Non-scaling FFAG classroom tutorial

1. The transfer matrix for a thin lens FODO, M_{FODO}, is given below

$$M_{FODO} = \begin{bmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -f^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ +f^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{bmatrix}$$

where f is the focal length of a thin quadrupole and L is the long drift length.

(a.) Carry out the matrix multiplication to find the transfer matrix for a thin lens FODO. Then find an expression for the phase advance per cell in terms of the f and L.

(b.) Assuming a linear non-scaling FFAG made up of FODO cells, with magnets of length 65 mm and long drift 200mm, what is the quadrupole gradient required to ensure the cell tune is 0.4 at 10 MeV/c? (Note that $1/f = k^*L_Q$ where k is the quadrupole strength and L_Q the length of each quadrupole).

(c.) Keeping the quadrupole gradient fixed, what will the cell tune be if the momentum doubles?

(d.) It can be shown that the natural chromaticity per cell in a FODO lattice is approximately given by $\xi_n = -\frac{1}{\pi}\tan\left(\frac{\psi_c}{2}\right)$ so long as $\psi_C \ll \pi$. Calculate the chromaticity at 20 MeV/c assuming a 42 cell ring. In how many turns will betatron oscillations decohere assuming a momentum spread of 0.5%.

2.

(a.) At 10 MeV/c what is the COD amplitude that results from a 0.5 mTm integrated stray field? Make use of the smooth-focusing betatron function R/Q, and assume a machine of average radius R=2.7m. Again assume a cell tune of 0.4.

(b.) R. Baartman (Proc. of FFAG04) derived a formula for the amplitude growth following integer resonance crossing

$$\Delta A = \frac{\pi}{\sqrt{Q_\tau}} \frac{R}{Q} \frac{B_n}{\bar{B}}$$

where Q_τ is ring tune change per turn, $\bar{B} = B\rho/R$ is the average radius and B_n is the Fourier component of the error field (assume this is the integrated stray field averaged over the ring circumference).

Again assuming a 0.5 mTm integrated stray field, what tune change per turn Q_τ is required to limit amplitude growth to 1mm when crossing Q=7 (which we assume corresponds to 19 MeV/c)?

Bonus question: By calculating the chromaticity and assuming a 42 cell thin lens FODO lattice, convert this result into the required momentum change per turn.