Jet Mass Dependent Fragmentation

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Motivation

- **Goal**

  Hadronisation inside fat jets

- **Proposed model**

  Statistical Model

- **Suggestion**

  Parametrise fragmentation functions as

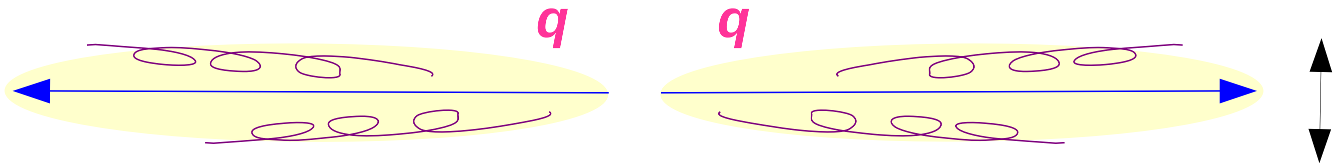
$$D\left[\, x = \frac{2\, P^{jet}_{\mu}\, p^{\mu}_{h}}{M^2_{jet}} \;,\; Q^2 = M^2_{jet} \,\right]$$

Energy fraction the hadron takes away in the frame co-moving with the jet

Fragmentation scale: jet mass

Ideal world: e^+e^- annihilations in the factorized picture

2 identical jets:



width:

$$\sim \sqrt{p_q^2} \approx 0 \ll M_{jet}$$

$$p_\mu^{q,\bar{q}} = (\sqrt{s}/2, 0,0, \pm\sqrt{s}/2)$$

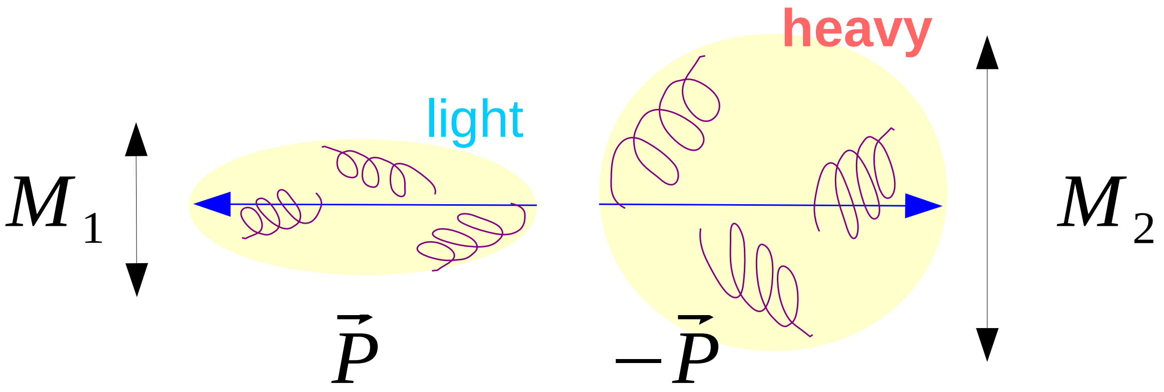
Problem: *P^2 ~ 0 quark* **produces a** ***heavy jet*** **of mass** $M \sim [0.1 - 0.5]\sqrt{s}$

- ***energy fraction*** **of the hadron takes away from the energy of the jet:**
- ***fragmentation scale:***

$$x = \frac{p_h^0}{\sqrt{s}/2}$$

$$Q \sim \sqrt{s}$$

Real world: *the 2 jets are not identical*



Energy-momentum **conservation:**

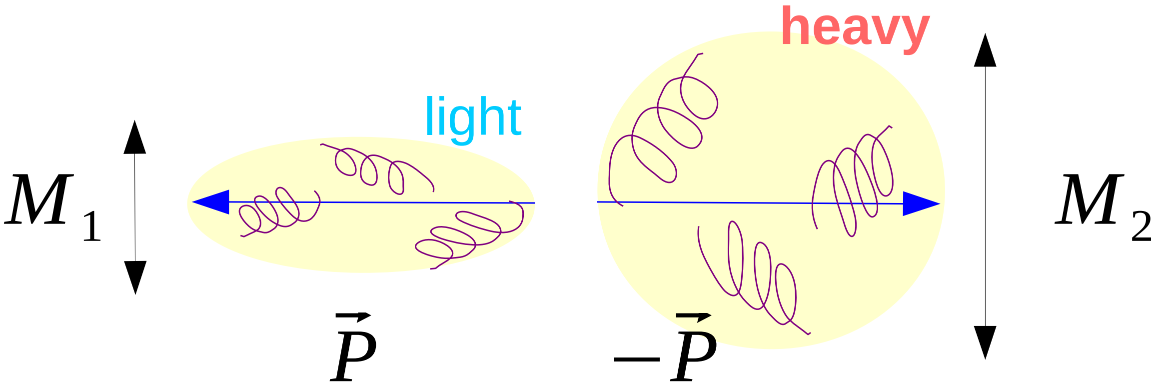
$$P_1^{\mu}=\left(P^0,0,0,|\mathbf{P}|\right)$$

$$P_2^{\mu}=\left(\sqrt{s}-P^0,0,0,-|\mathbf{P}|\right)$$

Problems:

- ***the energy*** **of a jet** $P^0 \neq \left(\sqrt{s}/2\right)$**, so** $x=\dfrac{p_h^0}{\sqrt{s}/2}$ **is** ***no longer the energy fraction*****, the hadron takes away from the energy of the jet.**
- ***fragmentation scale*** **is** ***no longer*** $\sqrt{s}/2$

Real world: the 2 jets are not identical



Energy-momentum conservation:

$$P_1^\mu = \left(P^0, 0, 0, |\mathbf{P}|\right)$$

$$P_2^\mu = \left(\sqrt{s} - P^0, 0, 0, -|\mathbf{P}|\right)$$

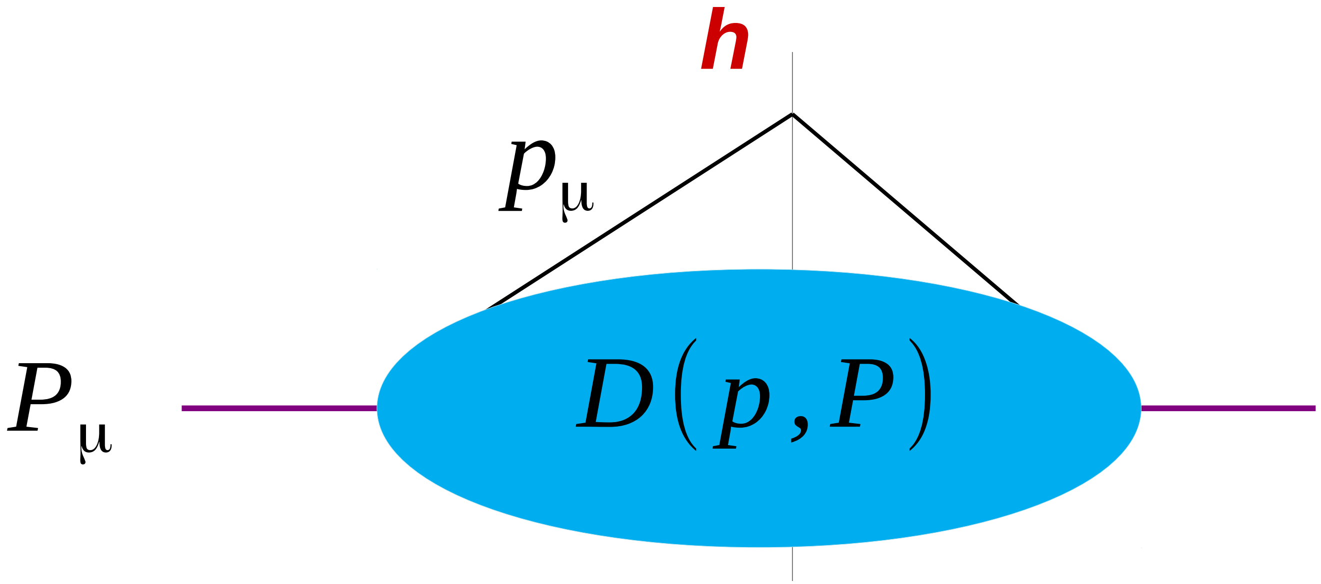
We propose to use:

- the *real energy fraction* the hadron takes away from the energy of the jet in the *frame co-moving* with jet:

- the *jet mass* as *fragmentation scale*:

$$\boxed{x = \frac{2\, p_h^\mu P_\mu^{jet}}{M_{jet}^2} \qquad Q \sim M_{jet}}$$

Natural variables?



