QCD Master Class 2017

18-24 June 2017
Saint-Jacut-de-la-Mer
Europe/Paris timezone

Theory of Quarkonium Production

Jianwei Qiu
Theory Center, Jefferson Lab

June 19-24, 2017

Lecture one/two

Theory Center

Jefferson Lab
EXPLORING THE NATURE OF MATTER

The plan for my eight lectures

- **The Goal:**

  To understand the theory of heavy quarkonium production, and strong interaction dynamics in terms of QCD

- **The Plan (approximately):**

  Inclusive production of a single heavy quarkonium

  The November Revolution

  Theoretical models and approximations

  Surprises and anomalies

  QCD factorization at the leading and next-to-leading power

  Five lectures

  Heavy quarkonium associate and in medium production

  Quarkonium associate production

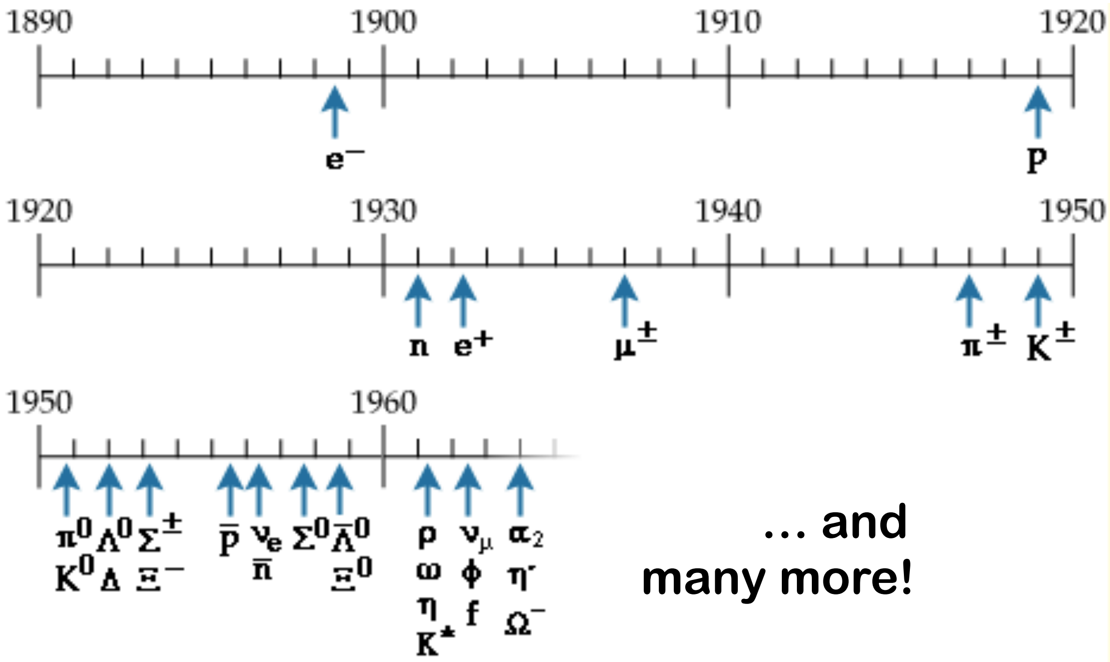
  Quarkonium production in a jet

  Quarkonium production in cold/hot medium

  Three lectures

New particles, new ideas, and new theories

- Early proliferation of new hadrons – "particle explosion":

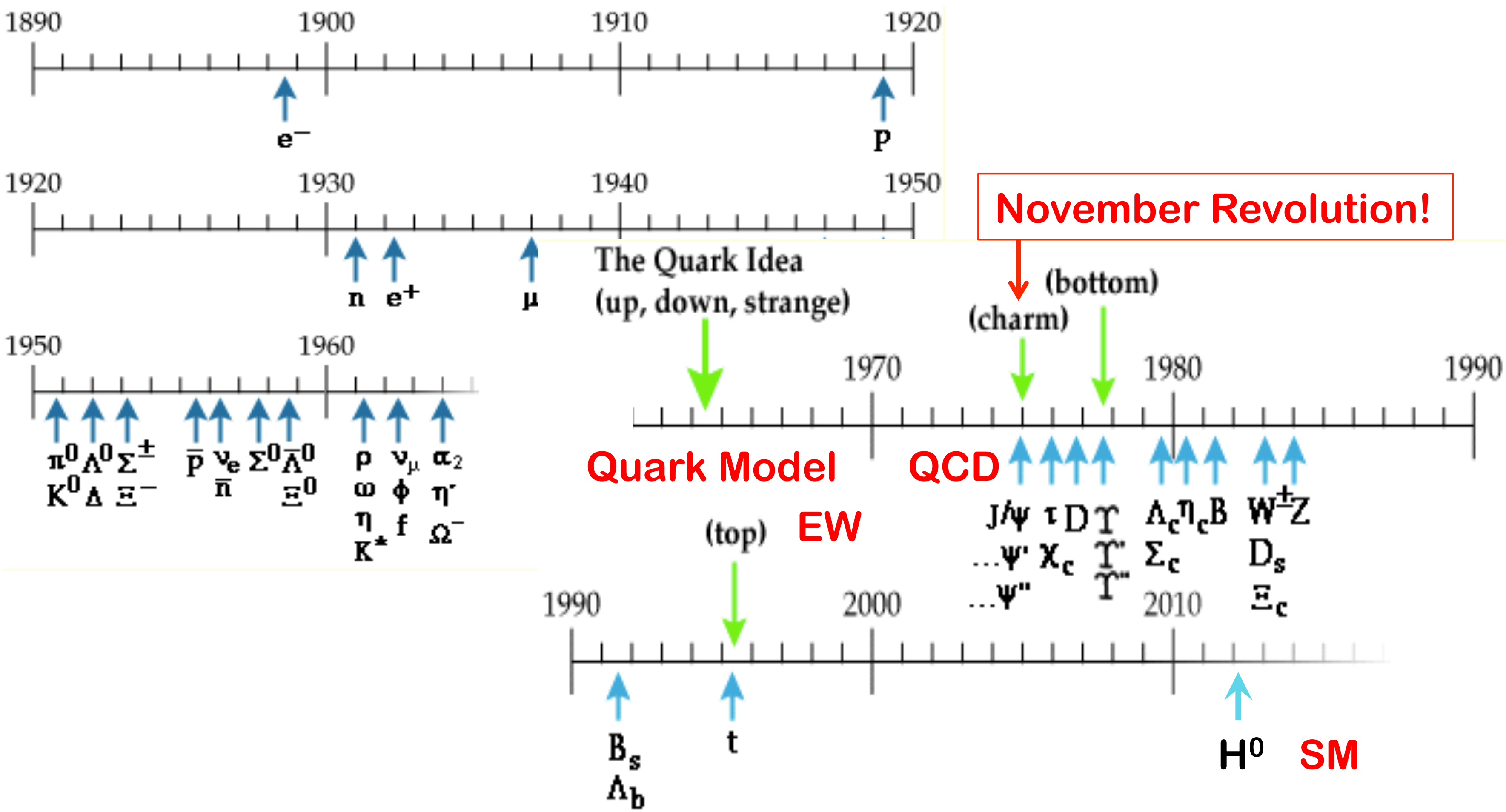


… and
many more!

The idea of "quark, flavor, color, …"
The Quark Model, …

New particles, new ideas, and new theories

- ❑ Proliferation of new particles – "November Revolution":

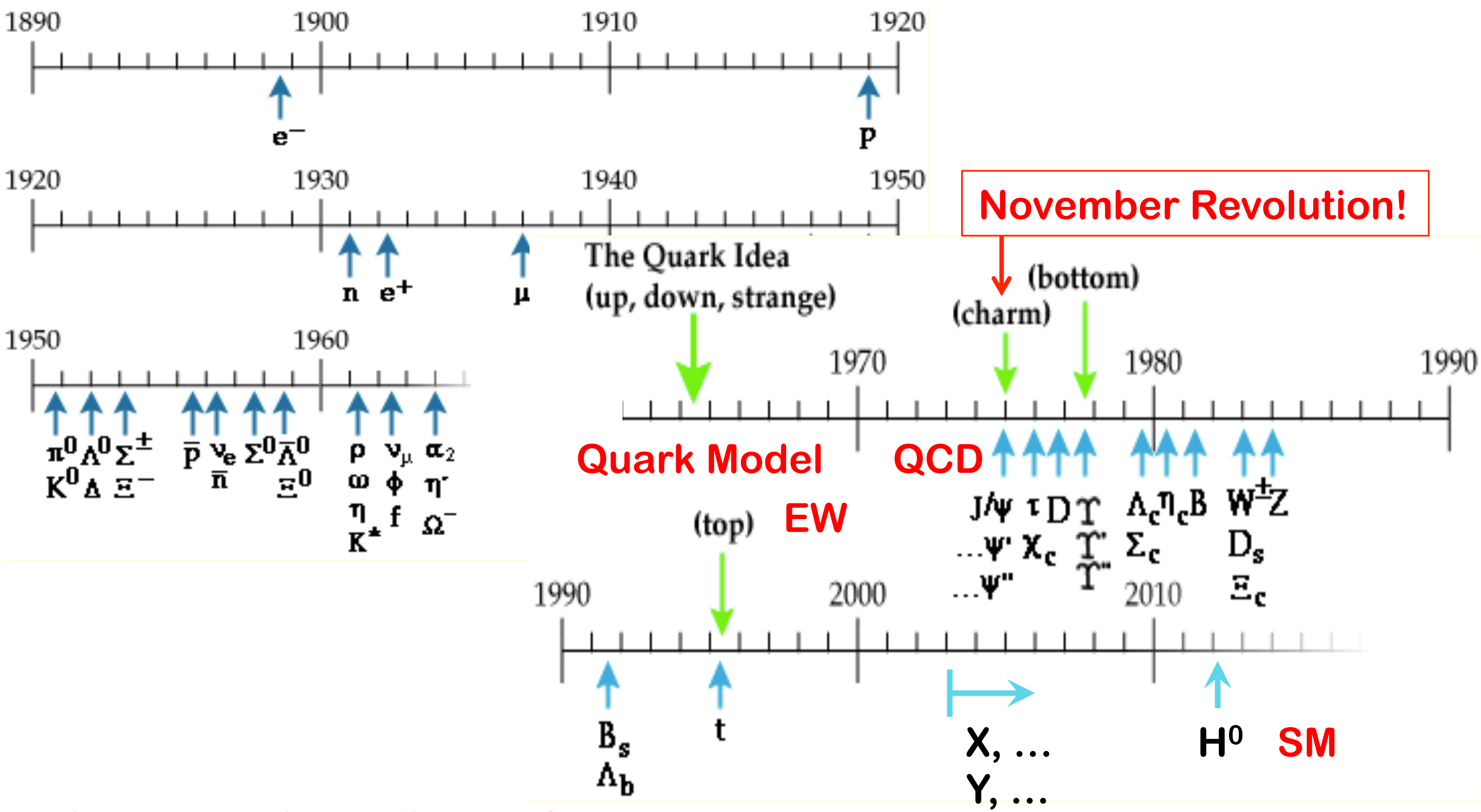


The idea of "heavy" flavor, …

Help the completion of Standard Model (SM), …

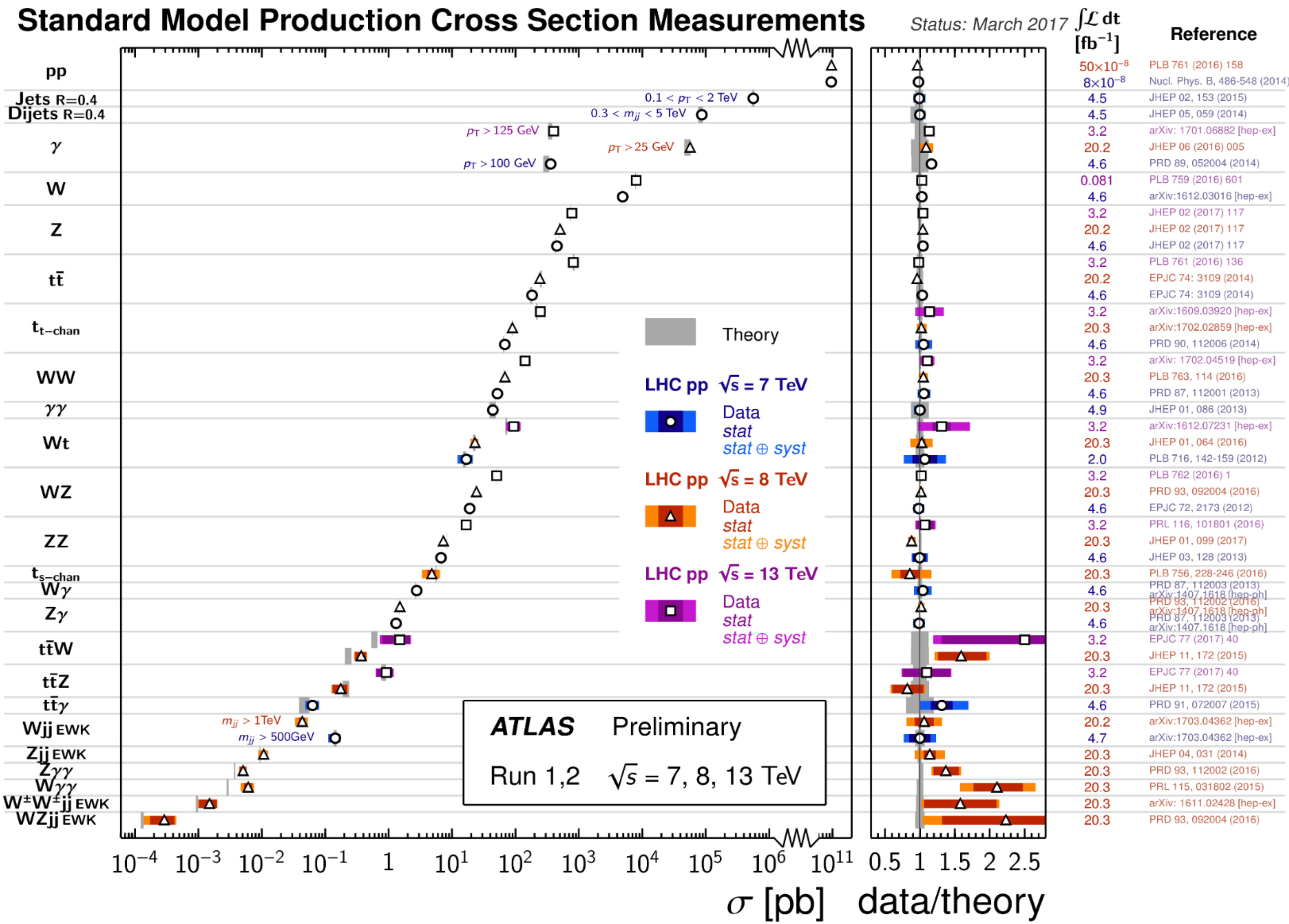
New particles, new ideas, and new theories

