Soft and Coulomb effects in top-quark pair production beyond NNLO

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— RWTH Aachen —

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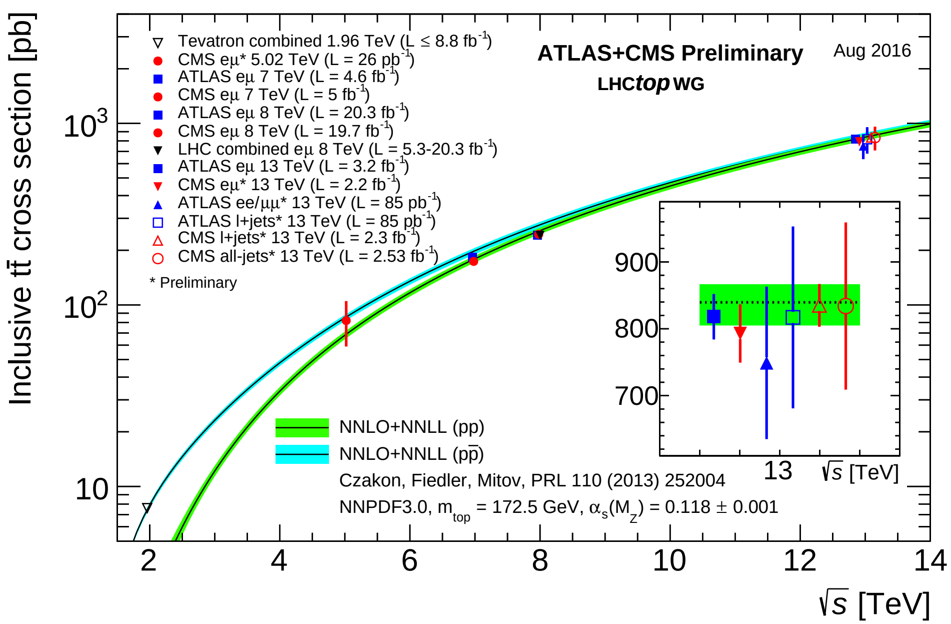
work in progress with Jan Piclum

Total $t\bar{t}$ cross section

Test of QCD and nature of top-quark:

- Sensitive to m_t, α_s, PDFs
  - pole mass $m_t = 173.8^{+1.7}_{-1.8}\mathrm{GeV}$ from σ_{tt} measurement (CMS 16)
- Experimental precision comparable to uncertainty of NNLO+NNLL prediction (Bärnreuther/Czakon/Fiedler/Mitov 12–13)



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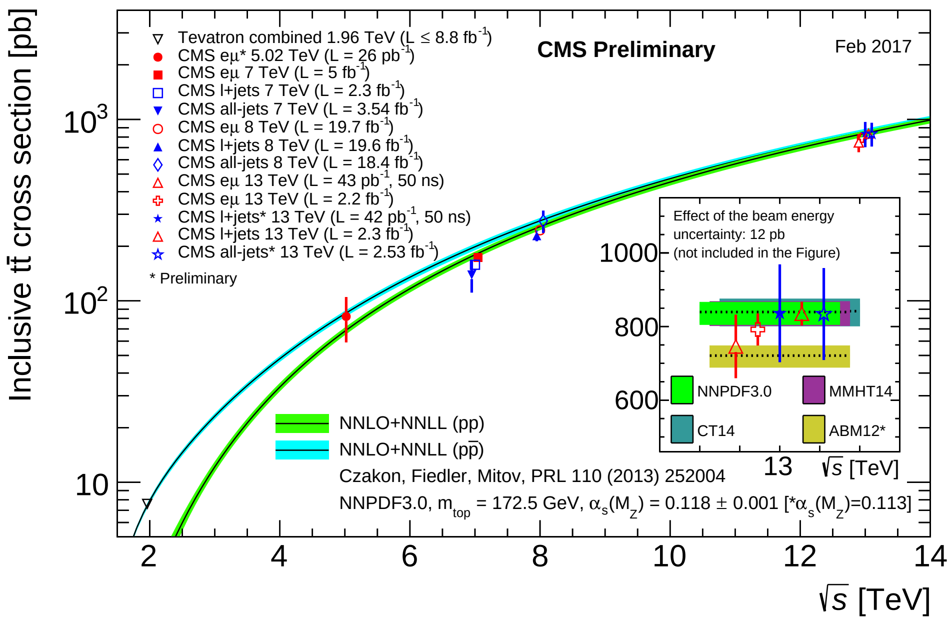

Resummation of threshold-enhanced corrections, $\beta=\sqrt{1-\frac{4m_t^2}{\hat{s}}}\to 0$

$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s\ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s\ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s\ln\beta)}_{\text{(NNLL)}} + \underbrace{\alpha_s^2 g_3(\alpha_s\ln\beta)}_{\text{(N}^3\text{LL)}} + \ldots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \Big\{\underbrace{1}_{\text{(LL,NLL)}} \;;\; \underbrace{\alpha_s, \beta}_{\text{(NNLL)}} \;;\; \underbrace{\alpha_s^2, \alpha_s\beta, \beta^2}_{\text{(NNLL', N}^3\text{LL)}} \;; \ldots\Big\} :$$

NNLL resummation

- Mellin-space resummation of threshold logarithms $\alpha_s \log\beta$

  (Czakon/Mitov/Sterman 09/Cacciari et al. 11)

  implemented in TOP++ (Czakon/Mitov)

- Threshold logs and Coulomb corrections α_s/β

  (Beneke/Falgari/(Klein)/CS 09/11)

  implemented in TOPIXS (Beneke et al.)

