Merging WW and WWj with MiNLO

Work done in collaboration with

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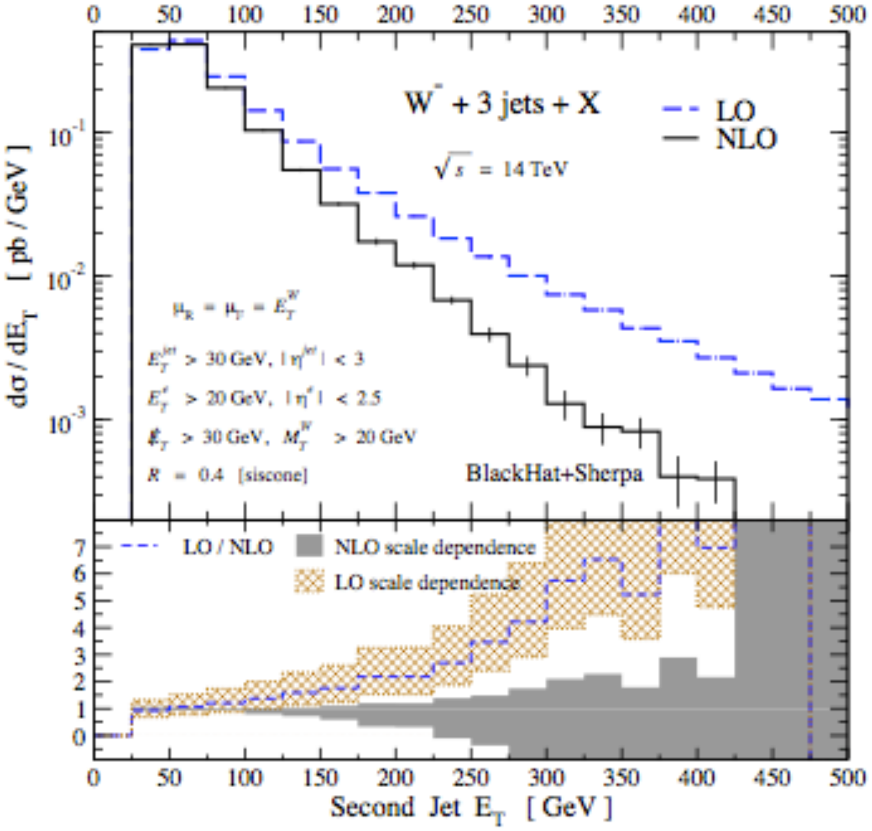
Scale choice at NLO

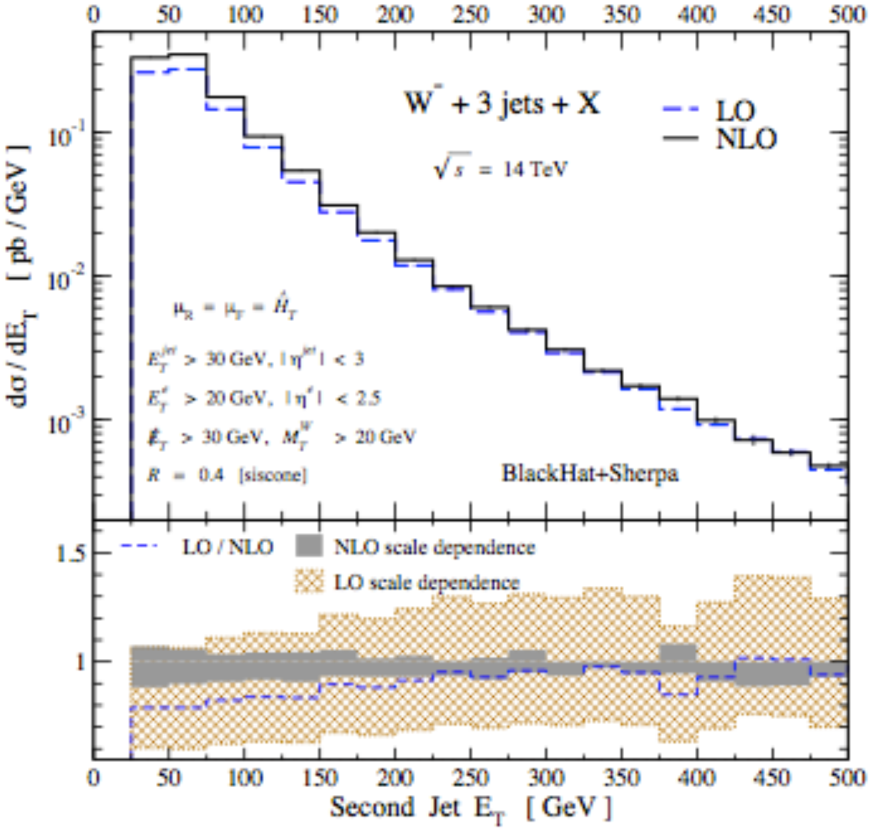
Even at NLO the scale choice is an issue. Different choices can lead to a different picture/contrasting conclusions: often a “good scale” is determined a posteriori, either by requiring NLO corrections to be small, or by looking where the sensitivity to the scale is minimised

