4-jet Production and Double Parton Scattering Effects in High Energy Factorization matched to Parton Showers

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Parton Showers, Event Generators and Resummation
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Work in collaboration with
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High Energy Factorization

High Energy Factorization *(Catani,Ciafaloni,Hautmann, 1991 / Collins,Ellis, 1991)*

$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2k_{1\perp}\, d^2k_{2\perp}\, \frac{dx_1}{x_1}\, \frac{dx_2}{x_2}\, f_g(x_1,k_{1\perp})\, f_g(x_2,k_{2\perp})\, \hat{\sigma}_{gg}\,(m,x_1,x_2,s,k_{1\perp},k_{2\perp})$$

where the f_g's are the gluon densities, obeying BFKL, BK, CCFM evolution equations.
Usual tool: Lipatov' *effective* (though not in the RG group sense) action
Lipatov, Nucl.Phys. B721 (1995) 111-135
Antonov, Cherednikov, Kuraev, Lipatov, Nucl.Phys. B452 (2005) 369-400

$$\mathcal{S}_{eff} = \mathcal{S}_{QCD} + \int d^4x\, \left\{ \mathrm{tr}\, \left[(W_-[v] - A_-)\, \partial_\perp^2 A_+ + (W_+[v] - A_+)\, \partial_\perp^2 A_- \right] \right\}$$

$$W_\pm[v] = -\frac{1}{g}\partial_\pm U[v_\pm] = v_\pm - g\, v_\pm \frac{1}{\partial_\pm} v_\pm + g^2\, v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm + \dots$$

$v_\mu \equiv -i\, t^a\, A^a_\mu$, gluon field $\quad A_\pm \equiv -i\, t^a\, A^a_\pm$, reggeized gluon fields

$$U[v_\pm] = \mathcal{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{x^\pm} dz^\pm\, v_\pm(z^\pm, x_\perp) \right)\,, \quad x_\perp = (x_\pm, \mathbf{x})$$

Sudakov parameterisation of initial state for for HEF:

$$k_1^\mu = x_1\, l_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\, l_2^\mu + k_{2\perp}^\mu\,, \quad l_i^2 = 0, \quad l_i \cdot k_i = 0, \quad k_i^2 = -k_{i\perp}^2, \quad i = 1,2$$

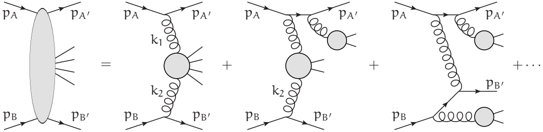
Gauge invariant amplitudes with off-shell gluons

Problem: general partonic processes must be described by gauge invariant amplitudes
$\Rightarrow$ ordinary Feynman rules are not enough !

THE IDEA:

on-shell amplitudes are gauge invariant, so off-shell gauge-invariant amplitudes could be got by embedding them into on-shell processes...

...first result...: 1) For off-shell gluons: represent g^* as coming from a $\bar{q}qg$ vertex, with the quarks taken to be on-shell



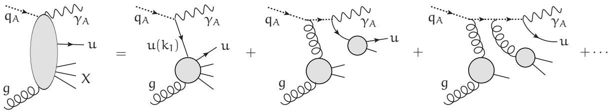
- embed the scattering of the off-shell gluons in the scattering of two quark pairs carrying momenta $p_A^\mu = k_1^\mu$, $p_B^\mu = k_2^\mu$, $p_{A'}^\mu = 0$, $p_{B'}^\mu = 0$
- Assign the spinors $|p_1\rangle, |p_1]$ to the A-quark and the propagator $\frac{i\,\not{p}_1}{p_1\cdot k}$ instead of $\frac{i\not{k}}{k^2}$ to the propagators of the A-quark carrying momentum k; same thing for the B-quark line.
- ordinary Feynman elsewhere and factor $x_1\sqrt{-k_\perp^2/2}$ to match to the collinear limit

K. Kutak, P. Kotko, A. van Hameren, JHEP 1301 (2013) 078

Gauge invariant amplitudes with off-shell quarks

... and second result:

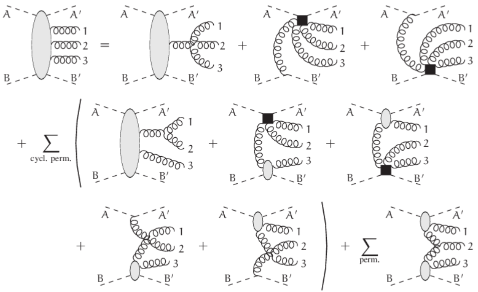
2) for off-shell quarks: represent q^* as coming from a $\gamma_A \bar{q}_A q$ vertex, where γ_A and $\bar{q}_A$ are on shell; γ_A is an artificial flavour-changing neutral boson coupling only to q !



- embed the scattering of the quark with whatever set of particles in the scattering of an auxiliary quark-photon pair, q_A and γ_A carrying momenta $p^\mu_{q_A} = k^\mu_1$, $p^\mu_{\gamma_A} = 0$
- Let q_A-propagators of momentum k be $\frac{i\,\not{p}_1}{p_1 \cdot k}$ and assign the spinors $|p_1\rangle, |p_1]$ to the A-quark.
- Assign the polarization vectors $\epsilon^\mu_+ = \frac{\langle q|\gamma^\mu|p_1]}{\sqrt{2}\langle p_1 q\rangle}$, $\epsilon^\mu_- = \frac{\langle p_1|\gamma^\mu|q]}{\sqrt{2}[p_1 q]}$ to the auxiliary photon, with q a light-like auxiliary momentum.
- Multiply the amplitude by $x_1\sqrt{-k^2_{1\perp}/2}$ and use ordinary Feynman rules everywhere else.

