Resummation for top quark pair production at the LHC at NNLO+NNLL′

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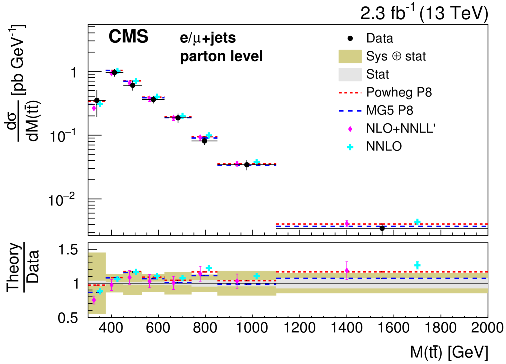
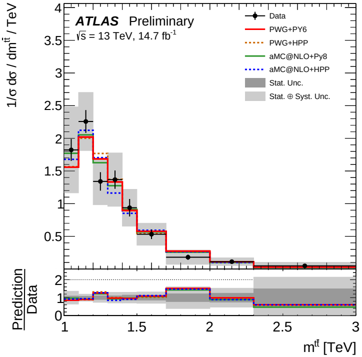
Outline

Top quarks at the LHC

LHC already beginning to probe high energy tails of $t\bar{t}$ distributions
[CMS-TOP-16-008] [ATLAS-CONF-2016-100]





Boosted regime not just a "corner of phase space"

Formalism

Consider $t\bar{t}$ production at hadron colliders.

$$i(p_1) + j(p_2) \to t(p_3) + \bar{t}(p_4) + X(p_X)$$

With $ij \in \{q\bar{q}, \bar{q}q, gg\}$ at leading order. QCD factorisation allows us to write the cross section for such processes as

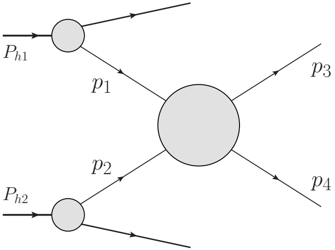
$$\frac{d^2\sigma(\tau)}{dM\, d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z, \mu_f) C_{ij}(z, M, m_t, \cos\theta, \mu_f)$$

$$\mathcal{L}_{ij}(y) = \int_y^1 \frac{dx}{x} \phi_i(x)\phi_j(y/x)$$

$$s = (P_{h1} + P_{h2})^2 \quad \hat{s} = (p_1 + p_2)^2$$

$$M_{t\bar{t}}^2 = (p_3 + p_4)^2$$

$$\tau = \frac{M_{t\bar{t}}^2}{s} \quad z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$



True threshold: $\tau \to 1$
Partonic threshold: $z \to 1$

Factorisation: Threshold

Perturbative expansions can be plagued by threshold plus distributions

$$\alpha_s^n \left[\frac{\ln^m(1-z)}{1-z} \right]_+ \quad 0 \le m \le 2n-1$$

The partonic cross section factorises in the $z \to 1$ limit.

$$\hat{s}, M_{tt}^2, m_t^2 >> \hat{s}(1-z)^2$$

In Mellin moment space [Kidonakis, Sterman, 9705234]
Using the SCET framework [Ahrens, Ferroglia, Neubert, Pecjak, Yang: 1003.5827]
Factorisation allows Resummation

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

$\mathbf{H}^m$, $\mathbf{S}^m$, matrices in colour space

$\mathbf{H}_{ij}$ - Hard Function. Related to virtual corrections
$\mathbf{S}_{ij}$ - Soft Function. Related to real emission of soft gluons.
Contains distributions singular in $(1-z)$.

