Two-loop calculation of the Higgs mass in general models and the Goldstone Boson Catastrophe

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Overview

- Why calculate higher-order corrections the Higgs mass?
- How we can calculate it for generic theories.
- The Goldstone Boson Catastrophe ...
- ... and how to avoid it.

The Higgs mass as a precision electroweak observable

Consider the current experimental accuracy of the Higgs mass measurement:

$$\text{ATLAS} + \text{CMS (Moriond 2015)} : \quad m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11\,(\text{syst.})$$

The uncertainty is tiny!
In the Standard Model:

- The Higgs mass is used to calculate the Higgs quartic coupling $\mathcal{L} \supset -\lambda|H|^2)$ (from [Butazzo *et al*, 1307.3536]):

$$\lambda(\mu = m_t) = 0.12604 + 0.00206\left(\frac{m_h}{\text{GeV}} - 125.15\right) - 0.00004\left(\frac{m_t}{\text{GeV}} - 173.34\right) \pm 0.00030$$

- Vital for stability analysis (also needed in principle for future triple/quartic Higgs coupling measurements).
- State-of-the-art computation includes most two-loop effects.

But in any BSM theory:

- If we can choose the quartic from top-down theory, have a prediction for the Higgs mass instead.
- But: the electroweak scale/Higgs mass is *very* sensitive to new physics – hence the hierarchy problem! – so the Higgs mass is a precision probe of new states.

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For many years the standard example has been the MSSM:

- Quartic predicted to be determined entirely by gauge couplings at tree level: $\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta$ in heavy M_H limit.
- Hence $\rightarrow m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim 10$ GeV $\rightarrow \delta m_h^2(\text{two loops}) \sim 15\% m_h^2$!
- While at three-loop order, have $\delta m_h \sim$ few hundred MeV, $\rightarrow \delta m_h^2(\text{three loops}) \lesssim 1\% m_h^2$

Much work has led to: full one-loop calculation, two loops full diagrammatic calculation for $\alpha_s \alpha_t$ only; effective potential approximation and gaugeless limit for (Yukawa coupling)^4 diagrams, and three-loop $\alpha_s^2 \alpha_t$.

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Calculations in models beyond the MSSM

With the LHC results questioning naturalness in the MSSM, there is an increased need to look at other models, either SUSY or non-SUSY.

Up until recently, calculations/tools for models beyond the MSSM were rare:

- [Degrassi and Slavich, 2009] calculated the complete one-loop corrections in the NMSSM, and the two-loop corrections proportional to $\alpha_s\alpha_t, \alpha_s\alpha_b$ in the effective potential approach.
- `NMSSMTools`, `NMSSMSOFTSUSY` implemented these and included the MSSM corrections proportional to the Yukawa couplings in the third family for the MSSM-like Higgs pairs.
- [Muhlleitner, Nhung, Rzehak, Walz, 2014] calculated the $\alpha_s\alpha_t, \alpha_s\alpha_b$ corrections for the CP-violating NMSSM at zero external momentum and will implement these in `NMSSMCALC`.
- [Braathen, MDG and Slavich, 2016] calculated the $\alpha_s\alpha_t$ contributions for Dirac gaugino models in the effective potential approach.

For other models, the only possibility to have a two-loop computation is `SARAH`:

- "Spectrum generator generator" that Calculates RGEs, full one-loop masses for all sparticles including momentum dependence, generates code for `SPheno`, `MicrOMEGAs`, `MadGraph`, ...
- I'll describe our two-loop computation of the Higgs mass.

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Calculation of the Higgs mass

The Higgs mass is corrected order by order through two effects:

1. Self energy corrections

$$m^2_{\text{pole}} = m^2_{\text{tree}} + \Pi(m^2_{\text{pole}})$$

2. Shifts to the minimum conditions: we define the potential in terms of real scalars with vevs v to be

$$\begin{aligned} V(v) =& V^{\text{tree}} + \Delta V \equiv \frac{1}{2} m^2_{\text{run}} v^2 + V^{\text{tree}}_{\lambda} + \Delta V \\ \rightarrow 0 =& m^2_{\text{run}} v + \left(\frac{\partial V^{\text{tree}}_{\lambda}}{\partial v} + \frac{\partial \Delta V}{\partial v} \right) \end{aligned}$$

If we take v as fixed to all orders (which is convenient since couplings depend on v) we must shift m^2_{run} so that

$$\rightarrow m^2_{\text{tree}} = m^2_{\text{run}} + \frac{\partial^2 V^{\text{tree}}_{\lambda}}{\partial v^2} = \left(\frac{\partial^2 V^{\text{tree}}_{\lambda}}{\partial v^2} - \frac{1}{v} \frac{\partial \Delta V^{\text{tree}}_{\lambda}}{\partial v} \right) - \frac{1}{v} \frac{\partial \Delta V}{\partial v}.$$

So we need the tadpole diagrams as well as self-energies to calculate the mass; note that if we took the masses fixed instead of the vevs we would still have a shift in the mass due to a shift in v (c.f. Higgs tree-level mass of $2\lambda v^2$).