K. Kutak, T. Salwa, A. van Hameren, Phys.Lett. B727 (2013) 226-233

One left issue: huge slowness for many legs



Scattering amplitudes in Yang-Mills theories via ordinary Feynman diagrams: soon overwhelming !

Number of Feynman diagrams at tree level on-shell in the color ordered representation:

# of gluons	4	5	6	7	8	9	10
# of diagrams	4	25	220	2485	34300	559405	10525900

And there are even more with the proposed method for amplitudes with off-shell particles due to the gauge-restoring terms.

Britto, Cachazo, Feng, Nucl.Phys. B715 (2005) 499-522

Britto, Cachazo, Feng, Witten, Phys.Rev.Lett. 94 (2005) 181602

BCFW: an analytic recursion for (almost) all massless QCD amplitudes

Amazingly simple recursive relation, now fully generalised to the off-shell case:

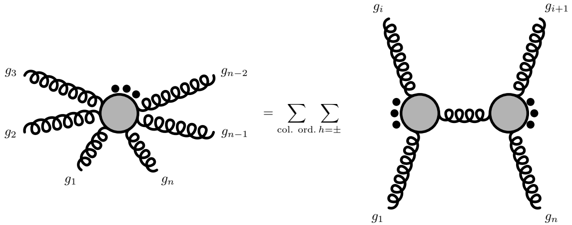
any tree-level color-ordered amplitude is the sum of residues of the poles it develops when it is made dependent on a complex variable as above.

Such residues are simply products of color-ordered lower-point amplitudes evaluated at the pole times an intermediate propagator.

Shifted particles are always on opposite sides of the propagator.

$$\mathcal{A}(g_1,\ldots,g_n)=\sum_{i=2}^{n-2}\sum_{h=+,-}\mathcal{A}(g_1,\ldots,g_i,\hat{P}^h)\,\frac{1}{(p_1+\cdots+p_i)^2}\,\mathcal{A}(-\hat{P}^{-h},g_{i+1},\ldots,g_n)$$

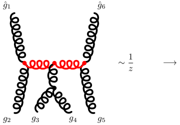
$$z_i=\frac{(p_1+\cdots+p_i)^2}{[1|p_1+\cdots+p_i|n\rangle}$$ location of the pole corresponding for the "i-th" partition

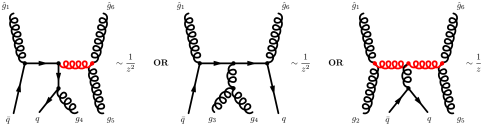


Natural to ask whether something like BCFW exists with off-shell particles.
For off shell gluons answer first given in *van Hameren, JHEP 1407 (2014) 138*
With off shell fermions *van Hameren, MS, JHEP 1507 (2015) 010; Kutak, van Hameren, MS JHEP 1702 (2017) 009*

$$\mathcal{A}(0) = \sum_{s=g,f} \left(\sum_{p} \sum_{h=+,-} \mathrm{A}^{s}_{p,h} + \sum_{i} \mathrm{B}^{s}_{i} + \mathrm{C}^{s} + \mathrm{D}^{s} \right) ,$$

- $\mathrm{A}^{g/f}_{p,h}$ are the same poles as in the original BCFW recursion for on-shell amplitudes: intermediate virtual gluon or fermion.
- $\mathrm{B}^{g/f}_{i}$ are due to the poles in auxiliary eikonal quark propagators.
- $\mathrm{C}^{g/f}$ and $\mathrm{D}^{g/f}$ show up us the first/last shifted particle is off-shell and their external propagator develops a pole. **External propagators for off-shell particles necessary to ensure** $\lim_{z\to\infty} \mathcal{A}(z) = 0$





A final full-fledged numerical implementation

- With growing number of legs, it is necessary to figure out practical ways to compute amplitudes efficiently. A promising possibility is the BCFW (Britto-Cachazo-Feng-Witten) recursion relation, originally discovered for on-shell QCD amplitudes and extended to off-shell gluon amplitudes in *A. van Hameren, JHEP 1407 (2014) 138*
- A general analysis extending the modified BCFW to amplitudes with fermion pairs has been developed in *A. van Hameren, MS JHEP 1507 (2015) 010* and *A. van Hameren, K. Kutak, MS, JHEP 1702 (2017) 009*
- Numerical implementation and cross-checks are done and always successful. A program exists implementing Berends-Giele recursion relation, *A. van Hameren, M. Bury, Comput.Phys.Commun. 196 (2015) 592-598*
- **The big player for phenomenology: *KaTie***, a parton level event generator for k_T-dependent initial states *A. van Hameren, arXiv:1611.00680*. Once interfaced with the **AvHlib** library by the same author and supplied with the desired TMDs, it can compute cross sections in HEF for any process in the Standard Model, providing automatised phase space optimisation (KALEU).

Introducing Double Parton Scattering

More formal approaches: Diehl, Ostermeier, Schäfer, JHEP 1203 (2012) 089
Diehl, Gaunt, Ostermeier, Plößl, Schäfer JHEP 1601 (2016) 076;
Diehl, Gaunt, Schönwald, arXiv:1702.06486

DPS $\equiv$ the simultaneous occurrence of two partonic hard scatterings in the same proton-proton collision

$$\sigma^D = \mathcal{S} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, b; t_1, t_2)\, \Gamma_{kl}(x_1', x_2', b; t_1, t_2)\, \hat{\sigma}(x_1, x_1')\, \hat{\sigma}(x_2, x_2')\, dx_1 dx_2 dx_1' dx_2' d^2 b$$

Usual assumption: separation of longitudinal and transverse DOFs:

$$\Gamma_{ij}(x_1, x_2, b; t_1, t_2) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F^{ij}(b) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F(b)$$

- Longitudinal correlations, most often ignored or assumed to be negligible, especially at small x : $D_h^{ij}(x_1, x_2; t_1, t_2) = D^i(x_1; t_1)\, D^j(x_2; t_2)$
- Transverse correlation, assumed to be independent of the parton species, taken into account via $\sigma_{eff}^{-1} = \int d^2 b\, F(b)^2 \approx (15mb)^{-1}$ (CDF, D0, LHCb ...)

