Progress towards including colour interference in parton showers

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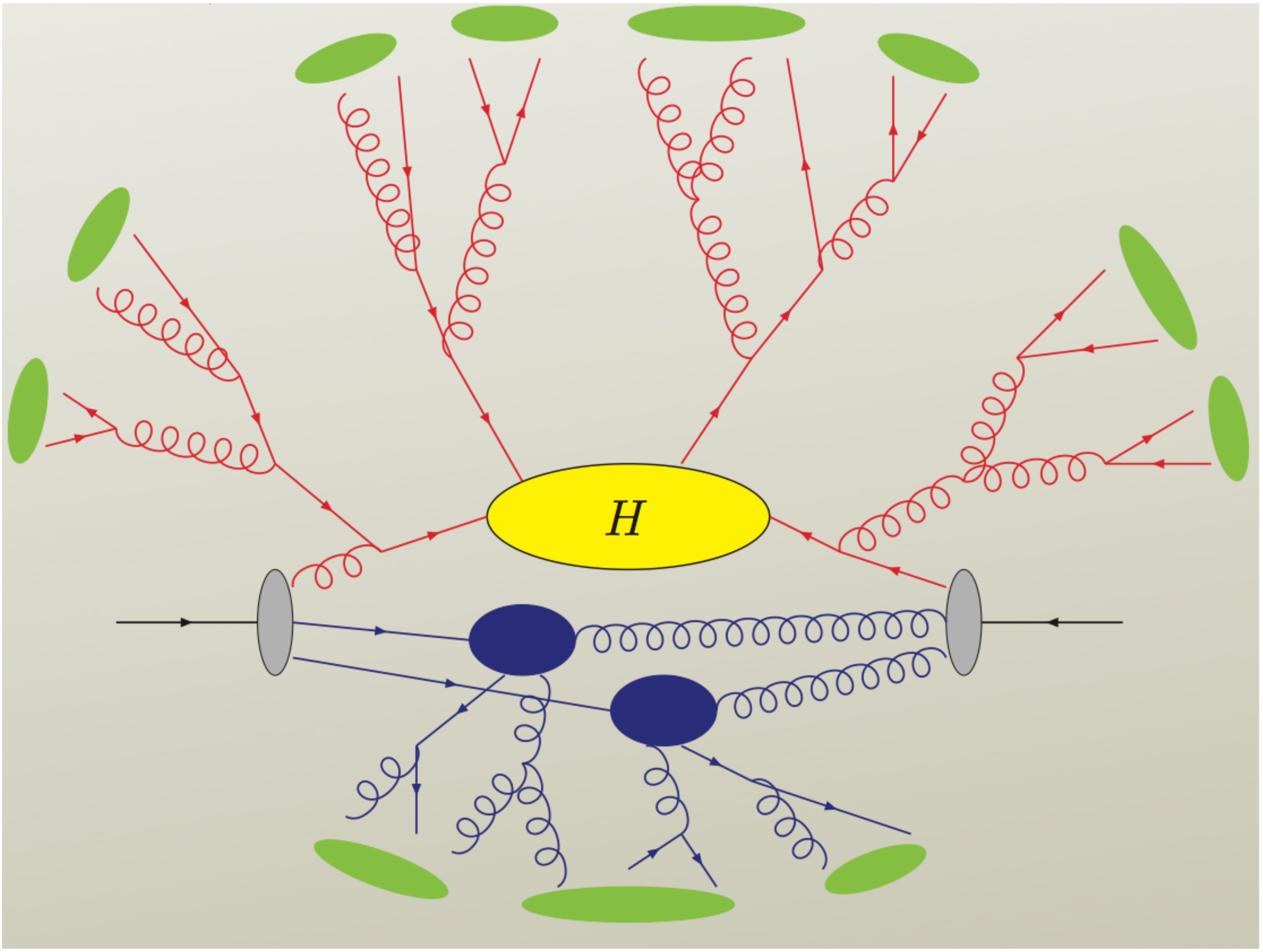
in collaboration with
Jeff Forshaw
Mike Seymour

JHEP 1512 (2015) 091
& Phys. Rev. Lett. 116, 212003

Epiphany
2017



H-H collision by Soper (CTEQ School)

In this talk: Focus in the colour interference of soft gluons for partons showers.

PDF …

Hard process (Q)

Parton Shower:
Approx: Collinear
+
Soft radiation
$Q \gg q^{\mu} \gg \Lambda_{QCD}$

Underlying event

Hadronization
…

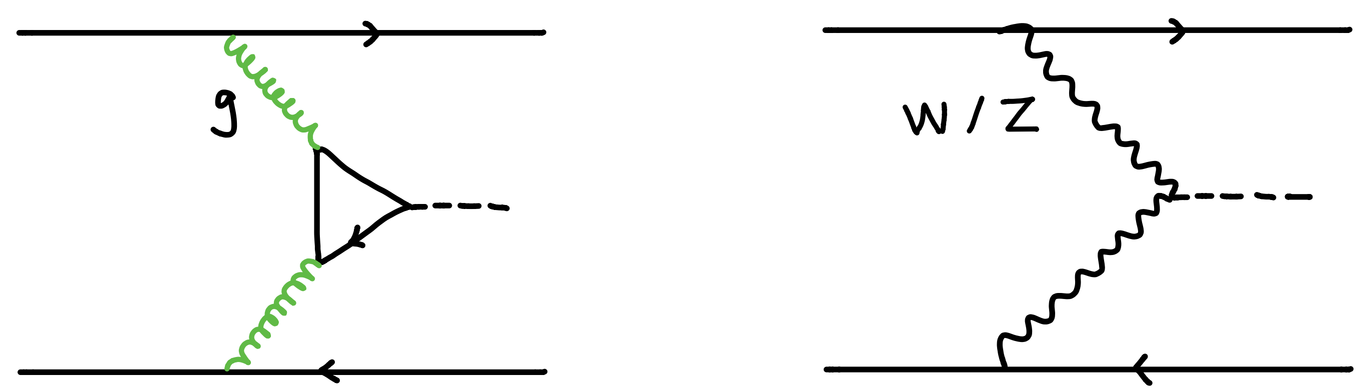
See also:
S. Plätzer, Eur. Phys. J. C 74, 2907 (2014).
Nagy and Soper, J. HEP. 07 (2015) 119
Caron-Huon arXiv:1501.03754
…

Motivation

- Is colour interference a relevant effect?
  - Definitely NO for "inclusive" observables!

