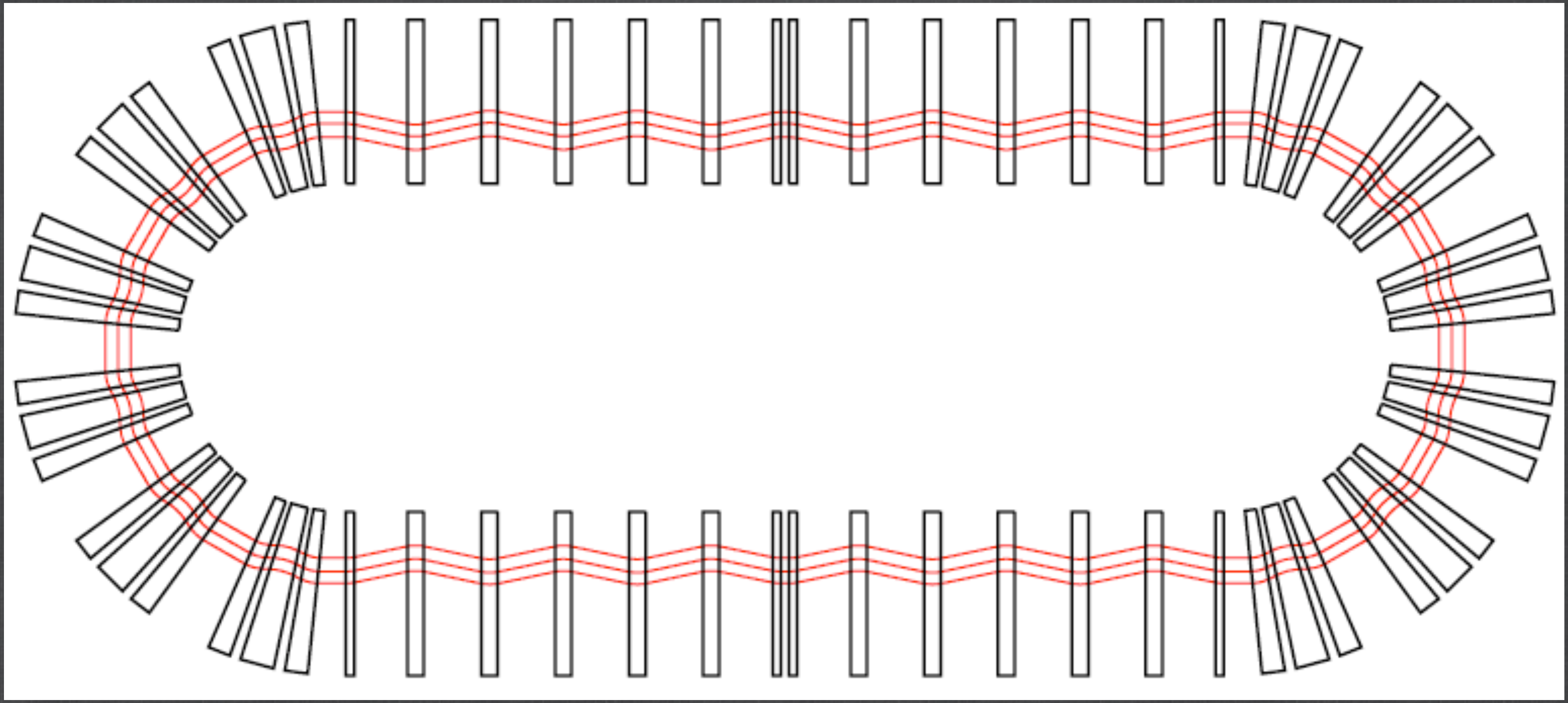
Insertions

Matching of different scaling FFAG cells



Insertions

Matching of different scaling FFAG cells

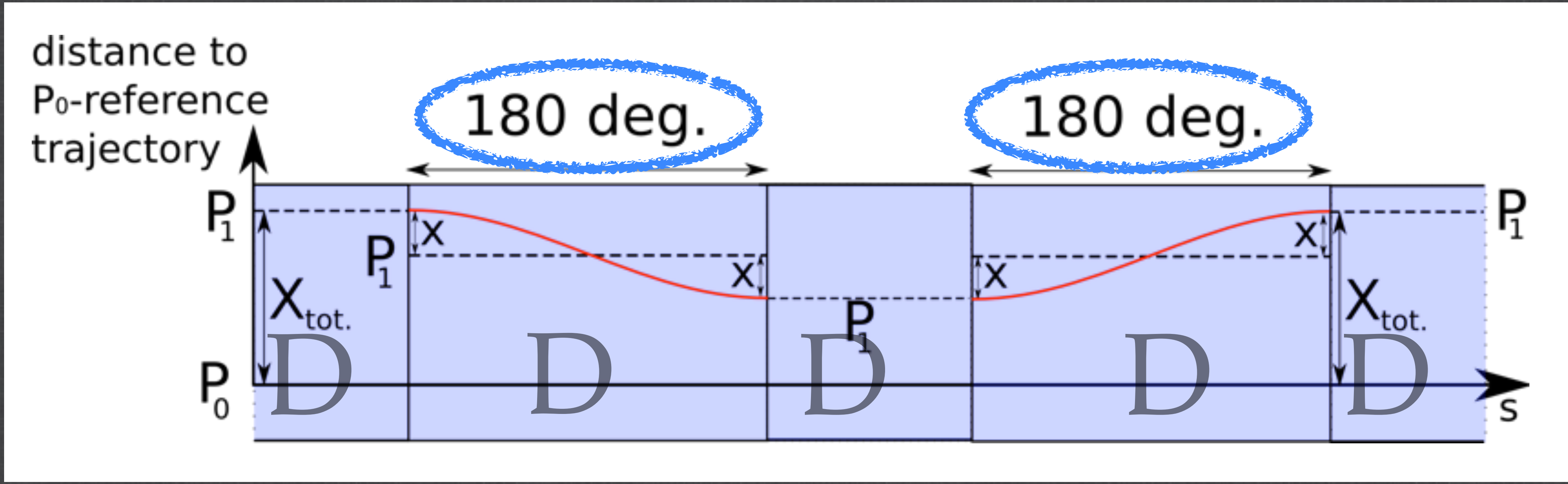