- **Proliferation of new particles – "November Revolution":**



Another particle explosion?
New understanding of QCD dynamics?
…

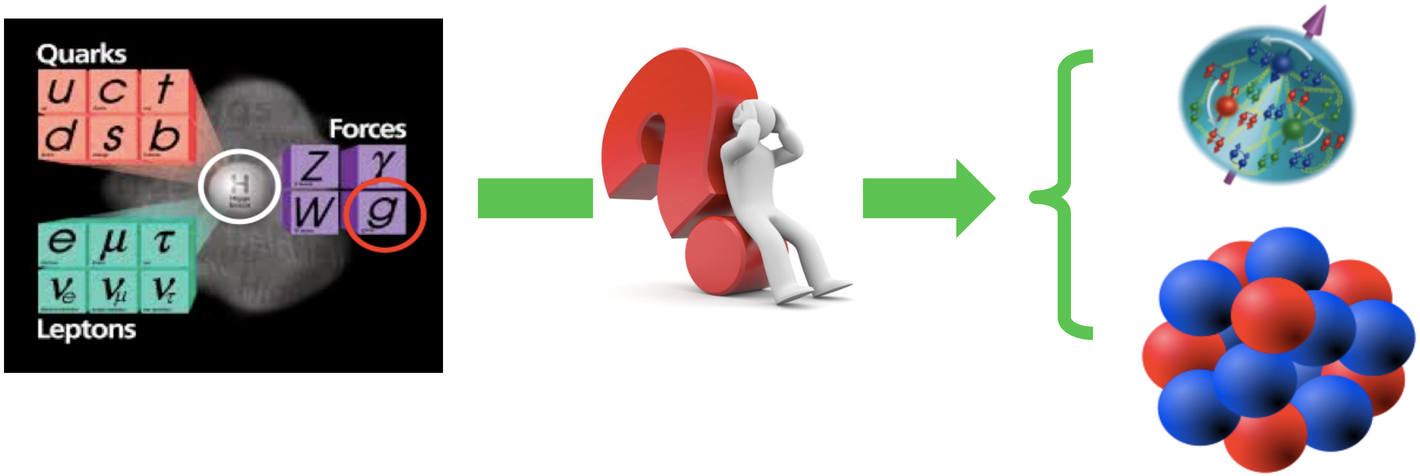
QCD – Final frontier of SM physics



SM: Electroweak processes + QCD perturbation theory works!

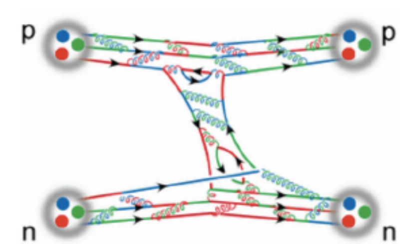
QCD – Final frontier of the SM physics

- How QCD works at low energy – the next QCD frontier!



- How hadrons are emerged from quarks and gluons?
- What is the quark/gluon structure of nucleon and nuclei?
- How does QCD make up the properties of hadrons?

  Their mass, spin, magnetic moment, …
- How does the nuclear force arise from QCD?



- …

Why QCD is so hard to deal with?

- It is strongly coupled – nonlinear + nonperturbative!
- It is relativistic – nontrivial QCD vacuum!
- No localized heavy mass/charge center – nucleus in an atom!
- Gluons are "dark" and carry "color" – intellectual challenge!

How to probe the quark-gluon dynamics, quantify the hadron structure, study the emergence of hadrons, …, if we cannot see quarks and gluons?

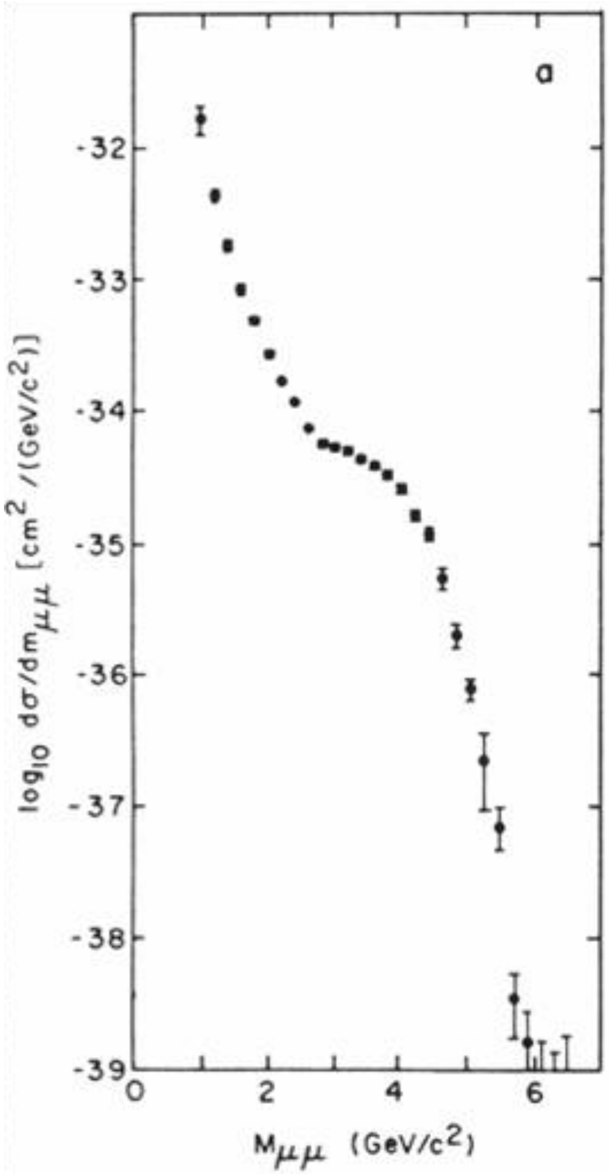
Heavy quarkonium:

- ✧ Heavy quark as relatively localized heavy mass/charge center
- ✧ Heavy quark in the pair's rest frame is almost non-relativistic
- ✧ Production of heavy quark pair could be perturbative
- ✧ Top decays too quickly, strange is too light, …

➡ Charmonium ($c\bar{c}$) + Bottomonium ($b\bar{b}$)

c	$1.0 - 1.4$ GeV
b	$4.0 - 4.5$ GeV

First hint of charm: The Lederman's Shoulder



Phys. Rev. Lett. 25, 1523 (1970)

Production of muon pairs at AGS, BNL

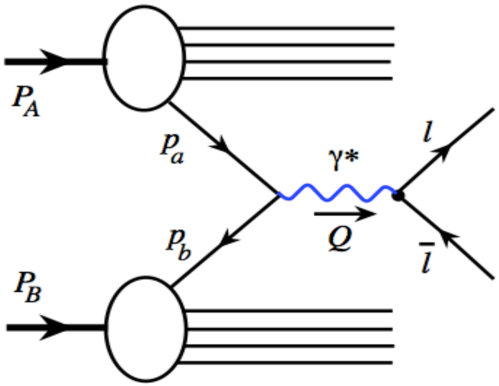
$$p(29\,\mathrm{GeV}) + U \Longrightarrow \mu^+\mu^-(M_{\mu\mu}) + X$$

"..., the cross section varies smoothly as

$$\frac{d\sigma}{dM_{\mu\mu}} \approx \frac{10^{-32}}{M_{\mu\mu}^5}\ \mathrm{cm}^2 \left(\frac{\mathrm{GeV}}{c}\right)^{-2}$$

and exhibits no resonant structure. ..."

Discovery of the Drell-Yan mechanism:



Phys. Rev. Lett. 25, 316 (1970), Erratum-ibid. 25 (1970) 902

First hint of charm: The Lederman's Shoulder

Phys. Rev. Lett. 25, 1523 (1970)

Production of muon pairs at AGS, BNL

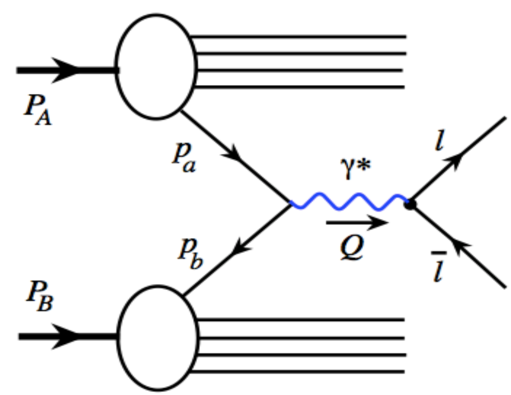
$$p(29\,\mathrm{GeV}) + U \Longrightarrow \mu^+\mu^-(M_{\mu\mu}) + X$$

"…, the cross section varies smoothly as

$$\frac{d\sigma}{dM_{\mu\mu}} \approx \frac{10^{-32}}{M_{\mu\mu}^5}\ \mathrm{cm}^2 \left(\frac{\mathrm{GeV}}{c}\right)^{-2}$$

and exhibits no resonant structure. …"

Discovery of the Drell-Yan mechanism:



Phys. Rev. Lett. 25, 316 (1970), Erratum-ibid. 25 (1970) 902

BUT, missed the discovery of J/Ψ!

November revolution (1974)

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1974

Experimental Observation of a Heavy Particle J†

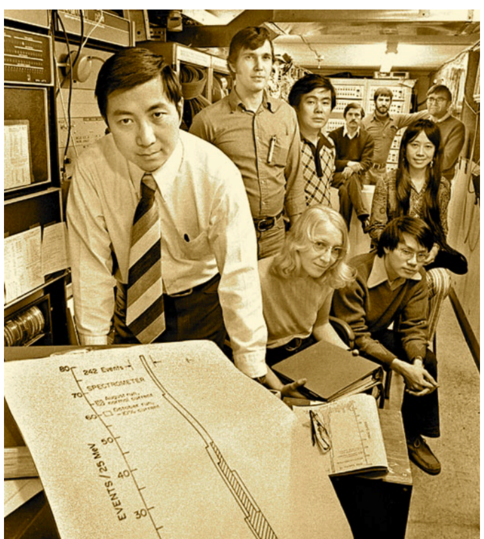
J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

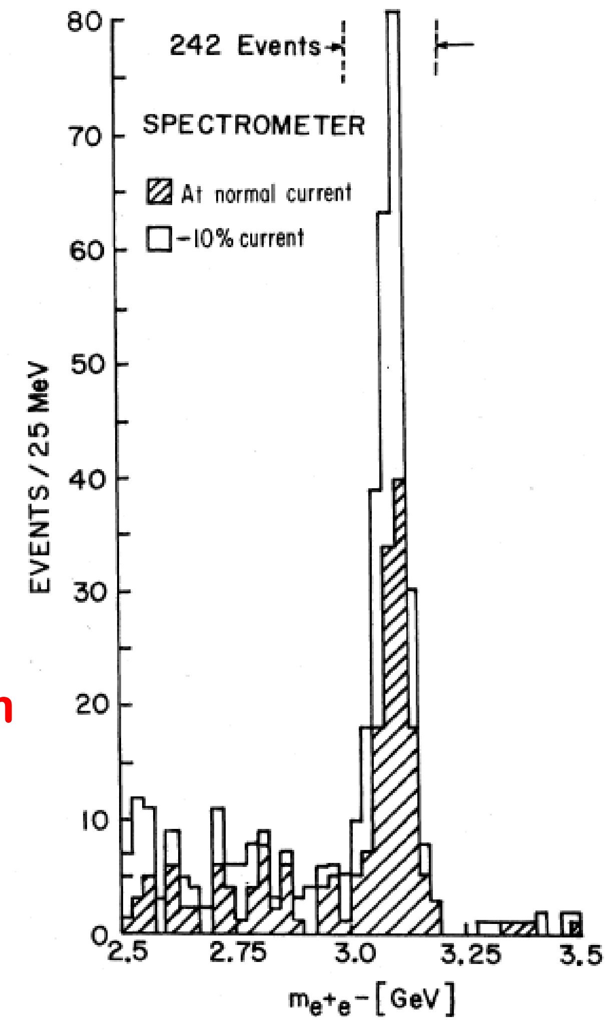
Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973

(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \rightarrow e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.