  - YES for particular "non-inclusive" observables of H-H collisions! For some of observables, it is necessary even to get leading logs right (e.g. GBJ hep /0604094 ; 0808.1269).

- Are such observables relevant in the search for new physics at the LHC? Yes, it is relevant to tag the (absence of) colour properties, e.g. (Phys. Lett., B696:87–91, 2011)



Notation/Essentials: 1-loop soft correction

$k^\mu \ll Q_{ij}$

Colour matrices

Hard subprocess vector (colour + spin)

$$ig_s^2 \mathbf{T}_i \cdot \mathbf{T}_j \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{-p_i \cdot p_j}{[p_j \cdot k \pm i0][-p_i \cdot k \pm i0][k^2 + i0]} \big|2\big\rangle$$

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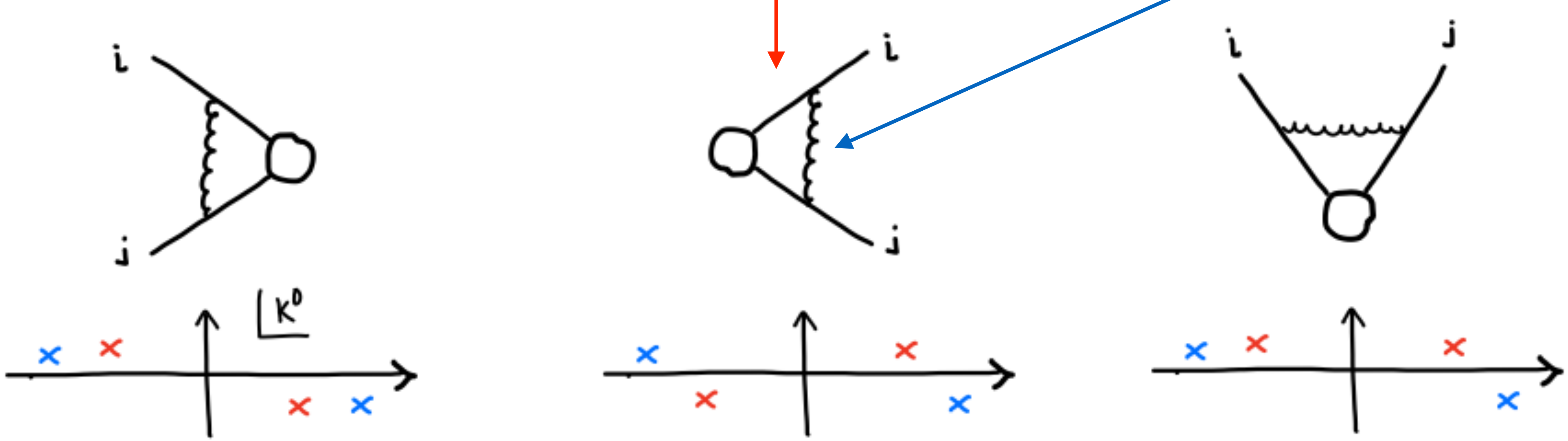


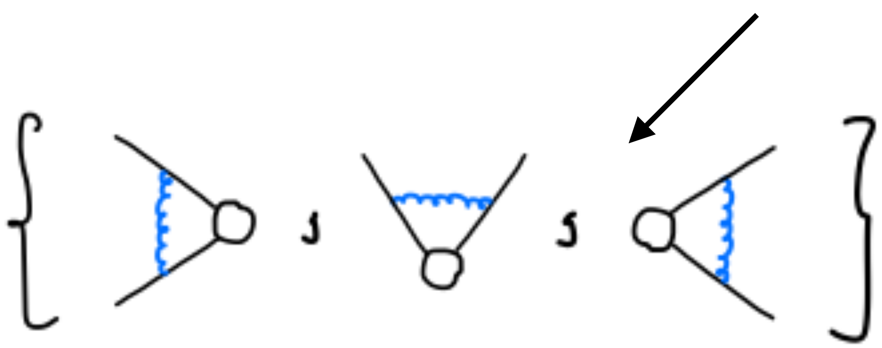
$i: \ -i0$
$j: \ -i0$

$i: \ +i0$
$j: \ +i0$

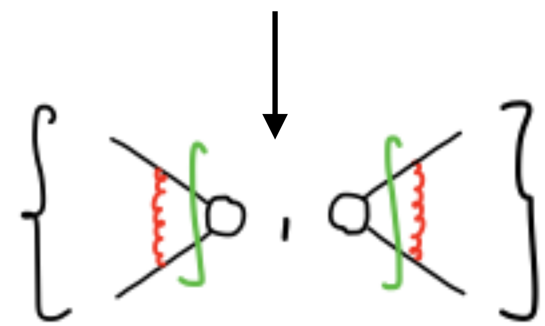
$i: \ -i0$
$j: \ +i0$

After contour integration:

$$\propto \mathbf{T}_i \cdot \mathbf{T}_j \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left[\frac{(2\pi)\delta(k^2)\theta(k^0)}{[p_j \cdot k][p_i \cdot k]} + i\tilde{\delta}_{ij} \frac{(2\pi)^2 \delta(p_i \cdot k)\delta(-p_j \cdot k)}{2[k^2]} \right] |2\rangle$$



