Hollowness in pp scattering

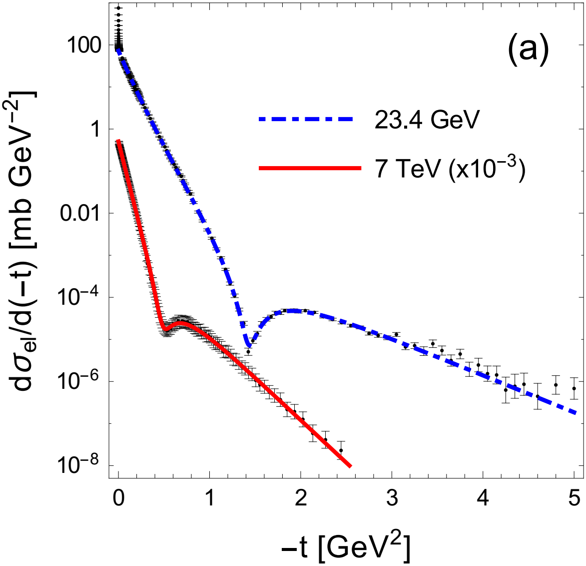
Wojciech Broniowski

Institute of Nuclear Physics PAN
Jan Kochanowski U., Kielce

XXIII Cracow Epiphany Conference
9-12 January 2017

Research with **Enrique Ruiz Arriola** [arXiv:1609.05597]

From ISR to TOTEM



Parametrization of the elastic pp scattering amplitude

Parametrization by [Fagundes 2013] based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

$$\frac{f(s,t)}{p} = \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} = \frac{i\sqrt{A}e^{\frac{Bt}{2}}}{\left(1-\frac{t}{t_0}\right)^4} + i\sqrt{C}e^{\frac{Dt}{2}+i\phi}$$

s-dependent (real) parameters are fitted separately to all known differential pp cross sections for $\sqrt{s} = 23.4$, 30.5, 44.6, 52.8, 62.0, and $7000\ \mathrm{GeV}$ with $\chi^2/\mathrm{d.o.f} \sim 1.2 - 1.7$

$$\frac{d\sigma_{\mathrm{el}}}{dt} = \frac{\pi}{p^2}|f(s,t)|^2, \quad \sigma_T = \frac{4\pi}{p}\mathrm{Im} f(s,0)$$

Eikonal approximation

$$f(s,t) = \sum_{l=0}^{\infty} (2l+1) f_l(p) P_l(\cos\theta)$$
$$= \frac{p^2}{\pi} \int d^2b \, h(\vec{b}, s) \, e^{i\vec{q}\cdot\vec{b}} = 2p^2 \int_0^{\infty} bdbJ_0(bq)h(b,s)$$

$t = -\vec{q}^2$, $q = 2p\sin(\theta/2)$, $bp = l + 1/2 + \mathcal{O}(s^{-1})$, $P_l(\cos\theta) \to J_0(qb)$

(would need 40000 partial waves at the LHC!)

In the impact-parameter representation

$$h(b,s) = \frac{i}{2p}\left[1 - e^{i\chi(b)}\right] = f_l(p) + \mathcal{O}(s^{-1})$$

The eikonal approximation works well for $b < 2$ fm and $\sqrt{s} > 20$ GeV

Procedure: $f(s,t) \to h(b,s) \to \chi(b)$...

Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

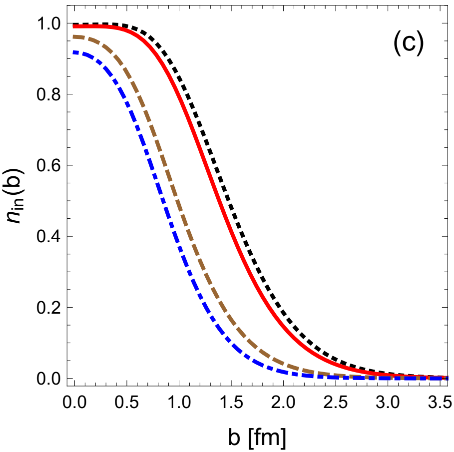
$$\begin{aligned}
\sigma_T &= \frac{4\pi}{p}\mathrm{Im} f(s,0) = 4p\int d^2b \mathrm{Im} h(\vec{b},s) = 2\int d^2b\left[1-\mathrm{Re}\, e^{i\chi(b)}\right] \\
\sigma_{\mathrm{el}} &= \int d\Omega |f(s,t)|^2 = 4p^2\int d^2b |h(\vec{b},s)|^2 = \int d^2b |1-e^{i\chi(b)}|^2 \\
\sigma_{\mathrm{in}} &\equiv \sigma_T - \sigma_{\mathrm{el}} = \int d^2b n_{\mathrm{in}}(b) = \int d^2b\left[1-e^{-2\mathrm{Im}\chi(b)}\right]
\end{aligned}$$