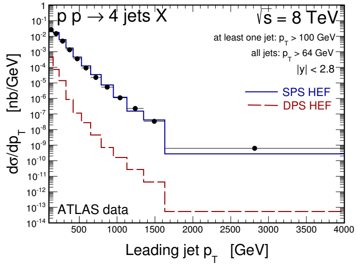
Usual final kind of crafty formula:

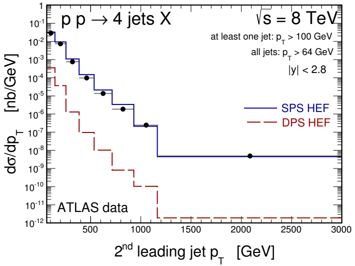
$$\begin{aligned} \sigma^D &= \frac{1}{\sigma_{eff}} \sum_{i_1,j_1,k_1,l_1;i_2,j_2,k_2,l_2} \frac{1}{1+\delta_{12}}\, \sigma(i_1 j_1 \to k_1 l_1) \times \sigma(i_2 j_2 \to k_2 l_2) \\ &\equiv \frac{1}{\sigma_{eff}} \sum_{A,B} \frac{\sigma^A \sigma^B}{1+\delta_{AB}} \end{aligned}$$

Validation with hard jets: differential distribution

Most recent ATLAS paper on 4-jet production in proton-proton collision:
ATLAS, JHEP 1512 (2015) 105

$$p_T \geq 100\,\text{GeV}\,, \quad \text{for leading jet}$$
$$p_T \geq 64\,\text{GeV}\,, \quad \text{for non leading jets}$$
$$|\eta| \leq 2.8\,, \quad R = 0.4$$





- All channels included and running α_s @ NLO
- Good agreement with data
- DPS effects are manifestly too small for such hard cuts: this could be expected.

DPS for 4-jets: the case of the CMS data

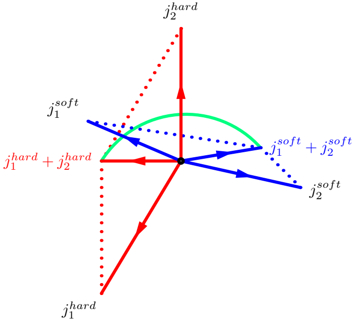
$p_T(1,2) \geq 50$ GeV , $p_T(3,4) \geq 20$ GeV , $|\eta| \leq 4.7$,

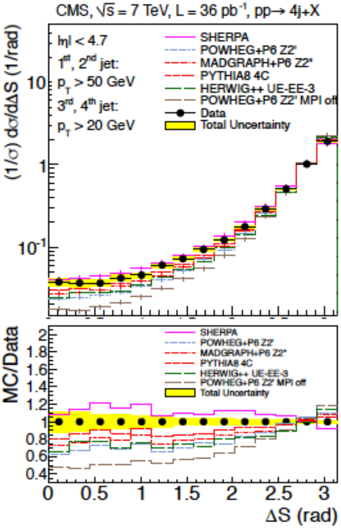
$$\Delta S = \arccos\left(\frac{\vec{p}_T(j_1^{\rm hard}, j_2^{\rm hard}) \cdot \vec{p}_T(j_1^{\rm soft}, j_2^{\rm soft})}{|\vec{p}_T(j_1^{\rm hard}, j_2^{\rm hard})| \cdot |\vec{p}_T(j_1^{\rm soft}, j_2^{\rm soft})|}\right)$$

$$\vec{p}_T(j_i, j_k) \equiv p_{T,i} + p_{T,j}$$

Expected to be flat for DPS

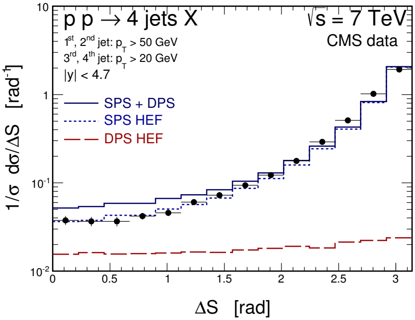
Not well described by available tools





ΔS in High Energy Factorization: the only-matrix-element prediction

No collinear MonteCarlo manages to really describe ΔS over the whole range.
Better job for the other two variables. What could HEF say about it ?



- We roughly describe the data via pQCD effects within our HEF approach which are (equally partially) described by parton-showers and soft MPIs by CMS. K. Kutak, R. Maciula, MS, A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175
- We seem to overshoot the data when adding DPS
- Natural to ask what happens when we include initial and final state radiation ⇒ we need to match parton-level k_T-factorization with parton showers.

Adding parton showers to High Energy Factorization

Matching the hard off-shell matrix elements with parton showers:

- Generate the hard matrix element in full High Energy Factorization: KaTie
- Perform backward evolution in order to have the transverse momentum in the hard matrix element unfolded to initial state radiation: CASCADE
- Add final state radiation and remnant treatment
- Reconstruct jets with anti-k_T algorithm: FastJet

Difference with respect to the collinear generators (MadGraph, Pythia, etc.):

One does not need to perform boosts and rotations of the hard matrix element in order to accomodate for the transverse momentum.