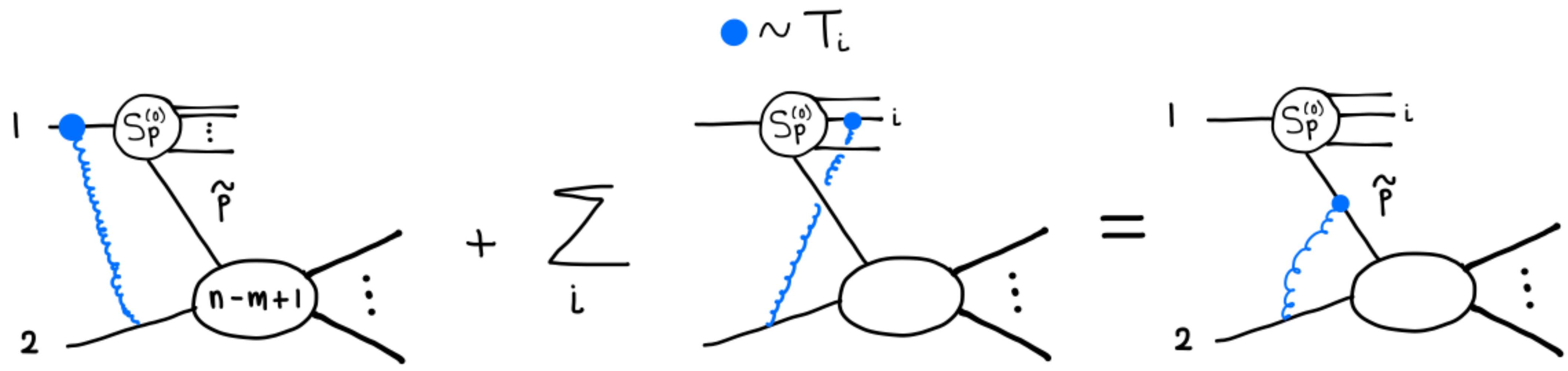
Exchange of an On-shell gluon: **Purely real**



Coulomb gluon: on-shell exchange **Purely imaginary**

$$\tilde{\delta}_{ij} = \begin{cases} 1 & \text{if } i,j \text{ in }, \\ 1 & \text{if } i,j \text{ out }, \\ 0 & \text{otherwise.} \end{cases}$$

Breakdown of colour coherence

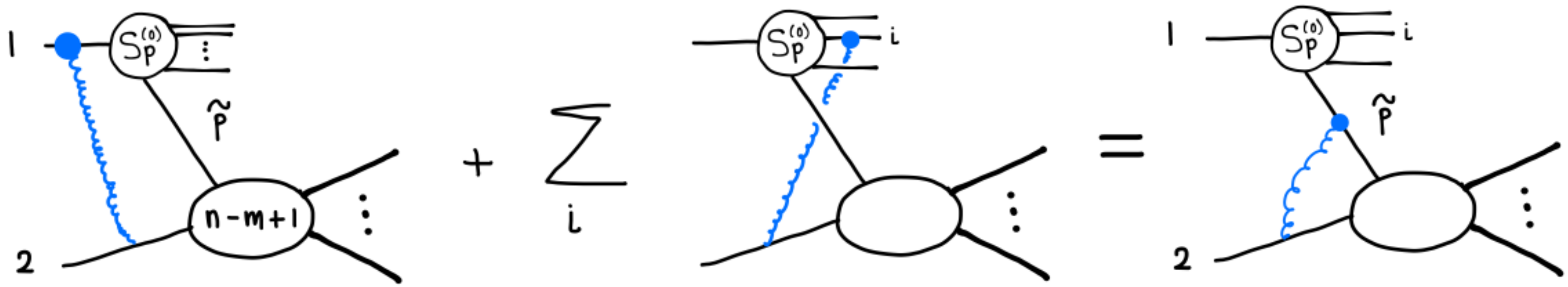


$$T_2\cdot\left(T_1+\sum_i T_i\right) Sp^{(0)}\, |n-m+1\rangle \;=\; Sp^{(0)} \left[T_2\cdot T_{\tilde p}\, |n-m+1\rangle\right]$$

Effectively, correction to

Breakdown of colour coherence

$\bullet \sim T_i$



$$T_2 \cdot \Big(T_1 + \sum_i T_i \Big) Sp^{(0)} |n\text{-}m\text{+}1\rangle = Sp^{(0)} \big[T_2 \cdot T_{\tilde{p}} |n\text{-}m\text{+}1\rangle \big]$$

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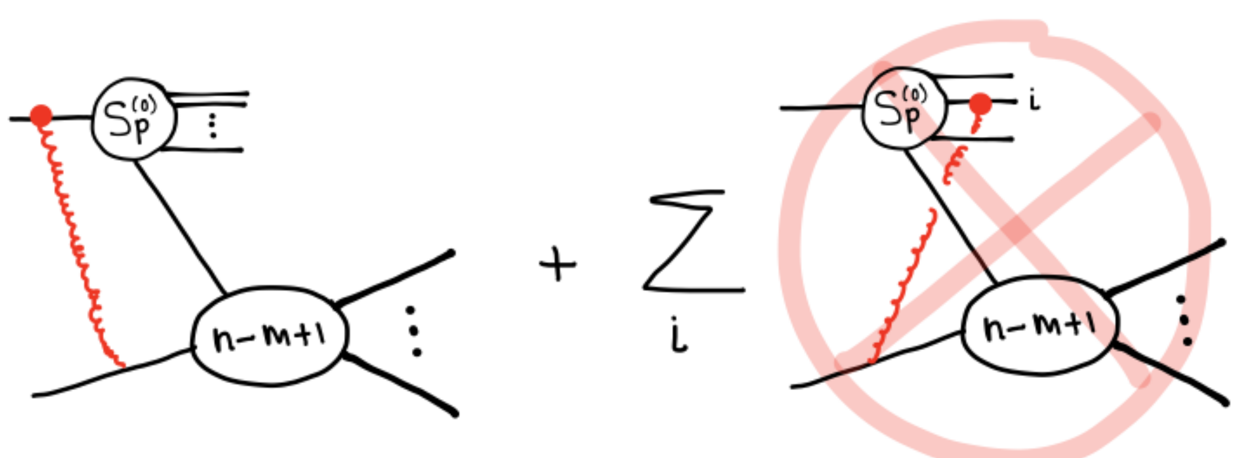