Lack of dilepton continuum is inconsistent with parton model, and Drell-Yan mechanism



November revolution (1974)

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1974

Discovery of a Narrow Resonance in e^+e^- Annihilation*

J.-E. Augustin,† A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie,† R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, and F. Vannucci‡

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

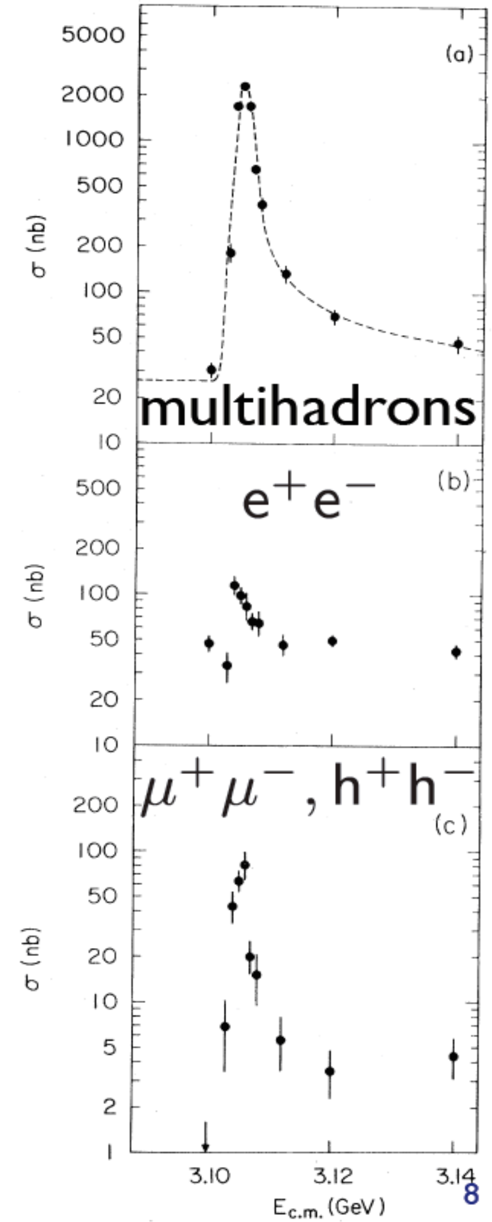
and

G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre,§ G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse

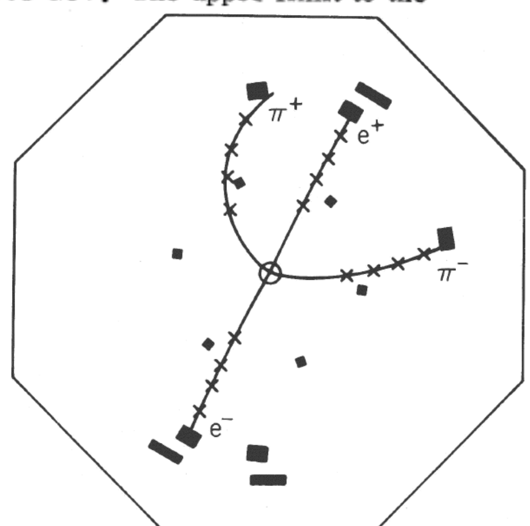
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

(Received 13 November 1974)

We have observed a very sharp peak in the cross section for $e^+e^- \to$ hadrons, e^+e^-, and possibly $\mu^+\mu^-$ at a center-of-mass energy of 3.105 ± 0.003 GeV. The upper limit to the full width at half-maximum is 1.3 MeV.



Same strong enhancement (peak) in multiple channels!



$$e^+e^- \to \psi' \to \pi^+\pi^- J/\psi$$

November revolution (1974)

Preliminary Result of Frascati (ADONE) on the Nature of a New 3.1-GeV Particle Produced in e^+e^- Annihilation*

C. Bacci, R. Balbini Celio, M. Berna-Rodini, G. Caton, R. Del Fabbro, M. Grilli, E. Iarocci, M. Locci, C. Mencuccini, G. P. Murtas, G. Penso, G. S. M. Spinetti, M. Spano, B. Stella, and V. Valente
The Gamma-Gamma Group, Laboratori Nazionali di Frascati, Frascati, Italy

and

B. Bartoli, D. Bisello, B. Esposito, F. Felicetti, P. Monacelli, M. Nigro, L. Paolufi, I. Peruzzi, G. Piano Mortemi, M. Piccolo, F. Ronga, F. Sebastiani, L. Trasatti, and F. Vanoli
The Magnet Experimental Group for ADONE, Laboratori Nazionali di Frascati, Frascati, Italy

and

G. Barbarino, G. Barbiellini, C. Bemporad, R. Biancastelli, F. Cevenini, M. Celvetti, F. Costantini, P. Lariccia, P. Parascandalo, E. Sassi, C. Spencer, L. Tortora, U. Troya, and S. Vitale
The Baryon-Antibaryon Group, Laboratori Nazionali di Frascati, Frascati, Italy
(Received 18 November 1974)

We report on the results at ADONE to study the properties of the newly found 3.1-BeV particle.

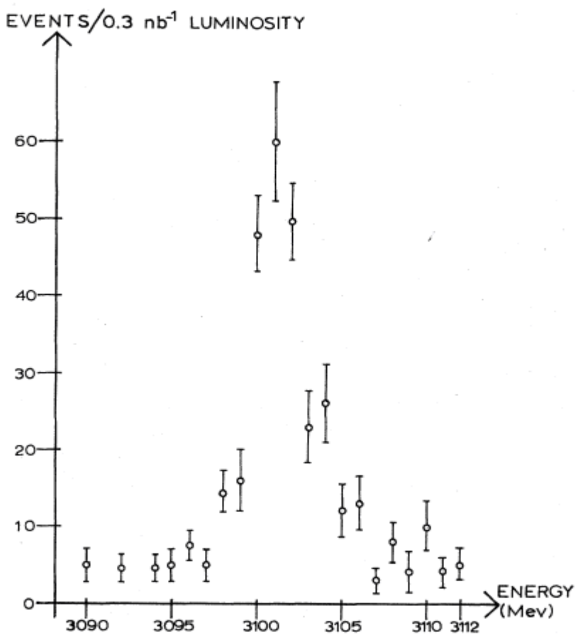


FIG. 1. Result from the Gamma-Gamma Group, total of 446 events. The number of events per 0.3 nb^{-1} luminosity is plotted versus the total c.m. energy of the machine.

Volume 53B, number 4 PHYSICS LETTERS 23 December 1974

A MEASUREMENT OF LARGE ANGLE e^+e^- SCATTERING AT THE 3100 MeV RESONANCE

DASP – Collaboration

W. BRAUNSCHWEIG, C.L. JORDAN, U. MARTYN, H.G. SANDER
D. SCHMITZ, W. STURM, W. WALLRAFF
I. Physikalisches Institut der RWTH Aachen

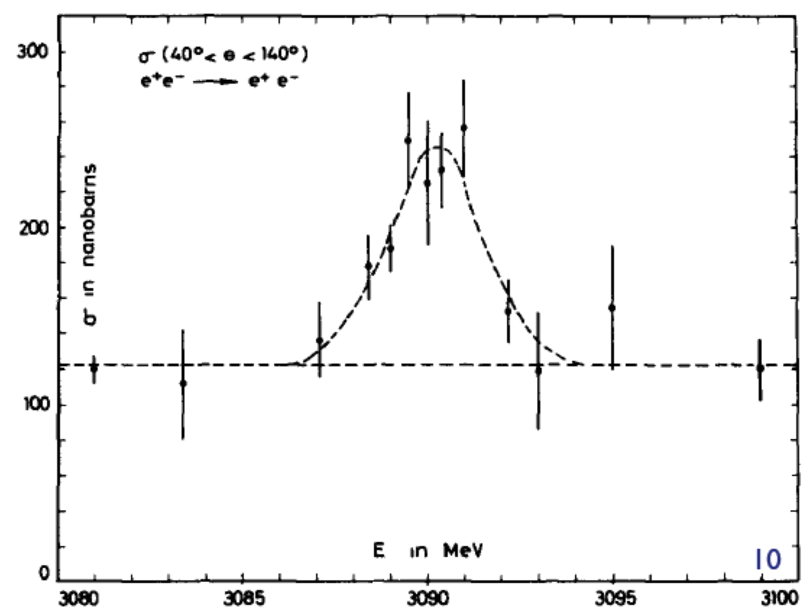
K. BERKELMAN*, D. CORDS, R. FELST, E. GADERMANN, G. GRINDHAMMER, H. HULTSCHIG, P. JOOS, W. KOCH, U. KÖTZ, H. KREHBIEL, D. KREINICK, J. LUDWIG, K.-H. MESS, K.C. MOFFEITT, D. NOTZ**, G. POELZ, K. SAUERBERG, P. SCHMÜSER, G. VOGEL, B.H. WIIK, G. WOLF
Deutsches Elektronen-Synchrotron DESY and II. Institut für Experimentalphysik der Universität Hamburg, Hamburg

G. BUSCHHORN, R. KOTTHAUS, U.E. KRUSE**, H. LIERL, H. OBERLACK, S. ORITO, K. PRETZL, M. SCHLIWA
Max-Planck-Institut für Physik und Astrophysik, München

T. SUDA, Y. TOTSUKA and S. YAMADA
University of Tokyo, Tokyo

Received 19 December 1974

Elastic e^+e^- scattering has been measured at total energies covering the newly found resonance at 3100 MeV. The angular distribution is consistent with spin-parity 1^-, and the cross section integrated over energy yields $\Gamma_{ee}^2/\Gamma_{tot}$ = 0.23 ± 0.05 keV for the resonance.



November revolution (1974)

Are the New Particles Baryon-Antibaryon Nuclei?

Alfred S. Goldhaber
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794*

and

Maurice Goldhaber
Physics Department, Brookhaven National Laboratory,† Upton, New York 11973
(Received 25 November 1974)

Baryon-antibaryon bound states and resonances could account for the new particles, as well as narrow states near nucleon-antinucleon threshold, which were reported earlier.

Interpretation of a Narrow Resonance in e^+e^- Annihilation*

Julian Schwinger
University of California at Los Angeles, Los Angeles, California 90024
(Received 25 November 1974)

A previously published unified theory of electromagnetic and weak interactions proposed a mixing between two types of unit-spin mesons, one of which would have precisely the characteristics of the newly discovered neutral resonance at 3.1 GeV. With this interpretation, a substantial fraction of the small hadronic decay rate can be accounted for. It is also remarked that other long-lived particles should exist in order to complete the analogy with ρ^0, ω, and φ.

November revolution (1974)

Possible Interactions of the *J* Particle*

H. T. Nieh
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794

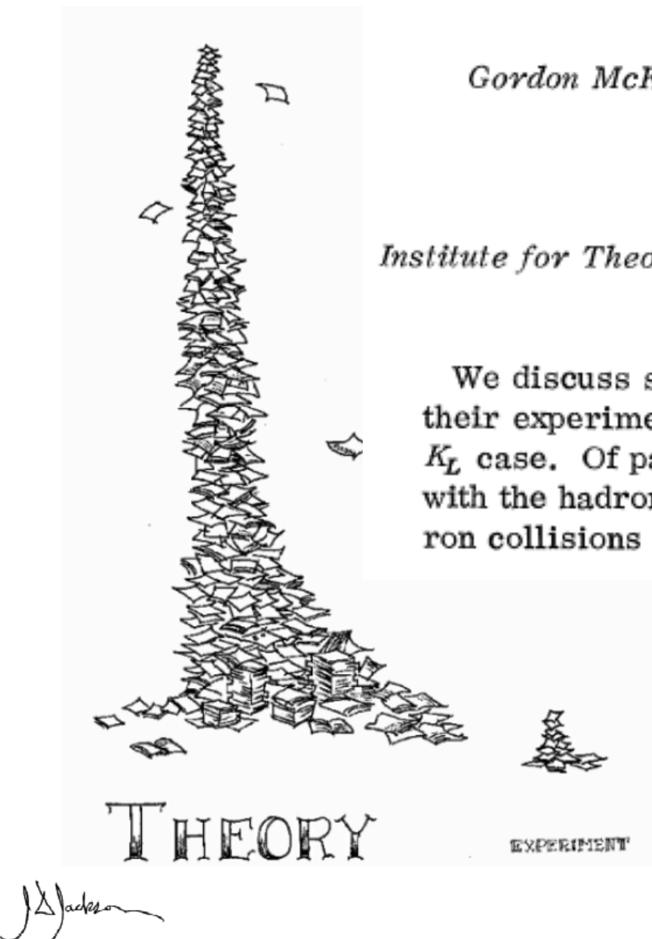
and

Tai Tsun Wu
Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts 02138

and

Chen Ning Yang
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794
(Received 25 November 1974)

We discuss some possible interaction schemes for the newly discovered particle J and their experimental implications, as well as the possible existence of two J^0's like the K_S-K_L case. Of particular interest is the case where the J particle has strong interactions with the hadrons. In this case J can be produced by associated production in hadron-hadron collisions and also singly in relative abundance in ep and μp collisions.



"The e+e- annihilation data contradict both the simple quark-parton model and the Bjorken scaling hypothesis. This has come as a shock for they were both doing so well ..."

B. Richter
ICHEP, London, 1974

November revolution (1974)

Search for charm

Mary K. Gaillard* and Benjamin W. Lee

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

Jonathan L. Rosner

University of Minnesota, Minneapolis, Minnesota 55455

A systematic discussion of the phenomenology of charmed particles is presented with an eye to experimental searches for these states. We begin with an attempt to clarify the theoretical framework for charm. We then discuss the $SU(4)$ spectroscopy of the lowest lying baryon and meson states, their masses, decay modes, lifetimes, and various production mechanisms. We also present a brief discussion of searches for short-lived tracks. Our discussion is largely based on intuition gained from the familiar —but not necessarily understood— phenomenology of known hadrons, and predictions must be interpreted only as guidelines for experimenters.