$\mu_R=\mu_F=E_T{}^W$



bad scale

$\mu_R=\mu_F=\hat{H}_T$



good scale

Scale choice at NLO

Often a “good scale” is determined a posteriori, either by requiring NLO corrections to be small, or by looking where the sensitivity to the scale is minimized

Reason: bad scale ➠ large logs ➠ large NLO, large scale dependence

But we also know that large NLO ~~➠~~ bad scale choice, since NLO corrections can have a “genuine” physical origin
(new channels opening up, Sudakov logarithms, color factors, large gluon flux ...)

Furthermore, double logarithmic corrections can never be absorbed by a choice of scale (single log). So a “stability criterion” can be misleading

Scale choice at LO

LO calculations in matrix elements generators that follow the CKKW procedure are quite sophisticated in the scale choice:

they use optimized/local scales at each vertex and Sudakov form factors at internal/external lines

Catani,Krauss, Kuehn, Webber '01
extension to hh collisions ***Krauss '02***

Reminder:
a Sudakov form factor encodes the probability of evolving from one scale Q_i to the next Q_j without branching above a resolution scale Q_0

Recap at CKKW procedure

The CKKW prescription in brief:

- use the k_t algorithm to reconstruct the most likely branching history
- evaluate each α_s at the local transverse momentum of the splitting
- for each internal line between nodes at scale Q_i and Q_j include a Sudakov form factor $\Delta_{ij}=D(Q_0,Q_i)/D(Q_0,Q_j)$ that encodes the probability of evolving from scale Q_i to scale Q_j without emitting. For external lines include the Sudakov factor $\Delta_i=D(Q_0,Q_i)$
- match to a parton shower to include radiation below Q_0

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Scale choice intertwined with inclusion of Sudakov form factors

MiNLO

MiNLO Born as an extension to NLO of the CKKW procedure, such that the procedure to fix the scales is unbiased and decided ***a priori***

In particular, the focus is on processes involving **many scales** (e.g. X+multi-jet production) and on soft/collinear branchings, i.e. on the region where it is more likely that associated jets are produced

Two observations

1. A generic NLO cross-section has the form

$$\alpha_{\mathrm{S}}^{n}\left(\mu_{R}\right) B+\alpha_{\mathrm{S}}^{n+1}\left(\mu_{R}\right)\left(V\left(Q\right)+n b_{0} \log \frac{\mu_{R}^{2}}{Q^{2}} B\left(Q\right)\right)+\alpha_{\mathrm{S}}^{n+1}\left(\mu_{R}\right) R$$

   Adopting CKKW scales at LO, this becomes naturally

$$\alpha_{s}(\mu_{1}) \ldots \alpha_{s}(\mu_{n}) B+\alpha_{s}^{n+1}(\mu_{R}^{\prime})\left(V(Q)+b_{0} \log \frac{\mu_{1}^{2} \ldots \mu_{n}^{2}}{Q^{2n}} B\right)+\alpha_{s}^{n+1}(\mu_{R}^{\prime\prime}) R$$

   and the scale choices μ_R’ and μ_R” are irrelevant for the scale cancelation

2. Sudakov corrections included at LO via the CKKW procedure lead to NLO corrections that need to be subtracted to preserve NLO accuracy

The MiNLO procedure

1. Find the CKKW n clustering scales $Q_1 < \ldots < Q_n$. Fix the hard scale of the process Q to the system invariant mass after clustering. Set Q_0 to Q_1 (inclusive on radiation below Q_1)
2. Evaluate the n coupling constants at the scales Q_i (times a factor to probe scale variation)
3. Set μ_R in the virtual to the geometric average of these scales and μ_F to the softest scale Q_1
4. Include Sudakov form factors for Born and virtual terms, and for the real term after the first branching
5. Subtract the NLO bit present in the CKKW Sudakov of the Born
6. Give a prescription for the $(n+1)^{th}$ power of α_s in the real and virtual terms

