Classroom - Scaling FFAGs

1. Find the relations between r_0, r_1, r_2, r_3, $\theta_{F/2}$, ρ_F, θ_D, ρ_D, L_O and $< r >= \frac{1}{2}(r_0 + r_3)$ that the closed orbit satisfies in DFD circular scaling FFAG cell (see Figure 1, half of the cell). Assume ρ_F and ρ_D are constant in the magnets. After linearisation, solve the system of equations. Find the parameters for the different angles at the entrance and exit of the magnets.

2. Find the transfer matrix of a DFD circular scaling FFAG cell.

3. Compute the tune for a machine of 12 cells, with $\beta_{F/2} = 5.1\,\text{deg.}$, $\beta_D = 5.6\,\text{deg.}$, $k = 7.6$. For a 150 MeV proton, $r_0 = 5.1\,\text{m}$ and the mean magnetic field in the F magnet $B_F = 1.6\,\text{T}$.

4. (Bonus) Find the stability region of such a lattice by varying the key parameters.

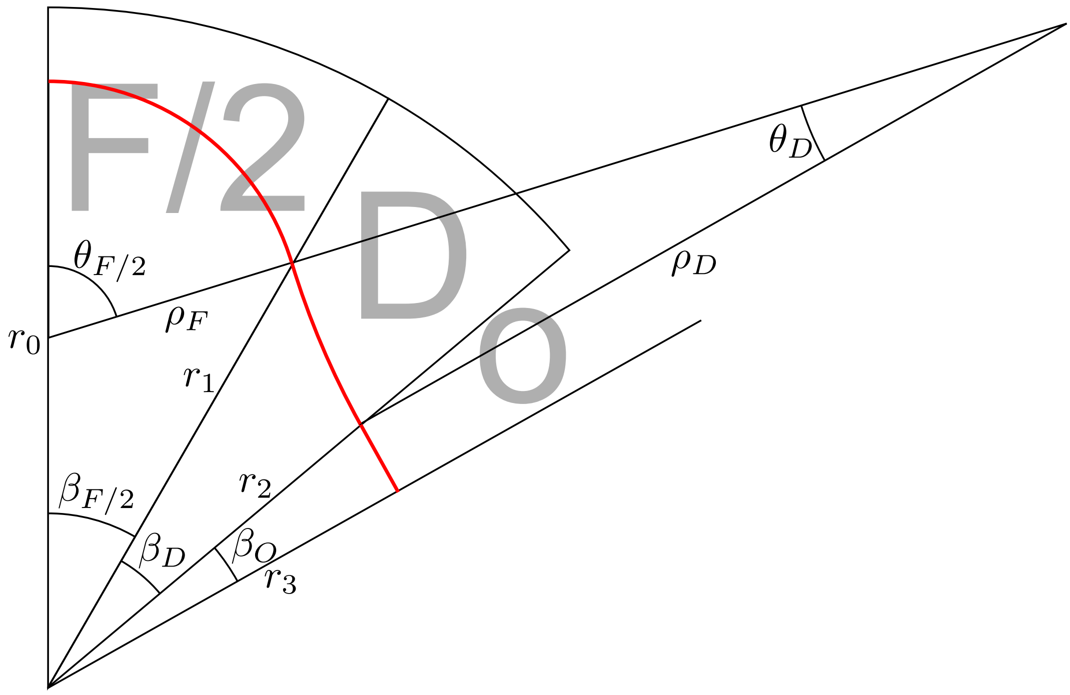

Figure 1: Closed orbit (red) in half of a DFD scaling FFAG cell.

Note: The linear transfer matrix of combined function magnets, such a scaling FFAG magnets, can be written as follows:
when $K > 0$:

$$\mathbf{M_{K>0}} = \begin{pmatrix} \cos(l\sqrt{K}) & \frac{1}{\sqrt{K}}\sin(l\sqrt{K}) \\ -\sqrt{K}\sin(l\sqrt{K}) & \cos(l\sqrt{K}) \end{pmatrix}, \tag{1}$$

and, when $K < 0$:

$$\mathbf{M_{K<0}} = \begin{pmatrix} \cosh(l\sqrt{-K}) & \frac{1}{\sqrt{-K}}\sinh(l\sqrt{-K}) \\ \sqrt{-K}\sinh(l\sqrt{-K}) & \cosh(l\sqrt{-K}) \end{pmatrix}. \tag{2}$$

The optical length l of the magnets is defined as $l = \rho\theta$. The coefficient K is defined in the horizontal plane as:

$$\begin{aligned} K_H &= \frac{|n|+1}{\rho_F^2} \quad \text{for F magnets} \\ K_H &= \frac{-|n|-1}{\rho_D^2} \quad \text{for D magnets,} \end{aligned} \tag{3}$$

and in the vertical plane as:

$$\begin{aligned} K_V &= \frac{-|n|}{\rho_F^2} \quad \text{for F magnets} \\ K_V &= \frac{|n|}{\rho_D^2} \quad \text{for D magnets,} \end{aligned} \tag{4}$$

where n is the effective field index calculated around the closed orbit.
The transfer matrix representing the effect of edge focusing in the horizontal plane writes:

$$\mathbf{M_{edge\ H}} = \begin{pmatrix} 1 & 0 \\ \frac{\tan\epsilon}{\rho} & 1 \end{pmatrix}, \tag{5}$$

with ϵ the edge angle ($\epsilon > 0$ is horizontally defocusing, $\epsilon < 0$ is horizontally focusing). In the vertical plane, the edge focusing is approximated by:

$$\mathbf{M_{edge\ V}} = \begin{pmatrix} 1 & 0 \\ -\frac{\tan\epsilon}{\rho} & 1 \end{pmatrix}. \tag{6}$$

Finally, the transfer matrix corresponding to drift spaces simply writes:

$$\mathbf{M_{drift}} = \begin{pmatrix} 1 & 0 \\ 0 & l \end{pmatrix}, \tag{7}$$

with l the drift length.