- Resummation for p_T, $M_{t\bar{t}}$ distributions (Kidonakis; Ahrens et al.;

  low p_T: Zhu et al; Catani et al; boosted tops: Ferroglia et al. ⇒ talk by Darren Scott)

Reduction of scale uncertainty from threshold resummation

$$\sigma_{t\bar{t}}^{\rm NNLO}(13{\rm TeV}) = 802.83^{+28.10(3.5\%)}_{-44.85(5.6\%)}{\rm pb} \Rightarrow \begin{cases} {\rm NNLL(top++)}: & 821.37^{+20.28(2.5\%)}_{-29.60(3.6\%)}{\rm pb} \\ {\rm NNLL(topixs)}: & 806.96^{+25.59(3.2\%)}_{-40.36(5.0\%)}{\rm pb} \end{cases}$$

`top++`: Mellin space resummation (Sterman 87; Catani/Trentadue 89)

- Includes 2-loop constant term H_2 in threshold expansion

$$\sigma_{t\bar{t}}^{\rm NLLL}|_{H_2=0} = 812.20 \text{ pb}$$

NNLL vs NNLO

Reduction of scale uncertainty from threshold resummation

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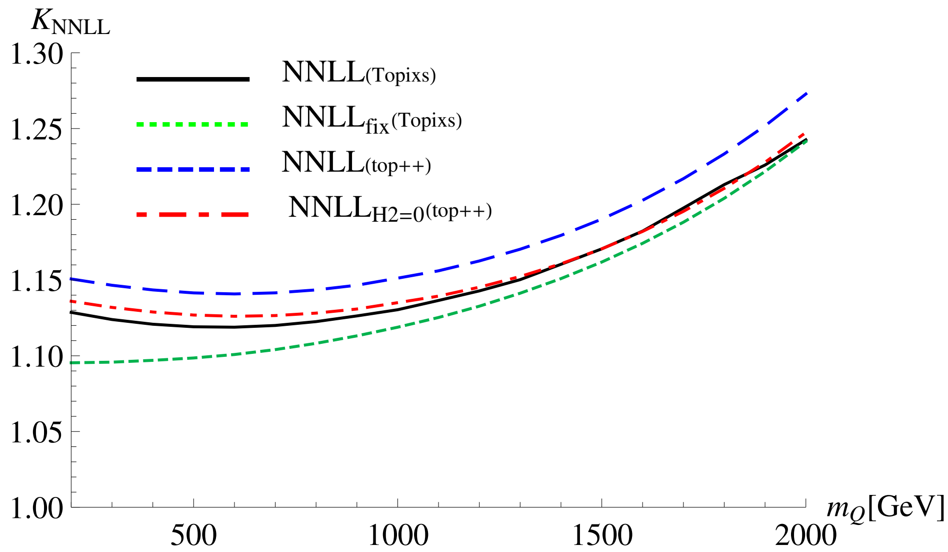
`topixs:` combined soft/Coulomb resummation

- RGE for momentum-space resummation (Becher/Neubert 06)
- dependence on scales μ_f, $\mu_h \sim 2M$: $\Delta_{\rm scale}\sigma_{t\bar{t}}^{\rm NNLL} = {}^{+15.64}_{-37.71}$ pb
- resummation uncertainty: choice of $\mu_s \sim M\beta^2$, kinematic ambiguities, higher-order terms: $\Delta_{\rm res}\sigma_{t\bar{t}}^{\rm NNLL} = {}^{+20.26}_{-14.37}$ pb
- Includes bound-state effects $\sigma_{t\bar{t}}^{\rm NNLL}|_{\rm BS} = 2.8$ pb

Heavy Quarks as test case for resummation methods



($K_{\mathrm{NNLL}} = \sigma^{\mathrm{NNLL}}/\sigma^{\mathrm{NLO}}$,

LHC $\sqrt{s} = 8$ TeV)

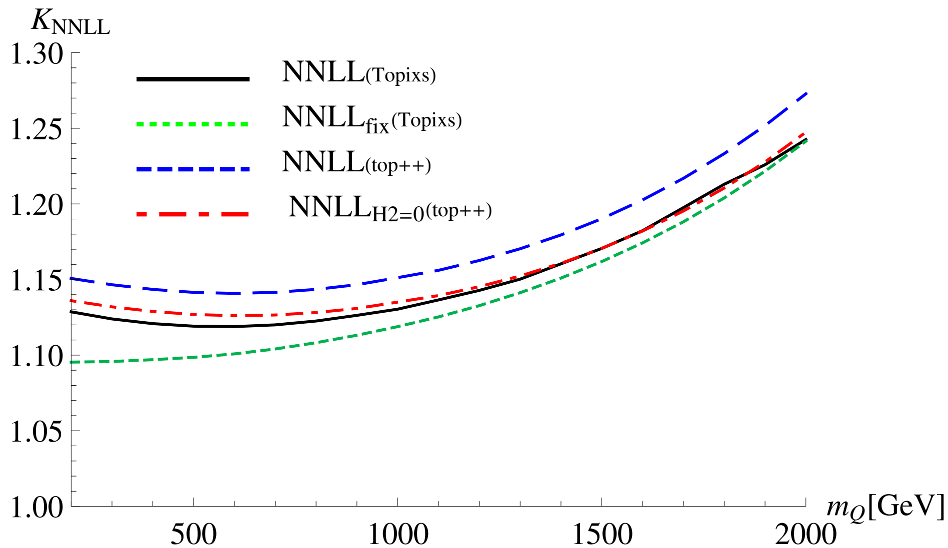
NNLL: momentum-space, running $\mu_s = 2m_Q\beta^2$ (Topixs default)