What invariants can we make out from P_μ and p_μ ?

- $p^2 \approx 0$
- $P^2 = \boxed{M_{jet}^2}$

- $$(P-p)^2 = M_{jet}^2 - 2P_\mu p^\mu = M_{jet}^2\left(1 - \boxed{\frac{2P_\mu p^\mu}{M_{jet}^2}}\right) = M_{jet}^2\left(1 - \boxed{x}\right)$$

- **Suggestion**

Parametrise fragmentation functions as

$$D\left[\; x=\frac{2\,P_{\mu}^{jet}\;p_{h}^{\mu}}{M_{jet}^{2}}\;,\;\; Q^{2}=M_{jet}^{2}\;\right]$$

Energy fraction the hadron takes away in the frame co-moving with the jet

Fragmentation scale: jet mass

These new variables, x and M_{jet} emerge naturally in a

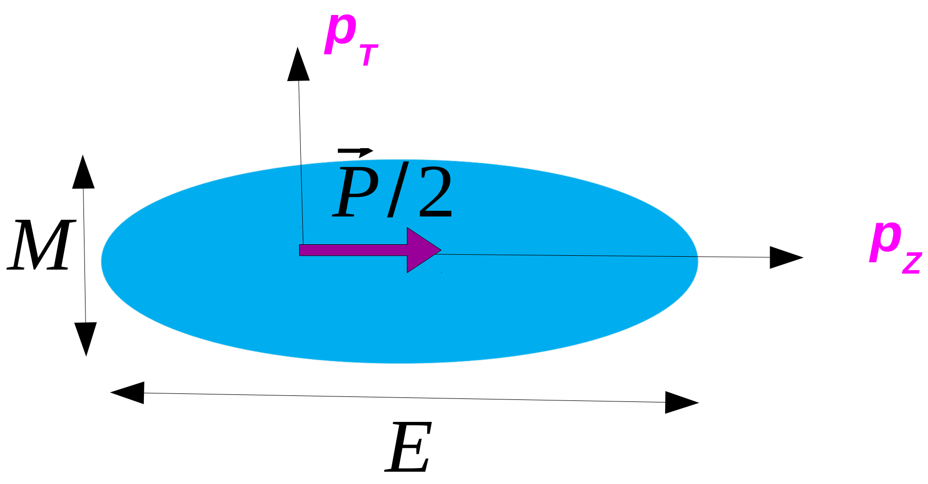
Statistical

Fragmentation

Model

Statistical ~ we *only* focus on the *phasespace*

The haron distribution in a jet of *n* hadron with total momentum *P*



$$p^0 \frac{d\sigma}{d^3p}^{n=fix} \propto (1-x)^{\boxed{n-3}}, \qquad x = \frac{P_\mu p^\mu}{M^2/2}$$

$$\boxed{P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r}$$

Problems

- ***Averaging over multiplicity fluctuations***

$$\boxed{p^0 \frac{d\sigma}{d^3p} = A\left[1+\frac{q-1}{\tau}x\right]^{-1/(q-1)}}$$

Refs.:

Urmossy et.al.,*PLB*, **701**: 111-116 (2011)

Urmossy et. al., *PLB*, **718**, 125-129, (2012)

Scale evolution

Approximations

Let the FF preserve its form:

$$D_{apx}(x,t) \;=\; A(t)\left(1+\frac{q(t)-1}{\tau(t)}\,x\right)^{-1/(q(t)-1)} \quad \textbf{with} \quad D(x,0) \;=\; A_0\left(1+\frac{q_0-1}{\tau_0}\,x\right)^{-1/(q_0-1)}$$

First step: ***Φ^3 theory***

Let us prescribe the approximations:

$$\int D_{apx}(x,t) \;=\; \int D(x,t)$$

$$\int x\,D_{apx}(x,t) \;=\; \int x\,D(x,t) \;=\; 1$$

(by definition)

$$\int x^2 D_{apx}(x,t) \;=\; \int x^2 D(x,t)$$

⇒

$$q(t)=\frac{\alpha_1(t/t_0)^{a1}-\alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1}-\alpha_4(t/t_0)^{-a2}}$$

$$\tau(t)=\frac{\tau_0}{\alpha_4(t/t_0)^{-a2}-\alpha_3(t/t_0)^{a1}}$$

$$a_1=\tilde{P}(1)/\beta_0, \quad a_2=\tilde{P}(3)/\beta_0$$

Urmossy, Z. Xu, arXiv:1606.03208

Fits

What dataset to analyse?

We have a haron distribution, which depends on $x = \frac{\boxed{P_\mu} p^\mu}{\boxed{M^2} /2}$

but, in case of available data, the *jet E* or *P fluctuate:*

- pp collisions: $\vec{P}$ is measured, $\boxed{E}$ fluctuates

- *$e^+e^- \rightarrow$ 2 jet:* both $\boxed{E \text{ and } \vec{P}}$ of the jets fluctuate

- *$e^+p \rightarrow$ 2 jet:* $\boxed{\vec{P}}$ of the jets fluctuate

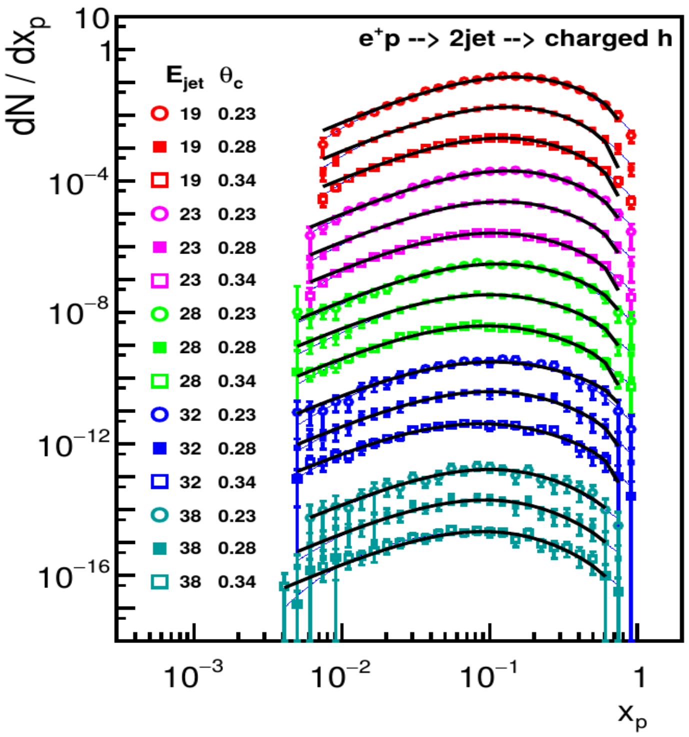
So, we *fit* a *characteristic/average jet mass* and extract the scale dependence of the parameters of the model

- *pp* **collisions**: $\vec{P}$ **is measured, $\boxed{E}$ fluctuates**

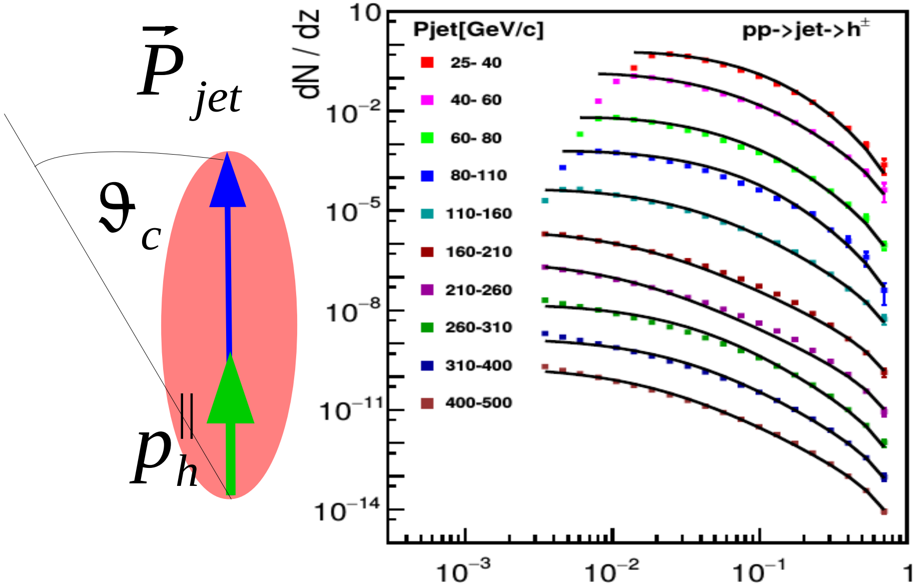
- *$e^{\pm}p \rightarrow 2$ jet:* $\boxed{\vec{P}}$ **of the jets fluctuate**