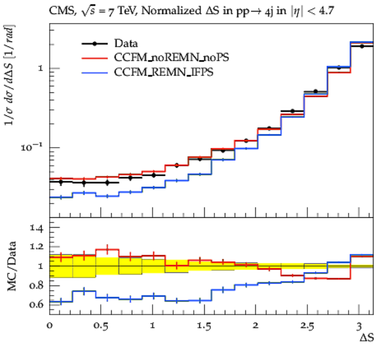
This is because this comes directly from the matrix element in a fully gauge invariant way. With respect to the fully collinear case, we include the additional hard dynamics coming form the transverse momentum.

Technical framework for phenomenology: what we have now

- **KaTie** (A. van Hameren) : https://bitbucket.org/hameren/KaTie complete Monte Carlo program for tree-level calculations of any process within the Standard Model; any initial-state partons on-shell or off-shell; numerical computation of helicity amplitudes
- u **and** d initial state quarks, final states with all the $N_f = 5$ lightest flavours.
- **Massless quarks approximation**, $m_{q/\bar{q}} = 0$.
- **KMR prescription** to generate the k_T-dependence from the collinear sets
- **CASCADE-2.4.07**: DGLAP/CCFM initial and final state parton showers (Hannes Jung et al.).
  Evolution performed with the CCFM $P_{gg}(z, k_T)$ and the DGLAP $P_{qq}(z)$, $P_{qg}(z)$, $P_{gq}(z)$: to be updated !
- **Running** α_s from the respective PDF sets
- **Scales:** $\mu_R = \mu_F \equiv \mu = \frac{H_T}{2} \equiv \frac{1}{2} \sum_i p_T^i$, (sum over final state particles)
- **We don't take into account correlations in DPDFs**:
  $D(x_1, x_2, \mu) = f(x_1, \mu)\, f(x_2, \mu)$.
  There are attempts to go beyond this approximation:
  Golec-Biernat, Lewandowska, Snyder, MS, Stasto, Phys.Lett. B750 (2015)
  Golec-Biernat, Stasto, Phys.Rev. D95 (2017) no.3, 034033: first full set of k_T-dependent DPDFs
  Rinaldi et al., JHEP 1412 (2014) 028, JHEP 1610 (2016) 063, Phys.Lett. B768 (2017) 270-273, Phys.Rev. D95 (2017) no.3, 034040

ΔS: HEF with DLC2016 plus CCFM parton showers

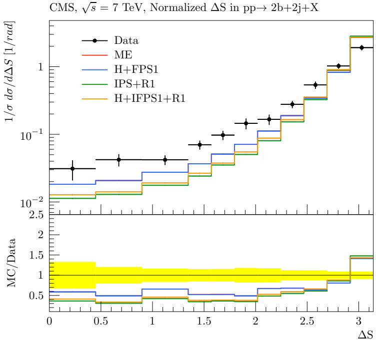
- We generate matrix elements without the restriction $k_{T,i}^2 < \mu^2$.
- Jets equally hard or harder than those from the hard matrix element can come from the showering.
- The predictions without parton showers roughly agrees with the data
- Once we include showers and full remnant treatment, we see that we recover a similar result as in the collinear case.
- We conclude that, in this ME+PS scenario, High energy Factorization seems to suggest the need for MPI's .
- **PRELIMINARY. UPDATES COMING SOON**



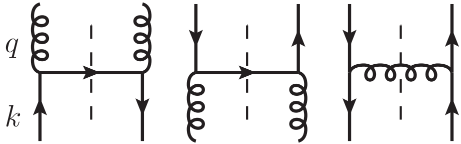
ΔS: HEF factorization with DLC2016 plus CCFM parton showers with two b-tagged jets

CMS collaboration, Phys.Rev. D94 (2016) no.11, 112005
$p_T > 20\text{GeV}, |\eta| < 2.4\,(\text{tagged}), 4.7\,(\text{untagged}), R = 0.3$

- We generate matrix elements without the restriction $k_{T,i}^2 < \mu^2$.
- Jets equally hard or harder than those from the hard matrix element can come from the showering.
- The predictions without parton showers now do not agree with the data
- Once we include showers and full remnant treatment, we are even more off.
- We conclude that, in this ME+PS scenario, High energy Factorization suggests the need for MPI's.
- **PRELIMINARY. UPDATES COMING SOON**



What will be next: improving the CCFM evolution



Real contributions to generalised TMD à la Curci-Furmanski-Petronzio
Hentschinski, Gituliar, Kutak, JHEP 1601 (2016) 181

- axial, light-cone gauge: collinear singularities only from upper propagators
- Checked equivalence between Lipatov's vertices and spinor helicity formalism in the cases above.

$$\Gamma^{\mu}_{q^*g^*q}(q,k,p') = igt^a\left(\gamma^{\mu} - \frac{n^{\mu}}{k\cdot n}\not{q}\right)$$

$$\Gamma^{\mu}_{g^*q^*q}(q,k,p') = igt^a\left(\gamma^{\mu} - \frac{p^{\mu}}{p\cdot q}\not{k}\right)$$

$$\Gamma^{\mu}_{q^*q^*g}(q,k,p') = igt^a\left(\gamma^{\mu} - \frac{p^{\mu}}{p\cdot p'}\not{k} + \frac{n^{\mu}}{n\cdot p'}\not{q}\right)$$

Trickier in the gluon case: check not only DGLAP, but also BFKL limit !