NNLL$_{\mathrm{fix}}$: momentum-space, fixed μ_s (Topixs)

NNLL (top++): Mellin-space (Cacciari et al. 11; Czakon/Mitov 11-13)

NNLL$_{H_2=0}$ (top++): Mellin-space, two-loop constant term set to zero

Heavy Quarks as test case for resummation methods



$(K_{\mathrm{NNLL}} = \sigma^{\mathrm{NNLL}}/\sigma^{\mathrm{NLO}}$,

LHC $\sqrt{s} = 8$ TeV)

⇒ resummation methods agree well for larger masses

- differences at m_t: estimate of resummation ambiguities
- main difference: treatment of $H_2 \Rightarrow \alpha_s^3 \log \beta^2$ terms (NNLL')

⇒ **Upgrade** `Topixs` to NNLL'/partial N³LL

- First step: expansion to N³LO

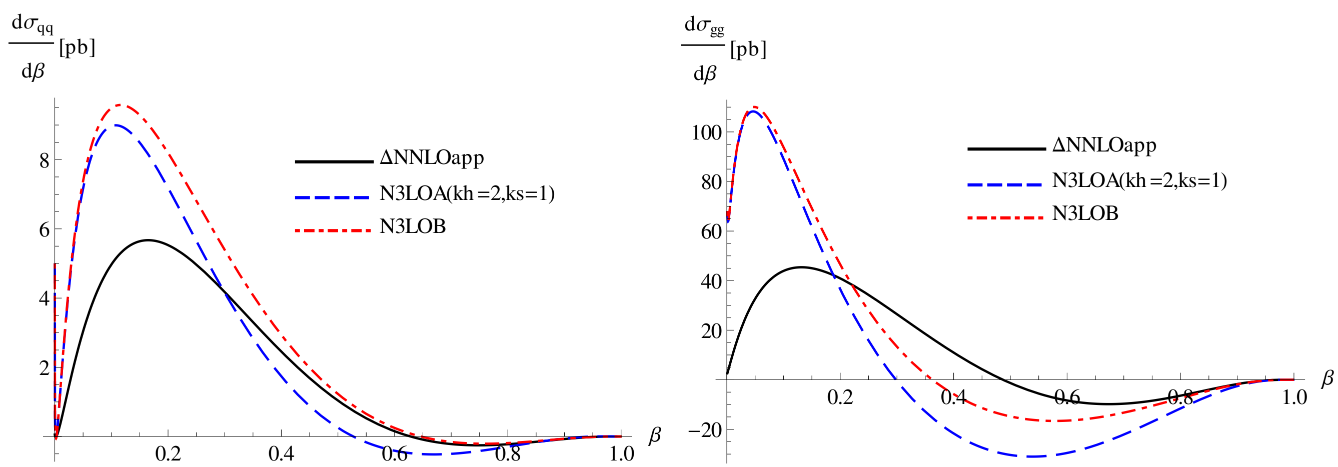
N3LO$_{\text{approx}}$?

Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g. (Beneke/Falgari/Klein/CS 11)

$$\Delta\sigma^{(3)}_{gg_8,\text{NNLL}} = \sigma^{(0)}_{gg_8} \frac{\alpha_s^3}{(4\pi)^3} \Big\{ 147456.\ln^6\beta - 169658.\ln^5\beta - 140834.\ln^4\beta + 524210.\ln^3\beta$$
$$+ \frac{1}{\beta}\left[-15159.7\ln^4\beta - 5364.82\ln^3\beta + 19598.9\ln^2\beta - 17054.7\ln\beta\right]$$
$$+ \frac{1}{\beta^2}\left[346.343\ln^2\beta + 522.978\ln\beta - 71.7884\right]\Big\} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

N^{3}LO$_A$: keep all terms, including μ_s, μ_h-dependence and constants

N^{3}LO$_B$: only keep terms known exactly



Numerical results: (MSTW08)

$$\sigma_{t\bar{t}}^{\text{NNLO}}(8\text{TeV}) = 239.18^{+\ 9.29(3.9\%)}_{-14.85(6.2\%)}\text{pb} \Rightarrow \begin{cases} \text{N3LO}_A: & 244.87^{+3.5(1.5\%)}_{-6.7(2.8\%)}(\mu_f) + {}^{+3.8}_{-12.1}(\mu_s)\text{pb} \\ \text{N3LO}_B: & 245.90^{+6.7(2.7\%)}_{-5.0(2.0\%)}\text{pb} \end{cases}$$

Other N^{3}LO approximations

- Using NNLL in one-particle inclusive kinematics (Kidonakis 14)

$$\sigma_{t\bar{t},1\text{PI}}^{\text{N3LOapp}}(8\text{TeV}) = 248^{+7(2.8\%)}_{-8(3.2\%)}\text{pb} \qquad \text{(MSTW08)}$$