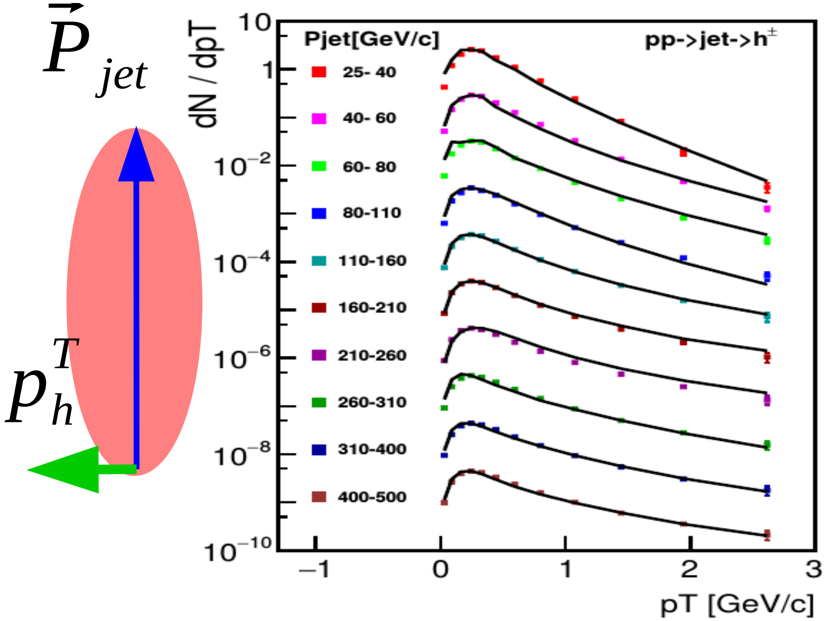
So, **we *fit* a *characteristic/average jet mass* and extract the scale dependence of the parameters of the model**

e^+P → 2 jets



PP → jets

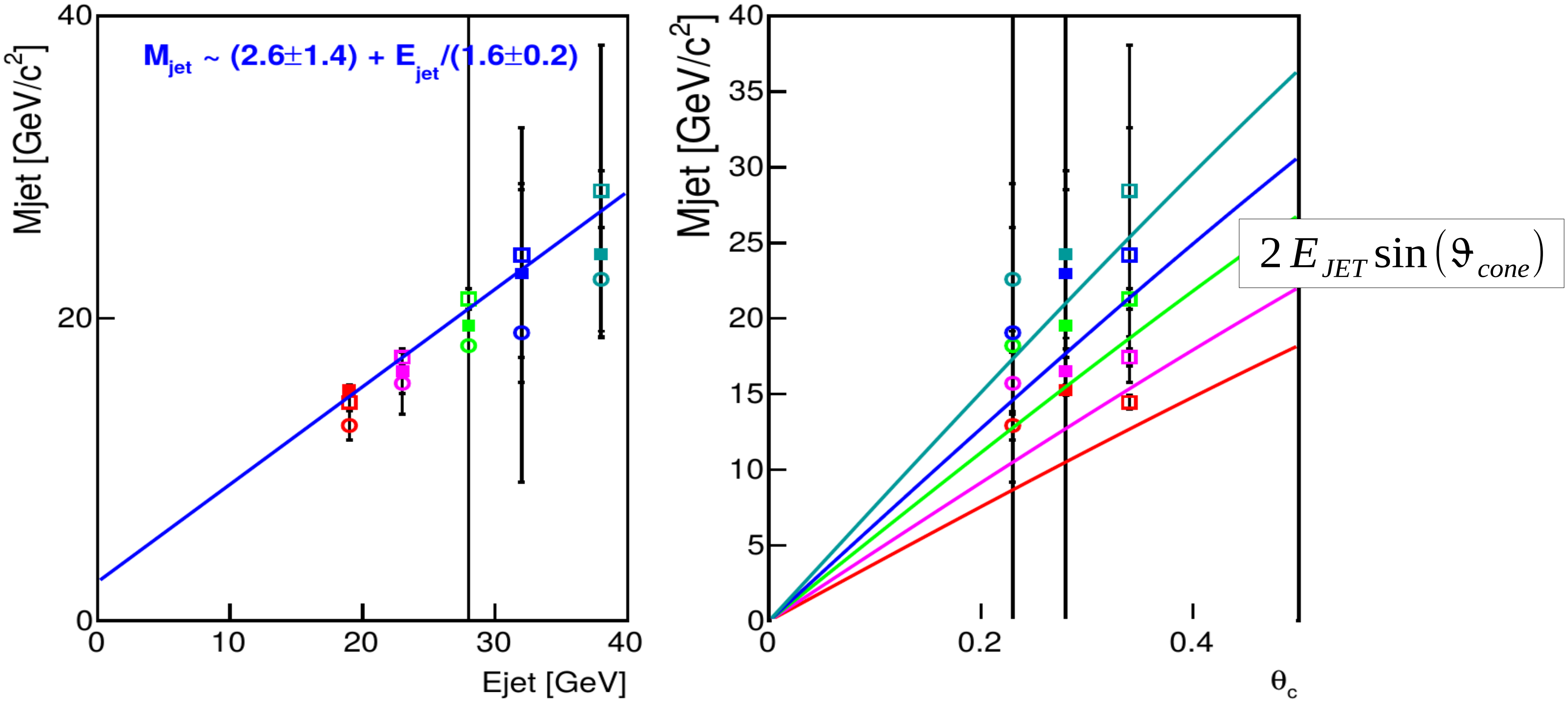




Urmossy, Z. Xu, arXiv:1606.03208

proc. of conf.: ***DIS2016***, arXiv:1605.06876

Fitted average characteristic jet mass



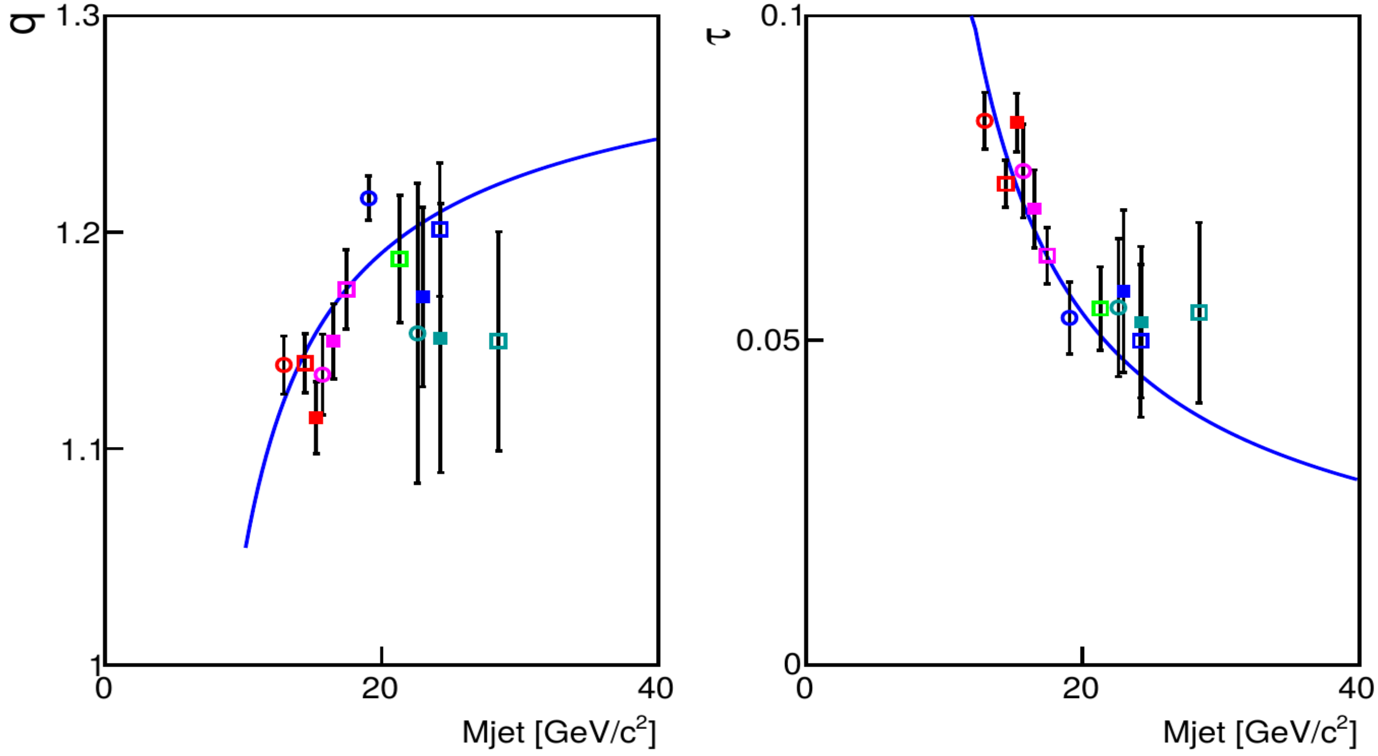
fitted $\langle M_{JET}\rangle = M_0 + E_{JET}/E_0$