Preprint, August 1974

$$\phi_c(c\bar{c}) \ :$$

$$M(\phi_c) \approx 3 \text{ GeV}$$

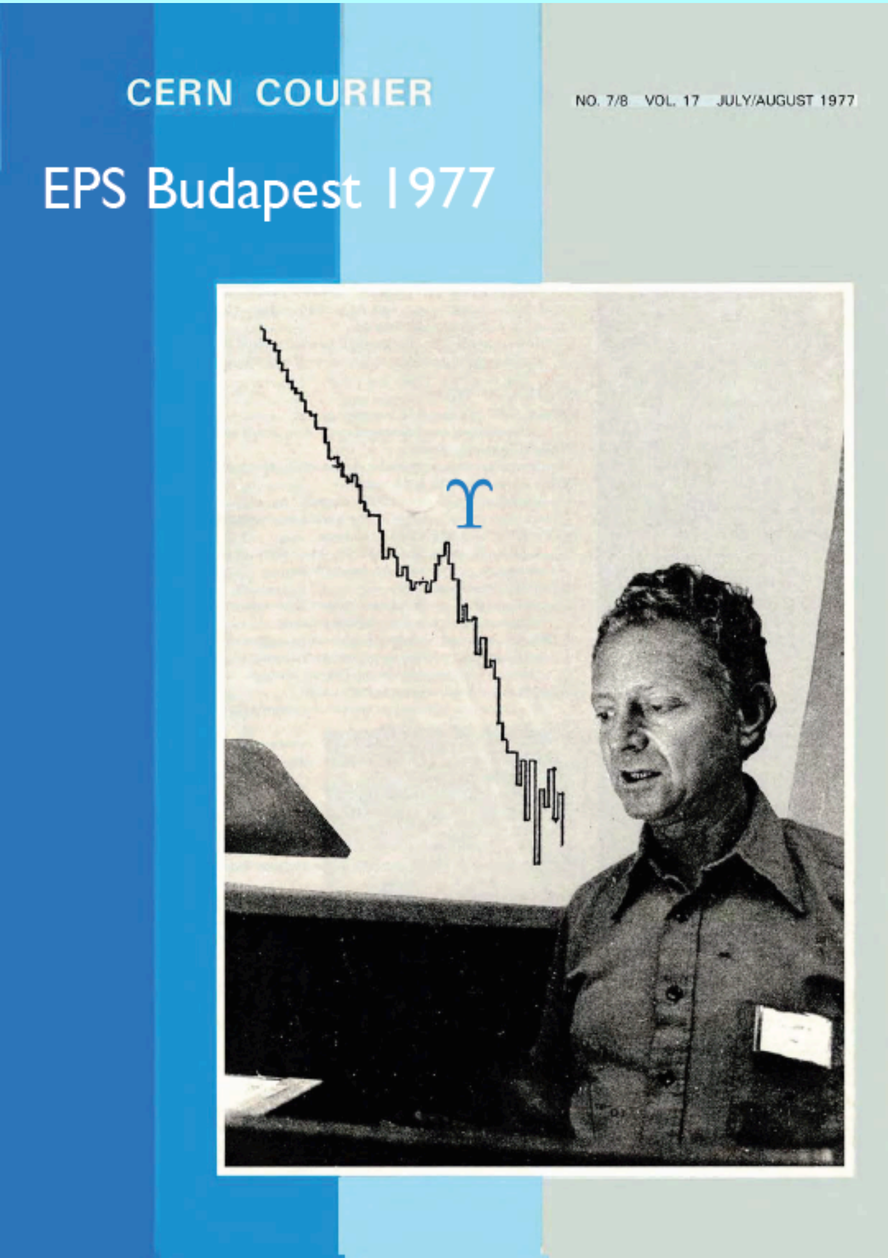
$$\Gamma(\phi_c) \approx 2 \text{ MeV}$$

$$\text{BR}(\phi_c \to e^+e^-) \approx 1 \ \%$$

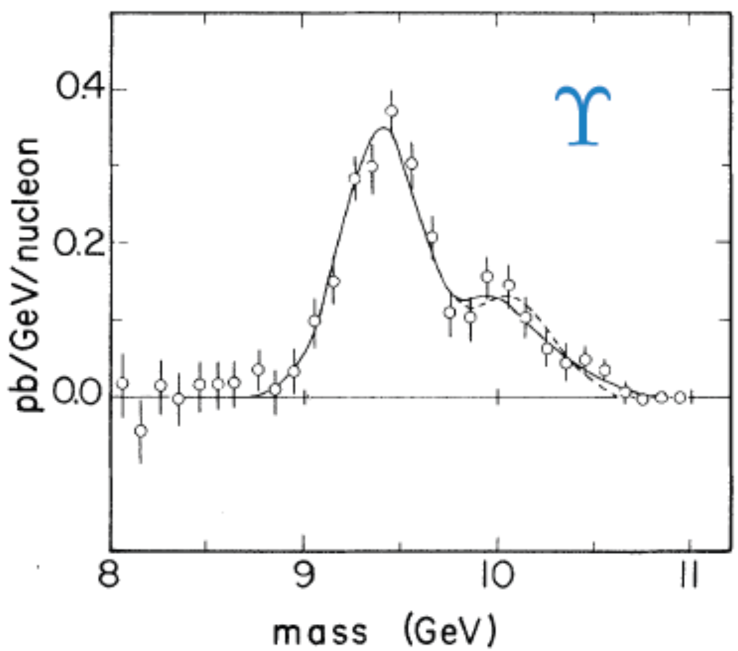
"… the discoveries of November 1974 were not just additions to our knowledge of Nature. Instead they signaled a change in our understanding of the structure of matter." – F. Gilman (11/84)

"The November revolution just set everything in motion toward the standard model that we have now." – J.D. Bjorken (11/84)

The third generation: bottom quark (1977)



FermiLab E288 fixed target experiment:



Leon Lederman, spokesperson,

Described the Upsilon discovery as

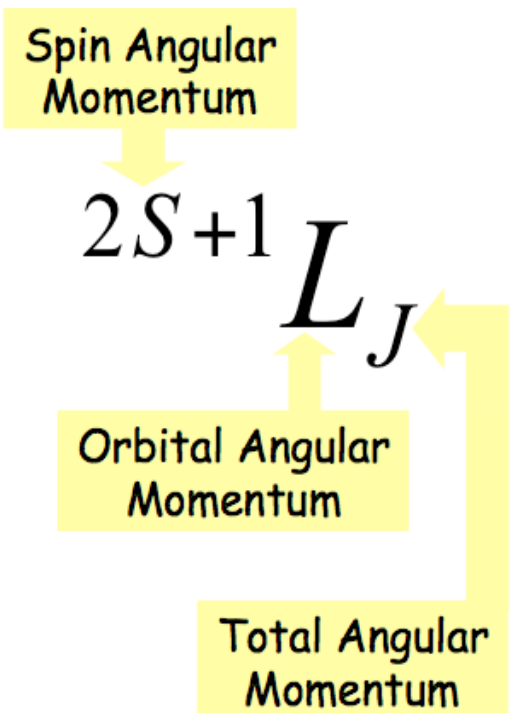
“one of the most expected surprises in particle physics”

Heavy quarkonium

- One of the simplest QCD bound states:

  Quark Model: $Q\bar{Q}$ **with** $Q = c, b$

- Quantum numbers – spectroscopy notation:

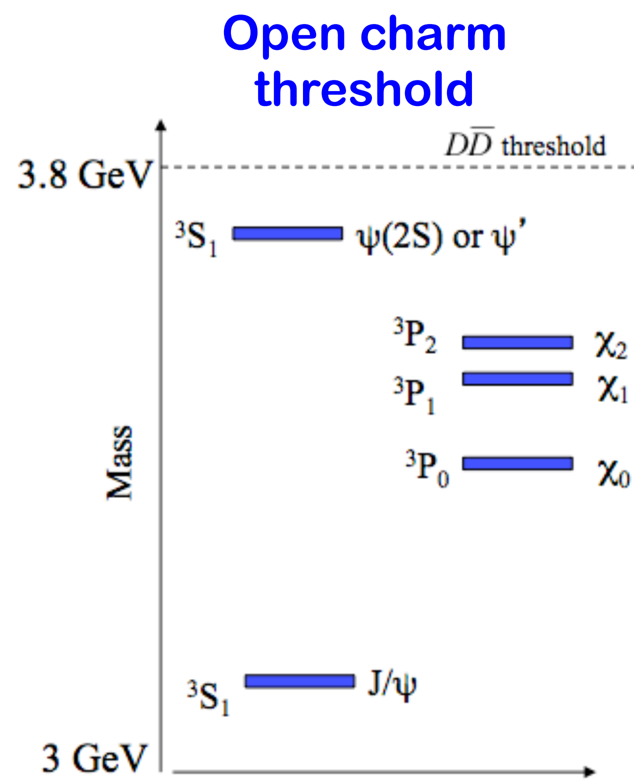


$$J/\psi = {}^3S_1$$

Means:
Quark Spin=1 (3 = 2 x 1 + 1)
Quark Orbital Ang. Mom. = 0
Total J/ψ Spin = 1

$$J^{PC} = 1^{--}$$

Means:
Total J/ψ Spin = 1
Parity is Odd
Charge Conjugation is Odd



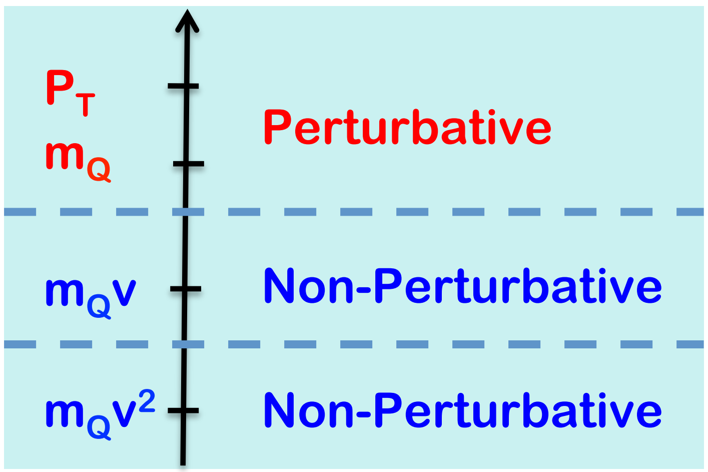
Heavy quarkonium

- **One of the simplest QCD bound states:**

  Localized color charges (heavy mass), non-relativistic relative motion

  Charmonium: $v^2 \approx 0.3$ **Bottomonium:** $v^2 \approx 0.1$

- **Well-separated momentum scales – effective theory:**



Hard — Production of $Q\bar{Q}$ **[pQCD]**

Soft — Relative Momentum **[NRQCD]**

$\Lambda_{\rm QCD}$

Ultrasoft — Binding Energy **[pNRQCD]**

- **Cross sections and observed mass scales:**

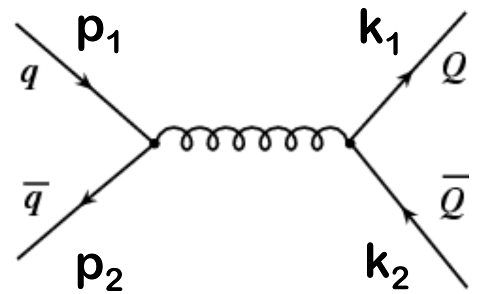
$$\frac{d\sigma_{AB\to H(P)X}}{dy dP_T^2} \qquad \sqrt{S}, \qquad P_T, \qquad M_H,$$

PQCD is "expected" to work for the production of heavy quarks

Difficulty: Emergence of a quarkonium from a heavy quark pair?

Leading order pair production

- Light quark annihilation:



- Partonic cross section:

$$|\bar{M}_{q\bar{q}\to Q\bar{Q}}|^2 = g^4\left(\frac{2}{9}\right)\frac{1}{\hat{s}^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right]$$
$$=g^4\left(\frac{2}{9}\right)\frac{2}{\hat{s}^2}\left[(m_Q^2-\hat{t})^2+(m_Q^2-\hat{u})^2+2m_Q^2\hat{s}\right]$$

$$\hat{s}=(p_1+p_2)^2$$
$$\hat{t}=(p_1-k_1)^2$$
$$\hat{u}=(p_2-k_1)^2$$

$$\frac{d\hat{\sigma}_{q\bar{q}}}{d\hat{t}}=\frac{1}{16\pi\hat{s}^2}|\bar{M}_{q\bar{q}\to Q\bar{Q}}|^2$$

Threshold constraint

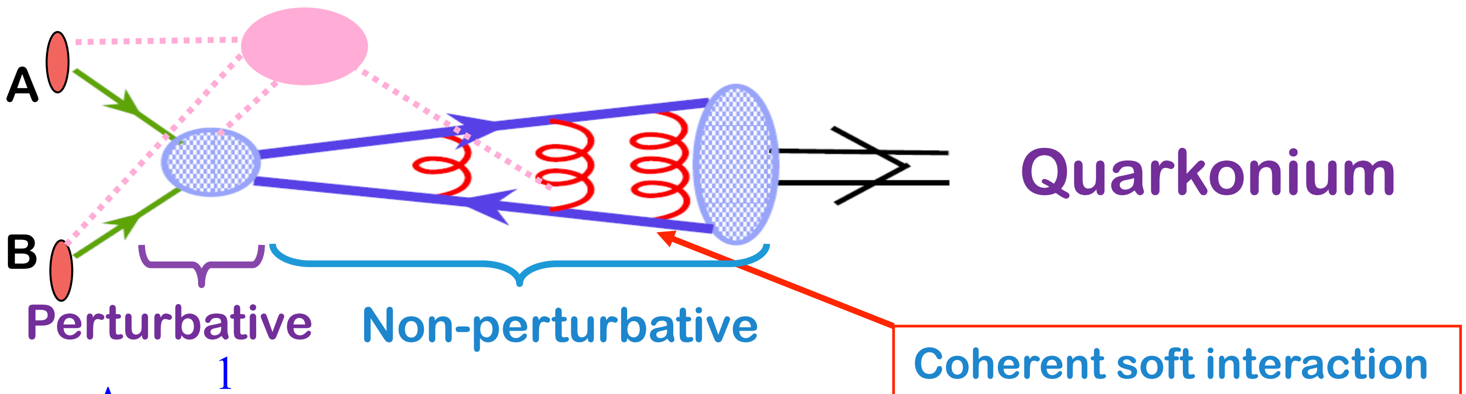
$$\hat{\sigma}_{q\bar{q}\to Q\bar{Q}}=\left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2}{3\hat{s}}\left[1+\frac{2m_Q^2}{\hat{s}}\right]\sqrt{1-\frac{4m_Q^2}{\hat{s}}}$$

$$\sigma_{AB}=\sum_{a,b}\varphi_{a/A}(x_1)\otimes\varphi_{b/B}(x_2)\otimes\hat{\sigma}_{ab\to Q\bar{Q}}$$

$$\hat{s}=x_1x_2S,\quad p_1=x_1P_A,\quad p_2=x_2P_B,\quad S=\left(P_A+P_B\right)^2$$