- Virtual contributions: in axial gauge and for off-shell vertices, they possibly imply computing integrals with more than one linear denominator.

To be provided: real and virtual P_{gg}, virtual P_{qq}
Hentschinski, Kusina, Kutak, MS, in preparation

What will be next: improving the CCFM evolution

Available: real contributions to the angular averaged P_{qg} , P_{gq} , P_{qq} TMD splitting functions: gauge invariant and correct DGLAP limits !

$$P_{qg}^{(0)}\left(z,\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2},\epsilon\right)=T_R\left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2+z(1-z)\,\mathbf{k}^2}\right)^2\left[z^2+(1-z)^2+4z^2(1-z)^2\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right]$$

$$P_{gq}^{(0)}\left(z,\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2},\epsilon\right)=C_F\left[\frac{2\tilde{\mathbf{q}}^2}{z|\tilde{\mathbf{q}}^2-(1-z)^2\mathbf{k}^2|}-\frac{\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2(2-z)+\mathbf{k}^2z(1-z^2))-\epsilon z(\tilde{\mathbf{q}}^2+(1-z)^2\mathbf{k}^2)}{(\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2)^2}\right]$$

$$P_{qq}^{(0)}\left(z,\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2},\epsilon\right)=C_f\left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2}\right)\left[\frac{\tilde{\mathbf{q}}^2+(1-z^2)\mathbf{k}^2}{(1-z)|\tilde{\mathbf{q}}^2-(1-z)^2\mathbf{k}^2|}+\frac{z^2\tilde{\mathbf{q}}^2-z(1-z)(1-3z+z^2)\mathbf{k}^2+(1-z)^2\epsilon(\tilde{\mathbf{q}}^2+z^2\mathbf{k}^2)}{(1-z)(\tilde{\mathbf{q}}^2+z(1-z)\mathbf{k}^2)}\right]$$

$$k_{in}^{\mu}=yp^{\mu}+k_{\perp}^{\mu}\,,\quad q_{out}^{\mu}=xp^{\mu}+q_{\perp}^{\mu}+\frac{q^2+\mathbf{q}^2}{2xp\cdot n}n^{\mu}\,,\quad \tilde{\mathbf{q}}=\mathbf{q}-\frac{x}{y}\mathbf{k}$$

To be provided: real and virtual P_{gg}, virtual P_{qq}
Hentschinski, Kusina, Kutak, MS, in preparation

Higher order corrections to 2-jet production

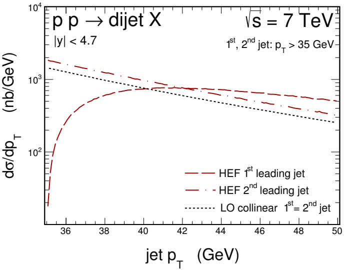


Figure: The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF.

NLO corrections to 2-jet production suffer from instability problem when using symmetric cuts: Frixione, Ridolfi, Nucl.Phys. B507 (1997) 315-333

Symmetric cuts rule out from integration final states in which the momentum imbalance due to the initial state non vanishing transverse momenta gives to one of the jets a lower transverse momentum than the threshold.

ATLAS data vs. theory (nb) @ LHC7 for 2,3,4 jets. Cuts are defined in Eur.Phys.J. C71 (2011) 1763; theoretical predictions from Phys.Rev.Lett. 109 (2012) 042001

#jets	ATLAS (nb)	LO (nb)	NLO (nb)
2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$1193(3)^{+130}_{-135}$
3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$54.5(0.5)^{+2.2}_{-19.9}$
4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$5.54(0.12)^{+0.08}_{-2.44}$

Reconciling HE and collinear factorizations : asymmetric p_T cuts

CMS collaboration : $\sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\ nb$

$p_T(1,2) \geq 50\text{GeV}\,, \quad p_T(3,4) \geq 20\text{ GeV}\,, |\eta| < 4.7\,, \quad \Delta R > 0.5$

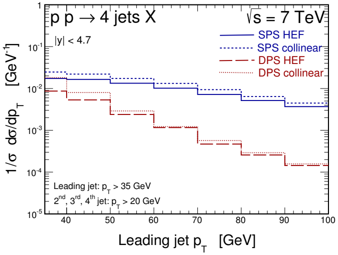
LO collinear factorization : $\sigma_{SPS} = 697\ nb\,, \quad \sigma_{DPS} = \mathbf{125\ nb}\,, \quad \sigma_{tot} = 822\ nb$

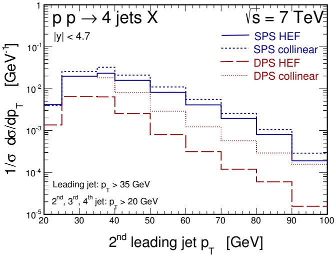
LO HEF k_T-factorization : $\sigma_{SPS} = 548\ nb\,, \quad \sigma_{DPS} = \mathbf{33\ nb}\,, \quad \sigma_{tot} = 581\ nb$

$p_T(1) \geq 35\text{GeV}\,, \quad p_T(2,3,4) \geq 20\text{ GeV}\,, |\eta| < 4.7\,, \quad \Delta R > 0.5$

LO collinear factorization : $\sigma_{SPS} = 1969\ nb\,, \quad \sigma_{DPS} = 514\ nb\,, \quad \sigma_{tot} = 2309\ nb$

LO HEF k_T-factorization : $\sigma_{SPS} = 1506\ nb\,, \quad \sigma_{DPS} = 297\ nb\,, \quad \sigma_{tot} = 1803\ nb$





DPS is competitive with SPS at low transverse momenta.

