Dire status report

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Parton Showers and Resummation 2017

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U.S. DEPARTMENT OF ENERGY
Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

Executive summary

Basic layout of Dire [Prestel,SH] arXiv:1506.05057

- Dipole-like parton shower, kernels as close as possible to DGLAP
- Partial fraction soft eikonal à la Catani-Seymour, evolve in dipole-k_T
- Two independent implementations (Pythia & Sherpa)
- Cross-validation at particle level

New developments [Krauss,Prestel,SH] any time soon

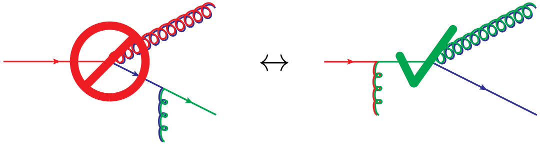
- MC counterterms implemented in Amegic & Comix
- MC@NLO matching & NLO subtraction available in Sherpa
- UNLOPS / MEPS@NLO merging being developed in Pythia / Sherpa
- 3-loop cusp term and integrated NLO DGLAP kernels included[1]
- Flavor-changing triple collinear splitting functions included

[1] Working on corrections to account for choice of dipole-k_T as evolution variable

Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved by gluons of wavelength larger than the dipole size
  $\rightarrow$ emission off combined mother parton instead

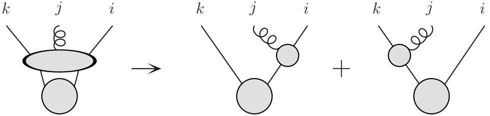


- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
  $\rightarrow$ Soft anomalous dimension halved due to reduced phase space
- Formerly implemented by angular ordering / angular veto
  [Webber at al.] hep-ph/0210213, [Sjöstrand et al.] hep-ph/0603175
- Alternative description in terms of color dipoles
  [Gustafsson,Pettersson] NPB306(1988)746, [Kharraziha,Lönnblad] hep-ph/9709424
  [Winter,Krauss] arXiv:0712.3913

The midpoint between dipole and parton showers

- Angular ordered / vetoed parton shower does not fill full phase space
  Dipole shower lacks parton interpretation $\rightarrow$ prefer alternative to both
- Can preserve parton picture by partial fractioning soft eikonal
  $\leftrightarrow$ soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \rightarrow \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j}$$



- "Spectator"-dependent kernels, singular in soft-collinear region only
  $\rightarrow$ capture dominant coherence effects (3-parton correlations)

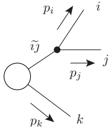
$$\frac{1}{1-z} \rightarrow \frac{1-z}{(1-z)^2 + \kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- For correct soft evolution, ordering variable must be identical
  at both "dipole ends" ($\rightarrow$ recover soft eikonal at integrand level)

The midpoint between dipole and parton showers

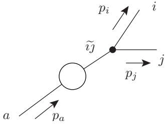
Choose parametrization such that soft term is $\frac{1-z}{(1-z)^2+\kappa^2}$ in all dipole types

(1) FF



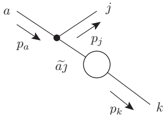
$$\kappa^2 = \frac{p_ip_j\, p_jp_k}{(p_{\widetilde{ij}}p_{\tilde{k}})^2} \qquad z_j = \frac{p_jp_k}{p_{\widetilde{ij}}p_{\tilde{k}}}$$

(2) FI



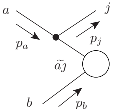
$$\kappa^2 = \frac{p_ip_j\, p_jp_a}{(p_{ij}p_a)^2} \qquad z_j = \frac{p_jp_a}{p_{ij}p_a}$$

(3) IF



$$\kappa^2 = \frac{p_ap_j\, p_jp_k}{(p_{jk}p_a)^2} \qquad z_j = \frac{p_jp_k}{p_{jk}p_a}$$

(4) II



$$\kappa^2 = \frac{p_ap_j\, p_jp_b}{(p_ap_b)^2} \qquad z_j = \frac{p_jp_b}{p_ap_b}$$