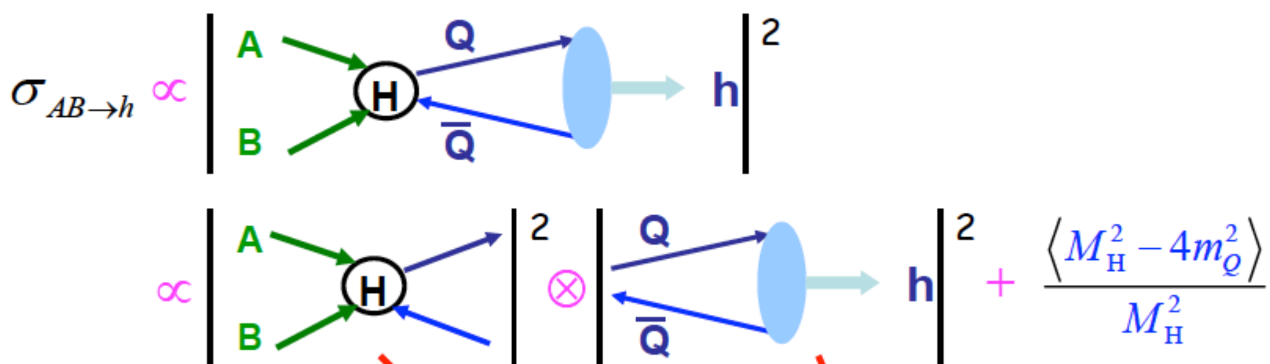
Basic production mechanism

- **Factorization is likely to be valid for producing the pairs:**
  - ✧ Momentum exchange is much larger than 1/fm
  - ✧ Spectators from colliding beams are "frozen" during the hard collision



$$\Delta r \le \frac{1}{2m_Q}$$

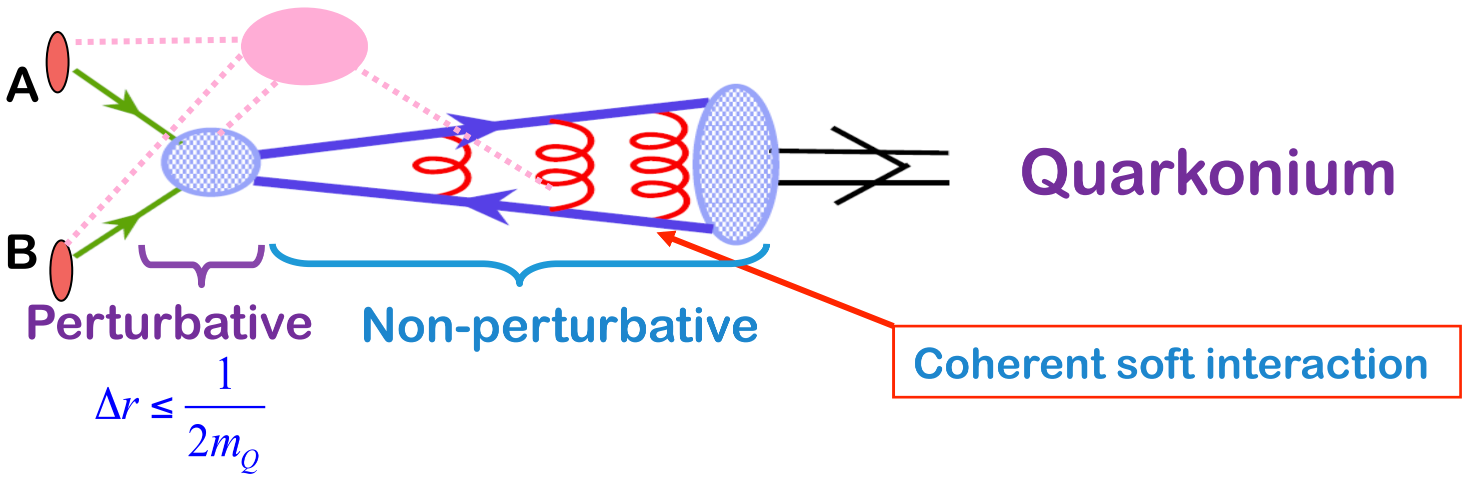
- **Approximation:**



$$\Rightarrow\ \sigma_{AB\to h} = \int dq^2\ \hat{\sigma}_{AB\to[Q\bar{Q}]}\left(m_Q^2, q^2\right)\ F_{[Q\bar{Q}]\to h}\left(q^2\right) + \ldots$$

Basic production mechanism

- Factorization is likely to be valid for producing the pairs:
  - ✧ Momentum exchange is much larger than 1/fm
  - ✧ Spectators from colliding beams are "frozen" during the hard collision



- Naïve factorization: on-shell pair + hadronization

$$\sigma_{AB\to J/\psi} = \sum_{[Q\bar{Q}(n)]} \int d\Gamma_{[Q\bar{Q}]}\, \hat{\sigma}_{AB\to[Q\bar{Q}(n)]}(p_Q, p_{\bar{Q}}) F_{[Q\bar{Q}(n)]\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

Models & Debates

⇔ Different assumptions/treatments on $F_{[Q\bar{Q}(n)]\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$

how the heavy quark pair becomes a quarkonium?

A long history for the production

- Color singlet model: 1975 –

  Einhorn, Ellis (1975),
  Chang (1980),
  Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- Color evaporation model: 1977 –

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- NRQCD model: 1986 –

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and α_s

- QCD factorization approach: 2005 –

  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …

  $P_T >> M_H$: M_H/P_T power expansion + α_s – expansion

  Unknown, but universal, fragmentation functions – evolution

- Soft-Collinear Effective Theory + NRQCD: 2012 –

  Fleming, Leibovich, Mehen, …

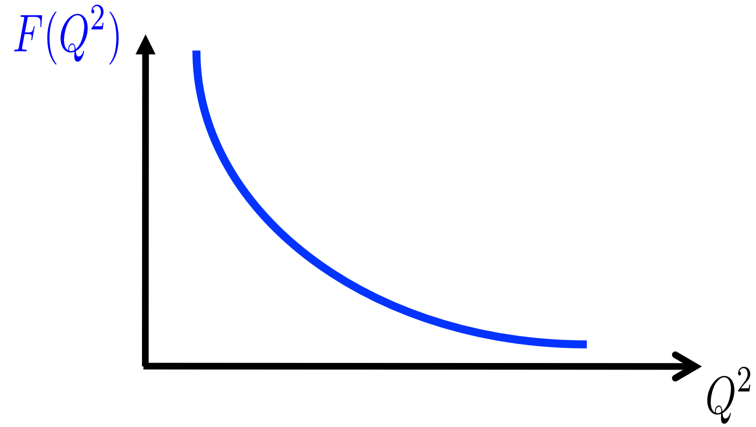
Color singlet model

❑ Assumptions:

- ✧ Only pairs with right quantum number can become quarkonia

$$\sigma_{AB\to J/\psi} = \sum_{[Q\bar{Q}(n)]} \int d\Gamma_{[Q\bar{Q}]}\, \hat{\sigma}_{AB\to[Q\bar{Q}(n)]}(p_Q, p_{\bar{Q}}) F_{[Q\bar{Q}(n)]\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

- ✧ The transition distribution is very narrowly peaked near the threshold!



❑ Model:

$$\hat{\sigma}(Q^2) \approx \hat{\sigma}(4m_c^2) + \frac{d\hat{\sigma}}{dQ^2}(Q^2 = 4m_c^2)(Q^2 - 4m_c^2) + \ldots$$

➡ $\int dQ^2 F(Q^2) \propto |\psi(0)|^2$ – fixed by decay

➡ $\sigma_{AB\to J/\psi} \propto \hat{\sigma}(Q^2 \approx 4m_c^2)\ |\psi(0)|^2$

No free parameter!

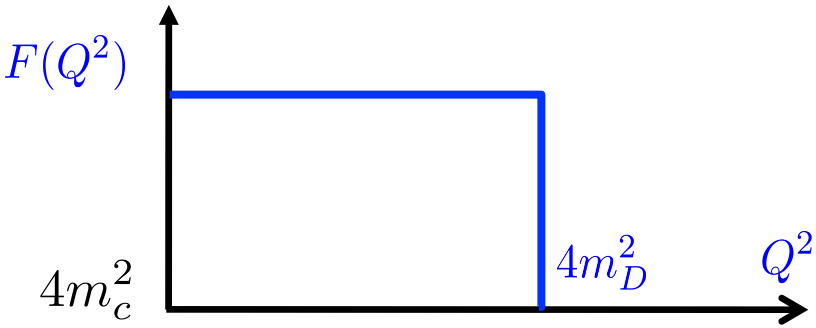
Color evaporation model

❑ Assumptions:

✧ All colored or color singlet pairs with invariant mass less then open charm threshold could become bound quarkonia

$$\sigma_{AB\to J/\psi} = \sum_{[Q\bar{Q}(n)]} \int d\Gamma_{[Q\bar{Q}]}\, \hat{\sigma}_{AB\to[Q\bar{Q}(n)]}(p_Q, p_{\bar{Q}}) F_{[Q\bar{Q}(n)]\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

✧ Threshold:



❑ Model:

$$F(Q^2) - \text{Constant!}$$

$$\sigma_{AB\to J/\psi} \approx f_{J/\psi} \int_{4m_c^2}^{4m_D^2} dQ^2 \left[\frac{d\sigma(Q^2)}{dQ^2}\right]$$

One constant per quarkonium state

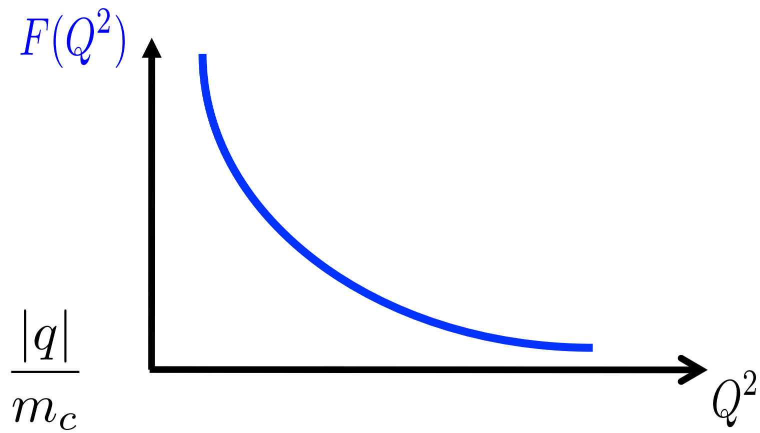
NRQCD model

❑ Assumptions:

✧ Steeply falling transition distribution, like color singlet model, but,

✧ All color $[Q\bar{Q}]$ states could contribute

✧ V-expansion: $v_{c\bar{c}}^2 \approx \dfrac{Q^2 - 4m_c^2}{4m_c^2}$ ➡ $v \sim \dfrac{|q|}{m_c}$



$$\sigma_{AB\to J/\psi} = \sum_{[Q\bar{Q}(n)]} \int d\Gamma_{[Q\bar{Q}]}\, \hat{\sigma}_{AB\to[Q\bar{Q}(n)]}(p_Q, p_{\bar{Q}}) F_{[Q\bar{Q}(n)]\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

❑ Model:

$$\sigma_{AB\to[Q\bar{Q}](n)}(q^2) \approx \sum_m \frac{\left[q^2\right]^m}{m!} \left[\frac{d}{dq^2}\right]^m \sigma_{AB\to[Q\bar{Q}](n)}(q^2 = 0)$$

➡ $$\sigma_{AB\to \mathrm{J}/\psi}(M_{\mathrm{J}/\psi}) = \sum_n \sigma_{[\mathcal{O}_n]}(4m_c^2 = M_{\mathrm{J}/\psi})\, \langle \mathcal{O}_n(0)\rangle$$

$$\langle \mathcal{O}(0)\rangle \propto \int dQ^2\, v_{c\bar{c}}^2\, F(Q^2)$$

Infinite number of parameters!