Insertions

Dispersion suppressor principle

a) Matching of a special momentum P_0.

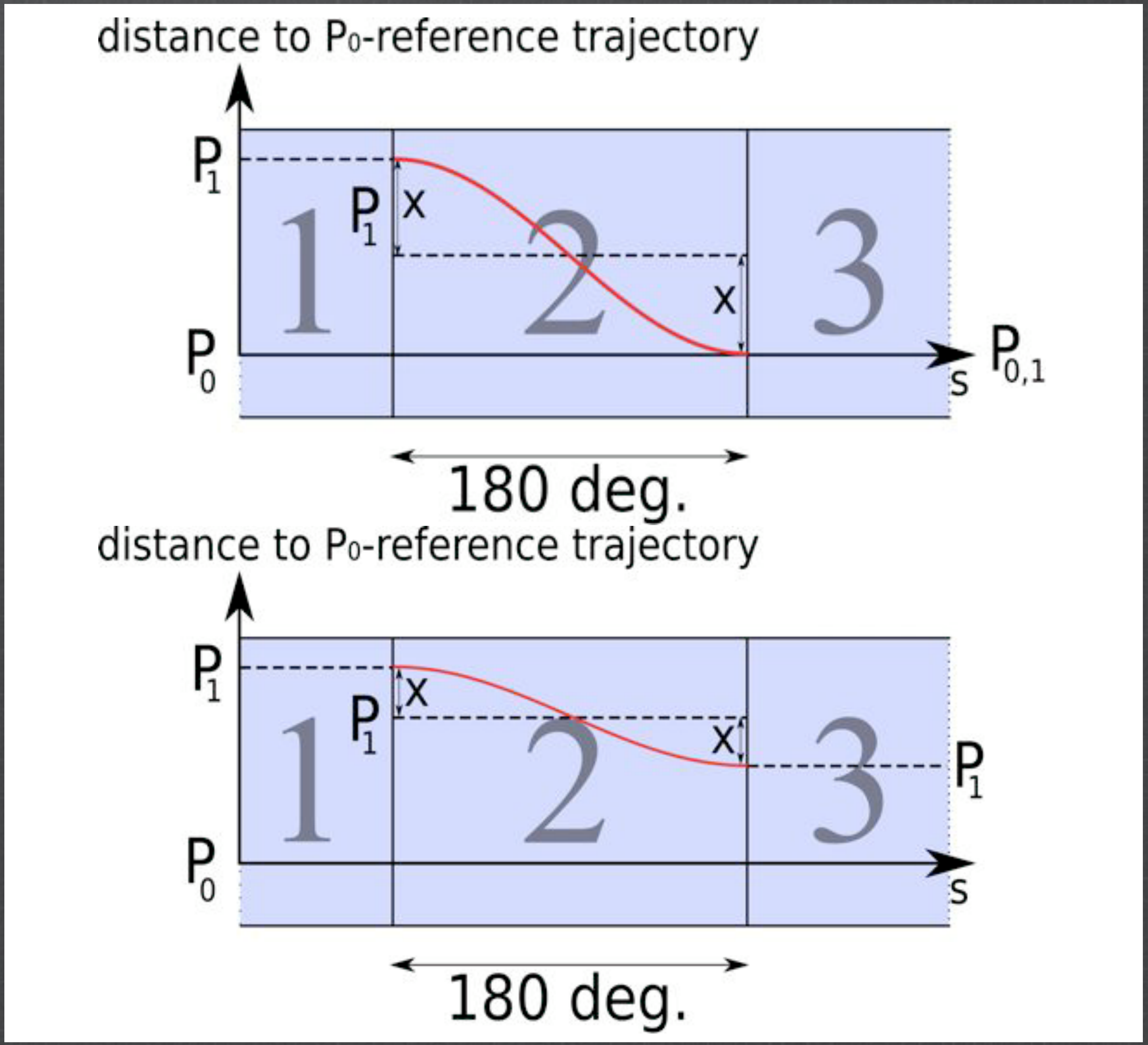
b) Matching of cell dispersions such as $D_2 = \frac{D_1 + D_3}{2}$