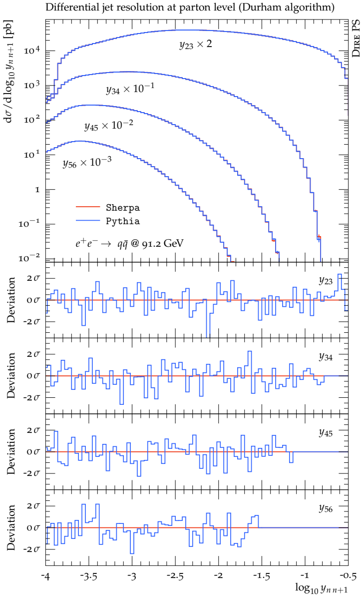
Preserve collinear anomalous dimensions & sum rules → splitting functions fixed

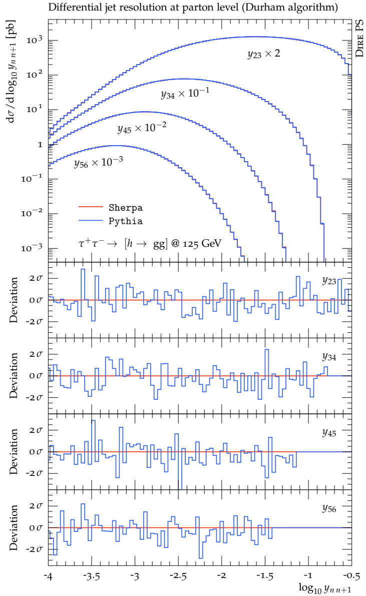
$$P_{qq}(z,\kappa^2) = 2\,C_F\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ - \frac{1+z}{2}\right] + \gamma_q\,\delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z)\right] + \gamma_g\,\delta(1-z)$$

$$P_{qg}(z,\kappa^2) = 2\,C_F\left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2}\right] \qquad P_{gq}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

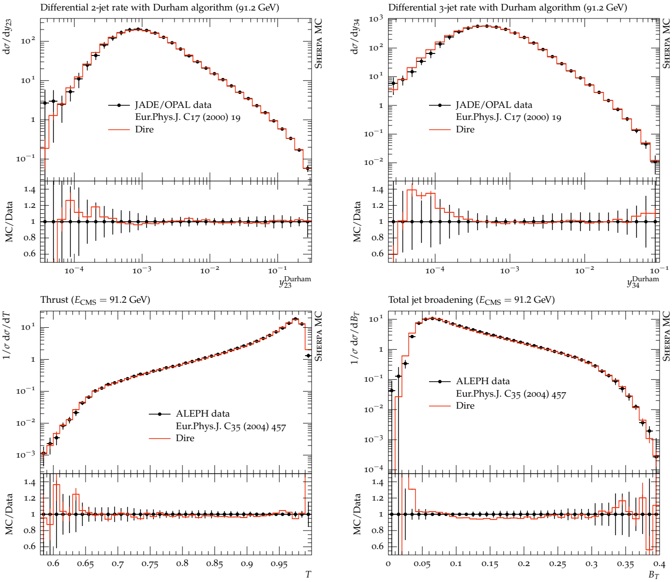
Validation in $e^+e^- \to$ hadrons





Comparison to LEP data

[Prestel,SH] arXiv:1506.05057



Including higher-order splitting functions

- NLO DGLAP kernels known since long

  [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
  [Floratos,Kounnas,Lacaze] NPB192(1981)417

  - Incorrect scheme for parton showers evolving in k_T, but
  - Correction terms straightforward ($\nearrow$ later)
- Focus on MC implementation technology first
  - Subtract 2-loop cusp term from NLO kernels and combine with LO soft (CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635)
    Include 3-loop term in the same way [Moch,Vermaseren,Vogt] hep-ph/0403192
  - Redefine time-like Sudakovs to recover NLO DGLAP evolution

    [Jadach,Skrzypek] hep-ph/0312355
  - Mostly negative NLO corrections require weighted veto algorithm