Predictive power ⟷ truncation in v-expansion

NRQCD model

- **NRQCD Lagrangian:** $\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \mathcal{L}_{\rm heavy} + \delta\mathcal{L}$

Caswell, Lepage, Phys. Lett. B, 1986
Bodwin, Braaten, Lepage, PRD, 1995

$$\mathcal{L}_{\rm light} = -\frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_{n_F=1}^{3 \text{ or } 4} \bar{q}\, i\not{D} q$$

$$\mathcal{L}_{\rm heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M}\right)\chi$$

Pauli spinor for antiquark (χ)

Pauli spinor for heavy quark (ψ)

$$\begin{aligned}\delta\mathcal{L}_{\rm bilinear} = &\frac{c_1}{8M^3}\left(\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi\right)\\ &+ \frac{c_2}{8M^2}\left(\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right)\\ &+ \frac{c_3}{8M^2}\left(\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right)\\ &+ \frac{c_4}{2M}\left(\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right),\end{aligned}$$

- **Limitation:**

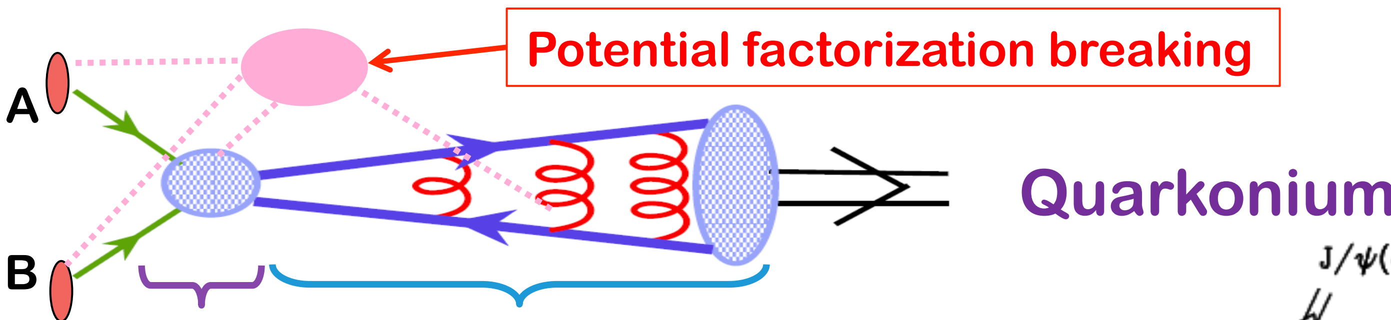
Powerful for a process with available kinetic energy: $Mv^2 \ll Mc^2$

- ✧ Formalism is ideal for heavy quarkonium decay — Bodwin, Braaten, Lepage, PRD, 1995
- ✧ Additional complications for production with $s \gg (2M)^2$

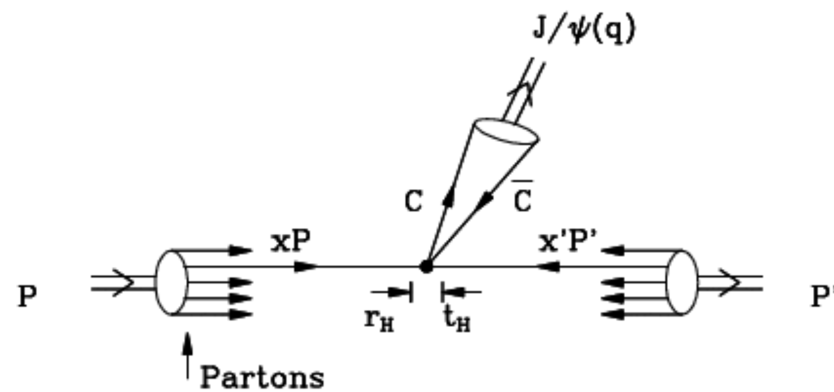
QCD factorization approach

❑ Concern:

All models, so far, assume that the hadronization of heavy quark pair to a physical quarkonium is independent of any collision environment



$$\Delta r \leq \frac{1}{2m_Q}$$



❑ Factorization:

$$E\frac{d\sigma_{AB\to J/\psi}}{d^3p} \approx \sum_n \mathcal{O}\left(\frac{1}{p_T}\right)^n \quad \text{for } p_T > m_Q \gg \Lambda_{\rm QCD}$$

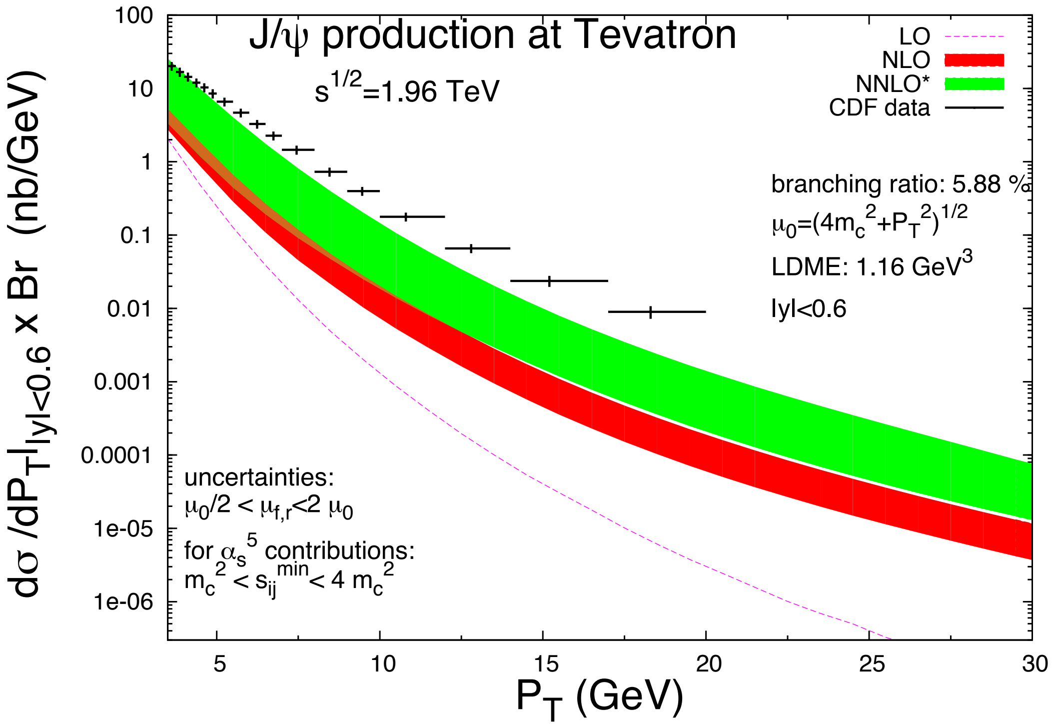
with the first, and the first subleading power term are factorizable!

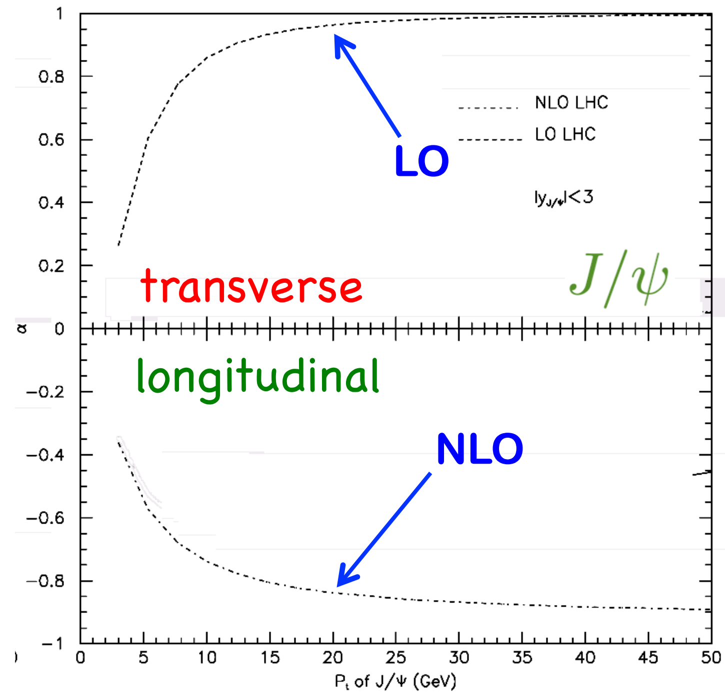
In this case, the hadrionization of the heavy quark pair is approximately process independent – NRQCD and other models could be used for …

Color singlet model (CSM)

- ❑ **Effectively No parameter:**

Campbell, Maltoni, Tramontano (2007), Artoisenet, Lansburg, Maltoni (2007), Artoisenet, et al. (2008)



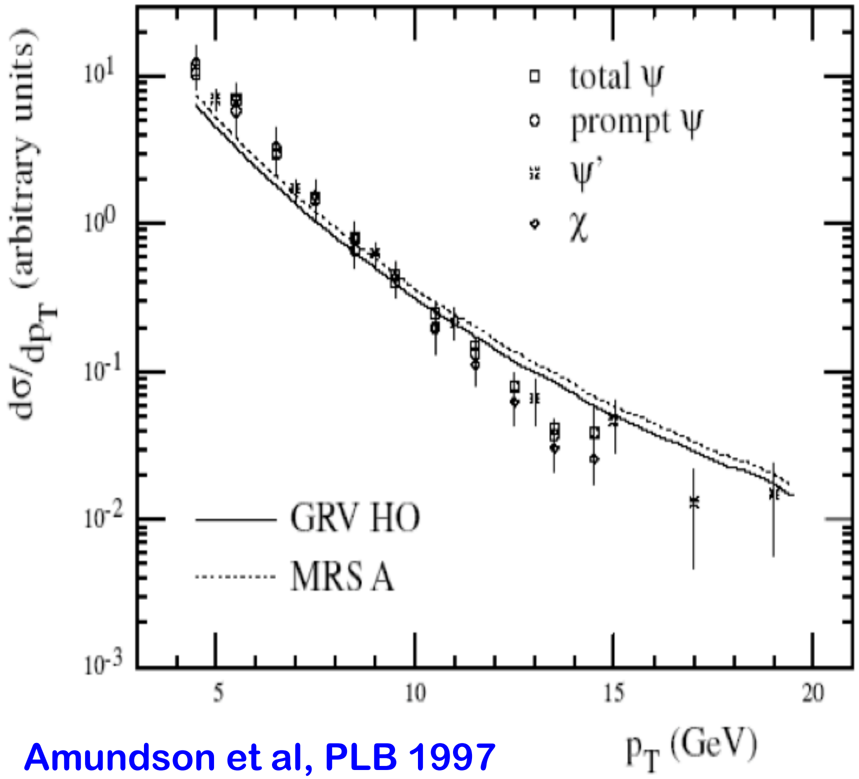


B. Gong et, al. PRL (2008)

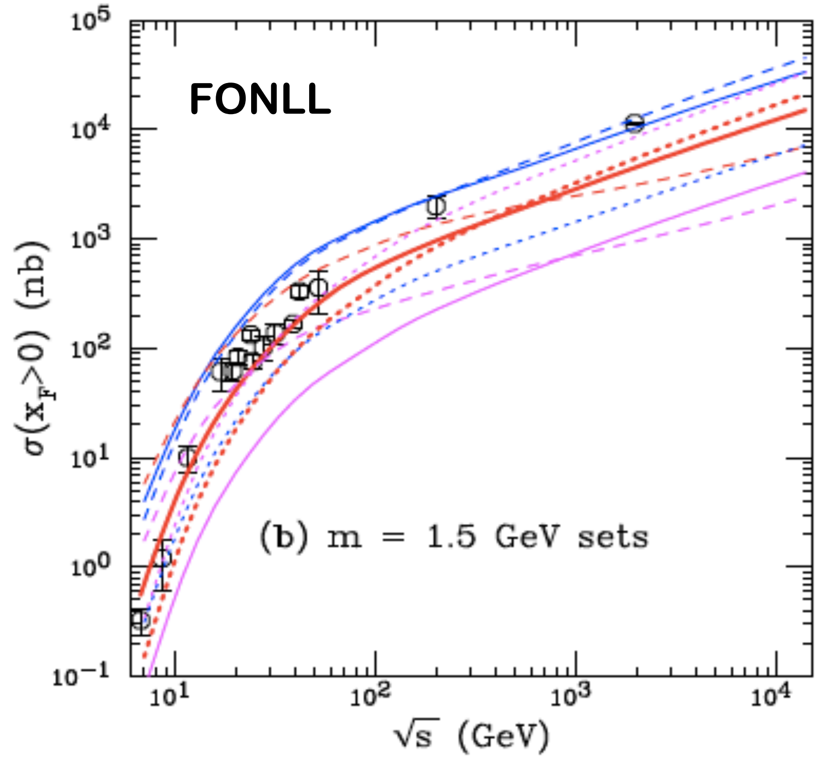
- ❑ **Issues:**
  - ✧ How reliable is the perturbative expansion?
  - ✧ S-wave: large corrections from high orders
  - ✧ P-wave: Infrared divergent – CSM is not complete

Color evaporation model (CEM)

- One parameter per quarkonium:



Amundson et al, PLB 1997

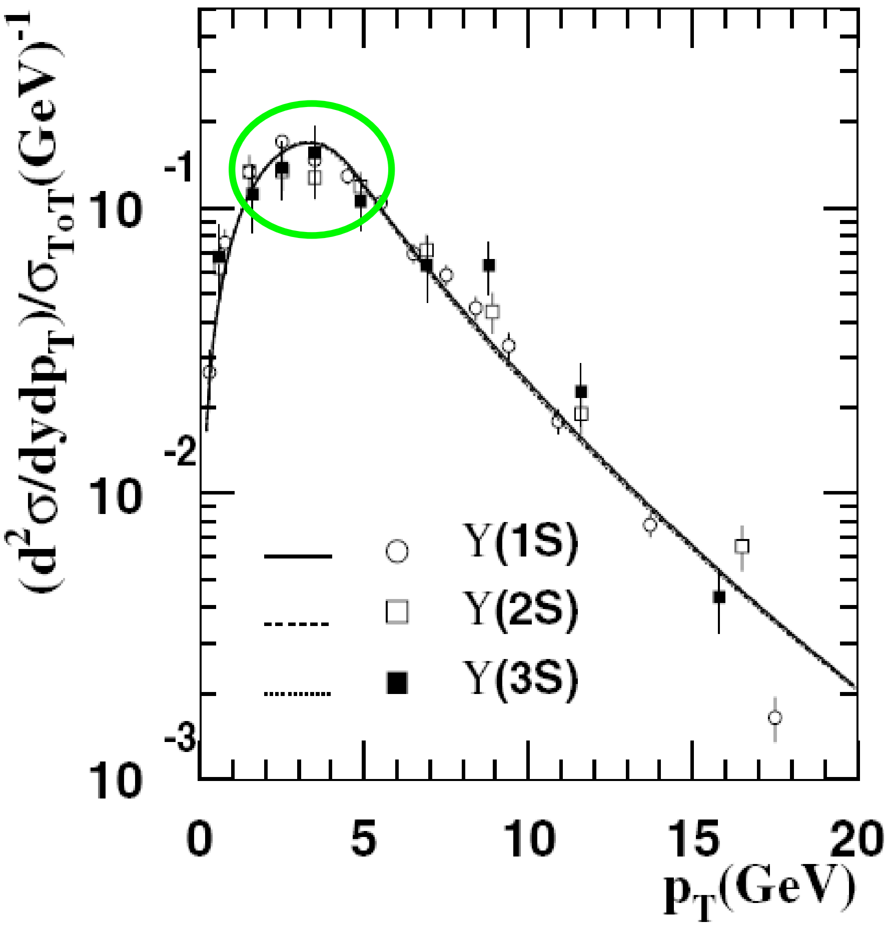


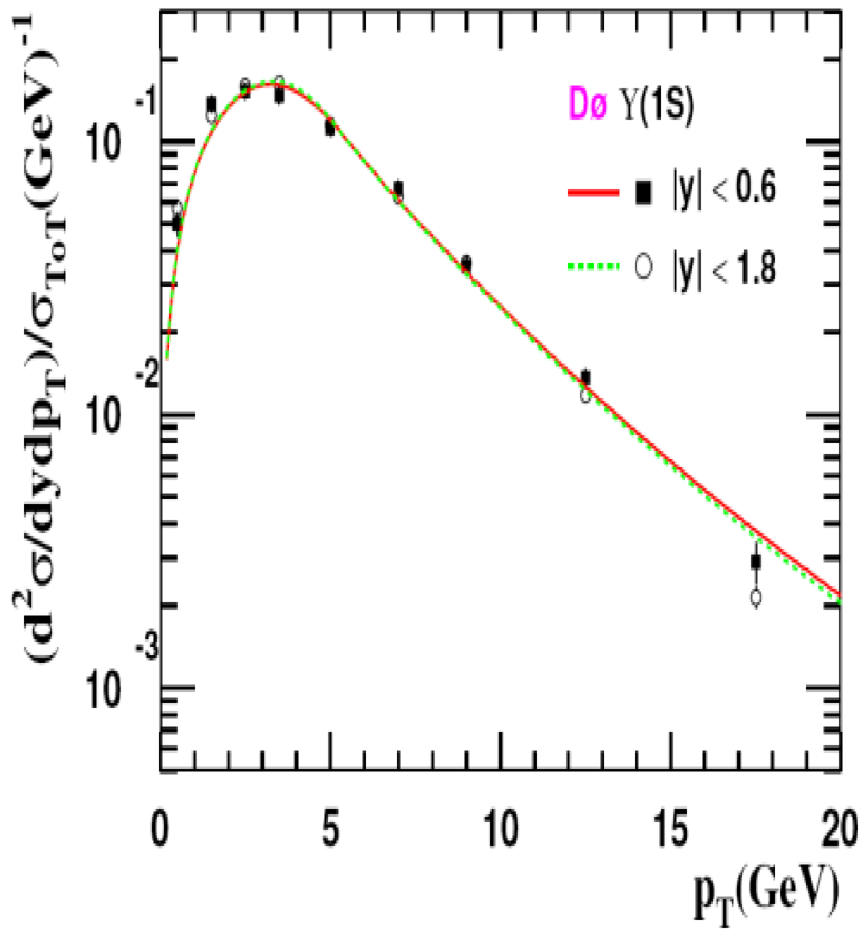
Nelson, Vogt, Frawley, PRC 2013

- Question:
  - Better p_T distribution – the shape?
  - Need intrinsic k_T – its distribution?

CEM: with resummation of shower logs

CDF Run-I **D0 Run-II**





- **Question:**

  Too hard p_T distribution – polarization?

Berger, Qiu, Wang, 2005

NRQCD – most successful so far

- **NRQCD factorization:**

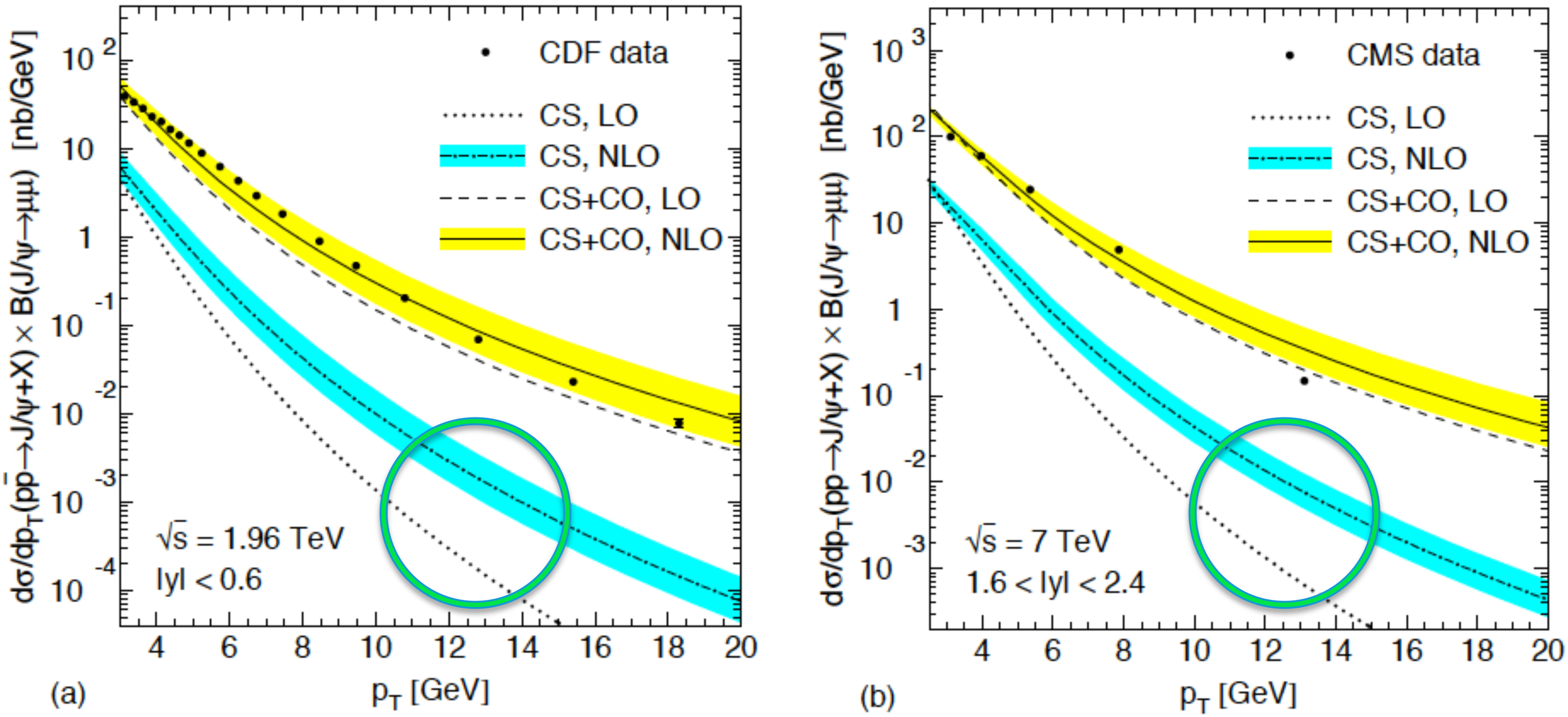
$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle$$

- ✧ **4 leading channels in v**

$${}^3S_1^{[1]},\ {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

- ✧ **Full NLO in α_s**

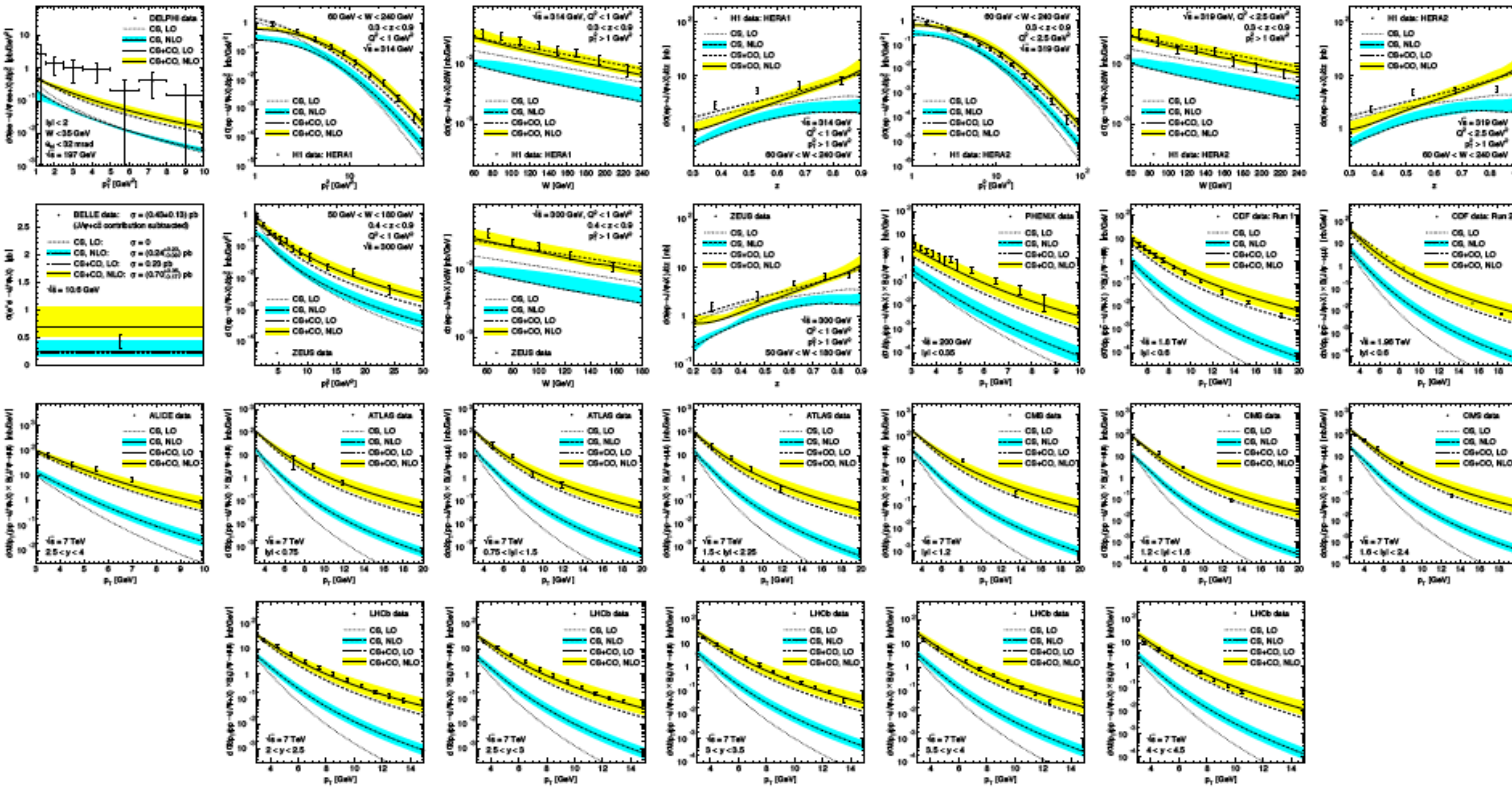
- **Phenomenology:**



- **Fine details – shape – high at large p_T?**

PRL 106, 022003 (2011)

NRQCD – global analysis



194 data points from 10 experiments, fix singlet $\langle O[^3S_1^{[1]}]\rangle = 1.32\ \mathrm{GeV}^3$

$\langle O[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\langle O[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

$\chi^2/d.o.f. = 857/194 = 4.42$

Butenschoen and Kniehl, arXiv: 1105.0820

Anomalies and surprises

- **Theory – the state of arts – NLO:**
  - **Very difficult to calculate, no analytical expression**

    → hard to obtain a clear physical picture on how various states of heavy quark pair are actually produced?