The inelasticity profile

$$n_{\mathrm{in}}(b) = 4p\mathrm{Im} h(b,s) - 4p^2|h(b,s)|^2$$

satisfies $0 \geq n_{\mathrm{in}}(b) \leq 1$ (unitarity)

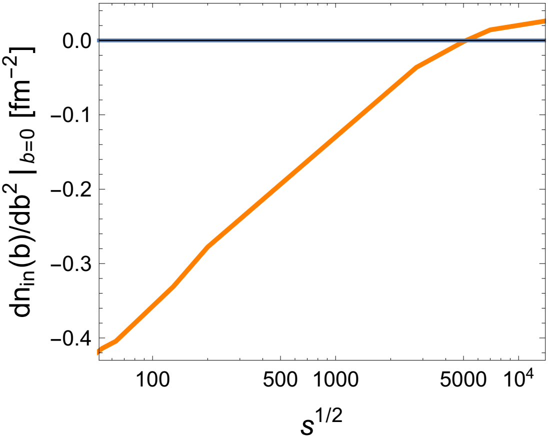
Dip (or flattening) in the inelasticity profile at $b = 0$



From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

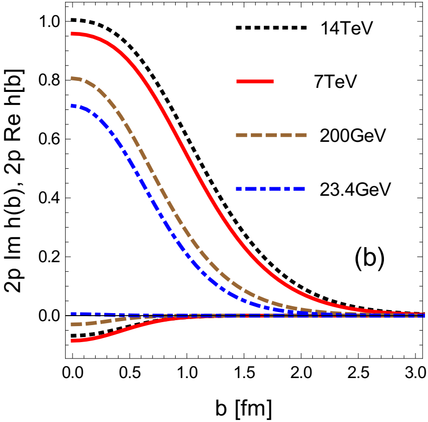
Dip: collisions more distractive at $b > 0$ than head-on!

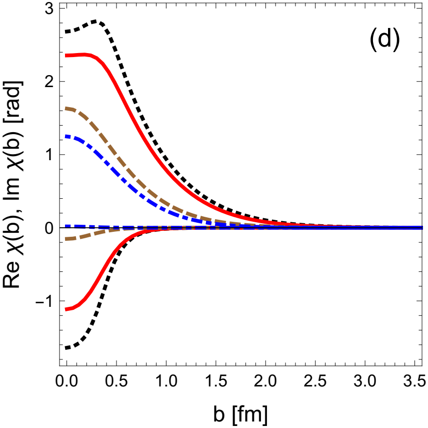
Slope of the inelasticity profile



Transition around $\sqrt{s} = 5$ TeV

Amplitude and eikonal phase





$$2p\,h(b) = i\left[1 - e^{i\chi(b)}\right]$$

(top curves - Im, bottom - Re)

The dip clearly visible in $\mathrm{Im}\chi(b)$ for the LHC

Importance of the real part of the eikonal phase

$$k(b) \equiv \mathrm{Im}[2p\,h(b)] = 1 - \cos\left(\mathrm{Re}[\chi(b)]\right) e^{-\mathrm{Im}[\chi(b)]}$$
$$\mathrm{Re}[2p\,h(b)] = \sin\left(\mathrm{Re}[\chi(b)]\right) e^{-\mathrm{Im}[\chi(b)]}$$

At the LHC $\mathrm{Re}[\chi(b)] < -\pi/2 \rightarrow \cos\left(\mathrm{Re}[\chi(b)]\right) < 0 \rightarrow k(b) > 1$

With the neglect of the small $\mathrm{Re}[2p\,h(b)])^2$ we have then from $n_{in}(b) = 2k(b) - k(b)^2$

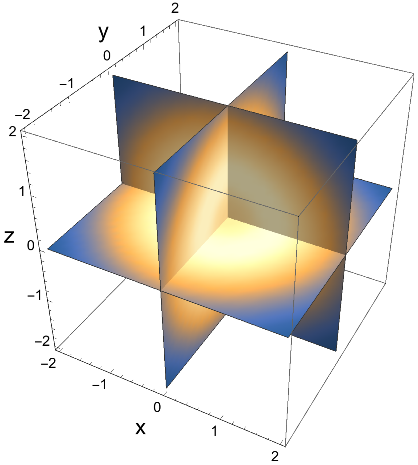
$$\frac{dn_{in}(b)}{db^2} = 2\frac{dk(b)}{db^2}[1 - k(b)] < 0$$