    [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
  - Flavor changing splitting functions require $2 \to 4$ transitions

    [Prestel,SH] any time soon
- Unsurprisingly the $2 \to 4$ technology took longest to develop

Connection to NLO DGLAP evolution

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- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}x\,D_a(x,t)}{\mathrm{d}\ln t}=\sum_{b=q,g}\int_0^1\mathrm{d}\tau\int_0^1\mathrm{d}z\,\frac{\alpha_s}{2\pi}\left[zP_{ab}(z)\right]_+\tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Define plus prescription $\left[zP_{ab}(z)\right]_+=\lim_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon)=P_{ab}(z)\,\Theta(1-z-\varepsilon)-\delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

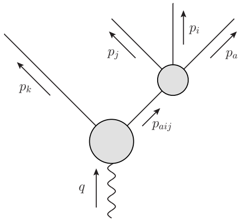
- Rewrite for finite ε

$$\frac{1}{D_a(x,t)}\frac{\mathrm{d}D_a(x,t)}{\mathrm{d}\ln t}=-\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)$$
$$+\sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is derivative of Sudakov $\rightarrow$ beyond LO forward Sudakovs must be computed with symmetry factors replaced by z [Jadach,Skrzypek] hep-ph/0312355

$q\bar{q}$ mixing and new splitting kernels

- New topology at NLO from $q \to \bar{q}$ and $q \to q'$ splittings
- Generic $1 \to 3$ process in parton shower $2 \to 4$ process in dipole(-like) shower
- First branching treated as soft gluon radiation, second as collinear splitting (to match diagrammatic structure)
- Requires new kinematics and suitable phase-space parametrization, set up similar to [Dittmaier] hep-ph/9904440



$2 \to 4$ kinematics mapping

- Define evolution & splitting variables

$$t = \frac{4\, p_j p_{ai}\, p_{ai} p_k}{q^2 - m_{aij}^2 - m_k^2}\;, \qquad z_a = \frac{2\, p_a p_k}{q^2 - m_{aij}^2 - m_k^2}$$

$$s_{ai} = 2\, p_a p_i + m_a^2 + m_i^2\;, \qquad x_a = \frac{p_a p_k}{p_{ai} p_k}$$

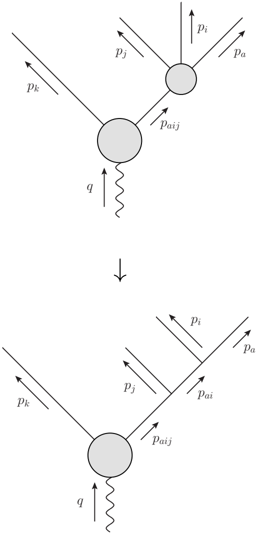
- First branching $(\widetilde{aij}, \tilde{k}) \to (ai, j, k)$ constructed with $m_{ai}^2 \to s_{ai}$, using
  [Catani,Dittmaier,Seymour,Trocsanyi] hep-ph/0201036
  [Prestel,SH] arXiv:1506.05057

$$y = \frac{t\, x_a/z_a}{q^2 - s_{ai} - m_j^2 - m_k^2}$$

$$\tilde{z} = \frac{z_a/x_a}{1-y}\,\frac{q^2 - m_{aij}^2 - m_k^2}{q^2 - s_{ai} - m_j^2 - m_k^2}\;.$$

- Second step now a decay $(ai, k) \to (a, i, k)$ can use CDST algorithm with

$$y' = \left[1 + \frac{z_a}{x_a}\,\frac{q^2 - m_{aij}^2 - m_k^2}{s_{ai} - m_a^2 - m_i^2}\right]^{-1}\;, \qquad \tilde{z}' = x_a$$