Fitted average jet mass is of the order of that used in DGLAP calcs.

$$\langle M_{JET}\rangle \sim 2\,E_{JET}\sin(\vartheta_{cone})$$

Urmossy, Z. Xu, arXiv:1606.03208

Scale evolution of the fit parameters

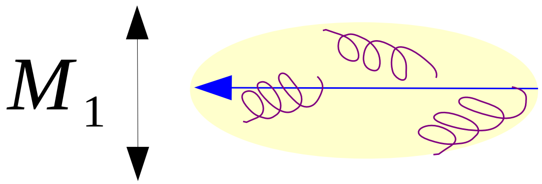
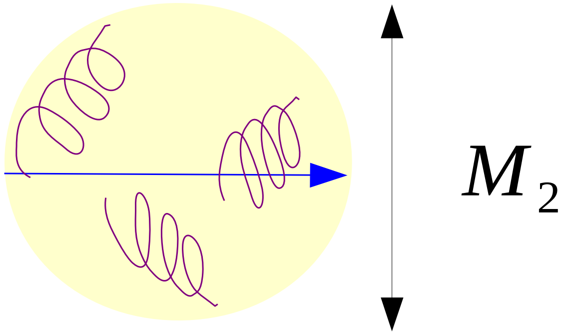


$$q(t)=\frac{\alpha_1(t/t_0)^{a1}-\alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1}-\alpha_4(t/t_0)^{-a2}} \qquad \tau(t)=\frac{\tau_0}{\alpha_4(t/t_0)^{-a2}-\alpha_3(t/t_0)^{a1}}$$

$$t=\ln\left(\boxed{M_{jet}^2}/\ \Lambda^2\right)$$

Urmossy, Z. Xu, arXiv:1606.03208

Interpretation of the results

	Inside *light jets*	**Inside *heavy jets***
		
The *fragmentation function:*	$D(x) \approx \exp\{-x/\tau\}$	$D(x) \approx \left(1+\frac{q-1}{\tau}x\right)^{-1/(q-1)}$
The *multiplicity distribution:*	$P(n) \approx \frac{(1/\tau)^n}{n!}e^{-1/\tau}$	$P(n) \approx \binom{n+r-1}{r-1}\tilde{p}^n(1-\tilde{p})^r$

$$\tilde{p} = (q-1)/(\tau+q-1) \qquad r = 1/(q-1)-3$$

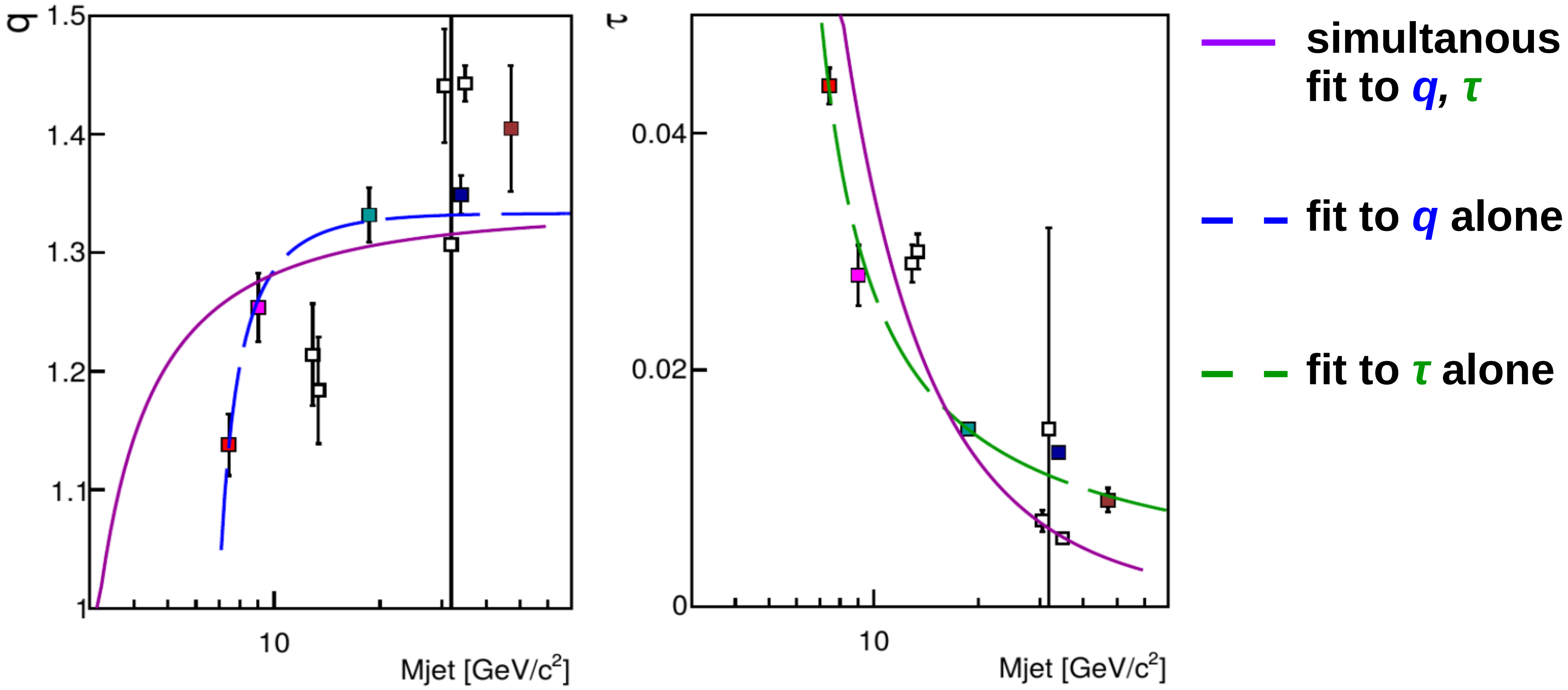
Interpretation of the results

Evolution of the mean *multiplicity* and its *dispersion:*

$$\langle n \rangle = \frac{4-3q_0}{\tau_0}\left(t/t_0\right)^{-a2} \sim \ln^a\left(M_{jet}\right)$$

$$\langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle \left[\frac{3-2q_0}{\tau_0}\left(t/t_0\right)^{a1} + 1 - \langle n \rangle \right]$$

Scale evolution of the fit parameters



Why does it look so messy? *Jet mass fluctuations spoil things?*

Urmossy, Z. Xu, proc. of conf.: ***DIS2016***, arXiv:1605.06876

Mass-averaged fits are *better*

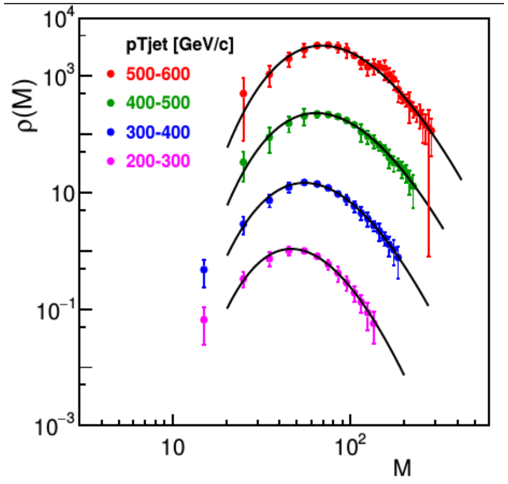
The *fragmentation function* is *jet mass dependent*