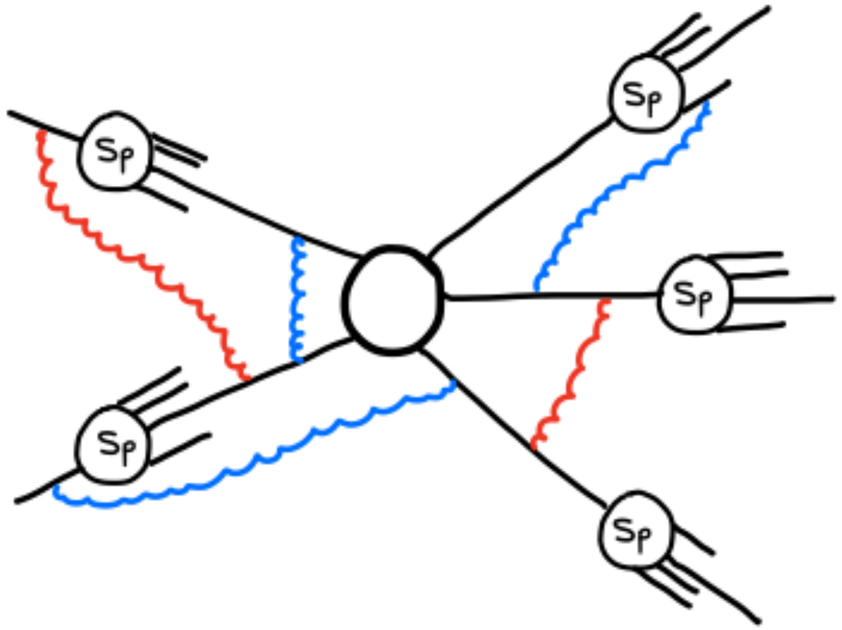
$$= T_1 \cdot T_2 \, Sp^{(0)} |n\text{-}m\text{+}1\rangle$$

Unavoidable colour interference

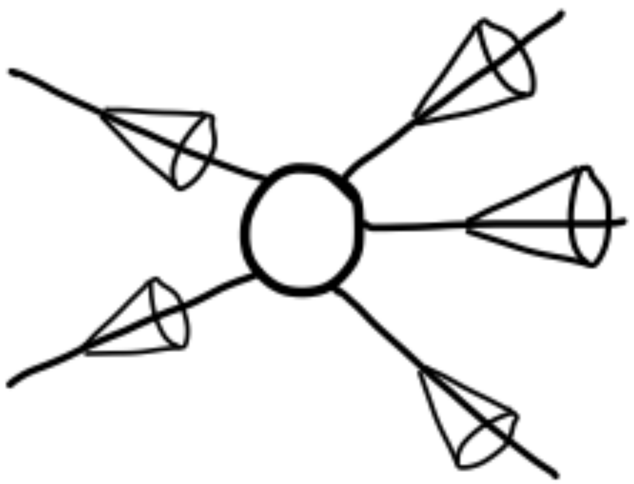
Coulomb gluons and (the lack of) coherence

JHEP 1207 (2012) 026 & JHEP 1207 (2012) 026

Coherence allows us to “unhook” on-shell gluons but it fails for Coulomb gluons.



instead of

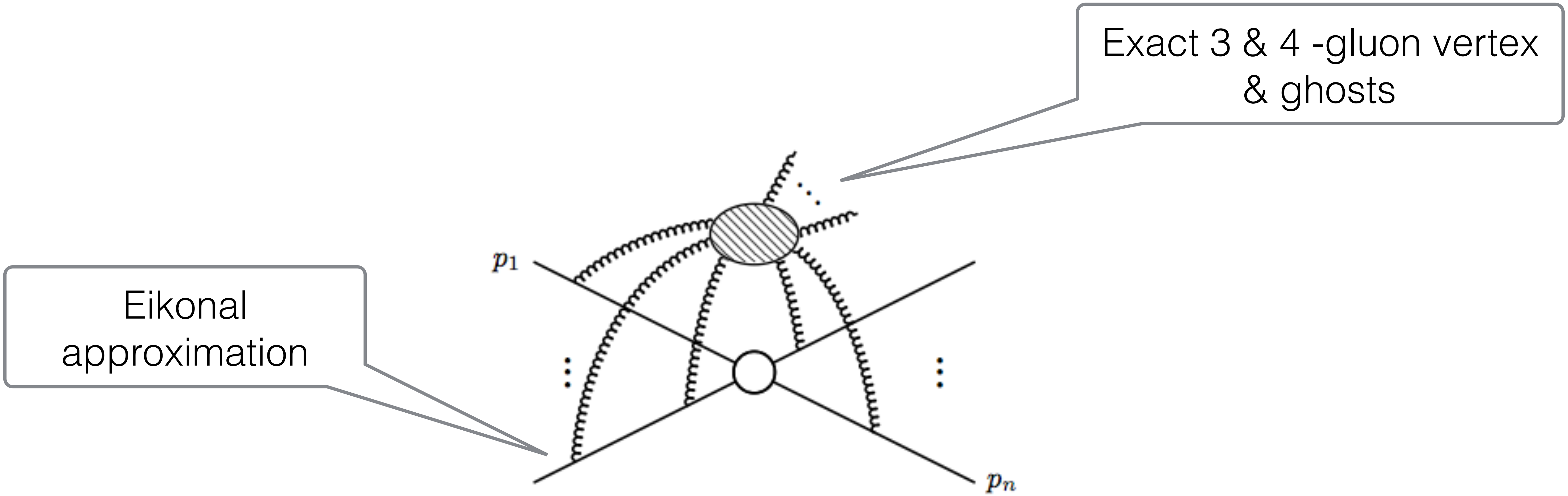


This exemplifies how colour interference becomes relevant for non-inclusive observables!

Can one make sense of these nested structure?