$2 \to 4$ phase space

- Phase space factorization derived similar to [Dittmaier] hep-ph/9904440
  $\to$ s-channel factorization over p_{aij}, subsequently over p_{ai}

$$\int \mathrm{d}\Phi(p_a,p_i,p_j,p_k|\,q) = \int \frac{\mathrm{d}s_{aij}}{2\pi} \int \mathrm{d}\Phi(p_{aij},p_k|\,q) \int \mathrm{d}\Phi(p_a,p_i,p_j|\,p_{aij})$$
$$= \int \mathrm{d}\Phi(\tilde{p}_{aij},\tilde{p}_k|\,q) \int \Big[\mathrm{d}\Phi(p_a,p_i,p_j|\,\tilde{p}_{aij},\tilde{p}_k)\Big]$$

- Obtain almost fully factorized form

$$\int \Big[\mathrm{d}\Phi(p_a,p_i,p_j|\,\tilde{p}_{aij},\tilde{p}_k)\Big] =$$
$$\left[\frac{J^{(1)}_{\mathrm{FF}}}{4(2\pi)^3}\int\frac{\mathrm{d}t}{t}\int \mathrm{d}z_a\int \mathrm{d}\phi_j\right]\left[\frac{1}{4(2\pi)^3}\int \mathrm{d}s_{ai}\int\frac{\mathrm{d}x_a}{x_a}\int \mathrm{d}\phi_i\, J^{(2)}_{\mathrm{FF}}\right] 2\,p_{ai}p_j$$

- Simple Jacobians that reduce to unity for massless partons

$$J^{(1)}_{\mathrm{FF}} = \frac{q^2-m^2_{aij}-m^2_k}{\sqrt{\lambda(q^2,m^2_{aij},m^2_k)}}\;, \qquad J^{(2)}_{\mathrm{FF}} = \frac{s_{aik}-s_{ai}-m^2_k}{\sqrt{\lambda(s_{aik},s_{ai},m^2_k)}}\;.$$

- Combination with massless matrix element in collinear limit leads to

$$\int \mathrm{d}\Phi(p_a, p_i, p_j, p_k|\, q)\, |M_{n+2}(a,i,j,k|\, q)|^2$$
$$= \int \frac{\mathrm{d}t}{t} \int \mathrm{d}z_a \int \mathrm{d}s_{ai} \int \frac{\mathrm{d}x_a}{x_a} \int \frac{\mathrm{d}\phi_i}{2\pi}\, \frac{2p_{ai}p_j}{s_{aij}}$$
$$\times \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{P_{(aij)a}(p_a,p_i,p_j)}{s_{aij}} \int \mathrm{d}\Phi(\tilde{p}_{aij}, \tilde{p}_k|\, q)\, |M_n(\widetilde{aij}, \tilde{k}|\, q)|^2$$

- Write as differential branching probability

$$\frac{\mathrm{d}\ln \Delta^{1\to 3}_{(aij)a}}{\mathrm{d}\ln t} = \int \mathrm{d}z_a\, z_a \int \mathrm{d}s_{ai} \int \frac{\mathrm{d}x_a}{x_a} \int \frac{\mathrm{d}\phi_i}{2\pi} \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{P_{(aij)a}(p_a,p_i,p_j)}{s^2_{aij}/\,2\,p_{ai}p_j}$$

- LO PS accounts for iterated collinear limit, hence we must subtract

$$\frac{\mathrm{d}\ln \Delta^{(1\to 2)^2}_{(aij)a}}{\mathrm{d}\ln t} = \int \mathrm{d}z_a\, z_a \int \frac{\mathrm{d}s_{ai}}{s_{ai}} \int \frac{\mathrm{d}\xi}{\xi} \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{\sum_{(ai)} P^{(0)}_{(aij)(ai)}(\xi) P^{(0)}_{(ai)a}(z_a/\xi)}{s_{aij}/\,2\,p_{ai}p_j}$$