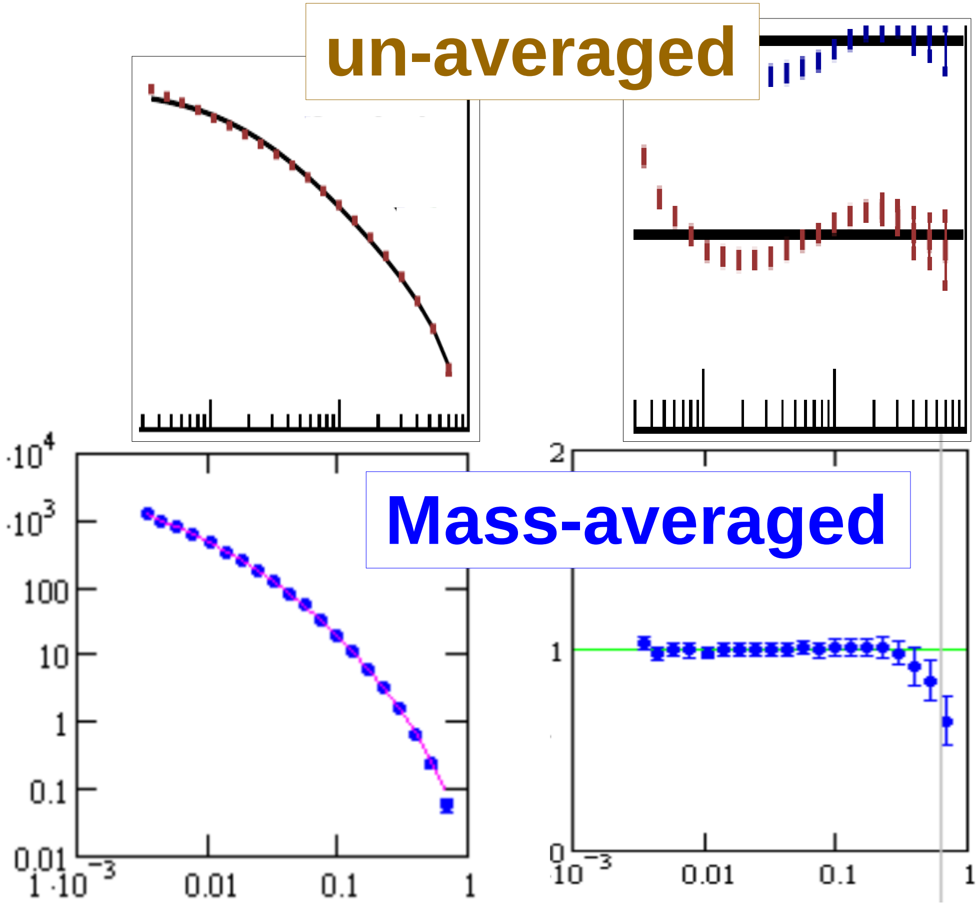
$$D(x,t) = \int_x^1 \frac{dz}{z} f(z,t) D_0(x/z)$$

$$t = \ln\left(M_{jet}^2 / \Lambda^2 \right)$$

The *jet mass* fluctuates as

$$\rho(M_{jet}) \sim \ln^b(M_{jet}/M_0)/M_{jet}^c$$




un-averaged

Mass-averaged

Does anybody know how to handle

Off-shell

Scale Evolution?

*with **DGLAP?***

*What are the **splitting functions?***

Conclusion

- ***It might be worthy not to neglect parton virtualities?***
- ***Suggestion***

It might be more suitable to

characterise JETs with their MASS

instead of thier P or E

Conclusion

- **Suggestion**

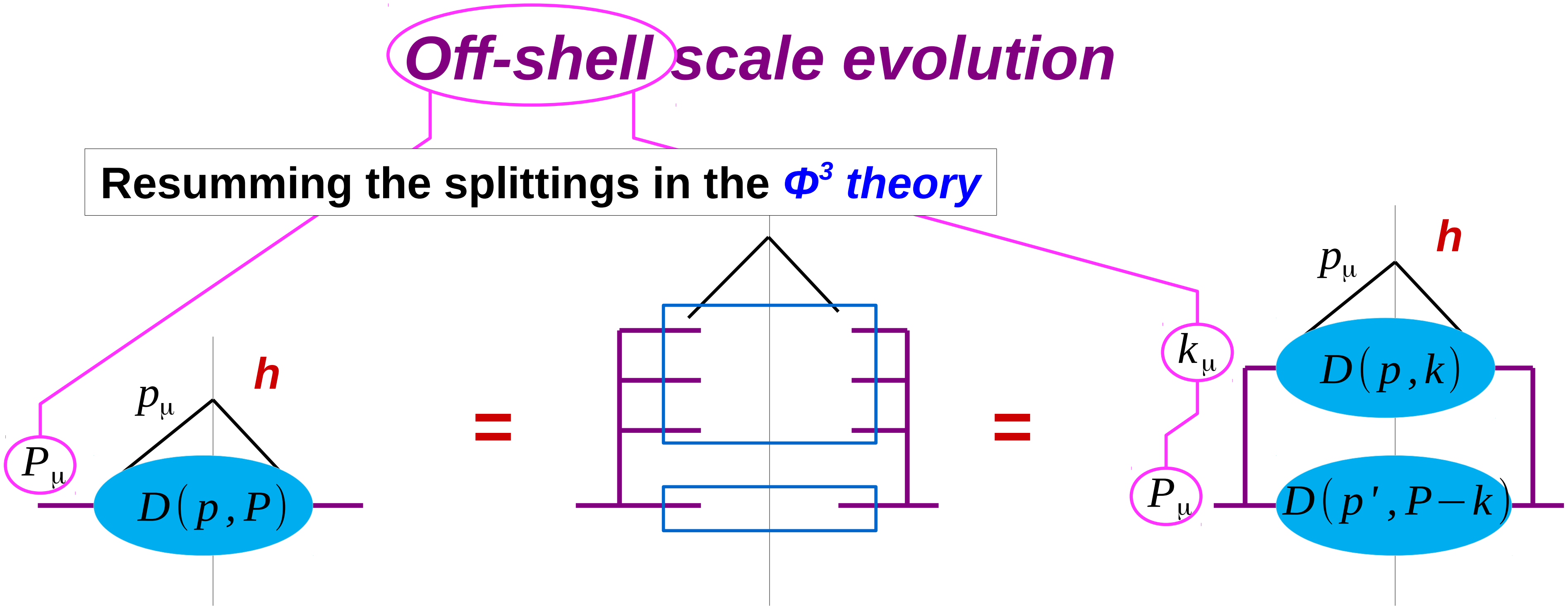
Parametrise fragmentation functions as

$$D\left[x=\frac{2\,P^{jet}_{\mu}\,p^{\mu}_{h}}{M^{2}_{jet}}\;,\; Q^{2}=M^{2}_{jet} \right]$$

Energy fraction the hadron takes away in the frame co-moving with the jet

Fragmentation scale: jet mass

Off-shell scale evolution



Resumming the splittings in the *Φ³ theory*

Thus, the equation for *D* is

$$D(p,P) \sim \int d^D k \frac{g^2(k^2)D(p,k)}{k^4(P-k)^4} \int d^D p' D(p',P-k)$$