- Combination of threshold resummation with subleading terms and large-x behaviour (Muselli et al. 15)

$$\sigma_{t\bar{t}}^{\text{N3LOapp}}(8\text{TeV}) = 253.98\text{pb} \pm 3.5\% \qquad \text{(NNPDF3.0)}$$

NNLO with "principle of maximal conformality":
scale choice minimizing μ_r-dependence (Brodsky/Wu 12; Wang et al. 17)

$$\sigma_{t\bar{t},\text{PMC}}^{\text{NNLO}}(8\text{TeV}) = 249\text{pb} \qquad \text{(CT14)}$$

Factorization of cross section (Beneke, Falgari, CS 09/10)

into hard function H, soft function W and potential function J

$$\Rightarrow \hat{\sigma}_{pp'\to t\bar{t}}|_{\hat{s}\to 4m_t^2} = \sum_{R=1,8} H_R(m_t,\mu) \int d\omega\, J_R(\underbrace{\sqrt{\hat{s}}-2m_t}_{E\approx m_t\beta^2} - \frac{\omega}{2}) W^R(\omega,\mu)$$

- derived using soft-collinear and non-relativistic effective field theories

  (for S-wave production and up to NNLL)

- can perform simultaneous summation of threshold Logs and Coulomb corrections
- soft function for single heavy particle in 1 or 8 representation

Resummation in momentum-space approach (Becher/Neubert 06)

- Evolve soft function from $\mu_s \sim m_t\beta^2$ to $\mu_f \sim m_t$
- Evolve hard function from $\mu_h \sim 2m_t$ to μ_f

Joint soft/Coulomb resummation for $\alpha_s \log\beta \sim 1$, $\frac{\alpha_s}{\beta} \sim 1$

- interplay of Coulomb $(\alpha_s/\beta)^n$and power corrections $\sim \beta^l$
- logarithmic NNLO contributions from $\alpha_s\beta$ potentials
- no $\alpha_s/\beta \times \alpha_s \log^{2,1}\beta \times \beta$ corrections
  to soft NNLL resummation for σ_{tot}, $d\sigma/dM_{t\bar{t}'}$ (Beneke/Falgari/CS 10)
- Known corrections relevant for N^{3}LO threshold expansion

$$\frac{\alpha_s^2}{\beta^2} \times \alpha_s\beta^2 \log\beta \sim \alpha_s^3 \log\beta$$

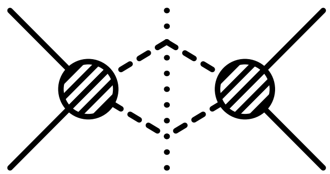
  - "next-to-eikonal" effects in DY/Higgs
    (Krämer/Laenen/Spira 98; Laenen et al. 10)
  - (ultra)-soft corrections as in $e^-e^+ \to t\bar{t}$ (Beneke/Kiyo 08)
- systematic treatment: extended factorization

$$\sigma = \sum_{ijklm} B_1^{(i)} B_2^{(j)} H^{(k)} \otimes W^{(l)} J^{(m)}$$

(Recent discussion in SCET: Larkoski et al. 14; Feige et al./Moult et al. 17)

Hard function

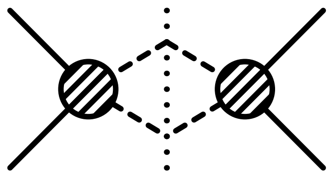
from matching coefficients of matrix element to EFT:

$H_i =$  $\sim |C^i|^2\,, \qquad \mathcal{A} = \sum_i C^{(i)} \langle \tilde{s}\tilde{s}' X|\mathcal{O}^{(i)}|pp'\rangle_{\text{EFT}}$

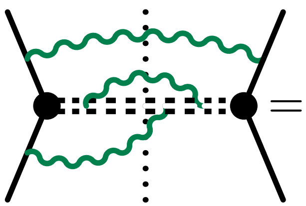
- one-loop H_i known (Czakon/Mitov 08; also Hagiwara et.al. 08)
- two-loop H_i from constant in NNLO threshold expansion
  (Bärnreuther/Czakon/Fiedler 13)
  if all other contributions to constant are known

Hard function

from matching coefficients of matrix element to EFT:

$H_i =$  $\sim |C^i|^2 \,,$ $\qquad \mathcal{A} = \sum_i C^{(i)} \langle \tilde{s}\tilde{s}' X | \mathcal{O}^{(i)} | p p' \rangle_{\text{EFT}}$

Soft function for single particle with total colour charge R:

${W_i}^R =$  $= \int \frac{dz^0}{4\pi} e^{i\omega z_0/2} \langle 0 | \overline{\mathsf{T}}[S_v^{(R_\alpha)} S_{\bar{n}}^{(r')\dagger} S_n^{(r)\dagger}](0) \mathsf{T}[S_n^{(r)} S_{\bar{n}}^{(r')} S_v^{(R_\alpha)\dagger}](z^0)|0\rangle$

with soft Wilson lines $S_n^{(r)}(x) = \mathsf{P} \exp\left[i g_s \int_{-\infty}^0 dt\, n \cdot A_s^a(x+nt) T^{(r)a} \right]$

- 1-loop soft function for arbitrary R (Beneke/Falgari/CS 09)
- 2-loop soft function for singlet (Belitzky 98;Becher/Neubert/Xu 07)
  octet (Czakon/Fiedler 13)