Insertions

Dispersion suppressor principle

Can be partial (bottom picture) or complete (top picture) dispersion suppression



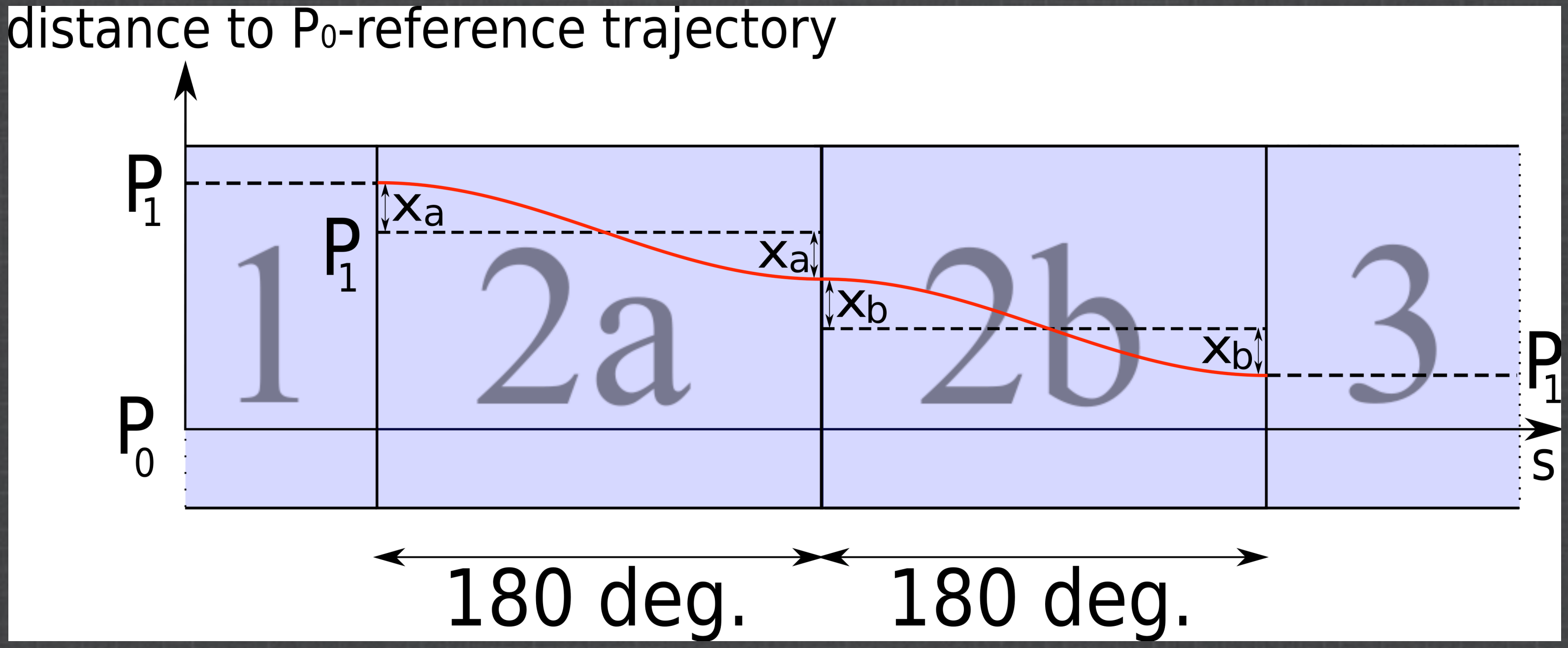
Insertions

Dispersion suppressor principle

Zero-chromatic system as long as amplitude detuning can be neglected.