Let us parametrise *D* as

$$D(p,P) \sim P^4 \boxed{\rho(P^2)} \boxed{f(p,P)}$$

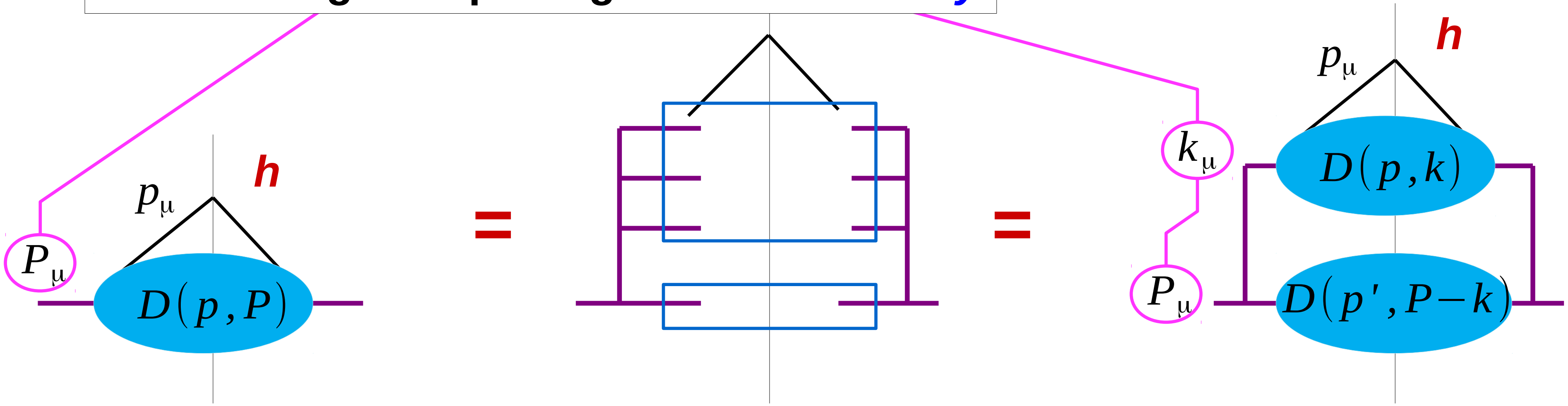
jet mass distribution

conditional probability of a *hadron* with *p* in a *jet* with *P*

$$\int d^D p f(p,P) = 1$$

Off-shell scale evolution

Resumming the splittings in the *Φ^3 theory*



Thus, we optain 2 equations for *f* and *ρ*

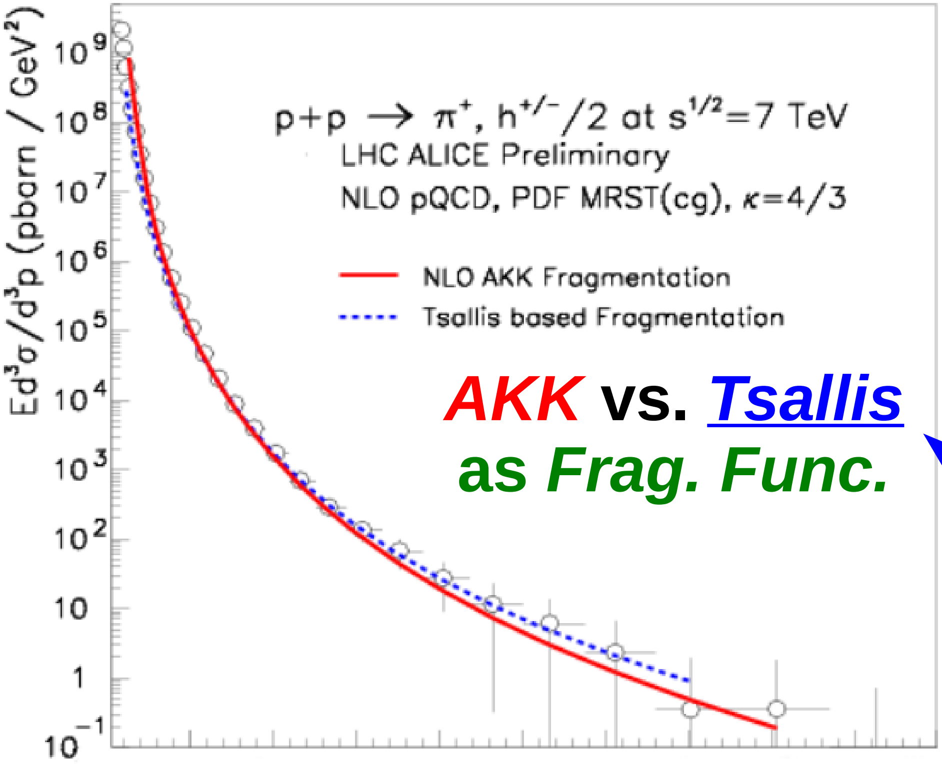
$$D(p,P)\ \sim\ \int d^D k\, \frac{g^2(k^2)\, D(p,k)}{k^4}\, \rho\left[(P-k)^2\right]$$

$$\rho\left(P^2\right)\ \sim\ \int d^D k\, g^2(k^2)\, \rho\left(k^2\right) \rho\left[(P-k)^2\right]$$

Back-up

Application in a pQCD calculation

π^+ spectrum in pp --> $\pi^\pm$X @ √s=7 TeV (NLO pQCD)



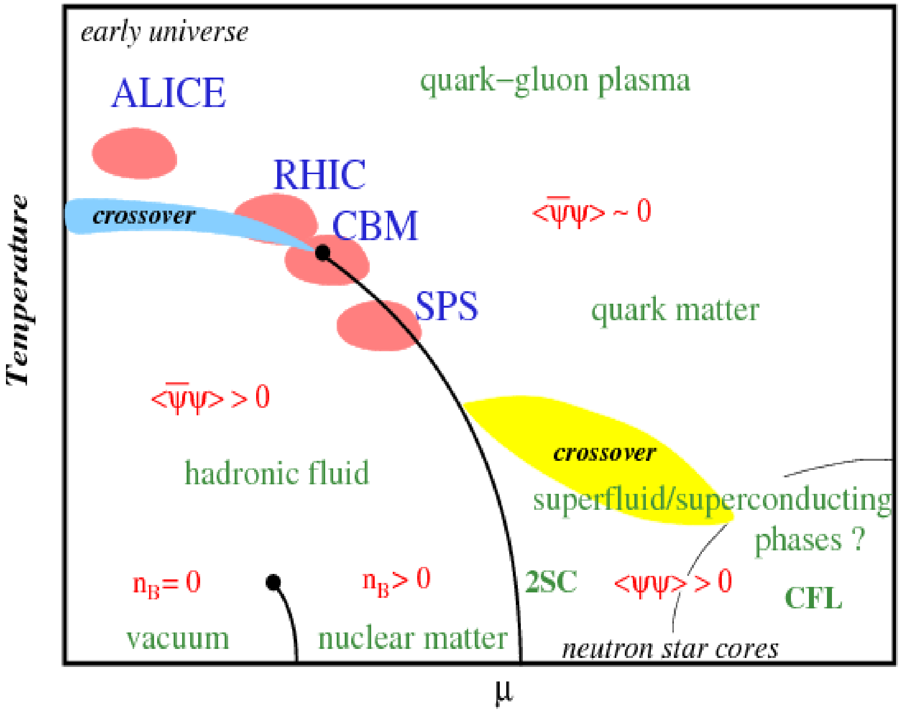
AKK vs. **Tsallis**
as Frag. Func.

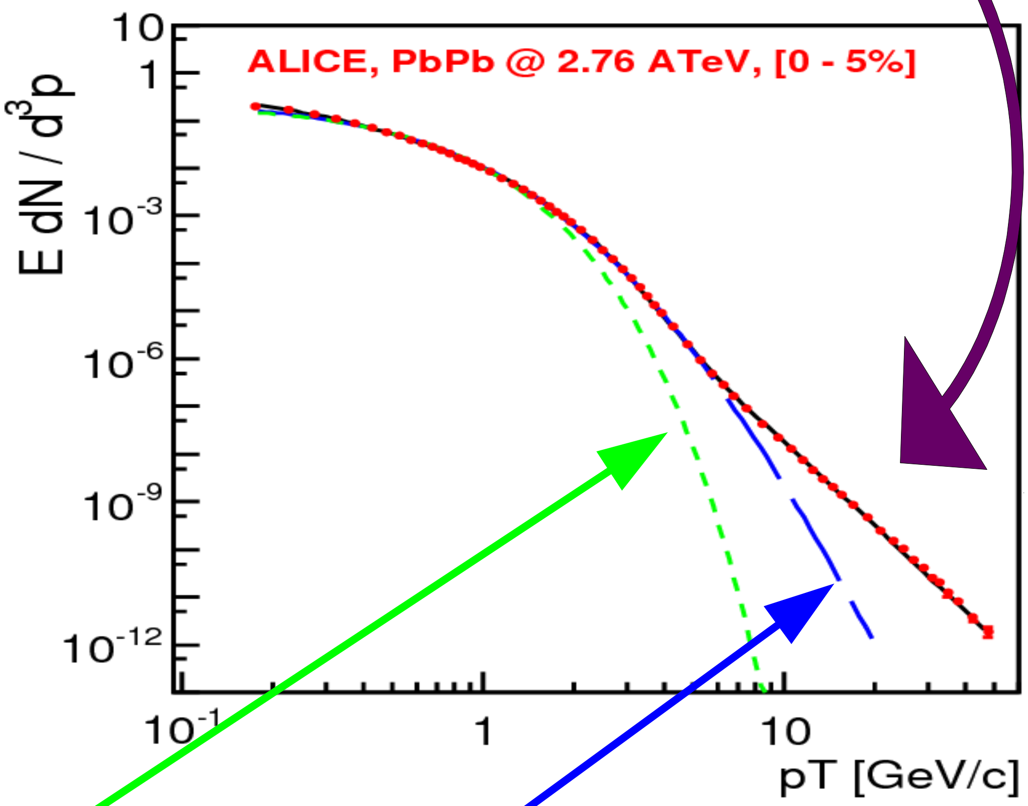
$$D_{p_i}^{\pi^+}(z) \sim \left(1 + (q_i - 1) z / T_i\right)^{-1/(q_i-1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