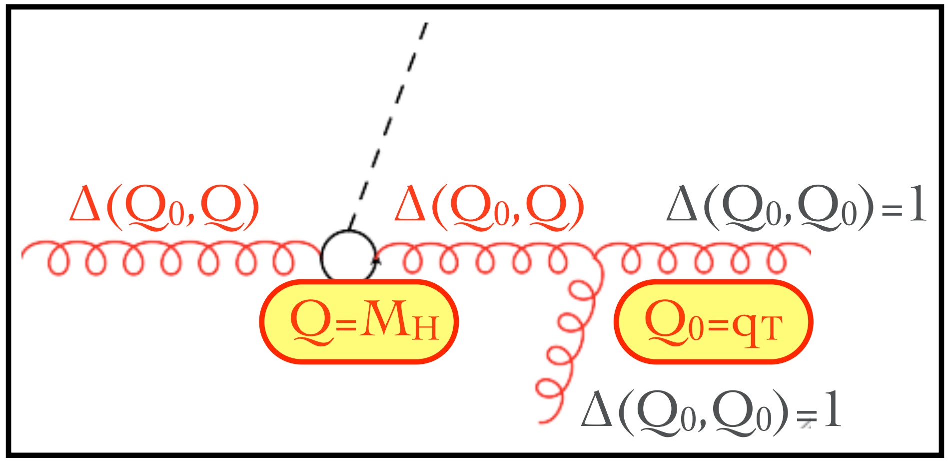
MiNLO in one equation

Example: take e.g. HJ

In POWHEG it is customary to discuss the B̄ function, which for HJ is defined as

$$\bar{B} = \alpha_s^3(\mu_R)\left[B + V(\mu_R) + \int d\Phi_{\rm rad}\, R\right]$$

With MiNLO this function becomes



$$\bar{B} = \alpha_s^2\left(M_H^2\right)\alpha_s\left(q_T^2\right)\Delta_g^2\left(M_H, q_T\right)\left[B\left(1 - 2\Delta_g^{(1)}\left(M_H, q_T\right)\right) + V(\mu') + \int d\Phi_{\rm rad}\, R\right]$$

Properties of MiNLO

MiNLO has the following properties

- the result is accurate at NLO, i.e. the scale dependence is NNLO
- the smooth behaviour of the CKKW scheme in the singular regions is preserved
- the X+n-jet cross-sections are finite even without jet cuts (do not need generation cuts or Born suppression factors)
- the procedure is simple to implement in any NLO calculation, i.e. the improvement requires only a very modest amount of work

Merging with MiNLO

Hamilton et al. 1212.4504

We have shown that it is possibly to modify the original MiNLO procedure in such a way that the X+1jet (X=H, Z, W) calculation, upgraded with MiNLO, is NLO accurate also for fully inclusive quantities
[e.g. you can look at the X transverse momentum or X rapidity (without any jet cut) and will get NLO accurate results]

This means that MiNLO applied on X+1jet merges the X+1jet and the inclusive calculations without using any merging scale (unlike most other approaches)

How does this work?

Sketch of the proof

Hamilton et al. 1212.4504

NNLL$_\Sigma$ Drell Yan q_T resummation at fixed rapidity can be written as

$$\frac{d\sigma}{dy dq_T^2} = \sigma_0 \frac{d}{dq_T^2}\Big\{ \left[C_{ga} \otimes f_{a/A}\right](x_A, q_T) \times \left[C_{gb} \otimes f_{b/B}\right](x_B, q_T) \times \exp \mathcal{S}(Q, q_T)\, \mathcal{F} \Big\} + R_f$$

Integrating in q_T one gets

$$\frac{d\sigma}{dy} = \sigma_0 \left[C_{ga} \otimes f_{a/A}\right](x_A, Q) \times \left[C_{gb} \otimes f_{b/B}\right](x_B, Q) + \int dq_T^2\, R_f + \ldots$$

i.e. the formula is NLO$^{(0)}$ accurate if O(α_s) corrections to the coefficient functions are included and R_f is LO$^{(1)}$ accurate

Now, need to show that if the derivative is taken explicitly, and some higher orders are neglected, NLO$^{(0)}$ accuracy is maintained.

Sketch of the proof

Taking the derivative one gets

Hamilton et al. 1212.4504

$$\sigma_0 \frac{1}{q_T^2} \left[\alpha_s,\, \alpha_s^2,\, \alpha_s^3,\, \alpha_s^4,\, \alpha_s L,\, \alpha_s^2 L,\, \alpha_s^3 L,\, \alpha_s^4 L\right] \exp \mathcal{S}\left(Q, q_T\right)$$

Sketch of the proof

Taking the derivative one gets

Hamilton et al. 1212.4504

$$\sigma_0 \frac{1}{q_T^2}\left[\underset{B_1}{\alpha_s,}\ \alpha_s^2,\ \alpha_s^3,\ \alpha_s^4,\ \alpha_s L,\ \alpha_s^2 L,\ \alpha_s^3 L,\ \alpha_s^4 L\right] \exp \mathcal{S}\left(Q, q_T\right)$$

Sketch of the proof

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$$\sigma_0 \frac{1}{q_T^2} \left[\underbrace{\alpha_s,}_{B_1} \underbrace{\alpha_s^2,}_{B_2} \alpha_s^3, \alpha_s^4, \alpha_s L, \alpha_s^2 L, \alpha_s^3 L, \alpha_s^4 L\right] \exp \mathcal{S}\left(Q, q_T\right)$$