→ several dispersion suppressors in cascade if the difference of dispersion is too large

$$D_{ini} + (-1)^{n+1} D_{fin} = 2 \sum_{i=1}^{n} (-1)^{i+1} D_i$$

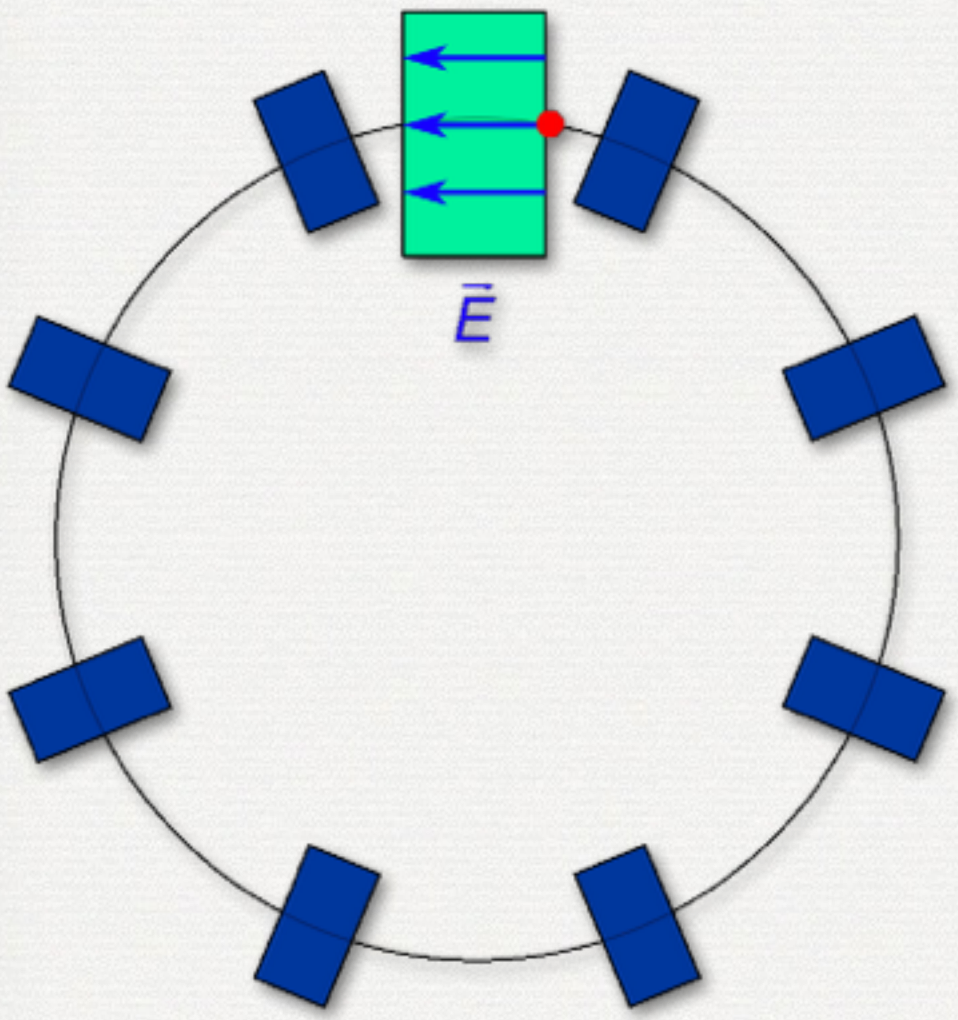


Acceleration

RF acceleration

Synchronisation condition:

$$f_{RF} = h \cdot f_{rev} \quad \text{with } h \in \mathbb{N}$$



Classical solutions:

- Frequency sweeping ➡ OK in scaling FFAGs with high field ferromagnetic core cavities.
- Constant RF frequency acceleration with isochronism (invariance with energy of f_{rev}) ➡ impossible in scaling FFAG