Factorisation: Booosted Tops

The boosted soft limit, $z \to 1$ and $M >> m_t$,

$$\hat{s}, t_1 >> m_t^2 >> \hat{s}(1-z)^2 >> m_t^2(1-z)^2$$

logs of the form $\ln(M/m_t)$ become important.
Further factorisaton in this limit. [Ferroglia, Pecjak, Yang: 1205.3662]

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

$$M^2 >> m_t^2$$

$$C_{ij} = C_D^2(m_t, \mu_f)\,\mathrm{Tr}\left[\mathbf{H}_{ij}(M, \mu_f, ..)\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), \mu_f, ...)\right] \otimes \mathbf{s}_D\left(m_t(1-z), \mu_f\right)$$
$$\otimes\, \mathbf{s}_D\left(m_t(1-z), \mu_f\right) \otimes c_{ij}^t(z, m_t, \mu_f) + \mathcal{O}(1-z) + \mathcal{O}\left(m_t/M\right)$$

Each of these functions known to two-loops

$\mathbf{H}$: [Glover et. al: '00-'01]
$\mathbf{S}$: [Ferroglia, Pecjak, Yang: 1207.4798]
s_D, C_D: [Melnikov, Mitov: 0404143], [Becher, Neubert: 0512208]

Mellin Space

For this talk, we are going to work in Mellin space. Convolutions become products

$$d\tilde{\sigma}(N) = \tilde{\mathcal{L}}(N)\tilde{C}(N)$$

where

$$\tilde{f}(N) = \int_0^1 dx\, x^{N-1} f(x) \qquad f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N} f(\tilde{N})$$

In Mellin space, the $z \to 1$ limit corresponds to $N \to \infty$

$$P_n(z) = \left[\frac{\ln^n(1-z)}{1-z}\right]_+ \qquad \bar{N} = N e^{\gamma_E}$$

$$\begin{aligned}
\mathcal{M}[P_0] &= -\ln\bar{N} + \mathcal{O}(1/N) \\
\mathcal{M}[P_1] &= \frac{1}{2}\left(\ln^2\bar{N} + \frac{\pi^2}{6}\right) + \mathcal{O}(1/N) \\
\mathcal{M}[P_2] &= -\frac{1}{3}\left(\ln^3\bar{N} + \frac{\pi^2}{2}\ln\bar{N} + 2\zeta(3)\right) + \mathcal{O}(1/N)
\end{aligned}$$

Mellin Space

Our factorisation formula becomes

$$C(N) = C_D^2(m_t, \mu_f)\,\mathrm{Tr}\left[\mathbf{H}(M, \mu_f, ..)\tilde{\mathbf{S}}\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, \mu_f, ...\right)\right]$$
$$\times\, \tilde{\mathbf{s}}_D^2\left(\ln\frac{m_t}{\bar{N}\mu_f}, \mu_f\right)\tilde{c}^t(\ln\bar{N}, m_t, \mu_f) + \mathcal{O}\left(1/N\right) + \mathcal{O}\left(m_t/M\right)$$

We now have single scale functions.

Aside: Heavy flavour matching coefficient, $\tilde{c}_{ij}^t$, introduces additional $\ln m_t$ dependence which is not resummed. We add such contributions in fixed order.

Resummed Results

One can derive and solve RG equations for each function.
The result can be written as,

$$
\begin{aligned}
C(N) = \exp\Bigg\{ & \frac{4\pi}{\alpha_s(\mu_h)}\left(g_1(\lambda_s,\lambda_f) + g_1^D(\lambda_{dh},\lambda_{ds},\lambda_f)\right) \\
& + \left(g_2(\lambda_s,\lambda_f) + g_2^D(\lambda_{dh},\lambda_{ds},\lambda_f)\right) + \ldots \Bigg\} \\
\times \mathrm{Tr}\Bigg[& \mathbf{u}(M,\cos\theta,\mu_h,\mu_s)\mathbf{H}(M,\cos\theta,\mu_h)\mathbf{u}^\dagger(M,\cos\theta,\mu_h,\mu_s) \\
& \times \tilde{\mathbf{S}}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2}, M, \cos\theta, \mu_s\right)\Bigg] C_D^2(m_t,\mu_{dh})\tilde{s}_D^2\left(\ln\frac{m_t^2}{\bar{N}^2\mu_{ds}^2}, \mu_{ds}\right)
\end{aligned}
$$

Where,

$$
\lambda_i = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\left(\frac{\mu_h}{\mu_i}\right) \qquad \mathbf{u}(M,\cos\theta,\mu_h,\mu_s) = \mathcal{P}\exp\left\{\int_{\mu_h}^{\mu_s}\frac{d\mu'}{\mu'}\gamma^h(M,\cos\theta,\alpha(\mu'))\right\}
$$

We can pick the scale for each function to free it of large logs.