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The effective potential approach

One significant simplification to calculations is to take $p^2 = 0$; this is then equivalent to taking

$$\Pi(0) = \frac{\partial^2 V}{\partial v^2}.$$

Hence the "effective potential limit." When the scale of new physics is above the electroweak scale this is a good approximation.
It leads to three different approaches:

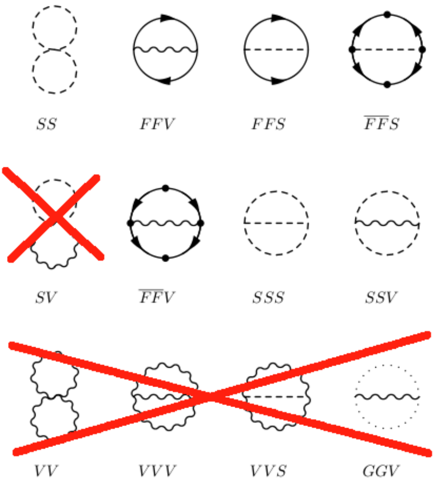
1. Calculate the effective potential as a function of the fields and take derivatives analytically. This works very well for simple models.
2. Numerically evaluate the effective potential as a function of the expectation values, and vary to determine the first and second derivatives; this is the simplest approach for more complicated models.
3. Perform the calculation of tadpoles and self-energies diagramatically and set $p^2 = 0$. This is still much simpler than calculating pole masses, because the loop functions are much simpler for zero external momentum – the two-loop integrals with external momentum do not generically have anlytic expressions and must be found as the solution to sets of differential equations (e.g. in `TSIL`).

The gaugeless limit

Another popular simplification for the two-loop calculation is the "gaugeless limit" where we set the coupling of all broken gauge groups to zero (i.e. $g_Y = g_2 = 0$):

- Neglect corrections proportional to α. In the MSSM these have been estimated/calculated to be typically $\mathcal{O}(100)$ MeV
- These corrections are of the same order as momentum effects.

This simplifies the set of diagrams and (in the MSSM) avoids problems with the Goldstone Boson Catastrophe



Generic calculations

- Some contributions of the effective potential are known for the Standard Model up to three and four loop order ...
- Otherwise it is only known in Landau gauge up to two loops. [S. Martin, 01] gave the expression in dimensional regularisation ($\overline{\text{DR}}$ and $\overline{\text{MS}}$) for generic theories.
- In the gaugeless limit this simplifies to:

$$
\begin{aligned}
V^{(2)}_{SSS} &= \frac{1}{12} (\lambda^{ijk})^2 f_{SSS}(m_i^2, m_j^2, m_k^2) \\
V^{(2)}_{SS} &= \frac{1}{8} \lambda^{iijj} f_{SS}(m_i^2, m_j^2) \\
V^{(2)}_{FFS} &= \frac{1}{2} |y^{IJk}|^2 f_{FFS}(m_I^2, m_J^2, m_k^2) \\
V^{(2)}_{\overline{FF}S} &= \frac{1}{4} y^{IJk} y^{I'J'k} M^*_{II'} M^*_{JJ'} f_{\overline{FF}S}(m_I^2, m_J^2, m_k^2) + \text{c.c.} \\
V^{(2)}_{SSV} &= \frac{g^2}{4} d(i) C(i) f_{SSV}(m_i^2, m_i^2, 0) \\
V^{(2)}_{FFV} + V^{(2)}_{\overline{FF}V} &= \frac{g^2}{2} d(I) C(I) F_{FV}(m_I^2, 0)
\end{aligned}
$$

- [Martin, '03] gave the two-loop scalar self-energies up to $\mathcal{O}(g^2)$ in gauge couplings (don't need g^4 in the gaugeless limit).
- In [MDG, Nickel, Staub 1503.03098] we calculated the tadpoles, and substantial simplifications for massless gauge fields.