Fixed frequency acceleration

Longitudinal equations of motion

$$\begin{cases} mc^2 \dfrac{\mathrm{d}\gamma}{\mathrm{d}\theta} = \dfrac{eV_0}{2\pi} \sin\phi \\ \dfrac{\mathrm{d}\phi}{\mathrm{d}\theta} = f_{RF} \cdot \dfrac{C}{\beta c} - h \end{cases}$$

m: rest mass
γ, β: Lorentz factors
C: Circumference for momentum p
V_0: Peak RF voltage over 1 turn
ϕ: RF phase

$$C = C_s e^{\left(\int_{p_s}^{p} \frac{\alpha}{p} \mathrm{d}p\right)} \text{ and } \alpha(p) = const.$$

$$\begin{cases} mc^2 \dfrac{\mathrm{d}\gamma}{\mathrm{d}\theta} = \dfrac{eV_0}{2\pi} \sin\phi \\ \dfrac{\mathrm{d}\phi}{\mathrm{d}\Theta} = h\left[\dfrac{\gamma}{\gamma_s}\left(\dfrac{\gamma^2 - 1}{\gamma_s^2 - 1}\right)^{\frac{\alpha - 1}{2}} - 1\right] \end{cases}$$

Fixed frequency acceleration

Longitudinal hamiltonian

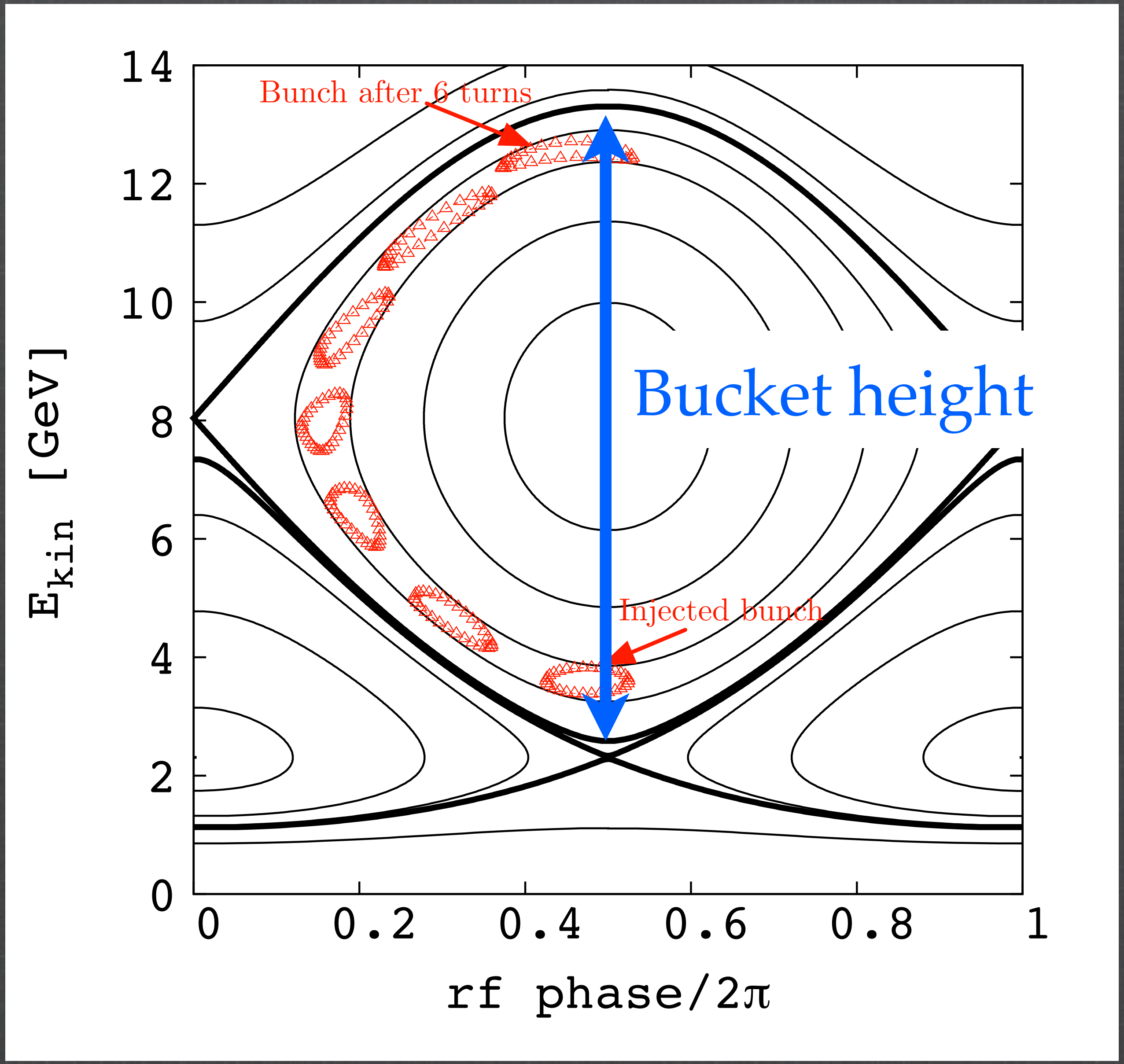
$$H(\phi, \gamma; \Theta) = h\left[\frac{1}{\alpha+1}\frac{\left(\gamma^2-1\right)^{\frac{\alpha+1}{2}}}{\gamma_s\left(\gamma_s^2-1\right)^{\frac{\alpha-1}{2}}} - \gamma\right] + \frac{eV_0}{2\pi mc^2}\cos\phi$$