Sketch of the proof

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Sketch of the proof

Taking the derivative one gets

Hamilton et al. 1212.4504

$$\sigma_0 \frac{1}{q_T^2}\left[\underset{B_1}{\boxed{\alpha_s,}}\ \underset{B_2}{\boxed{\alpha_s^2,}}\ \underset{\cdots}{\alpha_s^3,\ \alpha_s^4,}\ \underset{A_1}{\boxed{\alpha_s L,}}\ \alpha_s^2 L,\ \alpha_s^3 L,\ \alpha_s^4 L\right] \exp \mathcal{S}\left(Q, q_T\right)$$

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Sketch of the proof

Taking the derivative one gets

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$$\sigma_0 \, \frac{1}{q_T^2} \left[\underset{B_1}{\alpha_s}, \underset{B_2}{\alpha_s^2}, \underset{\cdots}{\alpha_s^3, \alpha_s^4}, \underset{A_1}{\alpha_s L}, \underset{\cdots}{\alpha_s^2 L}, \underset{C_1 \otimes C_1 \otimes A_1}{\alpha_s^3 L}, \alpha_s^4 L \right] \exp \mathcal{S}\left(Q, q_T\right)$$

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After integration with the Sudakov weight, the counting is set by $L \sim dL \sim 1/\sqrt{\alpha_s}$. So these terms contribute, e.g.

$$\sigma_0 \int dL \left[\alpha_s,\ \underset{\mathcal{O}(\alpha_s^{3/2})}{\underset{\Big\downarrow}{\alpha_s^2}},\ \alpha_s^3,\ \alpha_s^4,\ \alpha_s L,\ \alpha_s^2 L,\ \underset{\mathcal{O}(\alpha_s^2)}{\underset{\Big\downarrow}{\alpha_s^3 L}},\ \alpha_s^4 L \right] \exp \mathcal{S}\left(Q, q_T\right)$$

Need B₂ in Sudakov to reach NLO$^{(0)}$ accuracy

All higher order terms can be safety dropped maintaining NLO$^{(0)}$ accuracy

Sketch of the proof

Taking the derivative one gets

Hamilton et al. 1212.4504

$$\sigma_0 \frac{1}{q_T^2} \left[\underset{B_1}{\boxed{\alpha_s}},\, \underset{B_2}{\boxed{\alpha_s^2}},\, \underset{\cdots}{\alpha_s^3,\, \alpha_s^4},\, \underset{A_1}{\boxed{\alpha_s L}},\, \underset{\cdots}{\alpha_s^2 L},\, \underset{C_1 \otimes C_1 \otimes A_1}{\boxed{\alpha_s^3 L}},\, \underset{\cdots}{\alpha_s^4 L}\right] \exp \mathcal{S}\left(Q, q_T\right)$$

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Need B_2 in Sudakov to reach NLO$^{(0)}$ accuracy

All higher order terms can be safety dropped maintaining NLO$^{(0)}$ accuracy

Similarly, the scale in V and R gives the largest difference in the α_s^2L term, where they give an $\alpha_s^3L^2$ variation. This contributes O($\alpha_s^{3/2}$). So an effect of the same size to B_2.

Sketch of the proof

Conclusion:

Hamilton et al. 1212.4504

- ☛ The original MiNLO prescription is less than NLO accurate in the description of inclusive quantities, in that it neglects O($\alpha_s^{3/2}$) terms
- ☛ achieve NLO accuracy from XJ also for inclusive observables by
  - ✔ including the B_2 term in the Sudakov form factors
  - ✔ taking the scale in the coupling constant in the real, virtual and subtraction terms equal to the boson transverse momentum

Provided this is done, the XJ describes both X and X+j at NLO, i.e. merging of X and XJ is effectively achieved without doing any merging

NB: thus unlike other approaches, no merging scale is introduced

q.e.d.

Merging WW and WWjet

First, merging achieved only for single vector boson (X=H, Z, W).
In the following discuss extension to merging WW and WWjet

WW important both as signal and background

- interesting SM process on its own (some tension in the past with LHC data)
- background to H to WW
- anomalous couplings

Minlo: from Drell Yan to WW

Steps:

1. ex-novo WWJ POWHEG generator using to Madgraph / Gosam 2.0

Campbell et al 1201.547; Luisoni et al. 1306.2542; Cullen et al. 1404.7096

2. starting from B_2 for Drell-Yan extract B_2 for WW as follows

- for DY, the virtual corrections is proportional to the Born, and B_2 is just a number $B_2^{(W)}$
- for WW, this is no longer true $B_2^{(WW)} = B_2^{(WW)}(\Phi_{WW})$
- however one can observe that B2 as the form

$$B_2 = -2\gamma^{(2)} + \beta_0 C_F \zeta_2 + 2(2C_F)^2\zeta_3 + \beta_0 H_1(\Phi)$$