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Implementation

We implemented three different calculations of the Higgs mass in SARAH:

1. Calculate the effective potential and numerically take the first and second derivatives.
2. Numerically take the derivatives of the parameters entering the potential, and use analytic expressions for the derivatives of the loop functions themselves, e.g.

$$V^{(2\text{ loop})} \supset \sum_{ijk} (\lambda^{ijk})^2 f_{SSS}(m_i^2, m_j^2, m_k^2)$$

$$\rightarrow \frac{\partial V^{(2\text{ loop})}}{\partial v} \supset \sum_{ijk} 2\lambda^{ijk} \frac{\partial \lambda^{ijk}}{\partial v} f_{SSS}(m_i^2, m_j^2, m_k^2) + 3(\lambda^{ijk})^2 \frac{\partial f_{SSS}(m_i^2, m_j^2, m_k^2)}{\partial m_i^2} \frac{\partial m_i^2}{\partial v}$$

3. A diagrammatic calculation of the Higgs mass simplified to $p^2 = 0$ and the gaugeless limit.

Clearly only the diagrammatic approach will work for pseudoscalars or CPV, because both have fields defined to have zero vev. We can also (in principle) add momentum dependence by just changing the loop functions!
Why three methods? There are so few examples to test against!

SARAH: a tool for BSM model builders

So what is `SARAH` ?

- `Mathematica` package created by F. Staub (with now many contributions from K. Nickel and MDG).
- Takes an input model file for any SUSY or non-SUSY model.
- Specify: gauge groups, matter content, superpotential/couplings in Lagrangian.
- Relevant for this talk: spectrum generation with `SPheno`. Produces fortran code which compiles against the `SPheno` library to generate spectrum and precision observables etc for the model.
- Will calculate two-loop RGEs, one-loop masses for all particles in $\overline{\mathrm{DR}}'$ (SUSY) or $\overline{MS}$ (non-SUSY) models.
- Can specify input parameters at any scale: TeV, ~~SUSY~~ scale, GUT scale ...

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Advantages of the generic approach

Whereas previously computing some partial two-loop corrections was the subject of a paper or even a thesis, these can now be done with the press of a button:

- Any model (e.g. Dirac gauginos) can now be studied to the same precision as the MSSM.
- But in the NMSSM the corrections proportional to Yukawa couplings (i.e. λ^4, α_t^2, ...) had never been computed.
- Even in the MSSM, can have for the first time full family dependence (as opposed to third generation only).
- With CPV, there is an on-shell $\alpha_s \alpha_t$, α_t^2 calculation in the MSSM, [Hollik, Paßehr, 1401.8275,1409.1687]. But no $\overline{\text{DR}}$ computation existed!

For example, let us take the CPV-MSSM with

$$T_u^{3,3} = 0.9 A_0 e^{i\varphi_u}, \quad T_d^{3,3} = 0.064 A_0, \quad T_e^{3,3} = 0.05 A_0,$$

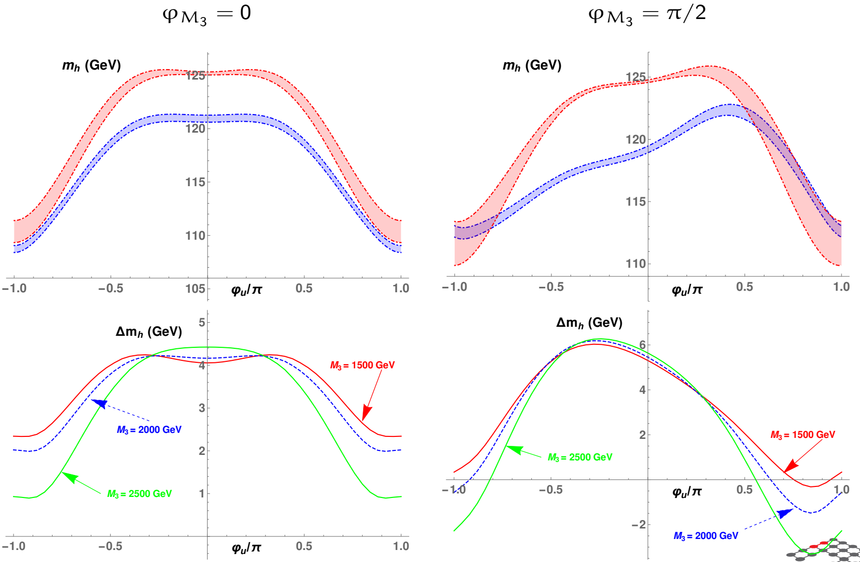
$$A_0 = 3800 \text{ GeV}, \quad \mu = 3800 \text{ GeV}, \quad \tan\beta = 5, \quad \text{Re}(B_\mu) = 10^6 \text{ GeV}^2 \left(\frac{\tan\beta}{1+\tan^2\beta}\right)$$