Valid for any momentum range

Exact map of hamiltonian contours
in longitudinal phase space!

Stationary bucket acceleration

Principle: use the synchrotron motion to accelerate beam inside a stationary rf bucket.

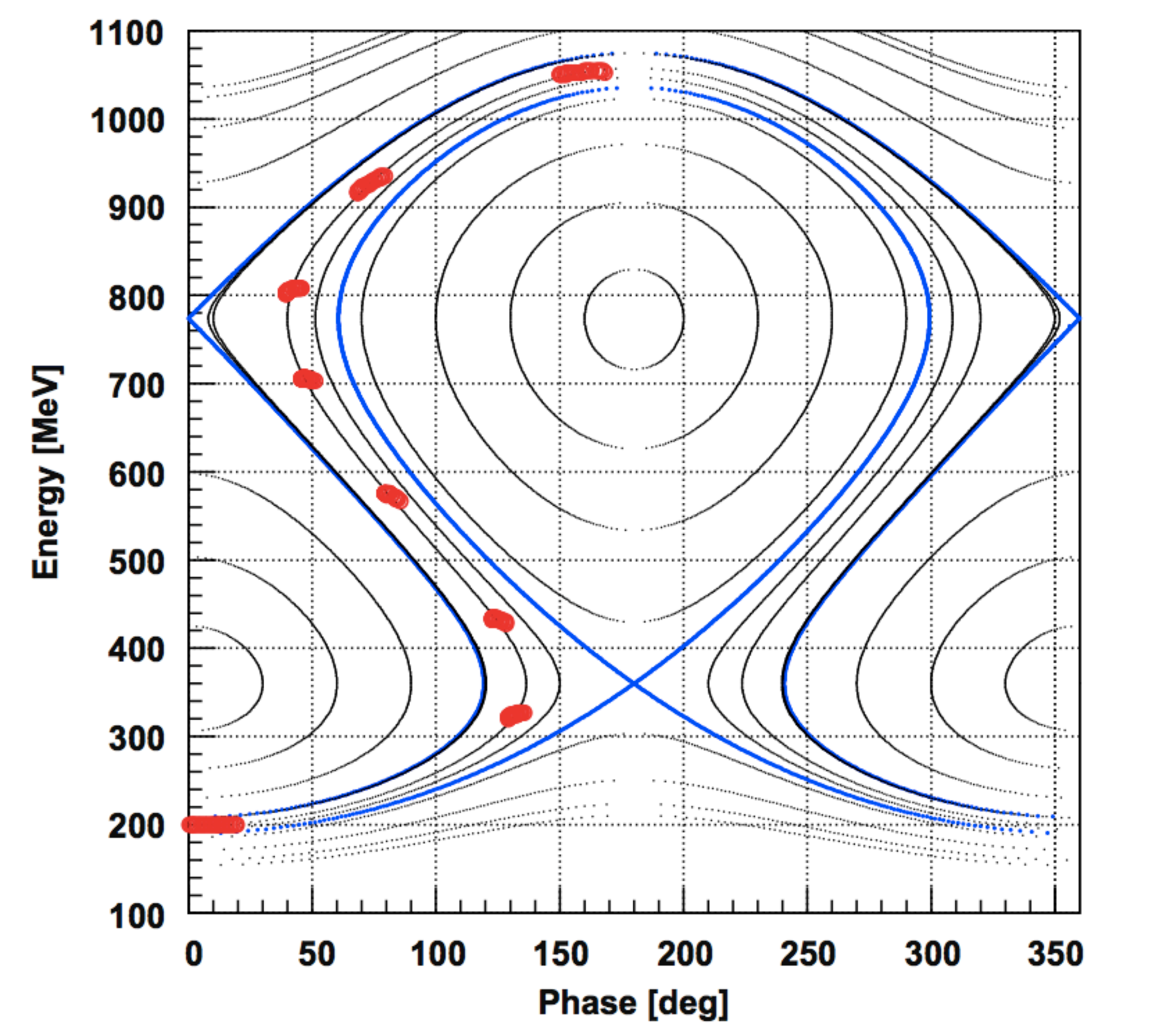


Useful in the relativistic region.