$\mu_h \sim M$, $\mu_s \sim M/\bar{N}$, $\mu_{dh} \sim m_t$ and $\mu_{ds} \sim m_t/\bar{N}$

Resummation accuracy

Schematically,

Boosted soft:

$$C(N) = \exp\left\{\frac{4\pi}{\alpha_s}g_1 + g_2 + \frac{\alpha_s}{4\pi}g_3 + ...\right\} \mathrm{Tr}\left[\mathbf{u}\,\mathbf{H}(\mu_h)\,\mathbf{u}^\dagger\,\tilde{\mathbf{S}}(\mu_s)\right] C_D^2(m_t, \mu_{dh})\tilde{s}_D^2\,(\mu_{ds})$$

Soft:

$$C(N) = \exp\left\{\frac{4\pi}{\alpha_s}g_1^m + g_2^m + \frac{\alpha_s}{4\pi}g_3^m + ...\right\} \mathrm{Tr}\left[\mathbf{u}\,\mathbf{H}^m(\mu_h)\,\mathbf{u}^\dagger\,\tilde{\mathbf{S}}^m(\mu_s)\right]$$

To achieve a given resummation accuracy

	g_i	γ_h	$\mathbf{H}^{(m)}, \widetilde{\mathbf{s}}^{(m)}, c_D, \widetilde{s}_D$
NLL	$g_1,\ g_2$	LO	LO
NNLL	$g_1,\ g_2,\ g_3$	NLO	NLO
NNLL′	$g_1,\ g_2,\ g_3$	NLO	NNLO

In this work we work to NNLL accuracy for the soft resummation and NNLL′ for the boosted soft resummation.

Mellin Inversion

To obtain results in momentum space, we need to invert the Mellin transform

$$\frac{d\sigma(\tau)}{dM\ d\cos\theta} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\ \tau^{-N}\frac{d\tilde{\sigma}(N)}{dM\ d\cos\theta}$$

With c to the right of all singularities. But our resummed coefficient function contains (exponentiated)

$$g_1(\lambda_s,\lambda_f) = \frac{\Gamma_0}{4\beta_0^2}\left[\lambda + (1-\lambda_s\ln(1-\lambda_s)+\lambda_s\ln(1-\lambda_f))\right] \quad \lambda_s = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0\ln\left(\frac{\mu_h}{\mu_s}\right)$$

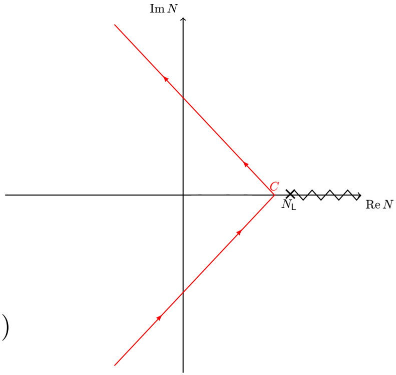
Since we pick $\mu_s \sim M/N$, pole at $\lambda_s = 1$

$$N_L = \exp\left(\frac{2\pi}{\alpha_s\beta_0}\right)$$

Minimal Prescription

- We need to select a method to deal with the Landau pole.
- We use the *Minimal Prescription*: Select our point on the real axis to be to the *left* of the Landau pole, but to the right of all other singularities in the integrand.
  [Catani, Mangano, Nason, Trentadue '96]

$$\frac{d\sigma(\tau)}{dM\ d\cos\theta} = \frac{1}{2\pi i}\int_{\mathsf{MP}_C} dN\ \tau^{-N}\mathcal{L}(N)C(N)$$