$$m^2_{\tilde{L},ii} = m^2_{\tilde{e},ii} = m^2_{\tilde{q},ii} = m^2_{\tilde{u},ii} = m^2_{\tilde{d},ii} = (3\times 10^3 \text{ GeV})^2, \quad i = 1,2$$

$$m^2_{\tilde{L},33} = m^2_{\tilde{e},33} = m^2_{\tilde{d},33} = (10^3 \text{ GeV})^2, \quad m^2_{\tilde{q},33} = (1.6\times 10^3 \text{ GeV})^2,$$

$$m^2_{\tilde{u},33} = (1.52\times 10^3 \text{ GeV})^2, \quad M_1 = 200 \text{ GeV}, \quad M_2 = 500 \text{ GeV}, \quad M_3 = M_3^0 e^{i\varphi_{M_3}}.$$

CPV-MSSM: top trilinear phase



The Goldstone Boson Catastrophe

But there is a technical barrier for any theory other than the gaugeless limit of the MSSM: the Goldstone Boson Catastrophe. Note that this includes the Standard Model where it was studied by [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]!

- Consider for simplicity the Abelian Goldstone Model of one complex scalar $\Phi = \frac{1}{\sqrt{2}}(v + h + iG)$ and tree-level potential

$$V = \mu^2|\Phi|^2 + \lambda|\Phi|^4.$$

- At tree level, the tadpole equation gives $\mu^2 + \lambda v^2 = 0$, and the masses are $m_G^2 = \mu^2 + \lambda v^2, M_h^2 = \mu^2 + 3\lambda v^2$.
- But we use $m_G^2 \equiv \mu^2 + \lambda v^2$ to calculate loops, and once we include loop corrections we have

$$0 = \mu^2 + \lambda v^2 + \frac{\partial \Delta V}{\partial v}$$

- ... hence $m_G^2 = \mathcal{O}(1 - \text{loop})$ and is of indefinite sign!
- In fact, at two loops we find (with $A(x) \equiv x(\log x/Q^2 - 1)$)

$$0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\Big[3A(m_h^2) + A(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log\frac{m_G^2}{Q^2}}{(16^2)^2}\Big[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{M_h^2} A(M_h^2)\Big] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}}$$

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Ways out

Up to now, to avoid the problem:

- Calculations in the Standard Model have used Feynman gauge. In general this is much more complicated and does not actually completely solve the problem!
- For the MSSM, using the gaugeless limit sets $\lambda = 0$ in the above, so $\frac{\partial m_G^2}{\partial v} = 0$ and, although m_G^2 can have either sign, there are no $\log m_G^2$ terms in the tadpoles or masses.
- Otherwise it is typical to simply ignore the phases introduced in the potential and/or try to find a renormalisation scale Q where $m_G^2 > 0$. But this is often impractical/impossible!
- In SARAH, we
  - (a) Write tree-level potential in the form
  $$V_0 \equiv \frac{1}{2} m_{ij}^2 S_i S_j + \tilde{V}_0 + \tilde{V}_0^D$$
  - (b) We retain the D-term part of the potential for the tadpole equations at two loops, even though we turn off g_1, g_2 in the couplings:
  $$\Delta m_{0,ii}^2 = -\frac{1}{v_i}\left(\partial_i \tilde{V}_0 + \partial_i \tilde{V}_0^D\right)) \quad \text{but} \quad \mathcal{M}_{0,ij}^2\big|_{\text{gaugeless}} \equiv m_{0,ij}^2 + \partial_i \partial_j \tilde{V}_0.$$
  - (c) This gives a shift in the masses proportional to g_1^2, g_2^2 so is fine given the precision we want. For example, in the NMSSM we find $m_G^2 = -M_Z^2 \cos^2 2\beta$ so we neglect the phases introduced in the potential.