(T. Planche's courtesy)

Serpentine acceleration

Principle: Bring closer the stationary buckets to open a path around them.



Useful in the non-relativistic region.

(E. Yamakawa's courtesy)

Harmonic number jump acceleration

Principle: change the harmonic number *h* by an integer number every turn

$$f_{RF} = h \cdot f_{rev} \quad \text{with } h \in \mathbb{N}$$

Required excursion:

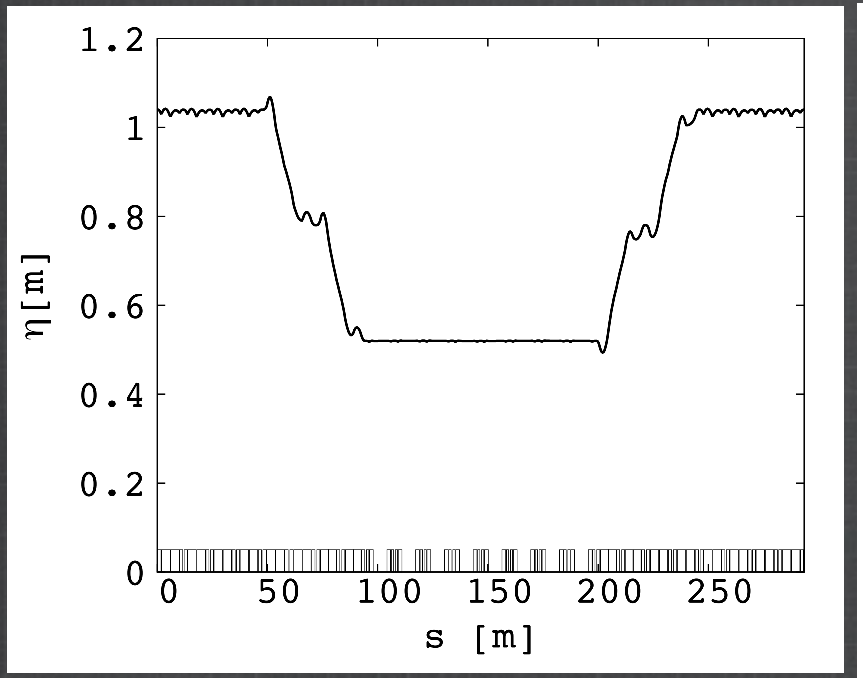
$$T(E_{i+1}) - T(E_i) = \frac{\Delta_i h}{f_{RF}} = \frac{2\pi\Delta_i R}{\beta c}$$