Strategy to get insight

1-loop diagrammatic calculation assuming that all gluons are soft (RAM, Forshaw, Seymour: PhD thesis, JHEP 1512 (2015) 091 & Phys. Rev. Lett. 116, 212003



Colour evolution approach

Our fixed order calculations suggest that the one-loop amplitude of a general hard scattering with N soft-gluon emissions (ordered in softness $q_i \lambda \sim q_{i+1}$) is

$$\left|n_N^{(1)}\right\rangle = \sum_{m=0}^{N}\sum_{i=2}^{p}\sum_{j=1}^{i-1} \mathbf{J}^{(0)}(q_N)\cdots\mathbf{J}^{(0)}(q_{m+1})\,\mathbf{I}_{ij}(q_{m+1}^{(ij)},q_m^{(ij)})\mathbf{J}^{(0)}(q_m)\cdots\mathbf{J}^{(0)}(q_1)\,|n_0^{(0)}\rangle$$
$$+\sum_{m=1}^{N}\sum_{j=1}^{n+m-1}\sum_{k=1}^{n+m-1} \mathbf{J}^{(0)}(q_N)\cdots\mathbf{J}^{(0)}(q_{m+1})\,\mathbf{I}_{n+m,j}(q_{m+1}^{(ij)},q_m^{(jk)})\,\mathbf{d}_{jk}(q_m)\mathbf{J}^{(0)}(q_{m-1})\cdots\mathbf{J}^{(0)}(q_1)|n_0^{(0)}\rangle,$$

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where the virtual insertion operator:

$$\mathbf{I}_{ij}(a,c) = \mathbf{I}_{ij}(a,b) + \mathbf{I}_{ij}(b,c), \qquad \mathbf{I}_{ij}(0,b) = \frac{\alpha_s}{2\pi}\frac{c_\Gamma}{\epsilon^2}\,\mathbf{T}_i\cdot\mathbf{T}_j\left[\left(\frac{b^2}{4\pi\mu^2}\right)^{-\epsilon}\left(1 + i\pi\epsilon\,\tilde{\delta}_{ij} - \epsilon\ln\frac{2p_i\cdot p_j}{b^2}\right)\right],$$

describes the non-emission evolution of partons i and j from b to a.

$$q^\mu = \alpha p_i^\mu + \beta p_j^\mu + (q_T^{(ij)})^\mu$$

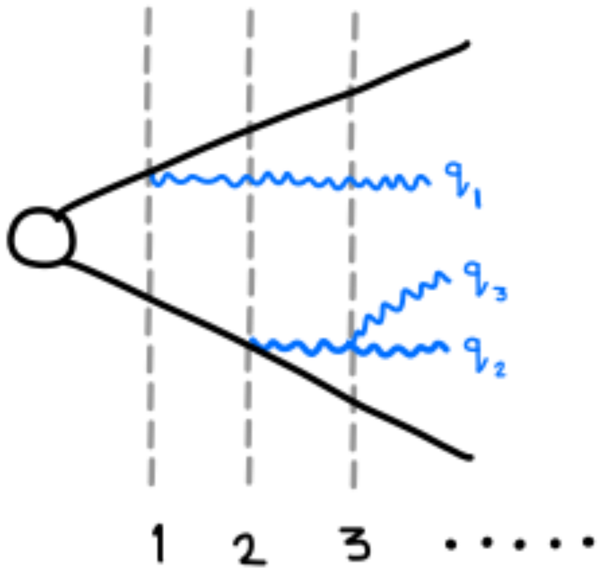
Key point: Ordering variable: dipole kT

$$\sum_j \mathbf{d}_{ij}(q) = \sum_j \mathbf{T}_j \frac{p_j\cdot\varepsilon}{p_j\cdot q} = \mathbf{J}^{(0)}(q)$$

- Gauge invariant.
- Correct IR poles
- Interpretation: ordered evolution similar to a shower!

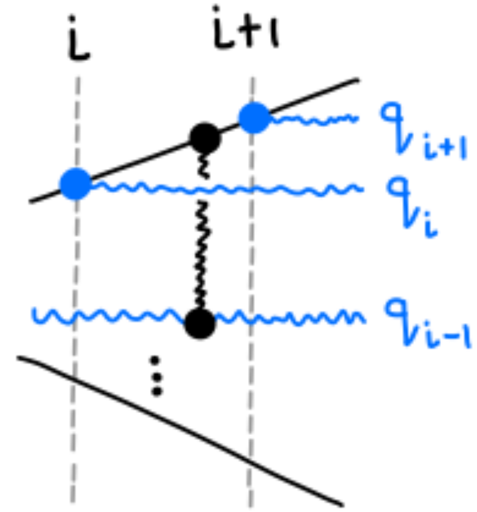
Diagrammatics of dipole: kT ordering

1.- Add N-emissions on external legs

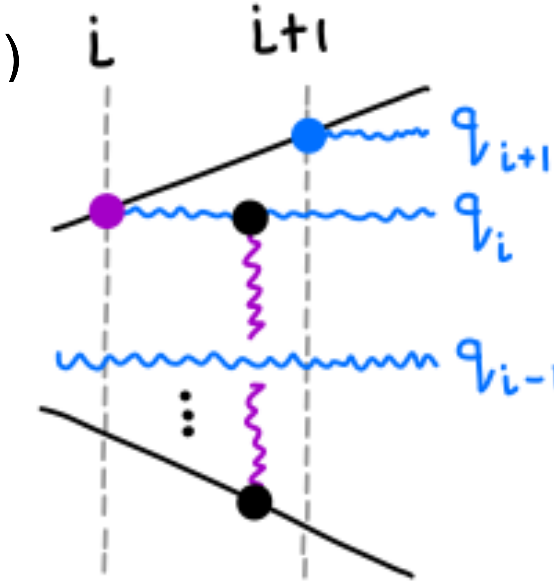


2.- Add virtual exchanges

Case a)



Case b)



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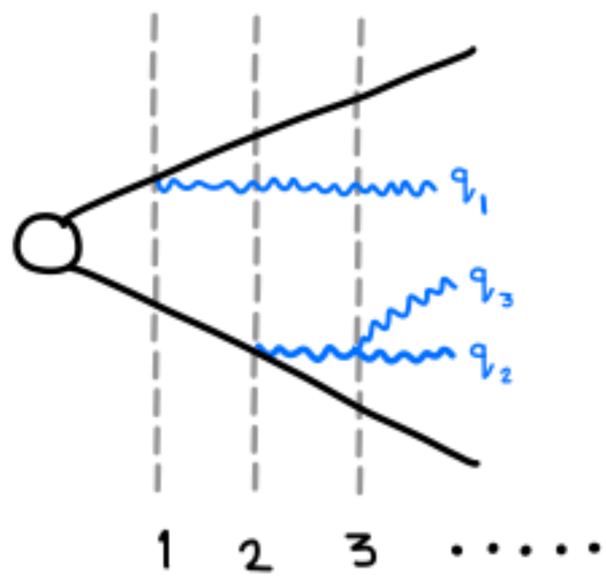