IR singularities of amplitude determine RGE of hard function

$$\frac{d}{d\ln\mu}H^{R,S}_{pp'}(\mu)=\left(\gamma_{\text{cusp}}(C_r+C_{r'})\ln\left(\frac{4m_t^2}{\mu^2}\right)+2(\gamma^p+\gamma^{p'}+\gamma^R_{H,s})+\gamma^{R,S}_J\right)H^R_{pp'}(\mu).$$

RGE for soft function

$$\frac{d}{d\log\mu}W_i^{R_\alpha}(z^0,\mu)=\left(2\gamma_{\text{cusp}}(C_r+C_{r'})\log\left(\frac{iz_0\bar\mu}{2}\right)-2(\gamma^{R_\alpha}_{H.s}+\gamma^r_s+\gamma^{r'}_s\right)W_i^{R_\alpha}(z^0,\mu)$$

Known input

- 3-loop γ_{cusp}, γ_s^r γ^p (Moch, Vermaseren, Vogt 04/05)
  2-loop $\gamma^{R_\alpha}_{H.s}$ (Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)
- 2-loop $\gamma_J^{R,S}$ from additional IR divergences from factorization of potential contributions (⇒ see below)

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Missing for N^3LL

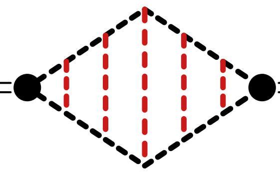
- 4-loop γ_{cusp} (large N_c result: Henn et al. 16)

  (usually approximated by Pade-approx.; not needed for N^3LO_{app})

- 3-loop $\gamma^{R_\alpha}_{H.s}$ (Massless result: Almelid/Duhr/Gardi 15)

- 3-loop $\gamma^{R,S}_J$ (colour singlet: Kniehl et al. 02/Hoang 03)

Potential function defined in terms of non-relativistic fields

$J_{R_\alpha}(E) =$  $= \int d^4z\, e^{iEz^0} \langle 0|[\psi^{(0)}\psi'^{(0)}](z^0) P^{R_\alpha} [\psi^{(0)\dagger}\psi'^{(0)\dagger}](0)|0\rangle$

$= 2\mathrm{Im}\, G_C^{R_\alpha}(0,0,E)$

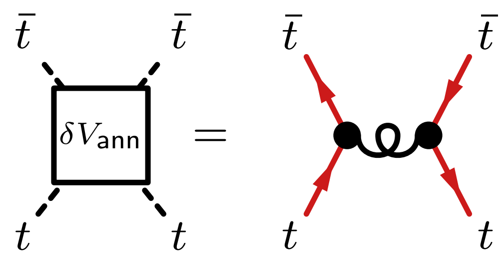
$\Rightarrow$ zero-distance Coulomb Green function resumms $\frac{\alpha_s}{\beta}$ corrections

Higher-order potentials

- one/two-loop Coulomb potential
- "non-Coulomb" potentials suppressed by $\alpha_s \frac{|\boldsymbol{q}|}{M}$, $\frac{\boldsymbol{q}^2}{M^2}$

  (Generalization from $e^-e^+ \to t\bar{t}$: Beneke et al. 2009)

- annihilation corrections from $t\bar{t} \to t\bar{t}$ scattering:



(reproduces result from Bärnreuther/Czakon/Fiedler 13)

Potential effects on $\log \beta$ terms in the NNLO cross section:

$$\Delta\hat{\sigma}^{(2)}_{\mathrm{nC+ann}} = \hat{\sigma}^{(0)}\,\alpha_s^2 D_{R_\alpha} \ln\left(\frac{\beta m_t}{\mu}\right)\left(C_A - 2D_{R_\alpha}(1+\nu^S_{\mathrm{spin}}) - \frac{1}{2}\nu^{R_\alpha,S}_{\mathrm{ann}}\right)$$

(Beneke et al. 09; Beneke/Piclum/CS/Wever 16)

explicit scale dependence (RGE in pNRQCD: Pineda 01)

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Expansion of NNLO potential function to $\mathcal{O}(\alpha_s^3)$

(using implementation of Beneke/Kiyo/Maier/Piclum 16)

$$\Delta J_R^{S(3)} \sim \alpha_s^3\left\{\frac{1}{\beta^2}\ln\left(\frac{\beta m_t}{\mu}\right), \frac{1}{\beta^2}, \frac{1}{\beta}\ln^2\left(\frac{\beta m_t}{\mu}\right), \frac{1}{\beta}\ln\left(\frac{\beta m_t}{\mu}\right), \frac{1}{\beta}\right\}$$

(for colour-singlet agreement with direct calculation by Kiyo/Maier/Maierhöfer/Marquardt 09)

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Contributions from $\mathrm{N^3LO}$ potential function relevant at $\mathrm{N^3LL}$

$$\Delta J^{S(3)}_{R,\mathrm{N^3LO}} \sim \alpha_s^3\left\{\ln^2\left(\frac{\beta m_t}{\mu}\right), \ln\left(\frac{\beta m_t}{\mu}\right)\ln\left(\frac{m_t}{\mu}\right), \ln\left(\frac{\beta m_t}{\mu}\right)\right\}$$

- Contributions from 2-loop potentials and ultrasoft corrections only known for colour singlet

Further effects

- No 3-loop Coulomb correction $\sim \alpha_s^3/\beta^3$ for $\Gamma_t \to 0$

  Careful treatment in distributional sense: (Beneke/Ruiz-Femenia 16)

$$\Delta J^{S(3)}_{R,\text{LO}}(E) = \alpha_3 \frac{m_t^3}{8} \zeta_3 \, \delta(E)$$

  Small correction to cross section: $\Delta\sigma = 0.18$ pb at 8 TeV.