Combining and Matching with NNLO results

We wish to combine the results from the two separate resummations and match these with recent NNLO calculations

[Czakon, Fiedler , Heymes, Mitov]

Matching:

$d\sigma_b \sim$ boosted soft factorisation

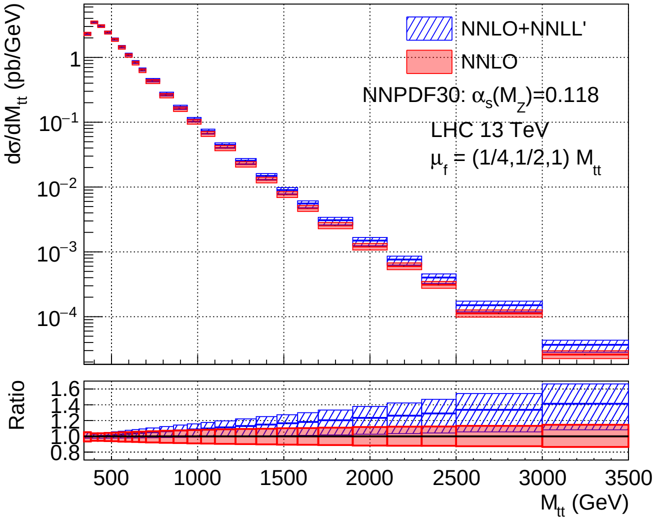
$d\sigma_{\text{Threshold}} \sim$ threshold factorisation

$$d\sigma^{\text{NNLO+NNLL'}} = \underbrace{d\sigma_b^{\text{NNLL'}}}_{\substack{\text{Missing parts subleading} \\ \text{in } m_t/M \text{ and } 1/\text{N}}} + \overbrace{\left(\underbrace{d\sigma_{\text{Threshold}}^{\text{NNLL}}}_{\substack{\text{Missing parts} \\ \text{subleading in } 1/N}} - \underbrace{d\sigma_b^{\text{NNLL}} \Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}}}_{\substack{\text{Removes double} \\ \text{counting}}} \right)}^{\substack{\text{Adds in parts subleading in } m_t/M \\ \text{but enhanced by } \ln N}}$$

$$+ \underbrace{\left(d\sigma^{\text{NNLO}} - d\sigma_{\text{``top line''}}^{\text{NNLL}} \Big|_{\substack{\text{NNLO} \\ \text{expansion}}} \right)}_{\substack{\text{Adds exact NNLO results,} \\ \text{avoiding double counting}}}$$