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Resummation of the Goldstone boson contribution

The solution proposed in [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14] for the Standard Model is to resum the Goldstone boson propagators – in the one-loop effective potential we make the substitution

$$V^{(1)}_{\text{eff}} \supset -\frac{i}{2}C\int d^d k \log(-k^2+m_G^2) \to -\frac{i}{2}C\int d^d k \log(-k^2+m_G^2+\Pi_{GG}(k^2))$$
$$\to -\frac{i}{2}C\int d^d k \log(-k^2+m_G^2+\Pi_{GG}(0)) + \ldots \equiv \frac{1}{16\pi^2} f(m_G^2+\Pi_{GG}(0)) + \ldots$$

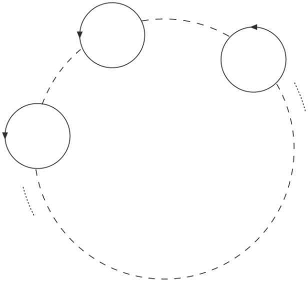
Since $f'(0)=0$ the tadpoles are then free of divergences; we then should use instead the resummed potential

$$\hat{V}_{\text{eff}} \equiv V_{\text{eff}} + \frac{1}{16\pi^2}\left[f(m_G^2+\Delta) - \sum_{n=0}^{l-1} \frac{\Delta^n}{n!}\left(\frac{\partial}{\partial m_G^2}\right)^n f(m_G^2)\right].$$

The two potentials only differ by terms of order $l+1$. The two papers then differ in how to proceed:

- [Elias-Miro, Espinosa, Konstandin, '14] proposed to use $\Delta_1 \equiv \Pi_g(0)$, defined in terms of the self energy excluding "soft" Goldstones.
- [Martin, '14] proposed to expand the potential at two loops as a series in m_G^2, and use this to define Δ_1:

$$V^{(2)} \equiv V^{(2)}|_{m_G^2=0} + \frac{1}{2}\Delta_1 A(m_G^2) + \frac{1}{2}\Omega m_G^2 + \mathcal{O}(m_G^4).$$



So why does this work?

- The daisy diagram on the right clearly contributes the most singular term at any fixed loop order
- $f(x) \equiv \frac{1}{4}x^2(\log x - \frac{3}{2})$ so $f(0) = 0$, whereas $f(m_G^2)$ contains a logarithmic term.
- The resummed potential becomes, at two-loop order:

$$\hat{V}_{\text{eff}} = V^{(2)}|_{m_G^2=0} + \frac{1}{2}\Omega m_G^2 + \frac{1}{16\pi^2} f(m_G^2 + \Delta)$$

- ... so the first derivative is also free of divergences:

$$\frac{\partial \hat{V}_{\text{eff}}}{\partial v} = \frac{\partial V^{(2)}|_{m_G^2=0}}{\partial v} + \frac{1}{2}\Omega \frac{\partial m_G^2}{\partial v} + \mathcal{O}(3\text{-loop})$$

On-shell scheme

In [Braathen, MDG, '16] we showed how to extend this to general models at two loops (and not just the gaugeless limit):

- In general the Goldstone bosons can mix with the other fields, or there can be several Goldstone bosons that mix $\rightarrow$ this needs special treatment but we can use the transformations of the broken symmetries to completely determine the Goldstone boson directions.
- It turns out that it is actually <u>much</u> simpler to use an <u>on-shell scheme</u> for the Goldstone boson; we make the formal substitution:

$$(m_G^2)^{\text{run.}} \equiv (m_G^2)^{\text{OS}} - \Pi_{GG}((m_G^2)^{\text{OS}})$$

  and then expand retaining only terms up to two loop order. This is equivalent to starting with the on-shell mass and changing the one-loop counterterms.
- We can do this <u>directly</u> in the tadpole equations – and also the self-energies! In the other approaches, it is necessary to take derivatives of mixing matrices etc ...
- For example, applying the above shift to the one loop tadpole gives a two-loop correction:

$$\frac{\partial V}{\partial v} \supset \frac{\lambda v}{16\pi^2} A(m_G^2) = \frac{\lambda v}{16\pi^2}\Big[\underbrace{A((m_G^2)^{\text{OS}}}_{\rightarrow 0} - \Pi_{GG}((m_G^2)^{\text{OS}}) \log \frac{(m_G^2)^{\text{OS}}}{Q^2} + \underbrace{\ldots}_{3-\text{loop}}\Big]$$

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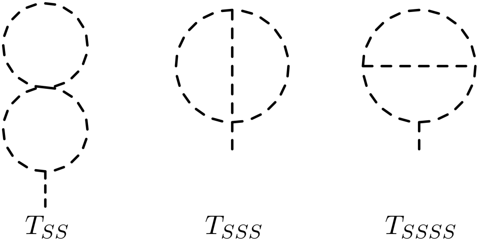
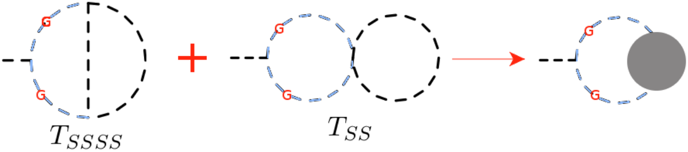
Illustration