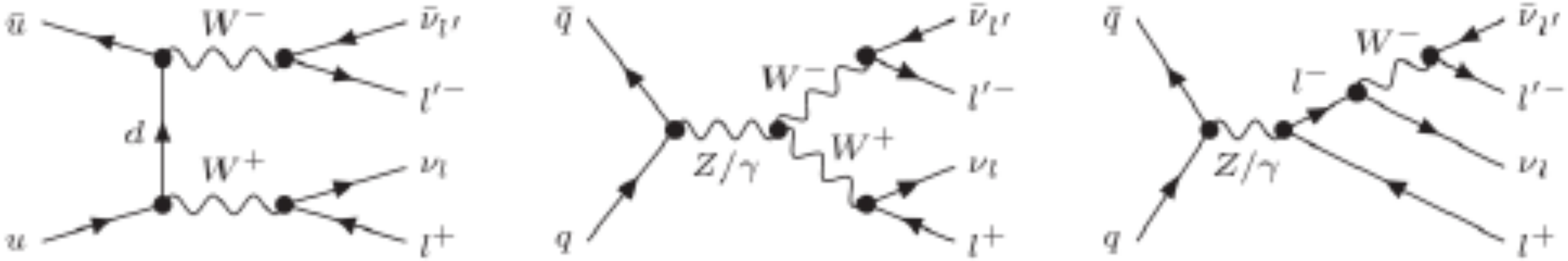
- from which one can derive

$$B_2^{(WW)} = B_2^{(W)} - \beta_0 H_1^{(W)} + \beta_0 H_1^{(WW)}$$

- process dependent part extracted on an event-by-event basis using a projection of Φ_{WWJ} to Φ_{WW} using FKS ISR mapping

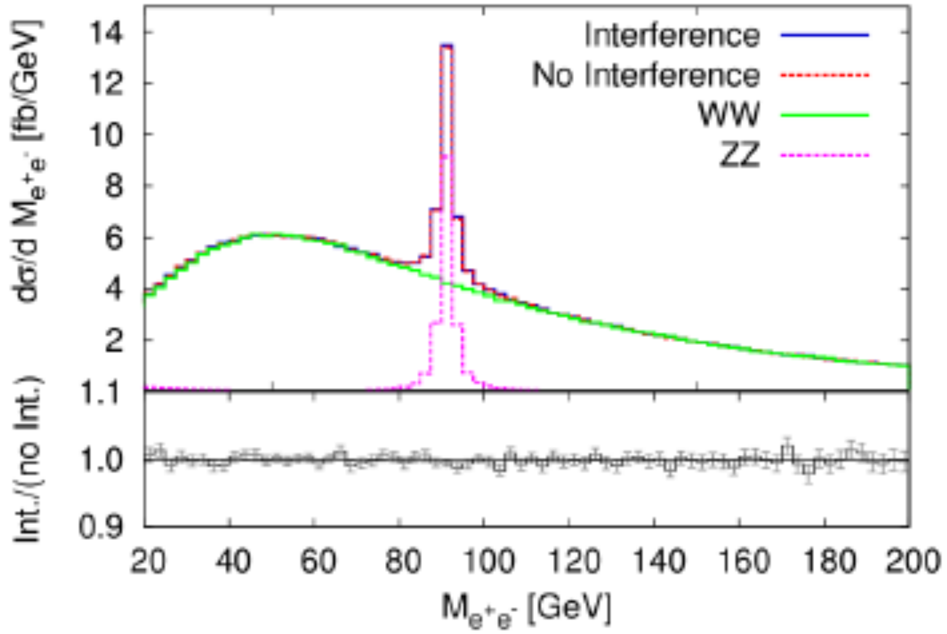
WWJ: technical details

- All off-shell and single resonant diagrams included. Full matrix elements with decays to four leptons



- Work on four-flavour scheme (4FS): no interference with Wt, $t\bar{t}$

- For same family leptons, ZZ → $l^+l^- \nu \bar{\nu}$ not included (considered part of ZZ)