Preliminary assessments of the potential of various asymmetric cuts

Below, HEF predictions with **DLC2016** without PS for 4 jets (no b-tagging).
Experimentally ideal: using particle flow jets, in order to optimally deal with high pile-up $\Rightarrow \boldsymbol{R = 0.4}\,, |\eta| < \mathbf{2.1}$.

Competing effects at work :

1. The lower the highest cut, the more DPS we can see (see PDFs)
2. As the spread in transverse momentum between jets wides up, the phase space is reduced
3. One has to be careful to impose an asymmetry in $\boldsymbol{p_T \geq 5}$ GeV, in order to tame extra logarithms *Alioli, Andersen, Oleari, Re, Smillie, Phys.Rev. D85 (2012) 114034*

- $\boldsymbol{p_T(1) \geq 40\ GeV\,, p_T(2) \geq 30\ GeV\,, p_T(3) \geq 20\ GeV\,, p_T(4) \geq 10\ GeV}$
  $\boldsymbol{\sigma_{SPS} = 2132nb\,, \sigma_{DPS} = 240nb \Rightarrow f_{DPS} \simeq\ 0.11}$
- $\boldsymbol{p_T(1) \geq 40\ GeV\,, p_T(2) \geq 25\ GeV\,, p_T(3) \geq 25\ GeV\,, p_T(4) \geq 10\ GeV}$
  $\boldsymbol{\sigma_{SPS} = 1571nb\,, \sigma_{DPS} = 151nb \Rightarrow f_{DPS} \simeq 0.10}$
- $\boldsymbol{p_T(1) \geq 35\ GeV\,, p_T(2) \geq 20\ GeV\,, p_T(3) \geq 20\ GeV\,, p_T(4) \geq 20\ GeV}$
  $\boldsymbol{\sigma_{SPS} = 512nb\,, \sigma_{DPS} = 104nb \Rightarrow f_{DPS} \simeq 0.17}$

Not very different for equal cuts on the second and third jet:
Better to stick to 2 rather than more different cuts because of point 3.
Showered prediction to be produced next.

Summary and perspectives

- **Previous results**: HEF smears out the DPS contribution to the cross section, pushing the DPS-dominance region to lower p_T, but **asymmetric cuts are in order**: initial state transverse momentum generates asymmetries in the p_T of final state jet pairs.
- Preliminary: ΔS variable, potential DPS smoking gun, does not really seem to do well without DPS, if we add with final state PS. With parton showers + remnant treatment: hardest k_T not always coming from the hard matrix element. Individual results with various TMDs to be matched to suitable showers, **with M. Bury, A. van Hameren, H. Jung and K. Kutak**
- It will be interesting to have an experimental analysis with asymmetric cuts, in order to enhance DPS. We are going to produce predictions. Without PS, some preliminary results were already presented in this talk. CMS potentially better than ATLAS thanks to particle flow reconstruction.
- **HEF is being pushed from various sides, among which**:
  - 1-loop amplitudes in HEF, **with A. van Hameren and O. Gituliar**
  - TMD splitting kernels improving the CCFM evolution, currently limited to the small and large-z components: **with K. Kutak, M. Hentschinski and A. Kusina** :
    based on the Curci-Furmanski-Petronzio approach, progress ongoing,
    $\Rightarrow$ **see Olek Kusina's talk next week at DIS**, based on
    Hentschinski, Gituliar, Kutak, JHEP 1601 (2016) 181
    Hentschinski, Kusina, Kutak, Phys.Rev. D94 (2016) no.11, 114013
    Hentschinski, Kusina, Kutak, MS, in preparation

Backup slides

Prescription for off-shell gluons: derivation 1

Auxiliary vectors $p_{3,4}$ (complex in general):
$$\begin{cases} p_3^\mu = \frac{1}{2}\langle p_2|\gamma^\mu|p_1] \\ p_4^\mu = \frac{1}{2}\langle p_1|\gamma^\mu|p_2] \\ p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0 \\ p_{1,2}\cdot p_{3,4} = 0\,, \quad p_1\cdot p_2 = -p_3\cdot p_4 \end{cases}$$

Auxiliary momenta:
$$\begin{cases} p_A^\mu = (\Lambda + x_1)p_1^\mu - \frac{p_4\cdot k_{1\perp}}{p_1\cdot p_2}p_3^\mu\,, \quad p_{A'}^\mu = \Lambda p_1^\mu + \frac{p_3\cdot k_{1\perp}}{p_1\cdot p_2}p_4^\mu \\ p_B^\mu = (\Lambda + x_2)p_2^\mu - \frac{p_3\cdot k_{2\perp}}{p_1\cdot p_2}p_4^\mu\,, \quad p_{B'}^\mu = \Lambda p_2^\mu + \frac{p_4\cdot k_{2\perp}}{p_1\cdot p_2}p_3^\mu \end{cases}$$

For any Λ:
$$\begin{cases} p_A^\mu - p_{A'}^\mu = x_1\, p_1^\mu + k_{1\perp}^\mu \\ p_B^\mu - p_{B'}^\mu = x_2\, p_2^\mu + k_{2\perp}^\mu \\ p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0 \end{cases}$$

Prescription for off-shell gluons: derivation 2

Momentum flowing through a propagator of an auxiliary quark line:

$$k^\mu = (\Lambda + x_k)p_1^\mu + y_k\, p_2^\mu + k_\perp^\mu$$

Final step: remove complex components taking the $\Lambda \to \infty$ limit.