The spectrum is not Boltzmann

Search for the critical point





T, μ **obtained by fitting spectra with**

Boltzmann $\sim \exp\left(-\dfrac{\epsilon-\mu}{T}\right)$

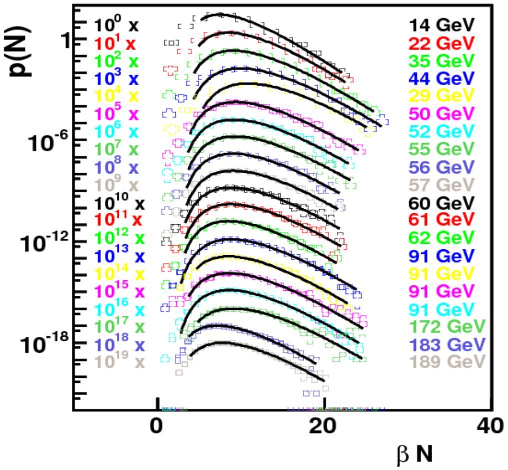
Tsallis $\sim \left(1+\dfrac{(q-1)\epsilon}{T}\right)^{-1/(q-1)}$

Goal: **to describe the dependence of *q, T* on *√s* and *centrality***

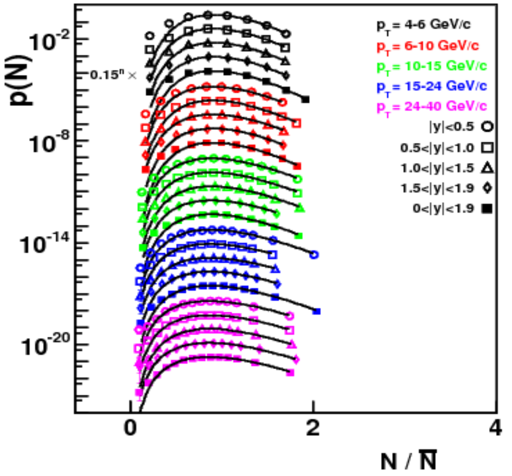
Idea of our statistical model is to combine

Negative Binomial hadron multiplicity distribution

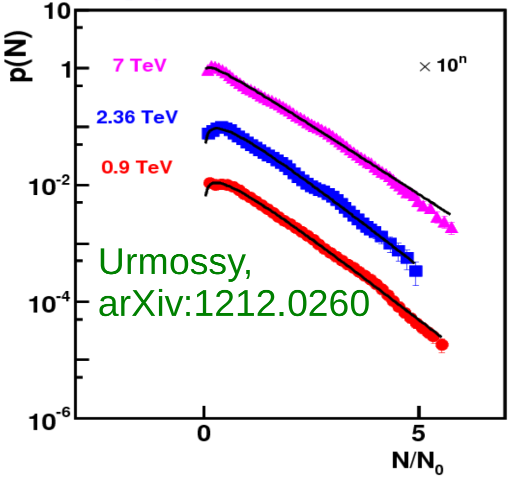
$e^-e^+ \to h^\pm$



pp → jets @ 7 TeV

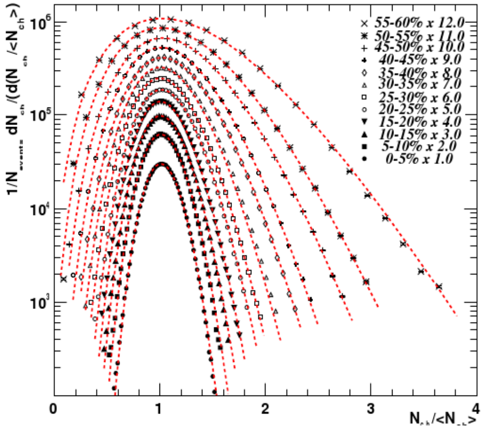


pp → h± @ LHC

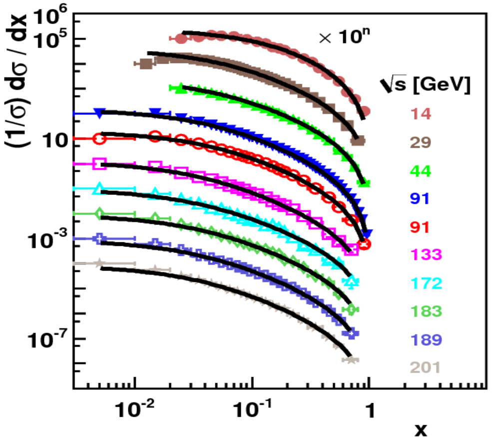


Urmossy, arXiv:1212.0260

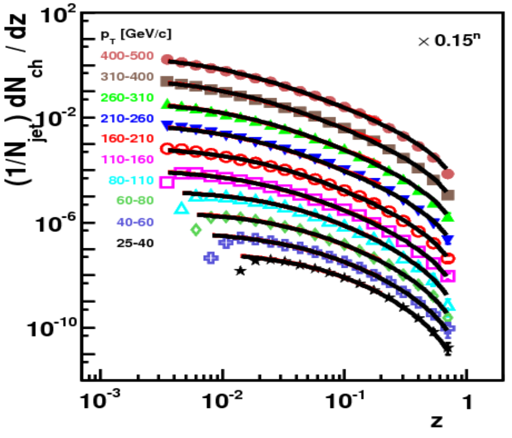
AuAu → h± @RHIC



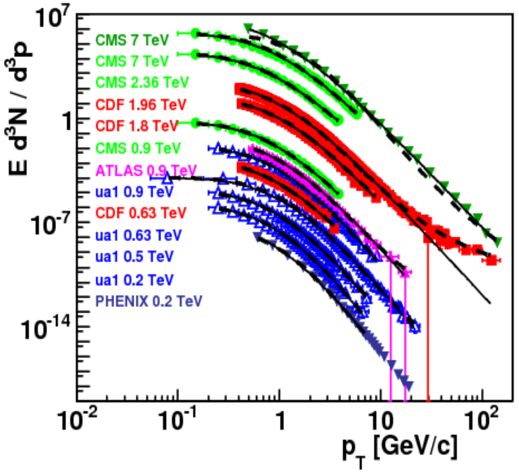
Power-law hadron spectra



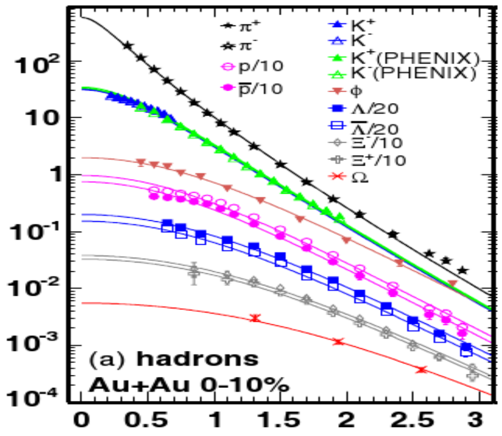
Urmossy et.al.,*PLB*, **701**: 111-116 (2011)



Urmossy et. al., *PLB*, **718**, 125-129, (2012)



Barnaföldi etal, *J. Phys.: Conf. Ser.*, **270,** 012008 (2011)



J. Phys. G: Nucl. Part. Phys. **37** 085104 (2010),

Statistical jet-fragmentation

The cross-section of the creation of hadrons $h_1, \dots, h_N$ in a jet of N hadrons

$$d\sigma^{h_1,\dots,h_N} = |M|^2 \delta^{(4)}\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\dots,h_N}$$

If *|M|* ≈ *constans*, *we arrive at a microcanonical ensemble*:

$$d\sigma^{h_1,\dots,h_n} \sim \delta\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\dots,h_n} \propto \left(P_{\mu} P^{\mu}\right)^{n-2} = M^{2n-4}$$