Collinear factorization and resummation

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- Simplest possible configuration $q \to q'$ [Catani,Grazzini] hep-ph/9908523

$$P_{qq'} = \frac{1}{2} C_F T_R \frac{s_{aij}}{s_{ai}} \left[-\frac{t_{ai,j}^2}{s_{ai} s_{aij}} + \frac{4\, z_j + (z_a - z_i)^2}{z_a + z_i} + (1 - 2\varepsilon) \left(z_a + z_i - \frac{s_{ai}}{s_{aij}} \right) \right]$$

where $(z_a + z_i)\, t_{ai,j} = 2(z_a s_{ij} - z_i s_{aj}) + (z_a - z_i) s_{ai}$

- Apparent collinear singularity in s_{ai} that cancels upon azimuthal averaging against iterated LO splitting
- But integrand locally divergent $\to$ not amenable to MC simulation
- Solved by subtraction of spin-correlated LO splitting functions [Somogyi,Trocsanyi,del Duca] hep-ph/0502226

$$P_{qg}^{\mu\nu} = C_F \left[-2 \frac{z}{1-z} \frac{k_T^\mu k_T^\nu}{k_T^2} + \frac{1-z}{2} \left(-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{np} \right) \right]$$

$$P_{gq}^{\mu\nu} = T_R \left[-g^{\mu\nu} + 4\, z(1-z) \frac{k_T^\mu k_T^\nu}{k_T^2} \right]$$

- Leads to additional subtraction term

$$\Delta P_{qq'} = C_F T_R \frac{4 z_a z_i z_j}{(1 - z_j)^3} \left(1 - 2 \cos^2 \phi \right) , \qquad \cos\phi = \frac{s_{ai} s_{jk} + s_{ak} s_{ij} - s_{aj} s_{ik}}{\sqrt{4\, s_{ai} s_{ak}\, s_{ij} s_{jk}}}$$

- Reference for $q \to q'$ upon integration over s_{ai}, x_a, ϕ_j given by NLO kernel

$$P_{qq'}(z) = C_F T_R \left[(1+z)\log^2 z - \left(\frac{8}{3}z^2 + 9z + 5\right)\log z + \frac{56}{9}z^2 + 4z - 8 - \frac{20}{9z} \right]$$

- So far we have

$$P_{qq'}(z) = -C_F T_R \left[5(1-z) + 2(1+z)\log z \right]$$

- The difference lies in $\mathcal{O}(\varepsilon^0)$ contributions from renormalization $\times$ LO

$$\Delta P^{(\mathrm{R})}_{qq'} = \int_{z_a} \frac{\mathrm{d}\xi}{\xi} C_F \left(\frac{1+(1-\xi)^2}{\xi} \log(\xi(1-\xi)) + \xi \right) P_{gq}(z_a/\xi)$$

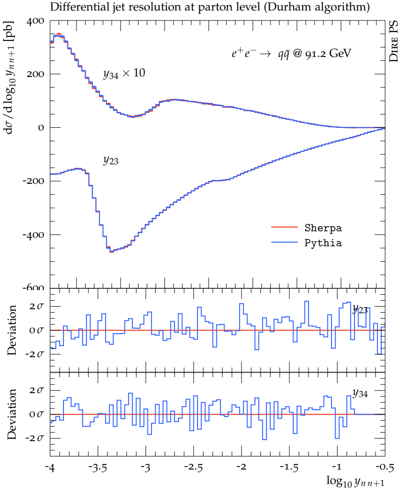
- ... and in integrating the iterated LO kernels over the D-dimensional $1 \to 3$ phase space as required for local subtraction vs. integrating over the D-dimensional $1 \to 2$ phase space as required for $\Delta P^{(\mathrm{R})}_{qq'}$

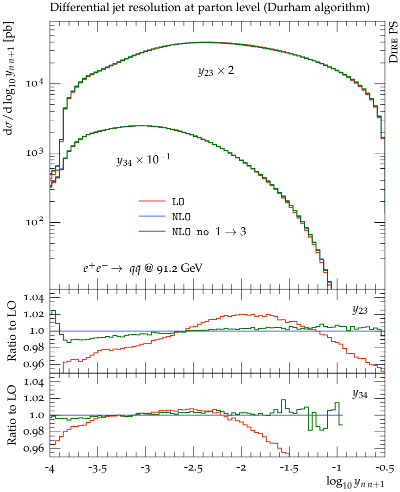
$$\int \frac{dx_a}{x_a} C_F T_R \left[\frac{1+z_j^2}{1-z_j} + \left(1 - \frac{2\,z_a z_i}{(z_a+z_i)^2}\right)\left(1 - z_j + \frac{1+z_j^2}{1-z_j}\right)\left(\log(z_a z_i z_j) - 1\right) \right]$$