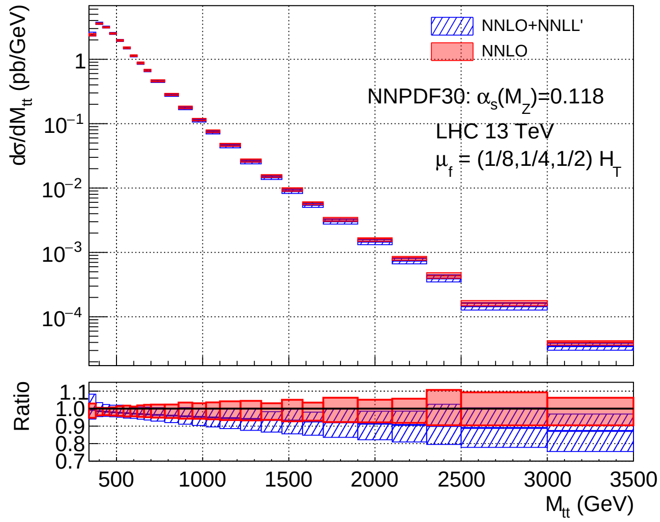
Distributions: M_{tt}

$\mu_f = M_{tt}/2$

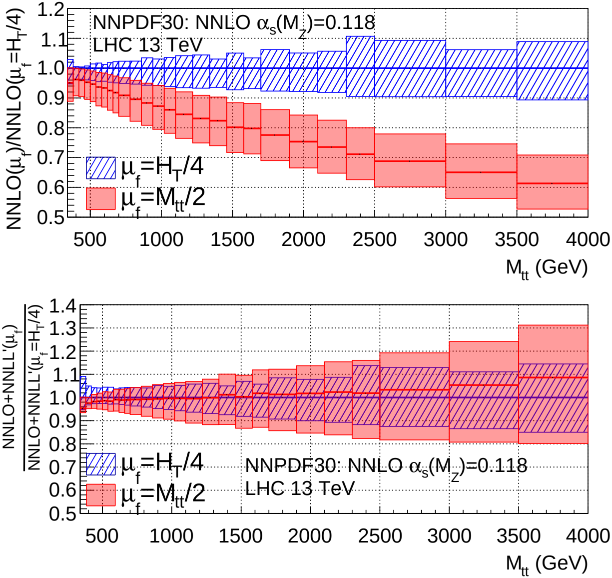


Distributions: M_{tt}

$$\mu_f = H_T/4, \left(H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2}\right)$$

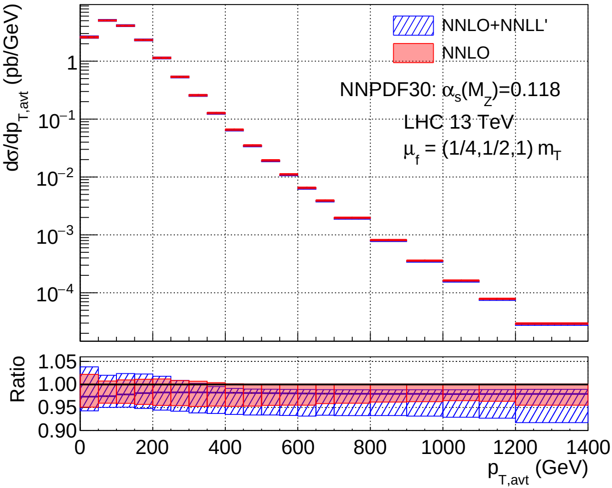


Distributions: M_{tt}



Distributions: p_T

$$\mu_f = m_T/2, \left(m_T = \sqrt{m_t^2 + p_{T,t}^2}\right)$$



Conclusions & Outlook

- Presented factorised differential cross sections:
  - Threshold resummation ($z \to 1$)
  - Boosted Soft resummation ($z \to 1$, $M_{tt} >> m_t$)
- Combined these and matched with fixed order NNLO results, NNLO+NNLL$'$
- Results for M_{tt} and p_T distributions at 13 TeV LHC
- Resummed results for the M_{tt} distributions are less sensitive to the scale choice

BACKUP SLIDES

Total Cross Section

We can also look at the effect on the total cross section

LHC 13 TeV	NNLO	NNLO+NNLL′
$\sigma(\mu_f = m_T)$	$791.8\ ^{+35.7}_{-49.0}$	$787.8\ 1^{+21.1}_{-0.00}$
$\sigma(\mu_f = m_T/2)$	$827.5\ ^{+9.28}_{-35.7}$	$808.9\ ^{+37.2}_{-21.1}$
$\sigma(\mu_f = M_{tt}/2)$	$779.4\ ^{+38.6}_{-50.4}$	$793.8\ ^{+24.4}_{-0.00}$
$\sigma(\mu_f = H_T/4)$	$828.0\ ^{+11.9}_{-36.6}$	$809.3\ ^{+39.8}_{-21.9}$
$\sigma(\mu_f = m_t)$	$802.7\ ^{+28.1}_{-45.30}$	—
$\sigma(\mu_f = m_t/2)$	$830.8\ ^{+0.00}_{-28.1}$	—

top++ can perform NNLL threshold resummation.