Thus, the haron distribution in a jet of *n* hadron is

$$p^0 \frac{d\sigma}{d^3 p}^{n=fix} \propto \frac{\Omega_{n-1}(P_{\mu} - p_{\mu})}{\Omega_n(P_{\mu})} \propto (1-x)^{n-3}, \qquad x = \frac{P_{\mu} p^{\mu}}{M^2/2}$$

**Energy of the hadron
in the co-moving frame**

Interpretation of q and τ

q measures *'deviation' from* the *exponential* distribution

$$\left[1+\frac{q-1}{\tau}x\right]^{-1/(q-1)} \rightarrow \exp\{-x/\tau\}$$

Equipartition

$$\langle p^0\rangle = d\tau\,\frac{M/2}{1-(d+2)(q-1)} \rightarrow d\tau(M/2) \quad \textbf{(if } q \rightarrow 1, \textbf{)}$$

$d=2$ **is the effective dimension of** $\frac{d^3\boldsymbol{p}}{p^0}$

Idea of our statistical model is to combine

Negative Binomial hadron multiplicity distribution

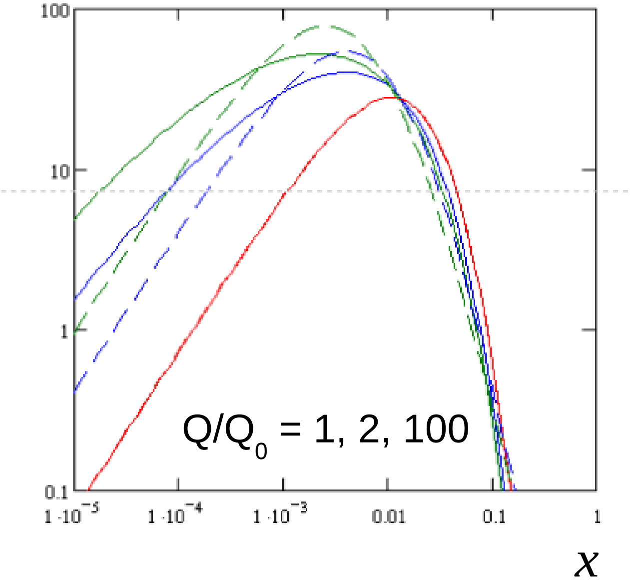
Power-law hadron spectra

Let us pick the simplest reaction: e^+e^- **annihilations**

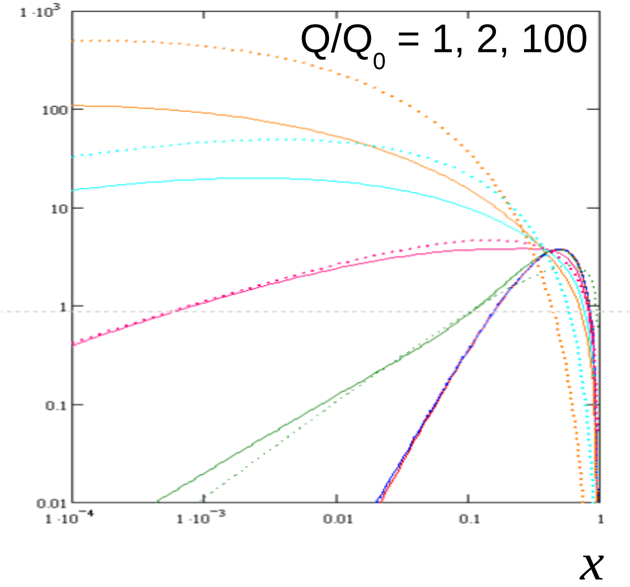
2.

How good is the approximation?

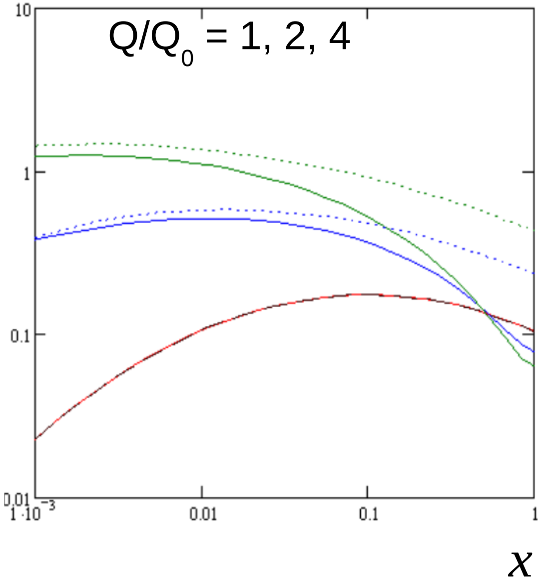
$$x\left(1+\frac{q-1}{\tau}x\right)^{-1/(q-1)}$$



$$x^b \ln^a(1/x)$$



Dist. Gauss in ln(1/x)



Requirement:

moments: $\langle x^j \rangle$ $\langle \ln^j(1/x) \rangle$ $\langle \ln^j(1/x) \rangle$

be equal in case of the shape preserving, approximate solution and the exact solution

What we have:

- **an *approximate* formula for the *fragmentation function* which *does not solve DGLAP***

$$D(x) \sim \left[1 + \frac{q-1}{\tau} x\right]^{-1/(q-1)}$$

- **Let us use *this ansatz* with *scale dependent parameters***

$$q, \tau \rightarrow q(t), \tau(t)$$

- **along with some other conjectures**

First step: in the Φ^3 theory

The Φ^3 theory case

Resummation **of branchings with** ***DGLAP***

$$\frac{d}{dt}D(x,t) = g^2\int_x^1 \frac{dz}{z}P(z)D(x/z,t), \qquad t = \ln(Q^2/\Lambda^2), \qquad g^2 = 1/(\beta_0 t)$$

with ***LO splitting function*****:** $P(z) = z(1-z)-\frac{1}{12}\delta(1-z)$

Let the non-perturbative input at starting scale Q_0 **be:** $D_0(x) = \left(1+\frac{q_0-1}{\tau_0}x\right)^{-1/(q_0-1)}$

The full solution is

$$\boxed{D(x,t) = \int_x^1 \frac{dz}{z}f(z,t)D_0(x/z)}$$

with

$$f(x) \sim \delta(1-x) + \sum_{k=1}^{\infty}\frac{b^k}{k!(k-1)!}\sum_{j=0}^{k-1}\frac{(k-1+j)!}{j!(k-1-j)!}x\ln^{k-1-j}\left[\frac{1}{x}\right]\left[(-1)^j+(-1)^k x\right]$$

$$b = \beta_0^{-1}\ln(t/t_0)$$

D **does not preserve its shape:**

$$\boxed{\int_x^1 \frac{dz}{z}f(z,t)\left(1+\frac{q_0-1}{\tau_0}\frac{x}{z}\right)^{1/(q_0-1)} \neq \left(1+\frac{q(t)-1}{\tau(t)}x\right)^{1/(q(t)-1)}}$$

It is only an approximation!

Urmossy, Z. Xu, arXiv:1606.03208

The solution is similar in QCD

Soultion in the Φ^3 theory (at LO splitting and 1-loop coupling) :

$$\frac{\tilde{D}(\omega,t)}{\tilde{D}(\omega,t_0)} = \exp\{b(t)\tilde{P}(\omega)\} \sim (t/t_0)^{\tilde{P}(\omega)/\beta_0} \qquad t=\ln\left(M_{jet}^2 \;/\; \Lambda^2 \right)$$

and q, τ are :

$$q(t)=\frac{\alpha_1(t/t_0)^{a1}-\alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1}-\alpha_4(t/t_0)^{-a2}} \qquad \tau(t)=\frac{\tau_0}{\alpha_4(t/t_0)^{-a2}-\alpha_3(t/t_0)^{a1}}$$

Soultion in QCD :

$$\frac{\tilde{D}_{q/g}^h(\omega,t)}{\tilde{D}_{q/g}^h(\omega,t_0)} = \exp\left\{\int_{t_0}^{t} dt'\gamma(\omega,t')\right\} \sim F(t/t_0,\omega) \qquad t=\ln\left(M_{jet}^2 \;/\; \Lambda_{QCD}^2 \right)$$

Thus, q, τ would look like :

$$q(t)=\frac{\alpha_1 F(t/t_0,1)-\alpha_2 F(t/t_0,3)}{\alpha_3 F(t/t_0,1)-\alpha_4 F(t/t_0,3)} \qquad \tau(t)=\frac{\tau_0}{\alpha_4 F(t/t_0,3)-\alpha_3 F(t/t_0,1)}$$