- P-wave contributions $\sigma^{(0)}_{gg}((t\bar{t})^P) \sim \beta^3$

  Coulomb corrections different from S-wave (Bigi/Fadin/Khoze 92)

  $\Rightarrow$ contributions $\sim \alpha_s^2 \times$ const., $\sim \frac{\alpha_s^3}{\beta}$ relative to leading S-wave

- Sub-leading soft corrections to DY/Higgs production:

  (Krämer/Laenen/Spira 96; Laenen et al. 10)

$$\left[\frac{\ln(1-x)}{1-x}\right]_+ \to \left[\frac{\ln(1-x)}{1-x}\right]_+ - \ln(1-x)$$

  enhancement by second Coulomb correction $\Rightarrow \sim \alpha_s^3 \ln\beta$ effect

  Numerical effect $< 1\text{pb}$ at 8TeV

Terms predicted by expansion of NNLL'/N³LL (preliminary)

$$\Delta\sigma^{(3)}_{gg_8,\text{NNLL'}} = \Delta\sigma^{(3)}_{gg_8,\text{NNLL}} + \sigma^{(0)}_{gg_8}\frac{\alpha_s^3}{(4\pi)^3}\Big\{ -2743.36\frac{1}{\beta} + (157.914 J^{S=0,(3)}_{L2,8} - 299282.)\ln^2\beta + (1137.63\ln rs + 46613 + 157.914 J^{S=0,(3)}_{L,8})\ln\beta\Big\}$$

$J^{S=0,(3)}_{L2,8}$, $J^{S=0,(3)}_{L,8}$: coefficients of unknown $\ln^{2/1} E$ terms in N³LO potential function

remaining dependence on soft scale $\mu_s = r_s m_t \beta^2$

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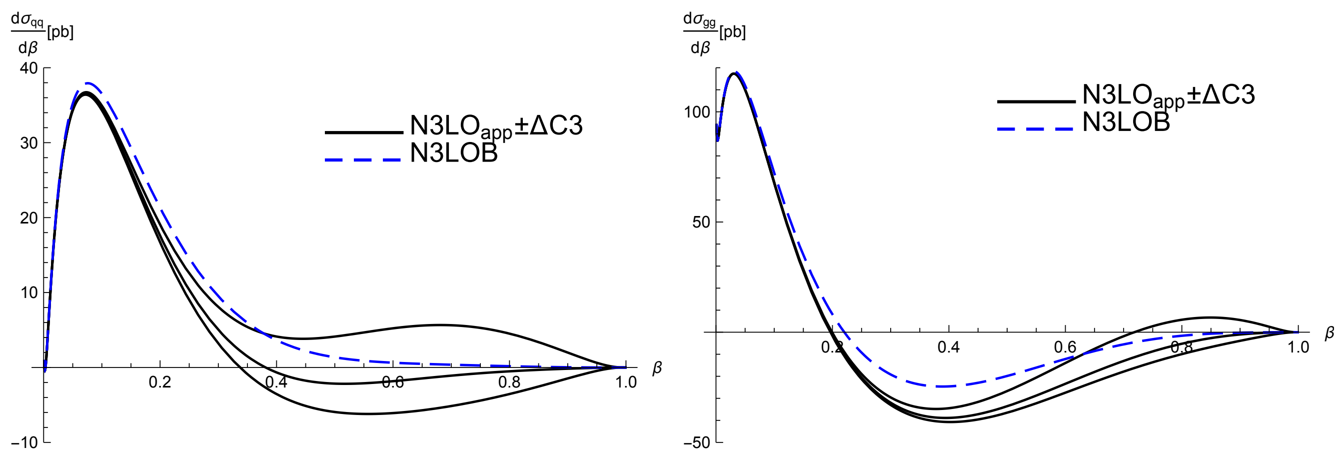


large cancellations ⇒ (accidentally) small leftover correction

$$\Delta\sigma^{\mathrm{N3LOapp}}_{t\bar{t}}(8\mathrm{TeV}) = -1.29\mathrm{pb} \qquad \text{(unknown constants zero)}$$