$$\sigma^{\text{NNLO+NNLL}}(\mu_f = m_t/2) = 827.7^{+0.0}_{-6.4}$$
$$\sigma^{\text{NNLO+NNLL}}(\mu_f = m_t) = 821.3^{+9.6}_{-0.0}$$

Parton Luminosity in Mellin Space

Our calculation requires parton luminosities in Mellin space. Normally given in momentum space.

$$\mathcal{L}(z)_{ij} = \int_z^1 \frac{dx}{x} \phi_{i/h_1}(x)\phi_{j/h_2}(z/x)$$

We approximate the luminosity in terms of Chebyshev polynomials
[Bonvini: 1212.0480] [Furmanski, Petronzio :164978]

$$\mathcal{L}(z) = \frac{1}{z} \sum_{i=0}^{n} (-2)^i \ln^i(z) \frac{1}{w_{min}^i} \sum_{k=i}^{n} \binom{i}{k} \tilde{c}_k$$

The Mellin transform gives

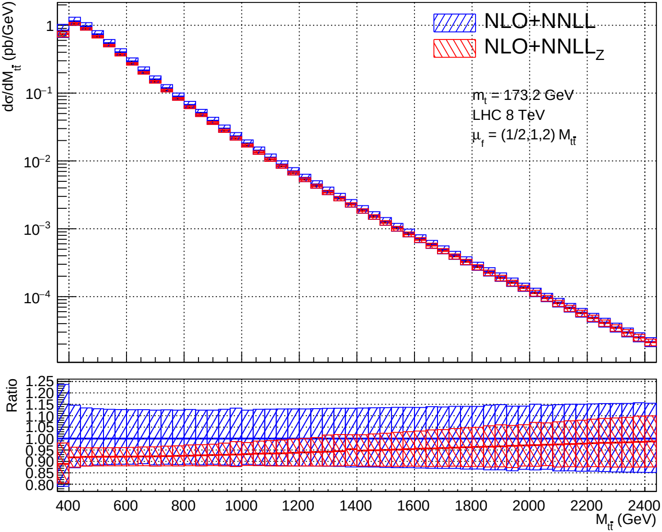
$$\mathcal{L}(N) = \int_0^1 \mathrm{d}z \; z^{N-1} \mathcal{L}(z) = \sum_{p=0}^{n} \frac{\bar{c}_p}{(N-1)^{p+1}}$$

where

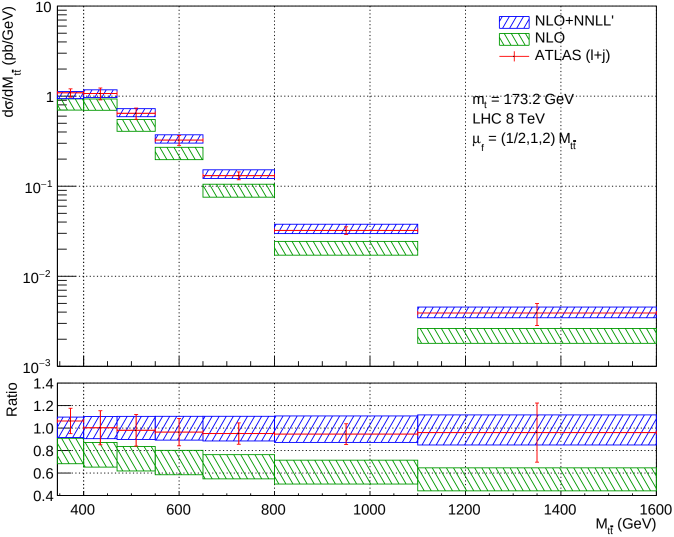
$$\bar{c}_p = \frac{2^p}{w_{min}^p} \sum_{k=p}^{n} \frac{k!}{(k-p)!} \tilde{c}_k$$

Momentum v Mellin Space: (old result)

MSWT2008 PDFs



Comparison with experimental data: (old result)



[ATLAS: 1511.04716]