Collinear factorization and resummation

- Contributions due to D-dimensionality of phase space must not spoil differential radiation pattern (we live in 4D)
- Hence simulate as endpoint contributions:
  - Generated using triple collinear phase space, but retroactively projected onto $s_{ai} = 0$
  - Guarantees phase-space coverage identical to fully differential simulation
  - Remainder taken care of by PS unitarity
- Despite subtraction terms not being accurate for current evolution variable, this is a general scheme for including NLO kernels in parton showers
- Most importantly, we never have to compute integrals analytically This will help with the complicated initial-state dipole phase space
- Natural extension to kernels with virtual corrections $\rightarrow$ "MC@NLO" inside the shower

Preliminary results





- $1 \to 3$ emission test (one only)
- NLO vs LO comparison

Dire outlook

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So far

- Developed MC algorithm to implement $2 \to 4$ splittings that recovers integrated NLO splitting functions for $q \to q'$ / $q \to \bar{q}$
- Added integrated splitting splitting functions in all other cases
- Cross-validated all implementations in FF/FI branchings

To-do list

- Triple collinear splitting function implementation for IF/II branchings
- Addition of corrections to account for evolution in dipole-k_T
- Implementation of $2 \to 4$ splittings for all NLO kernels

Negative branching "probabilities" in Markov Chains

- Problem in NLO splitting kernels, sub-leading color terms, etc. lies in negative weights → no-emission probability *locally* exceeds unity
- Recall standard veto algorithm: $\mathcal{P}_{\rm no}(t,t') = \exp\{F(t) - F(t')\}$
  Exact MC solution $t = F^{-1}\big[\,F(t') + \log R\,\big]$, R – random number
- Don't want or can't compute $F(t) = -\int_t {\rm d}\bar{t} f(\bar{t})$,
  instead find simple function $g(t) > f(t)$ with integral $G(t)$
- Generate points according to $g(t)$ and accept with $f(t)/g(t)$

Standard probability for one acceptance with n rejections

$$\frac{f(t)}{g(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{g(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\, g(t)\, \exp\left\{-\int_t^{t_1} {\rm d}\bar{t}\, g(\bar{t})\right\} \prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t'} {\rm d}t_i \left(1 - \frac{f(t_i)}{h(t_i)}\right) g(t_i)\, \exp\left\{-\int_{t_i}^{t_{i+1}} {\rm d}\bar{t}\, g(\bar{t})\right\}\right]$$

$$w(t,t_1,\ldots,t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)}\, \frac{g(t_i) - f(t_i)}{h(t_i) - f(t_i)}$$

Negative branching "probabilities" in Markov Chains

Looks trivial, surprisingly it's not: It allows to

- Resum sub-leading color terms in MC@NLO and POWHEG
  [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220
- Implement higher-order splitting functions in parton showers
  [Krauss,Prestel,SH] any time soon
- Use PDFs with negative values in parton showers
  [Prestel,SH] arXiv:1506.05057
- Enhance branching probabilities in parton showers
  [Schumann,Siegert,SH] arXiv:0912.3501, [Lönnblad] arXiv:1211.7204
- Reweight parton showers [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
  [Mrenna,Skands] arXiv:1605.08352, [Bothmann,Schönherr,Schumann] arXiv:1606.08753

Dire as an NLO subtraction method and MC@NLO

[SH] TBP?

- Can view new shower model as modification of CS subtraction
- IR counterterms computed and implemented in Sherpa (improved cancellation in $pp \to h + j$ due to regulated $1/z$ terms)
- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms

  [Krauss,Siegert,Schönherr,SH]
  arXiv:1111.1220, arXiv:1208.2815
- Dire modified CS subtraction automatically available for MC@NLO matching
- Interesting differences due to evolution variables and kernels