Estimate of $C^{(3)}$ by varying hard/soft scales: ${}^{+6.1}_{-2.8}\mathrm{pb}$

Estimate $J^{S,(3)}_{L2,R}$ from e^-e^+ results: effect $< 1\mathrm{pb}$

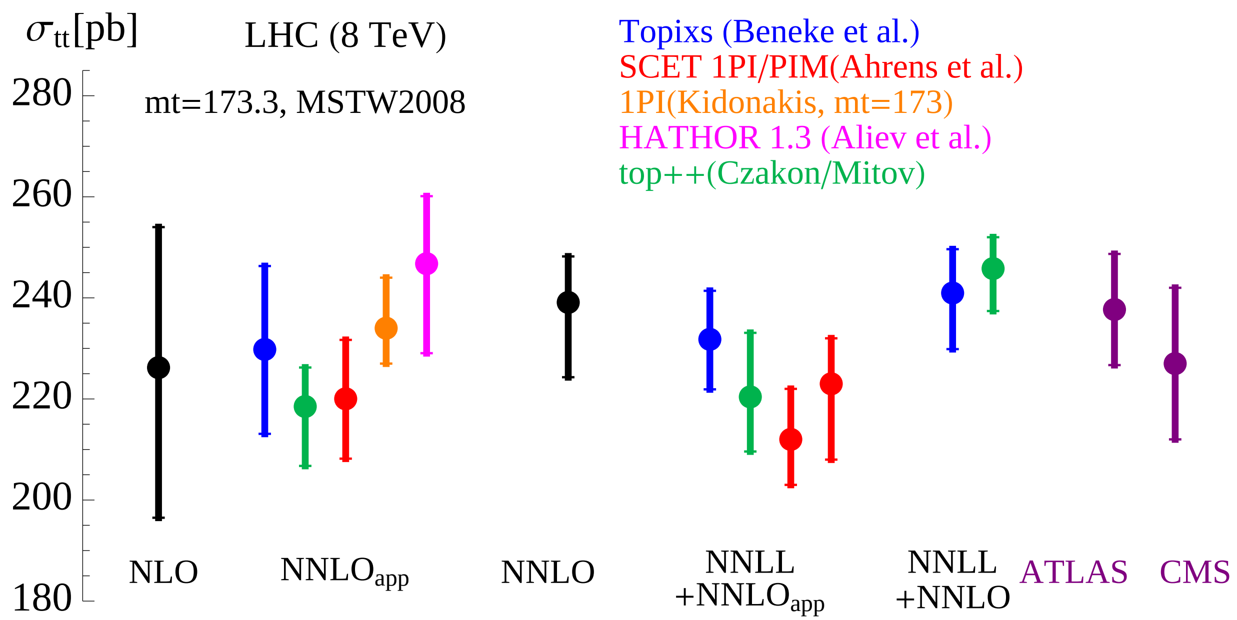
Summary and outlook

- **Experimental accuracy** of $\sigma_{t\bar{t}}$ comparable to NNLO+NNLL prediction
- construct partial N^{3}LL; approximate N^{3}LO
  - unknown: 3-loop anomalous dimensions, logarithmic terms in N^{3}LO Coulomb Green function
  - complete determination $\alpha_s^3 \ln^{2,1} \beta$ terms requires control over kinematically suppressed contributions
- **N^{3}LO$_{\mathrm{app}}$ results**
  - strong cancellations in integral over β

  $\Rightarrow$ corrections sensitive to power suppressed effects
  - estimate size of possible N^{3}LO corrections $\sim 5\mathrm{pb}$ at $8\mathrm{TeV}$.
- **Outlook**
  - estimate of uncertainties, μ_f dependence
  - implement NNLL'/N^{3}LL$_{\mathrm{part}}$ resummation.

Comparison of different approximations (excluding PDF+α_s uncertainties)

- $\pm 5\%$ scale uncertainty at NNLO; ± 3–4% at NNLL



Scale-dependence of approximate N³LO cross section

$$\hat{\sigma}^{(3),\text{app}}_{pp',R}(\beta,\mu_f) = \hat{\sigma}^{(0)}_{pp',R}\left(\frac{\alpha_s(\mu_f)}{4\pi}\right)^3 \sum_{m=0}^{3} f^{(3,m)}_{pp'(R)} \ln^m\left(\frac{\mu_f}{m_t}\right)$$

Obtained in two ways:

- Expansion of resummation formula
- Direct computation in $x \to 1$ limit of splitting functions

$$f^{(3,3)}_{pp} = \frac{1}{3}\left[8\beta^{(0)} f^{(2,2)}_{pp} - 2\overline{f}^{(2,2)}_{pp} \otimes P^{(0)}_{p/p}\right]$$

$$f^{(3,2)}_{pp} = 4\beta^{(0)} f^{(2,1)}_{pp} + 3\beta^{(1)} f^{(1,1)}_{pp} - \overline{f}^{(2,1)}_{pp} \otimes P^{(0)}_{p/p} - \overline{f}^{(1,1)}_{pp} \otimes P^{(1)}_{p/p}$$

$$f^{(3,1)}_{pp} = 8\beta^{(0)} f^{(2,0)}_{pp} + 6\beta^{(1)} f^{(1,0)}_{pp} + 4\beta^{(2)} f^{(0,0)}_{pp} - \overline{f}^{(2,0)}_{pp} \otimes P^{(0)}_{p/p} - \overline{f}^{(1,0)}_{pp} \otimes P^{(1)}_{p/p} - \overline{f}^{(0,0)}_{pp} \otimes P^{(2)}_{p/p}$$

with $P_{p/\tilde{p}}(x) \approx \left(2\Gamma^r_{\text{cusp}}(\alpha_s)\frac{1}{[1-x]_+} + 2\gamma^{\phi,r}(\alpha_s)\delta(1-x)\right)\delta_{p\tilde{p}}$

$(\overline{f}(z) \otimes P) = \frac{1}{\sqrt{1-z}}\int_z^1 \frac{dx}{x}\sqrt{1-\frac{z}{x}}\, f(\frac{z}{x})P(x)$

P-wave processes

P-wave contributions to $gg \to t\bar{t}$ in $R = 1, 8_s$ colour representations:

$$\sigma^{R(0)}(gg \to (t\bar{t})^{^3P_0}) = \sigma^{R(0)}(gg \to (t\bar{t})^{^1S_0})\ \beta^2,$$

$$\sigma^{R(0)}(gg \to (t\bar{t})^{^3P_2}) = \sigma^{R(0)}(gg \to (t\bar{t})^{^1S_0})\ \frac{4}{3}\beta^2.$$