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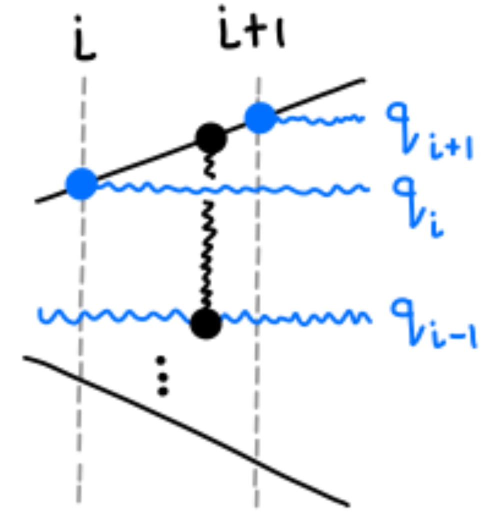
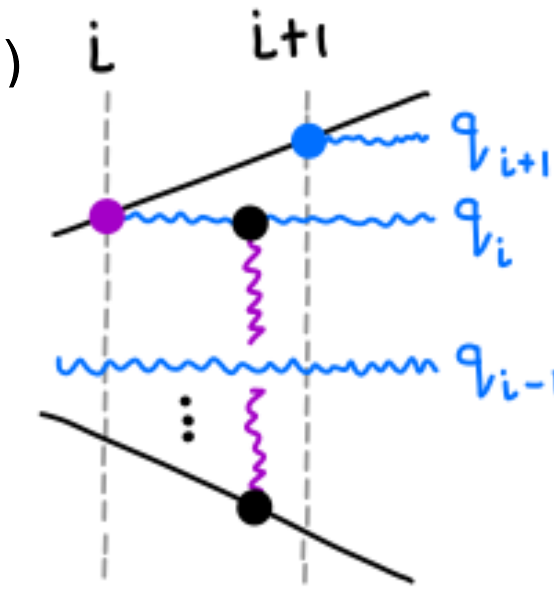
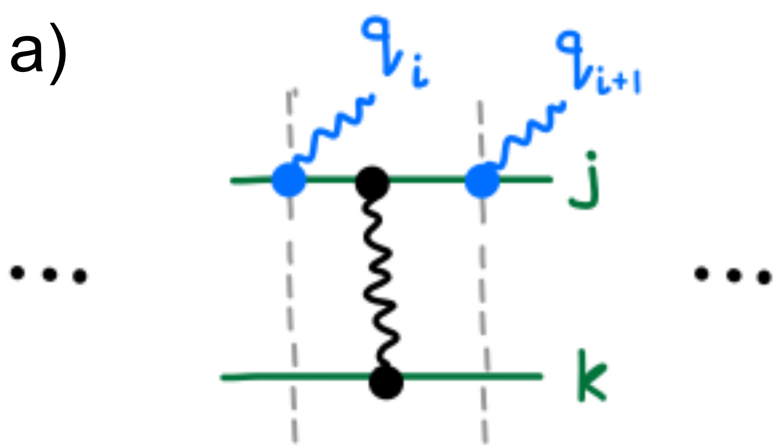


Case b)



and apply effective rules:

a)



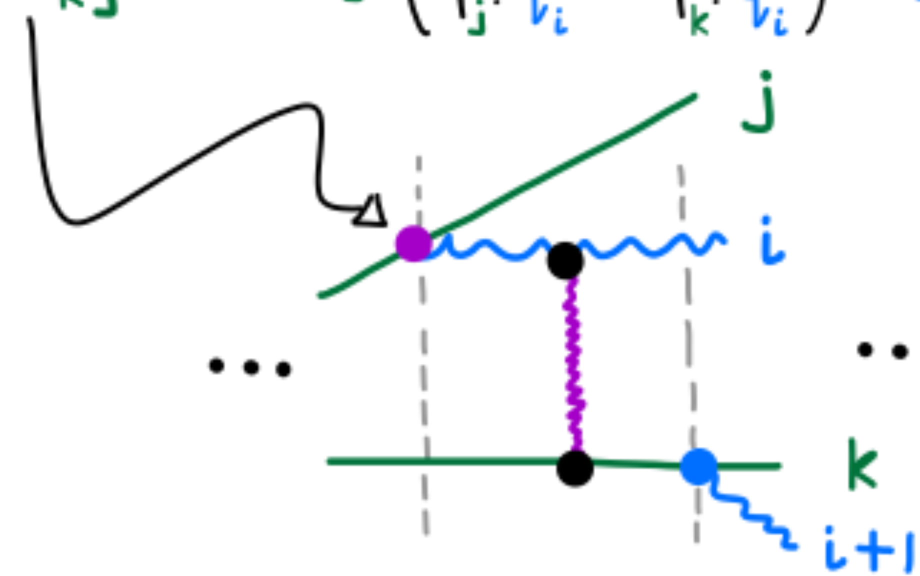
$$= I_{jk}\left(q_{i+1}^{(jk)},\, q_i^{(jk)}\right)$$

$$= T_j \frac{p_j \cdot \varepsilon_i}{p_j \cdot q_i}$$

b)