To see why this works, let us look at the scalar-only case. There are three classes of tadpole diagrams:

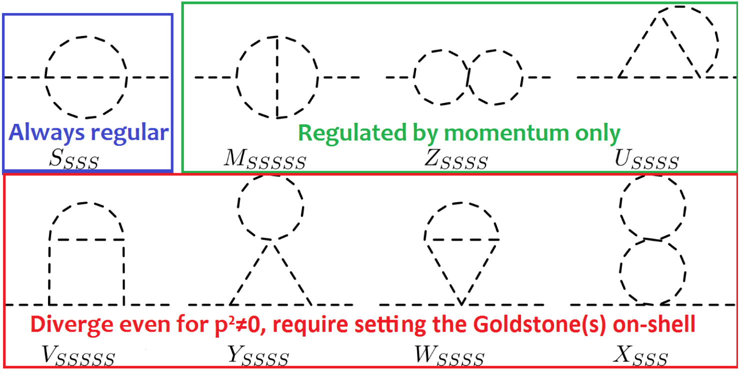


We find that the divergences only come from the T_{SS} and T_{SSSS} topologies, and they correspond to a Goldstone self-energy as a subdiagram and exactly cancel out against the on-shell shift:



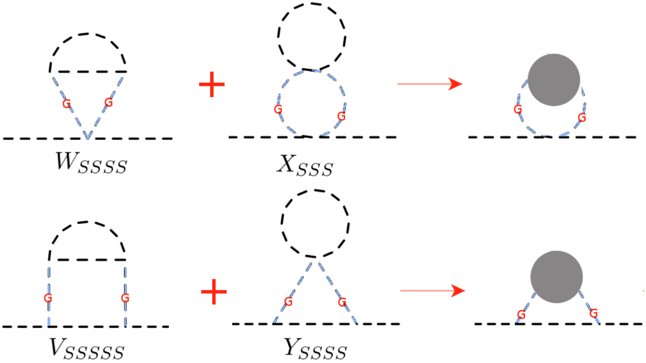
Mass diagrams

We also find that we can apply our on-shell scheme to the cancellation of divergences in self-energies! This seemed hopeless in the former approaches ... We can divide the topologies into three categories:



Mass diagram divergences

Again we find that the divergences in m_G^2 arise from Goldstone boson propagator subdiagrams:



... and once more the one-loop shifts from our on-shell scheme exactly cancel the divergences.

Generalised effective potential limit

Finally, since we see that there are classes of diagrams that are divergent when the $p^2 \equiv s \neq 0$ and the Goldstone bosons are on-shell, the obvious response is that we cannot avoid using momentum dependence – but this is computationally expensive. Instead, we can expand the self-energies as:

$$\Pi^{(2)}_{ij}(s) = \frac{\overline{\log}(-s)}{s}\Pi^{(2)}_{-1\,l,ij} + \frac{1}{s}\Pi^{(2)}_{-1,ij} + \Pi^{(2)}_{l^2,ij}\overline{\log}^2(-s) + \Pi^{(2)}_{l,ij}\overline{\log}(-s) + \Pi^{(2)}_{0,ij} + \sum_{k=1}^{\infty}\Pi^{(2)}_{k,ij}\frac{s^k}{k!}$$

If we discard all terms $\mathcal{O}(s)$ and higher, we have a generalised effective potential approximation! We can find closed forms for the singular terms, e.g.

$$U(0,0,0,u) = (\overline{\log}\,u - 1)\,\overline{\log}(-s) - \frac{\pi^2}{6} + \frac{5}{2} - 2\,\overline{\log}\,u - \frac{1}{2}\overline{\log}^2 u + \mathcal{O}(s).$$

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Conclusions

- The computation implemented in `SARAH` gives results for general models that are comparable to any other calculation in the literature.
- The Goldstone Boson Catastrophe represents an obstacle; in practice it results in spikes of the Higgs mass in parameter space scans.
- We now have a general solution at two loops, for both tadpoles and masses.
- In particular, since non-SUSY models don't have D-term potentials, the two-loop routines as they are in `SARAH` always suffer from this problem: but the new results will open up these models e.g. two-Higgs-doublets etc.
- Future directions will include the full electroweak corrections ...