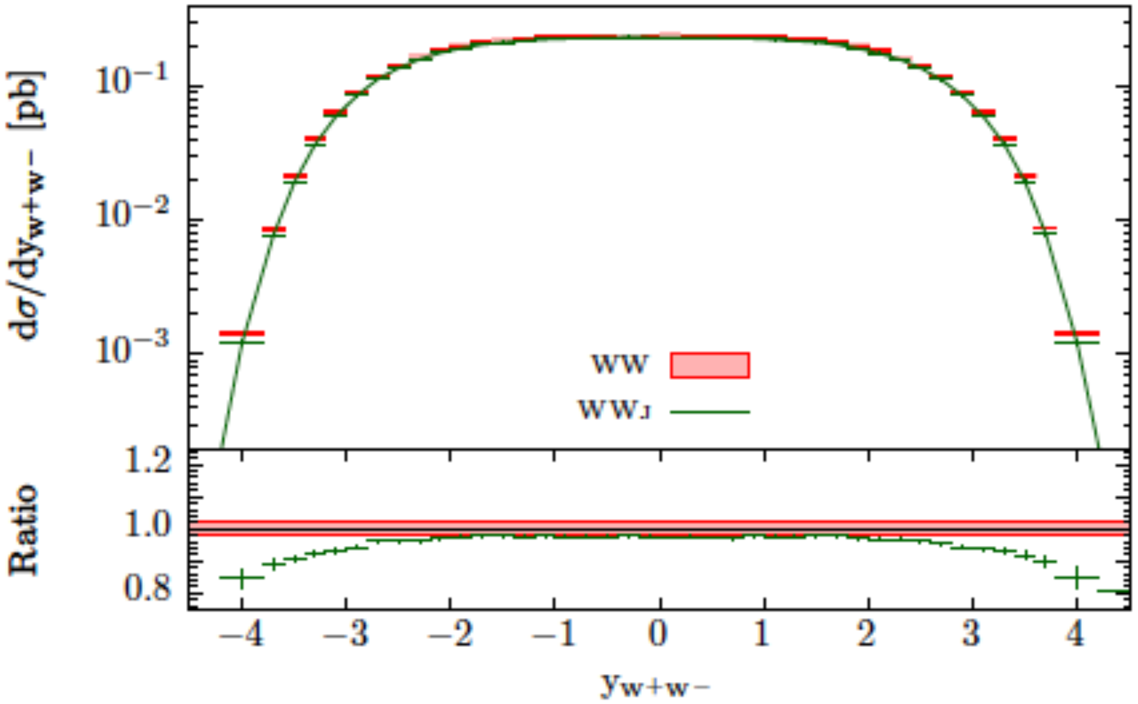


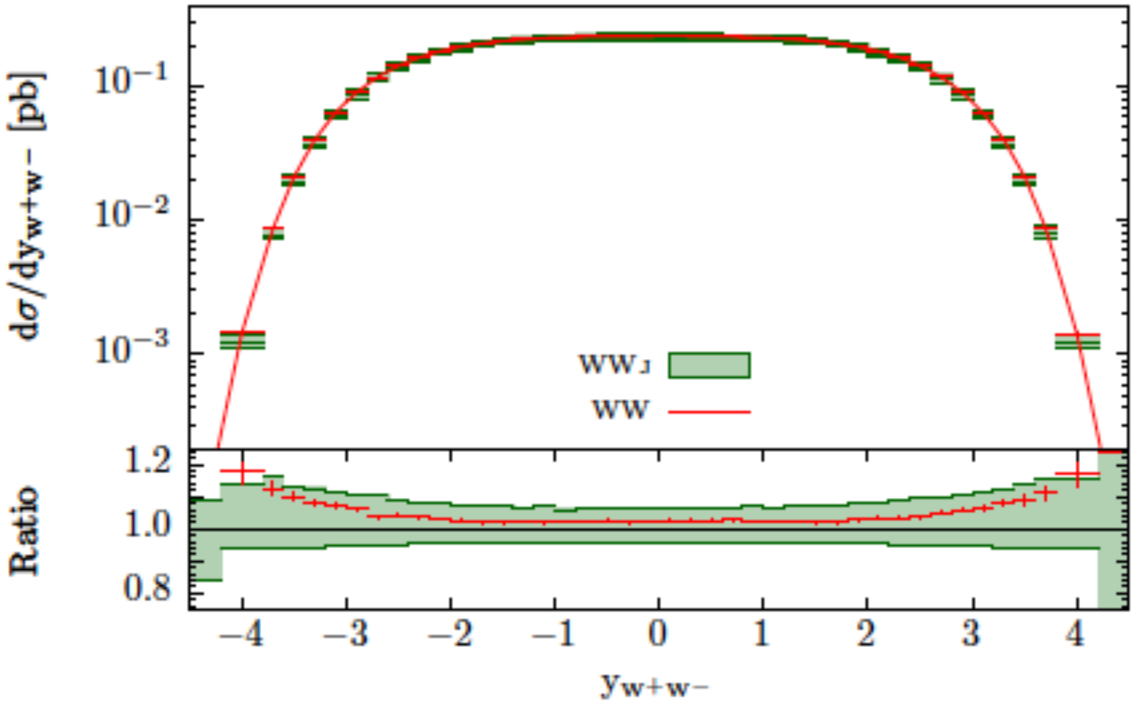
- Also (tiny) interference with ZZ and WW not included