Gauge invariance

$$= d_{kj}(i) = T_j\left(\frac{p_j}{p_j \cdot q_i} - \frac{p_k}{p_k \cdot q_i}\right)\cdot \varepsilon_i$$



$$= I_{q_i k}\left(q_{i+1}^{(ik)},\, q_i^{(jk)}\right)$$

Non-emission evolution operator

On-shell

Coulomb

$$\mathbf{I}_{ij}(a,b) = \frac{\alpha_s}{2\pi}\mathbf{T}_i\cdot\mathbf{T}_j c_\Gamma \int \mathrm{d}(k^{(ij)})^2 (k^{(ij)})^{-2\epsilon}\left[\int_{-\ln\sqrt{2}p_j^-/k^{ij}}^{\ln\sqrt{2}p_i^+/k^{ij}} \mathrm{d}y \frac{p_i\cdot p_j}{2[p_j\cdot k][p_i\cdot k]} - \frac{i\pi\delta_{ij}}{(k^{(ij)})^2}\right]$$

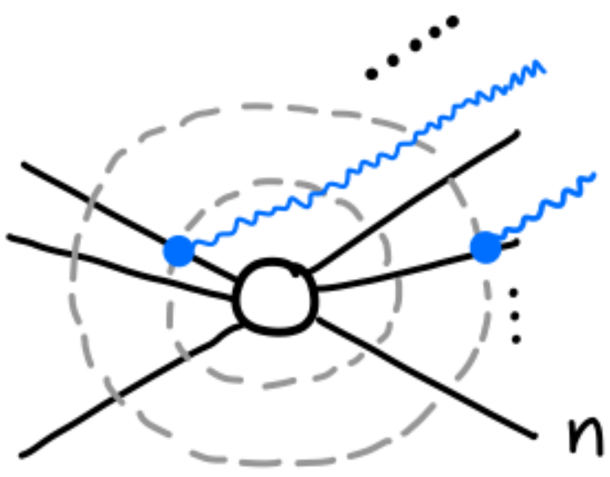
$$\times\,\theta(a<k^{(ij)}<b)$$

This is the same one-loop operator that appears at one-loop but kT ordered!

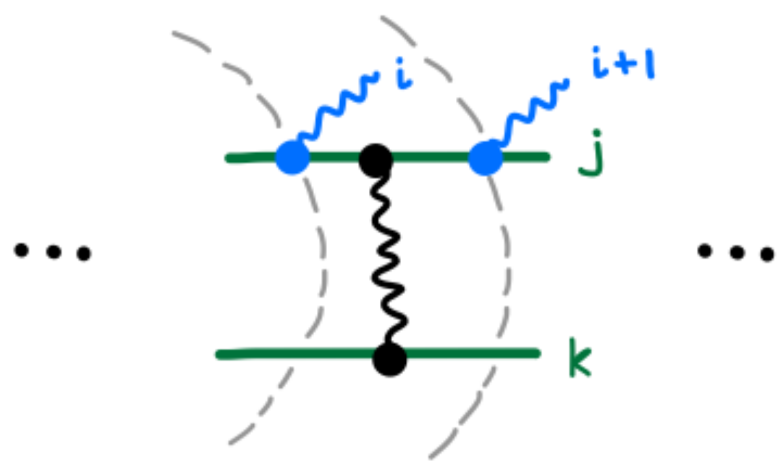
$$k^\mu = \alpha p_i^\mu + \beta p_j^\mu + (k^{(ij)})^\mu$$

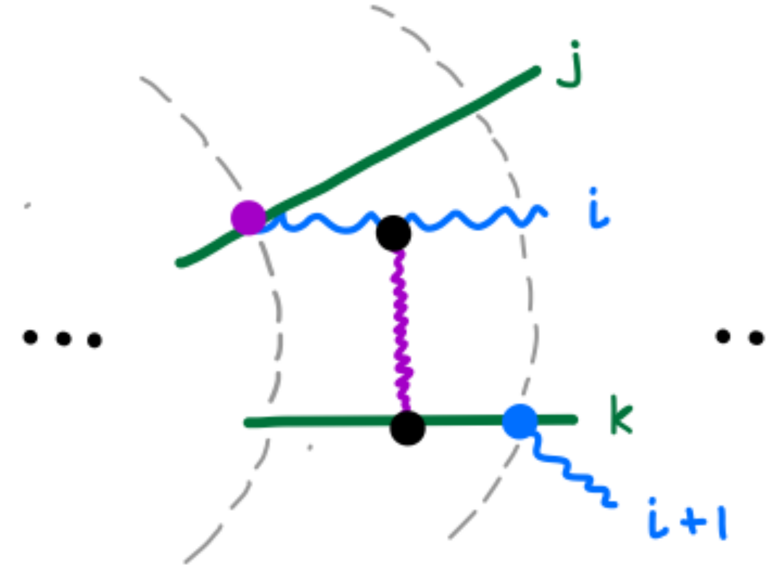
Diagrammatics of dipole kT evolution

For a general scattering $|n\rangle$ we need spheres



The effective rules are the same:

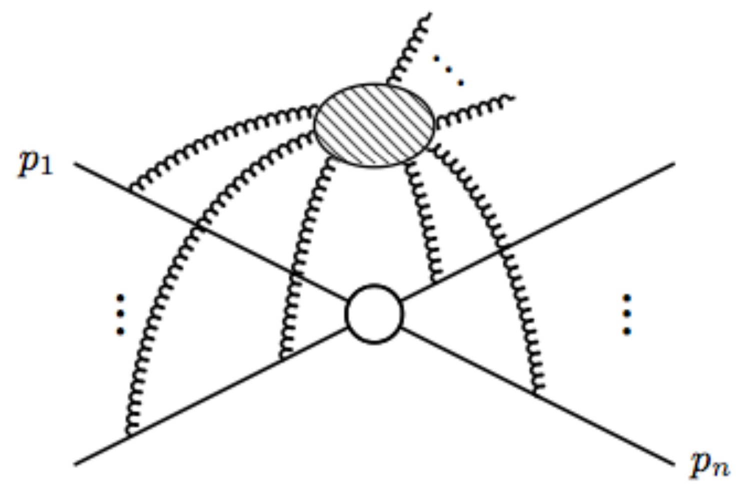




Summary / Conclusion

- Colour-interference can play an essential role for particular non-inclusive observables.