LO-Coulomb Green function for P-waves: (Bigi/Fadin/Khoze 92)

$$J^P_{R_\alpha}(E) = m_t E \left(1 + \frac{(\alpha_s D_{R_\alpha})^2 m_t}{4E}\right) J_{R_\alpha}(E)$$

$$= m_t^4 \left(\frac{E}{m_t}\right)^{3/2} \left[1 + \frac{\alpha_s(-D_R)}{2}\sqrt{\frac{m_t}{E}} + \frac{\alpha_s^2 D_R^2 (3+\pi^2)}{12}\frac{m_t}{E} + \frac{\alpha_s^3 \pi (-D_R)^3}{8}\left(\frac{m_t}{E}\right)^{-3/2} \ldots\right]$$

$\Rightarrow$ contributions $\sim \alpha_s^2 \times$ const., $\sim \frac{\alpha_s^3}{\beta}$ relative to leading S-wave

- NLL resummation sufficient for N^3LO_{app}
- no formal proof for NNLL resummation (see Falgari/CS/Wever 12)

Subleading PNRQCD and SCET interactions:

$$\psi^\dagger \vec{x}\cdot\vec{E}_{us}(x_0,0)\psi'^{\,\dagger}\,,\qquad \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, gF^{\rm us}_{\mu\nu} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi\,\ldots$$

Soft gluons not decoupled by field redefinitions.

(Higher order interactions can be treated as perturbation to LO Lagrangian $\Rightarrow$ factorized cross section with higher order soft and potential functions $\hat{\sigma}=\sum_a H^{(a)}\otimes W^{(a)}\otimes J^{(a)}$

Recent discussion in SCET: Larkoski et al. 14; Feige et al./Moult et al. 17)

Not relevant for $\sigma_{\rm tot}$ at NNLL (Beneke, Czakon,Falgari, Mitov, CS 09)

Known corrections relevant for N^{3}LO threshold expansion

$$\frac{\alpha_s^2}{\beta^2}\times\alpha_s\beta^2\log\beta\sim\alpha_s^2\log\beta$$

- "next-to-eikonal" effects in DY/Higgs

  (Krämer/Laenen/Spira 98; Laenen et al. 10)

- (ultra)-soft corrections in $e^-e^+\to t\bar{t}$ (Beneke/Kiyo 08)

Potential function

Resummation of $\frac{\alpha_s}{\beta}$ corrections: (Fadin, Khoze 87; Peskin, Strassler 90)

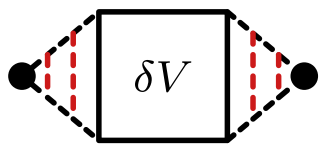
solve NR-Schrödinger equation for Green's function

$$-\left(\frac{\vec{\partial}_r^{\,2}}{2m_r}+E\right)G^{(0)}_{R_\alpha}(E,\vec{r},\vec{r}\,')-\frac{\alpha_s D_{R_\alpha}}{r}G^{(0)}_{R_\alpha}(E,\vec{r},\vec{r}\,')=(2\pi)^3\delta^3(\vec{r}-\vec{r}\,')$$

with Coulomb coefficients $D_1=-C_F$; $D_8=\frac{1}{2}(C_A-2C_F)=\frac{1}{2N_C}$

NLO potential function from perturbation theory

$$\delta G^{(1)}_{R_\alpha}(0,0,E)=$$



$$=\int d^3z\, G^{(0)}_{R_\alpha}(0,\vec{z},E)\,(i\delta V^{R_\alpha}(\vec{z}))\,iG^{(0)}_{R_\alpha}(\vec{z},0,E)$$

- all terms $\alpha_s(\alpha_s/\beta)^n$

NNLO Green function: (using implementation of Beneke/Kiyo/Maier/Piclum 16)

- double/single insertions of NLO/NNLO potentials
- all terms $\alpha_s^2(\alpha_s/\beta)^n$, $\alpha_s^2\ln\beta(\alpha_s/\beta)^n$

NNLO potential function explicitly scale-dependent:

$$\frac{d}{d\ln\mu} J_R^S(E) = -\gamma_J^{R,S} J_R^S(E)$$

$$\gamma_J^{R,S(1)} = -(4\pi)^2 D_R \left(2D_R\left(\nu_{\rm spin}^S + \frac{5}{4}\right) + \frac{\nu_{\rm ann}^{R_\alpha,S}}{2} + b_1^R\right)$$

$\mathcal{O}(\alpha_s^2)$ limit of NLL anomalous dimension in pNRQCD (Pineda 01)

Expansion of NNLO potential function to α_s^3

$$\Delta J_{R,NNLO}^{S(3)}(E) = J^{(0}(E)\frac{\alpha_s^3(\mu)}{4\pi}\left\{\frac{m_t}{E}\frac{D_R^2}{6}\left[\pi^2\left(2\beta_0 L_E + a_1\right) - 12\beta_0\zeta_3\right]\right.$$
$$\left. + \sqrt{\frac{m_t}{E}} D_R\left[-\frac{1}{2}\beta_0^2 L_E^2 + \frac{1}{8}\left(\gamma_J^{R,S(1)} - 2\beta_1 - 4a_1\beta_0\right) L_E + \text{const.}\right]\right\}$$

with $L_E = -\frac{1}{2}\ln\left(\frac{4Em_t}{\mu^2}\right)$