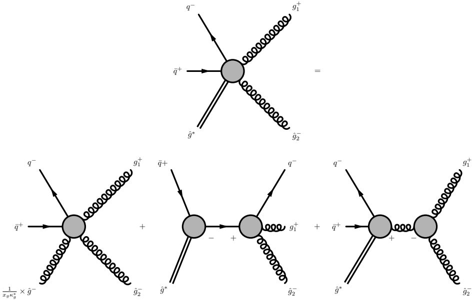
$$\frac{\not{k}}{k^2} = \frac{(\Lambda + x_k)\not{p}_1 + y_k\,\not{p}_2 + \not{k}}{2(\Lambda + x_k)y_k p_1 \cdot p_2 + k_\perp^2} \overset{\Lambda\to\infty}{\longrightarrow} \frac{\not{p}_1}{2\, y_k p_1 \cdot p_2} = \frac{\not{p}_1}{2p_1 \cdot k}$$

Results are in agreement with the ones gotten from Lipatov's effective action

Lipatov Nucl.Phys. B452 (1995) 369-400

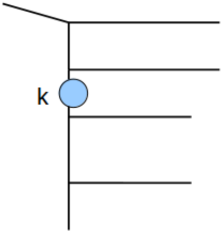
Antonov, Lipatov, Kuraev, Cherednikov, Nucl.Phys. B721 (2005) 111-135

A 5-point amplitude via BCFW in 3 simple contributions



$$\mathcal{A}(g^*, \bar{q}^+, q^-, g_1^+, g_2^-) = \frac{1}{\kappa_g^*} \frac{[\bar{q}1]^3 \langle 2g \rangle^4}{[\bar{q}q]\langle g|\not{p}_2 + \not{k}_g|1]\langle 2|\not{k}_g \; (\not{k}_g + \not{p}_2)\,|g]\langle 2|\not{k}_g|\bar{q}]}$$
$$+ \frac{1}{\kappa_g} \frac{1}{(k_g + p_{\bar{q}})^2} \frac{[g\bar{q}]^2 \langle 2q \rangle^3 \langle 2|\not{k}_g + \not{p}_{\bar{q}}|g]}{\langle 1q \rangle \langle 12 \rangle \left\{ (k_g + p_{\bar{q}})^2 [\bar{q}g]\langle 2q \rangle - \langle 2|\not{k}_g + \not{p}_{\bar{q}}|g]\langle q|\not{k}_g|\bar{q}] \right\}}$$
$$+ \frac{\langle gq \rangle^3 [g1]^4}{\langle \bar{q}q \rangle [12][g2]\langle q|\not{p}_1 + \not{p}_2|g]\langle g|\not{p}_1 + \not{p}_2|g]\langle g|\not{k}_g + \not{p}_2|1]}$$

Our PDFs: KMR prescription



Survival probability without emissions

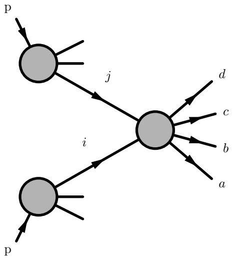
Kimber, Martin, Ryskin, Phys.Rev. D63 (2001) 114027 :

$$
\begin{aligned}
T_s(\mu^2, k^2) &= exp\Big(- \int_{\mu^2}^{k^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \\
&\qquad \times \sum_{a'} \int_0^{1-\Delta} dz' P_{aa'}(z') \Big) \\
\Delta &= \frac{\mu}{\mu + k}, \quad \mu = \text{hard scale} \\
\mathcal{F}(x, k^2, \mu^2) &\sim \partial_{\lambda^2} \left(T_s(\lambda^2, \mu^2)\, x\, g(x, \lambda^2) \right) \Big|_{\lambda^2 = k^2}
\end{aligned}
$$

First prescription: DLC2016 (Double Log Coherence), derived from CT10nlo and used in
K. Kutak, R. Maciula, MS, A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175

Second prescription to get TMD PDFs from the same Sudakov also shown in this talk, together with assessment of the sensitivity to the underlying collinear PDFs.
Martin, Ryskin, Watt Phys.Rev. D70 (2004) 014012,
Erratum: Phys.Rev. D70 (2004) 079902

4-jet production: Single Parton Scattering (SPS)



We take into account all the (according to our conventions) 20 channels.

Here q and q' stand for different quark flavours in the initial (final) state.

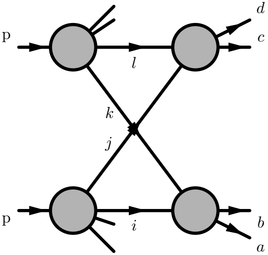
We do not introduce K factors, amplitudes@LO.

$\sim$ 95 % of the total cross section

There are 19 different channels contributing to the cross section at the parton-level:

$$gg \to 4g\,, gg \to q\bar{q}\,2g\,, qg \to q\,3g\,, q\bar{q} \to q\bar{q}\,2g\,, qq \to qq\,2g\,, qq' \to qq'\,2g\,,$$
$$gg \to q\bar{q}q\bar{q}\,, gg \to q\bar{q}q'\bar{q}'\,, qg \to qgq\bar{q}\,, qg \to qgq'\bar{q}'\,,$$
$$q\bar{q} \to 4g\,, q\bar{q} \to q'\bar{q}'\,2g\,, q\bar{q} \to q\bar{q}q\bar{q}\,, q\bar{q} \to q\bar{q}q'\bar{q}'\,, q\bar{q} \to q'\bar{q}'q'\bar{q}'\,,$$
$$q\bar{q} \to q'\bar{q}'q''\bar{q}''\,, qq \to qqq\bar{q}\,, qq \to qqq'\bar{q}'\,, qq' \to qq'q\bar{q}\,,$$