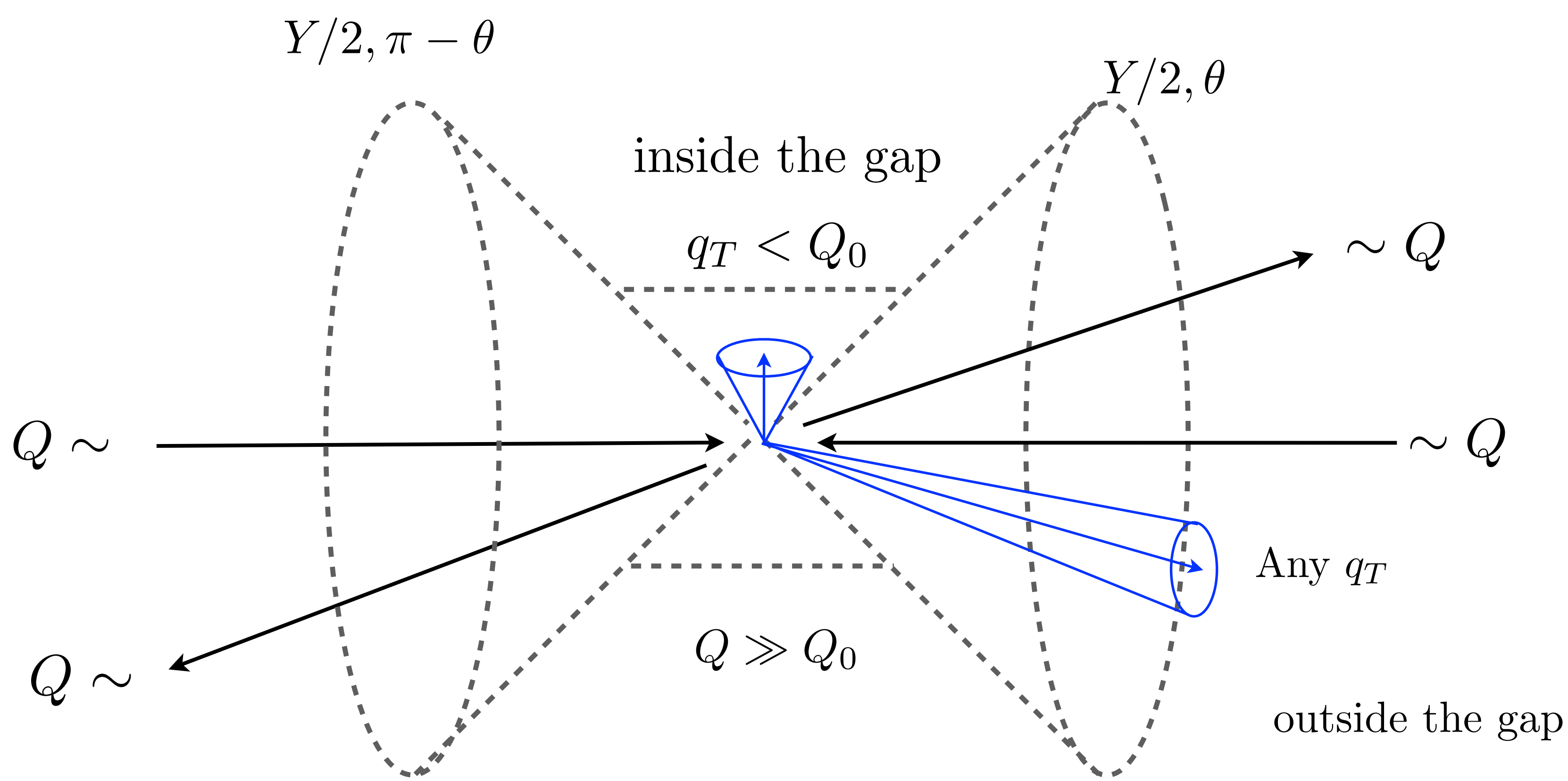
- Can be incorporated at *amplitude level* as an evolution in dipole transverse momentum. Making sense of



- Future: Monte Carlo Parton Shower implementation for general observables.

Concrete case: gaps-between-jets

(Forshaw, Kyrieleis & Seymour hep /0604094 ; /0808.1269)



Soft corrections

$$\sigma_m = \int |\mathcal{M}(q_1, \ldots, q_m)|^2 \, \mathrm{dPS}$$

$$Q_0 \ll q_i \ll Q$$

(On-shell gluons)

$$\sim \alpha_s^n \ln^n \left(\frac{Q^2}{Q_0^2} \right)$$

Super-leading logs
(On-shell + Coulomb gluons)

$$\sim \alpha_s^3 \ln^4 \left(\frac{Q^2}{Q_0^2} \right), \alpha_s^4 \ln^5 \left(\frac{Q^2}{Q_0^2} \right)$$

Origin: lack of coherence (strict factorisation).

Loop-expanded approach

The dipole kt evolution approach is equivalent to a Catani-Grazzini loop-expansion (RAM , Forshaw & Seymour hep-ph/0007142):

$$\left|n_N\right\rangle = (g_s\mu^{\epsilon})^N \, \mathbf{J}(q_N)\cdots\mathbf{J}(q_1)\left|n_0\right\rangle$$

$$\mathbf{J} = \mathbf{J}^{(0)} + \mathbf{J}^{(1)}, \qquad \left|n\right\rangle = \left|n_0^{(0)}\right\rangle + \left|n_0^{(1)}\right\rangle,$$

$$\mathbf{J}^{(1)}(q_a) = \frac{1}{2}\sum_{j\neq i}^{n+a-1} \frac{\alpha_s}{2\pi}\frac{c_\Gamma}{\epsilon^2}\left(\frac{(-p_i\cdot q_a - i0)(-p_j\cdot q_a - i0)}{(-p_i\cdot p_j - i0)8\pi}\right)^{-\epsilon} \mathbf{T}_{q_a}\cdot\mathbf{T}_i \;\mathbf{d}_{ij}(q_a).$$

Dipole kT

Manifestly, analytic function of Lorentz invariants! This approach was studied for e+e- annihilation (Feige & Schawartz PhD thesis & Phys.Rev. D90 (2014)) and is used in a recent resummation of non-global logs (1501.03754).