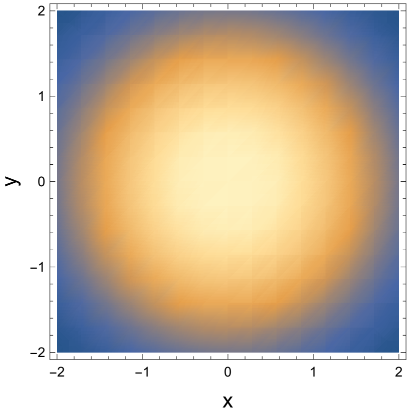
$\rightarrow$ minimum develops at $b = 0$

Glauber (1959): *The eikonal phase is additive in scattering of composite objects. The (potentially small) eikonal phases of the constituents may add up to a large eikonal phase of the whole proton.* Quantum interference is essential

2D vs 3D opacity – geometric idea

Projection of 3D on 2D covers up the hollow: $f(x, y, z)$ vs $\int_{-\infty}^{\infty} dz f(x, y, z)$





The hollow is covered up

Equivalent optical potential – invariant mass method

Phenomenological method [Allen, Payne, Polyzou 2000] introduces the total squared mass operator for the pp system:

$$\mathcal{M}^2 = P^\mu P_\mu \stackrel{CM}{=} 4(p^2 + M_N^2) + \mathcal{V}$$

P^μ – total four-momentum, p – CM three-momentum of each nucleon, M_N – nucleon mass, $\mathcal{V}$ – invariant interaction, determined in the CM frame by matching in the non-relativistic limit to a non-relativistic potential, i.e., $\mathcal{V} = 4U = 4M_N V$. Relativistic Schrödinger equation $\hat{\mathcal{M}}^2\Psi = s\Psi$ transforms into an equivalent non-relativistic Schrödinger equation

$$(-\nabla^2 + U)\Psi = (s/4 - M_N^2)\Psi$$

with the reduced potential $U = M_N V = \mathrm{Re}U + i\mathrm{Im}U$ (to be determined by inverse scattering)

No complication of spin/noncentrality (5 complex Wolfenstein amplitudes)

Eikonal limit and optical potential

As in WKB, we solve $-\hbar^2\nabla^2\Psi = 2m(E-V)\Psi$ with $\Psi = Ae^{iS/\hbar}$

$$(\nabla S)^2 - \cancel{i\hbar\nabla^2 S} = 2m(E-V)$$

$$\nabla S/\hbar = \sqrt{p^2 - 2mV/\hbar^2}$$

For $p \gg$ other scales

$$S/\hbar = pz - \frac{m}{\hbar^2 p}\int_{-\infty}^{z} dz' V(z')$$

Inverse scattering and optical potential

Hence in the eikonal approximation one has

$$\Psi(\vec{b}, z) = \exp\left[ipz - \frac{i}{2p}\int_{-\infty}^{z} U(\vec{b}, z')dz'\right]$$

$$\chi(b) = -\frac{1}{2p}\int_{-\infty}^{\infty} U(\sqrt{b^2+z^2})dz = -\frac{1}{p}\int_{b}^{\infty} \frac{rU(r)\,dr}{\sqrt{r^2-b^2}}$$

is the (complex) eikonal phase [Glauber 1959]. This Abel-type equation can be inverted:

$$U(r) = M_N V(r) = \frac{2p}{\pi}\int_{r}^{\infty} db \frac{\chi'(b)}{\sqrt{b^2-r^2}}$$

On-shell optical potential

From the definition of the inelastic cross section

$$\sigma_{\rm in} = -\frac{1}{p} \int d^3x \, {\rm Im}\, U(\vec{x}) |\Psi(\vec{x})|^2$$

$\to$ density of inelasticity is proportional to the absorptive part of the optical potential times the square of the modulus of the wave function. One can identify the *on-shell optical potential* (related to Bethe-Salpeter methods)

$${\rm Im}\, W(\vec{x}) = {\rm Im}\, U(\vec{x}) |\Psi(\vec{x})|^2$$

Upon z integration,

$$-\frac{1}{p} \int dz {\rm Im}\, W(\vec{b}, z) = n_{in}(b)$$

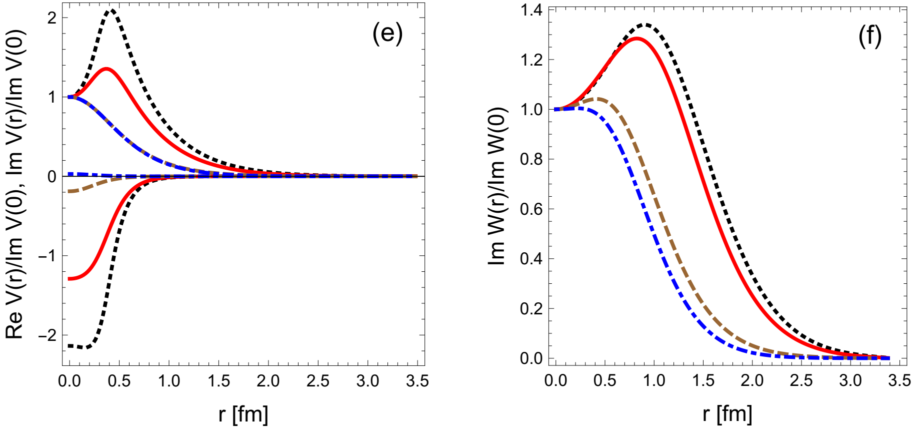
Inversion yields

$${\rm Im} W(r) = \frac{2p}{\pi} \int_r^\infty db \frac{n'_{in}(b)}{\sqrt{b^2 - r^2}}$$