4-jet production: Double parton scattering (DPS)



$$\sigma = \sum_{i,j,a,b;k,l,c,d} \frac{\mathcal{S}}{\sigma_{eff}} \sigma(i,j \to a,b)\,\sigma(k,l \to c,d)$$

$$\mathcal{S} = \begin{cases} 1/2 & \text{if} \quad i\,j = k\,l \quad \text{and} \quad a\,b = c\,d \\ 1 & \text{if} \quad i\,j \neq k\,l \quad \text{or} \quad a\,b \neq c\,d \end{cases}$$

$$\sigma_{eff} = 15\,mb\,,\text{(CDF, D0 and LHCb collaborations)}\,,$$

Experimental data may hint at different values of σ_{eff}; main conclusions not affected

In our conventions, 9 channels from $2 \to 2$ SPS events,

$$\begin{aligned} \#1 &= gg \to gg\,, & \#6 &= u\bar{u} \to d\bar{d} \\ \#2 &= gg \to u\bar{u}\,, & \#7 &= u\bar{u} \to gg \\ \#3 &= ug \to ug\,, & \#8 &= uu \to uu \\ \#4 &= gu \to ug\,, & \#9 &= ud \to ud \\ \#5 &= u\bar{u} \to u\bar{u} \end{aligned}$$

$\Rightarrow$ 45 channels for the DPS; only 14 contribute to $\geq 95\%$ of the cross section :

$$(1,1),(1,2),(1,3),(1,4),(1,8),(1,9),(3,3)$$
$$(3,4),(3,8),(3,9),(4,4),(4,8),(4,9),(9,9)$$

Pinning down double parton scattering: $\Delta\phi_3^{min}$ - azimuthal separation

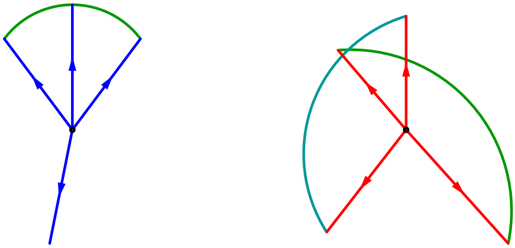


Figure: Left: typical 4-particle final state topology associated with SPS. Right: typical 4-particle final state topology generated by DPS. No way, in the latter case, to get a $\Delta\phi_{\mathbf{3}}^{min}$ below

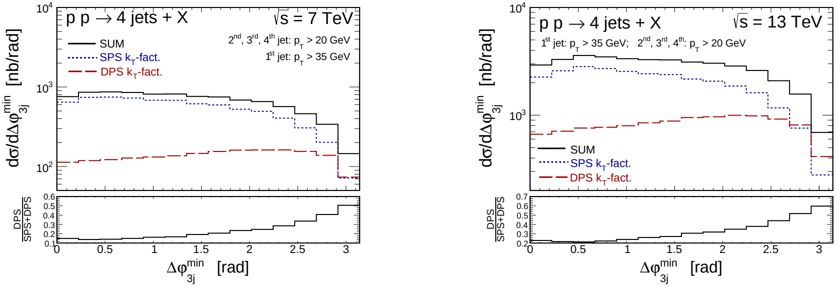
Minimum total distance in azimuthal angle between triplets of jets:

$$\Delta\phi_{\mathbf{3}}^{min} = min_{i,j,k[\mathbf{1},\mathbf{4}]} \left(\left|\phi_i - \phi_j\right| + \left|\phi_j - \phi_k\right| \right) \; , \quad i \neq j \neq k$$

Almost back-to-back topologies clearly favour high values of this variable !
$\Rightarrow$ DPS is expected to push up the cross section in the high-$\Delta\Phi_3^{min}$ region

Pinning down double parton scattering: $\Delta\phi_3^{min}$ - azimuthal separation

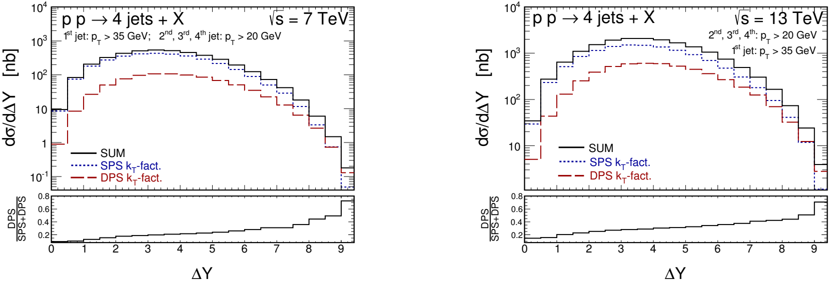
K. Kutak, R. Maciula, MS, A. Szczurek, A. van Hameren
Phys.Rev. D94 (2016) no.1, 014019



- Definition: $\Delta\phi_3^{min} = min_{i,j,k[1,4]} \left(|\phi_i - \phi_j| + |\phi_j - \phi_k|\right)$, $\quad i \neq j \neq k$
- Proposed by ATLAS in *JHEP 12 105 (2015)* for high p_T analysis
- High values favour DPS, because there is no way to construct a low value from a (nearly) back-to-back configuration.
- For $\Delta\phi_3^{min} \geq 2\pi/3$ the total cross section is heavily affected by DPS at 13 TeV.

Pinning down double parton scattering: large rapidity separation

K. Kutak, R. Maciula, MS, A. Szczurek, A. van Hameren
Phys.Rev. D94 (2016) no.1, 014019



- It is interesting to look for kinematic variables which could make DPS apparent.
- The maximum rapidity separation in the four jet sample is one such variable, especially at 13 GeV.
- for $\Delta Y > 6$ the total cross section is dominated by DPS.