$$\Rightarrow \quad \Delta_i R = \beta \Delta_i h \frac{\lambda_{RF}}{2\pi}$$

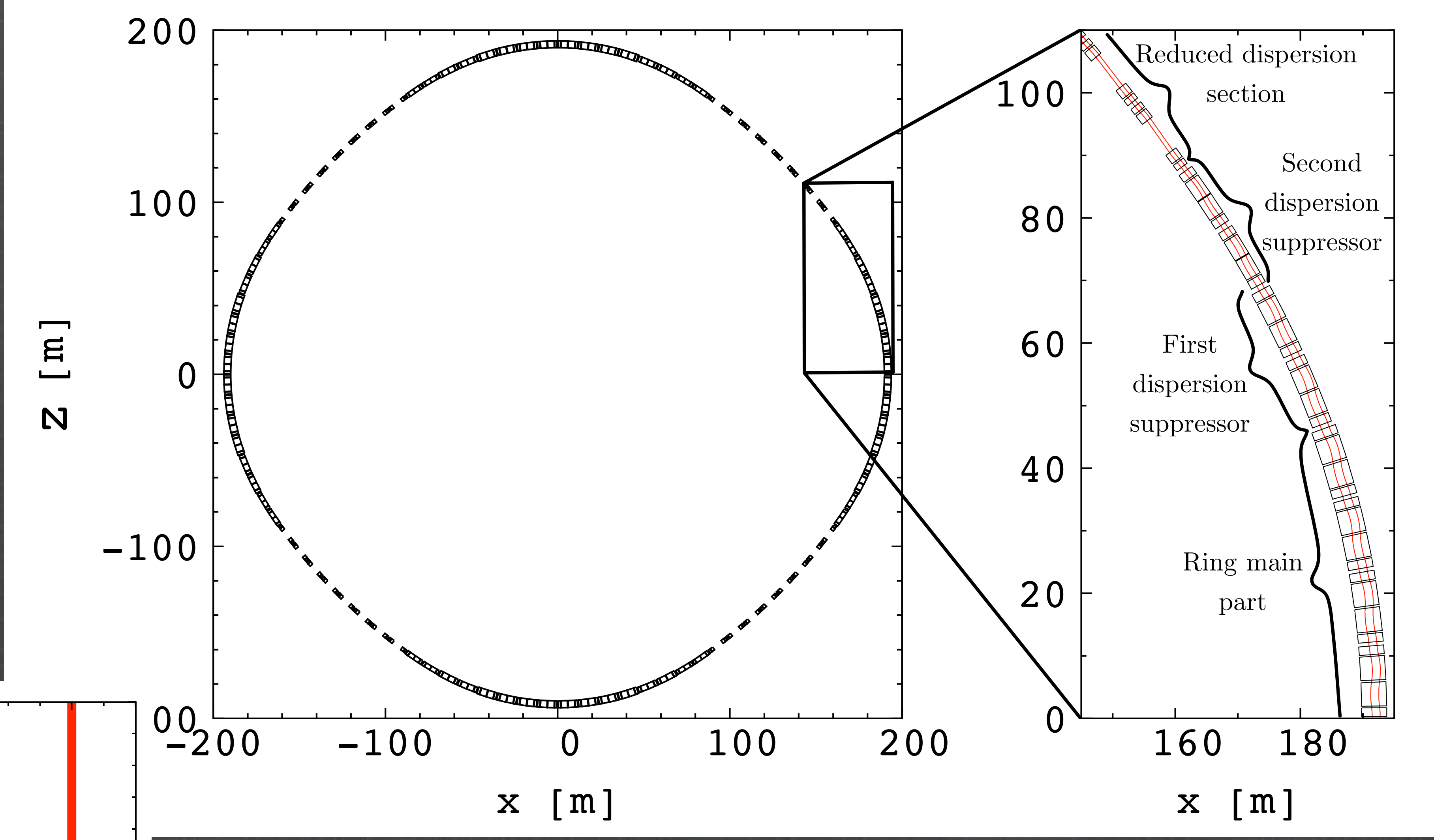
In relativistic region (β~1), insertions with reduced excursions are necessary to install cavities.

Harmonic number jump lattice

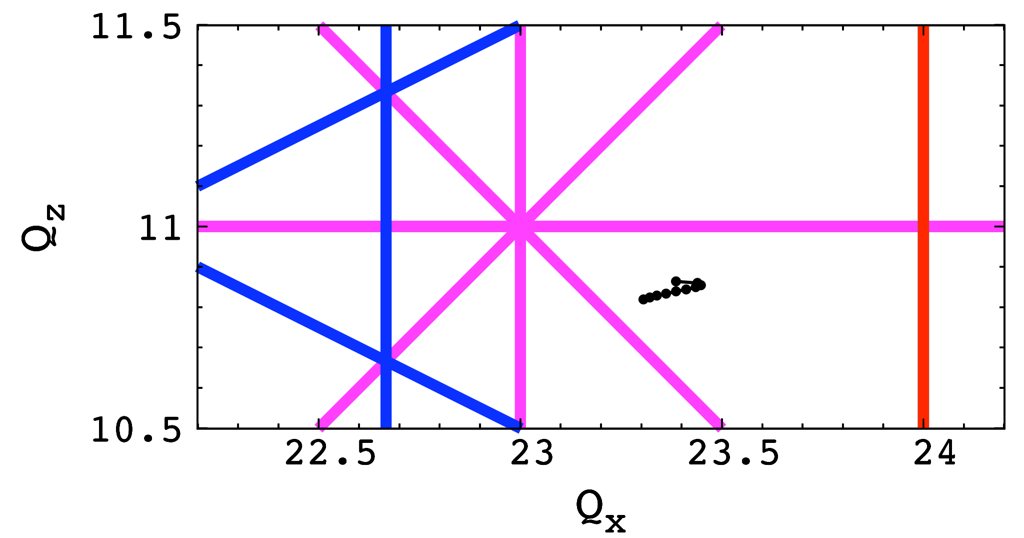
3.6-12.6 GeV muon HNJ ring



Dispersion (1/2 ring)



Layout



Tune spread over the momentum range