Results of inverse scattering

exp. amplitude → eikonal phase → $U(r) = M_N V(r)$

exp. amplitude → inelasticity profile → $\mathrm{Im} W(r)$



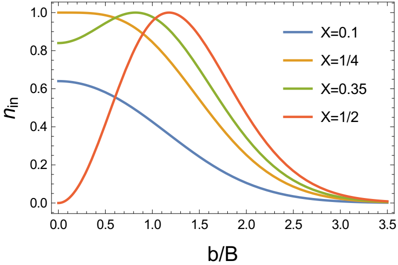
From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

Large dip in the absorptive parts, in $W(r)$ starts already at RHIC!

Gaussian model of Dremin (2014)

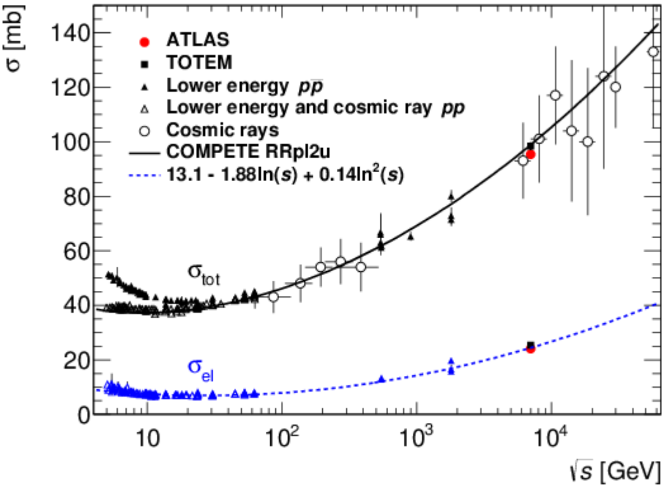
$$2p\mathrm{Im}h(b) \equiv k(b) = 4Xe^{-b^2/(2B^2)}, \quad \mathrm{Re}h(b) = 0, \quad X = \sigma_{el}/\sigma_T$$

$$n_{in}(b) = 2k(b) - k(b)^2 = 8Xe^{-b^2/(2B^2)} - 16X^2e^{-b^2/B^2}$$



- $X > 1/4$: $n_{in}(b)$ has a maximum at $b_0 = \sqrt{2}B\log(4X) > 0$, with $k(b_0) = 1$
- $X = 1/2$: black disk limit
- $W(r)$ develops a dip when $X > \sqrt{2}/8 = 0.177$

Cross sections



$\sigma_{\rm el}$ grows relatively faster than $\sigma_{\rm tot}$

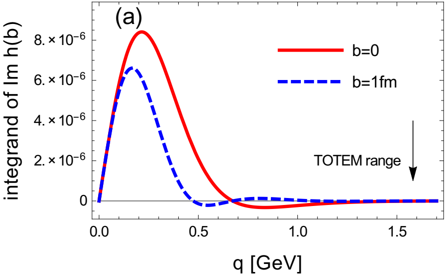
$\to$ ratio X goes above 1/4 as s increases!

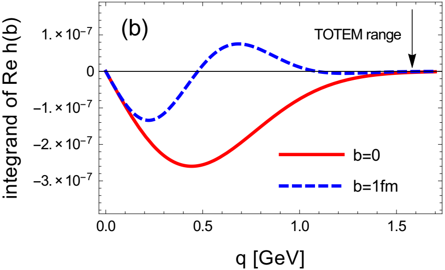
Conclusions

- Hollowness transition inferred from the parametrization of the data, seen in $n_{in}(b)$ for $s > 5$ TeV
- Quantum effect related to compositeness of the proton, rise of $2p\mathrm{Im}h(b=0)$ above 1
- 2D $\to$ 3D magnifies the effect (flat in 2D $\to$ hollow in 3D), interpretation via optical potential in a relativized problem
- Effect impossible to obtain classically by folding the absorptive parts from uncorrelated constituents
- Hot-spot model [Alba Soto+Albacete 2016] – a dynamic realization

- Qualitatively similar hollowness effect appears in low-energy ($\sim$500 keV) n-A scattering – less absorption for head-on collisions than for peripheral!

Fourier-Bessel transform





(TOTEM extends far enough)