- Since fermion loop contributions are numerically tiny (1-2%) but slow down the code, option to run excluding them

WWJ-MiNLO: results I

WW generator vs WWJ-MiNLO generator



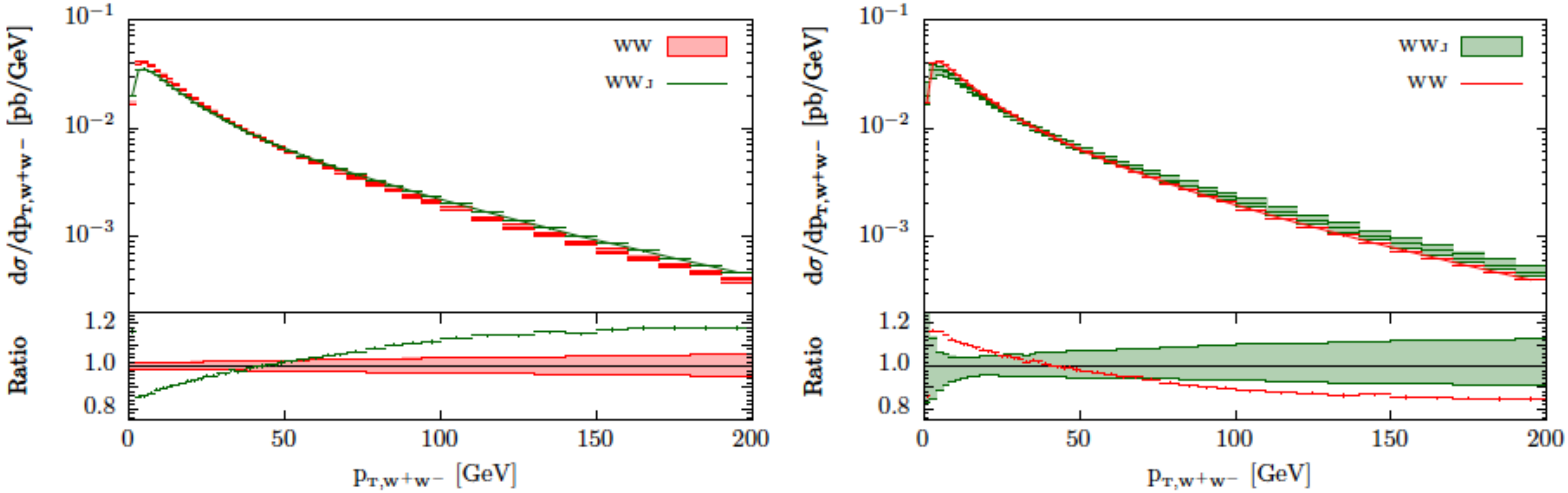


- agreement on total cross-section at 4% level (WWJ-MiNLO has larger uncertainty bands)
- shape difference in y_{WW} at large rapidity correlated to differences in the $p_{T,WW}$ spectrum (see next slide)

Uncertainty band in all plots: vary μ_R, μ_F by factor 2 up and down avoiding μ_R/μ_F = 1/4, 4 (7 scales)

WWJ-MiNLO: results II

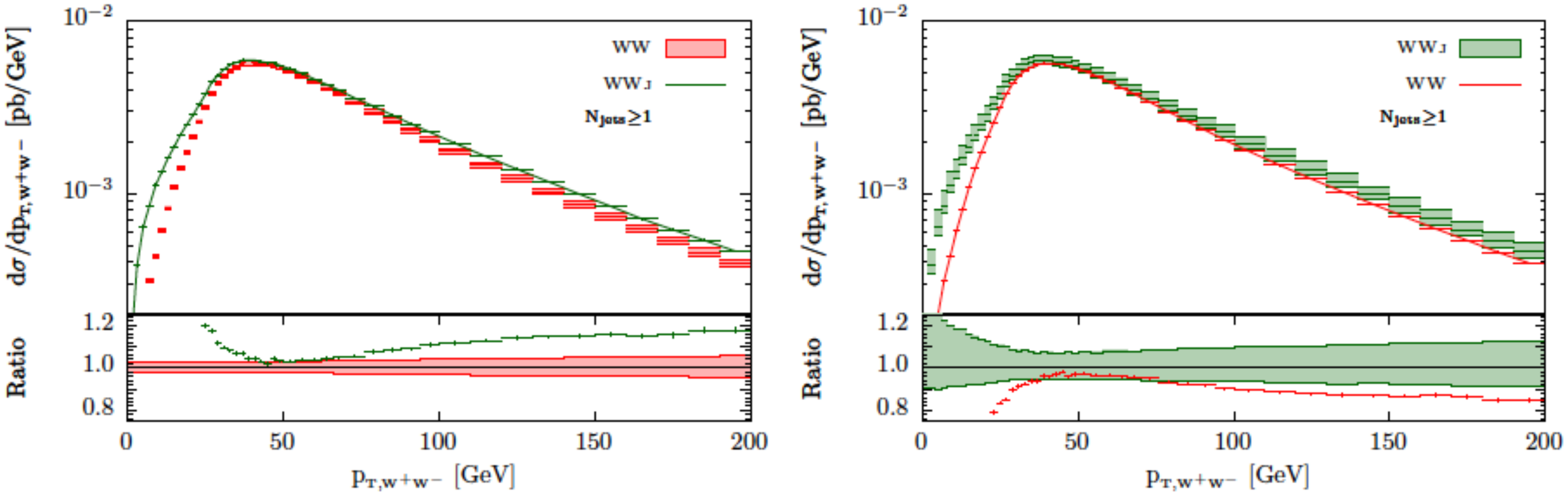
WW generator vs WWJ-MiNLO generator



- NLO correction sizeable in spectrum at high pt
- in small pt region, different terms included in the two approaches, in particular non-zero contribution from two emission matrix element, which is missing in WW
- uncertainties in WW are underestimated

WWJ-MiNLO: results III

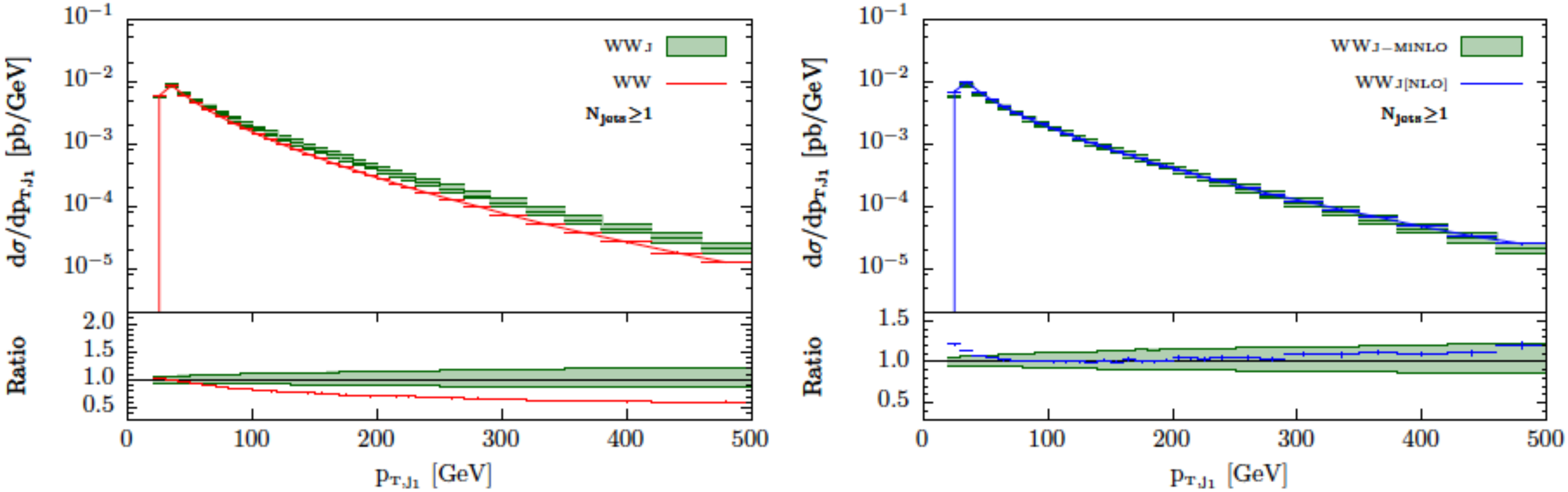
WW generator vs WWJ-MiNLO generator
including explicit jet requirement pt,j > 25 GeV (R = 0.4)



- below 25 GeV, need at least two emissions: from Parton Shower in WW, LO in WWJ
- WWJ NLO accurate above 25 GeV (nicely reflected in uncertainty band), while WW only LO throughout (uncertainty underestimated)
- large NLO K-factor

WWJ-MiNLO: results IV

WW generator vs WWJ-MiNLO generator and WWJ at pure NLO including explicit jet requirement pt,j > 25 GeV (R = 0.4)



- small differences between WWJ and WWJ-MiNLO because of Sudakov form factors and scale choices, while WW underestimates hardness of spectrum

Conclusions

Recent non-trivial application of ‘improved’ MiNLO-method to the process pp → WW + jet

- clear improvement over WW generator (better uncertainty estimate, better control of hard regions)
- the code is ready to be made available
- pp → ZZ + jet can be done in a similar way
- NNLOPS for WW+jet feasible, but hard because of complicated Born phase space
- including gg → WW channels can be done too