  - **For some channels, NLO corrections are orders larger than LO**

    → questions whether higher order contributions are negligible, while it is extremely difficult, if not impossible, to go beyond the NLO

- **Comparison with data:**
  - **Quarkonium polarization – "ultimate" test of NRQCD!**

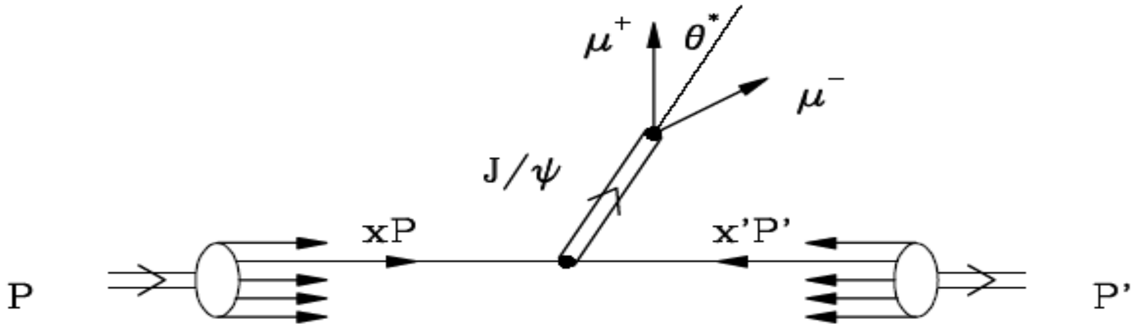
    → Clear mismatch between theory predictions and data

  - **Universality of NRQCD matrix elements – predictive power!**

    → Clear tension between different data sets, e^+e^-, ep, pp, …

Heavy quarkonium polarization

- Measure angular distribution of $\mu^+\mu^-$ in J/ψ decay



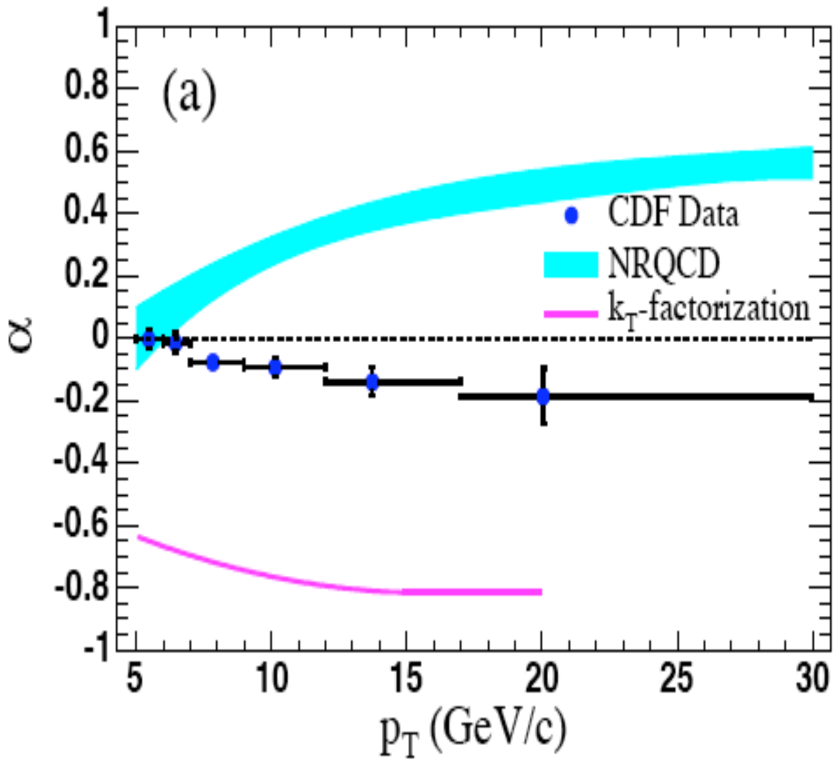
- Normalized distribution – integrate over φ:

$$I(\cos\theta^*) = \frac{3}{2(\alpha+3)}\,(1+\alpha\cos^2\theta^*)$$

$$\alpha = \begin{cases} +1 & \text{fully transverse} \\ 0 & \text{unpolarized} \\ -1 & \text{fully longitudinal} \end{cases}$$

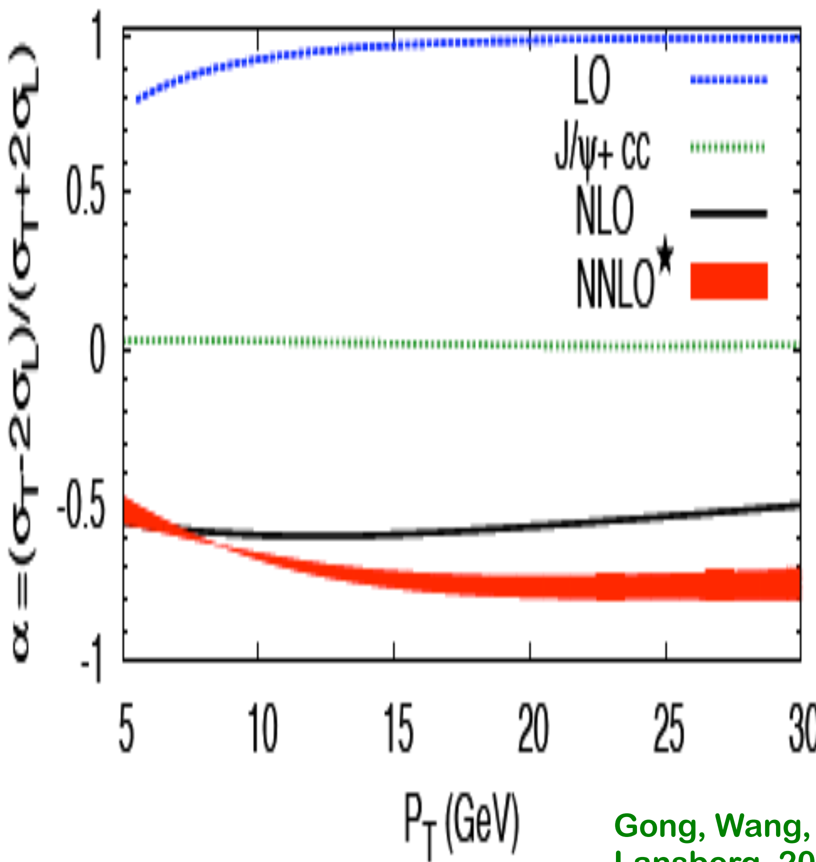
Anomalies from J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

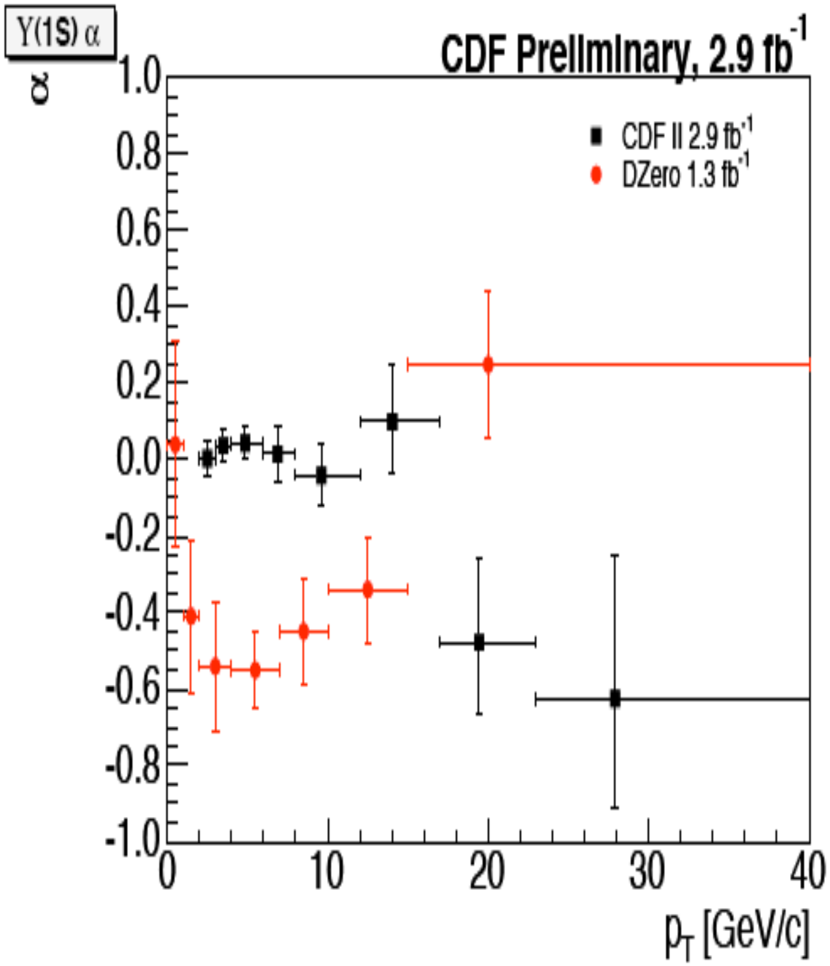
CSM

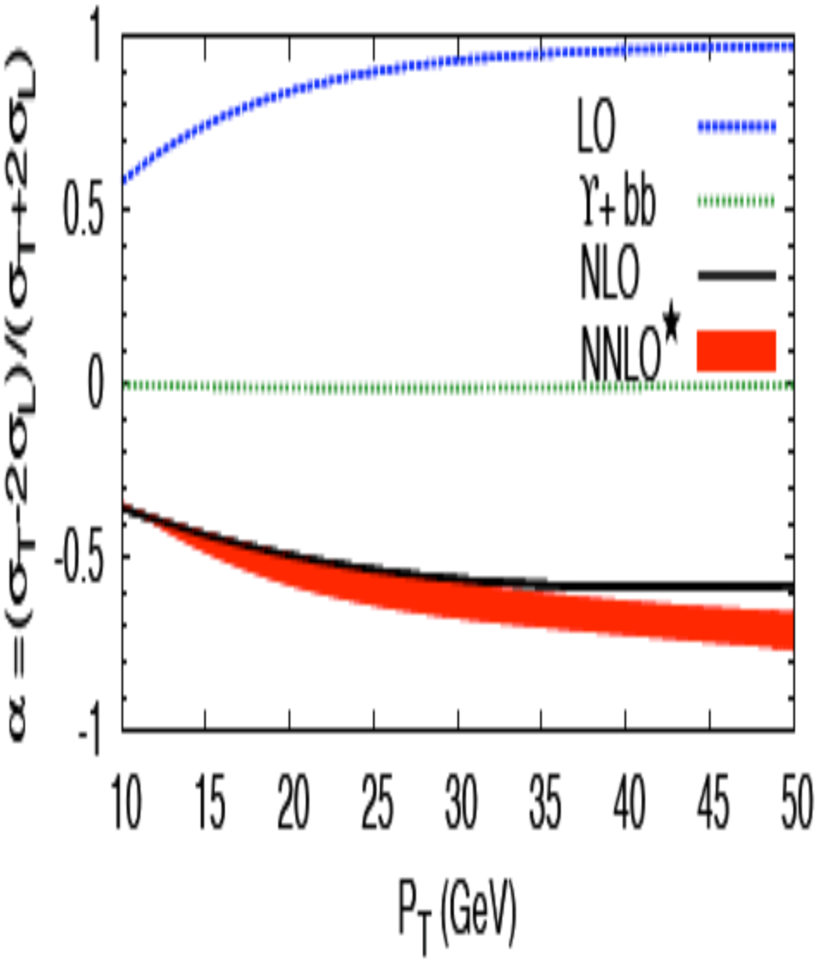


Gong, Wang, 2008
Lansberg, 2009

- ✧ **NRQCD:** Dominated by color octet – NLO is not a huge effect
- ✧ **CSM:** Huge NLO – change of polarization?

Confusions from Upsilon polarization





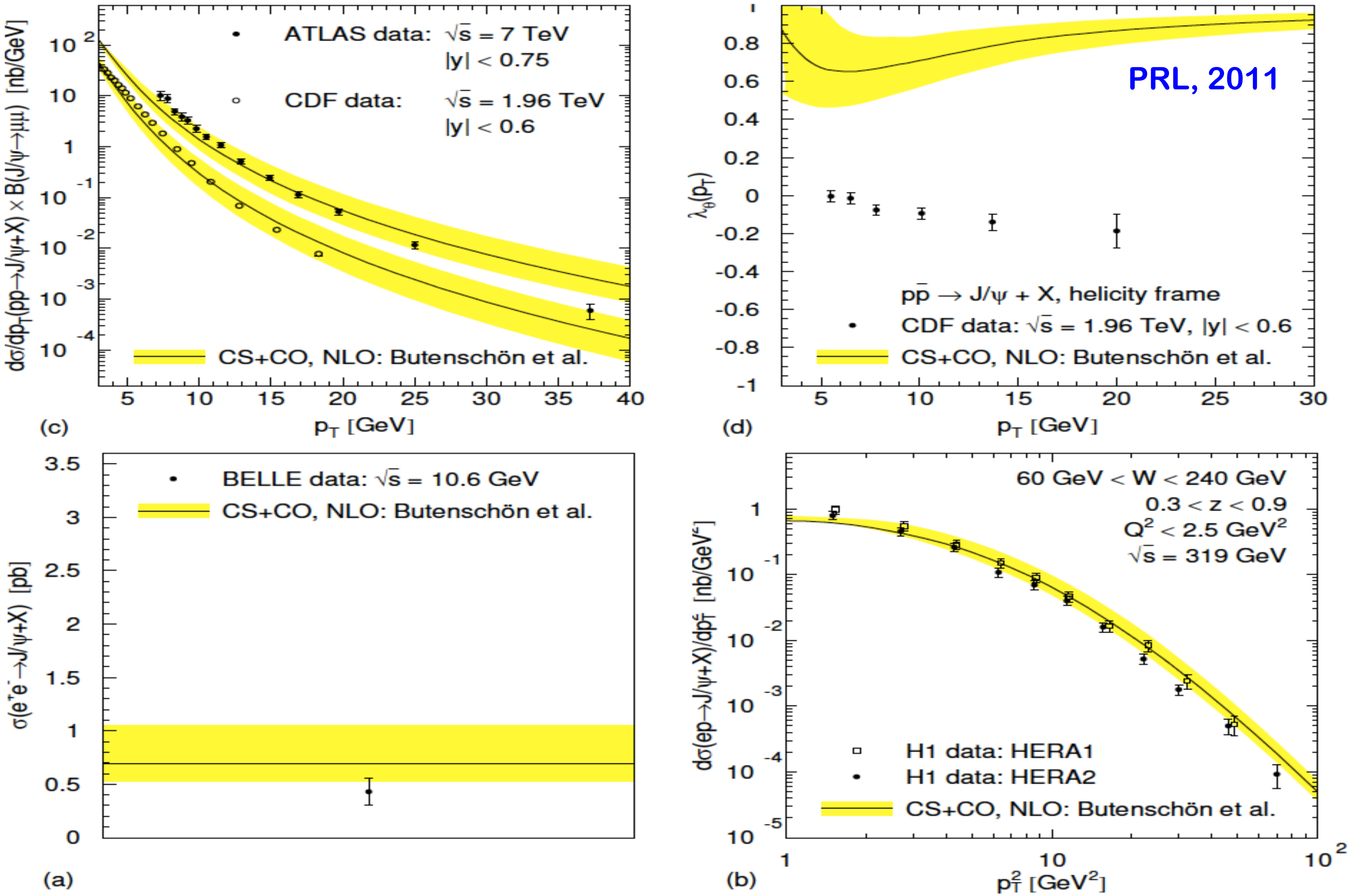
Resolution between CDF and D0?

Gong, Wang, 2008

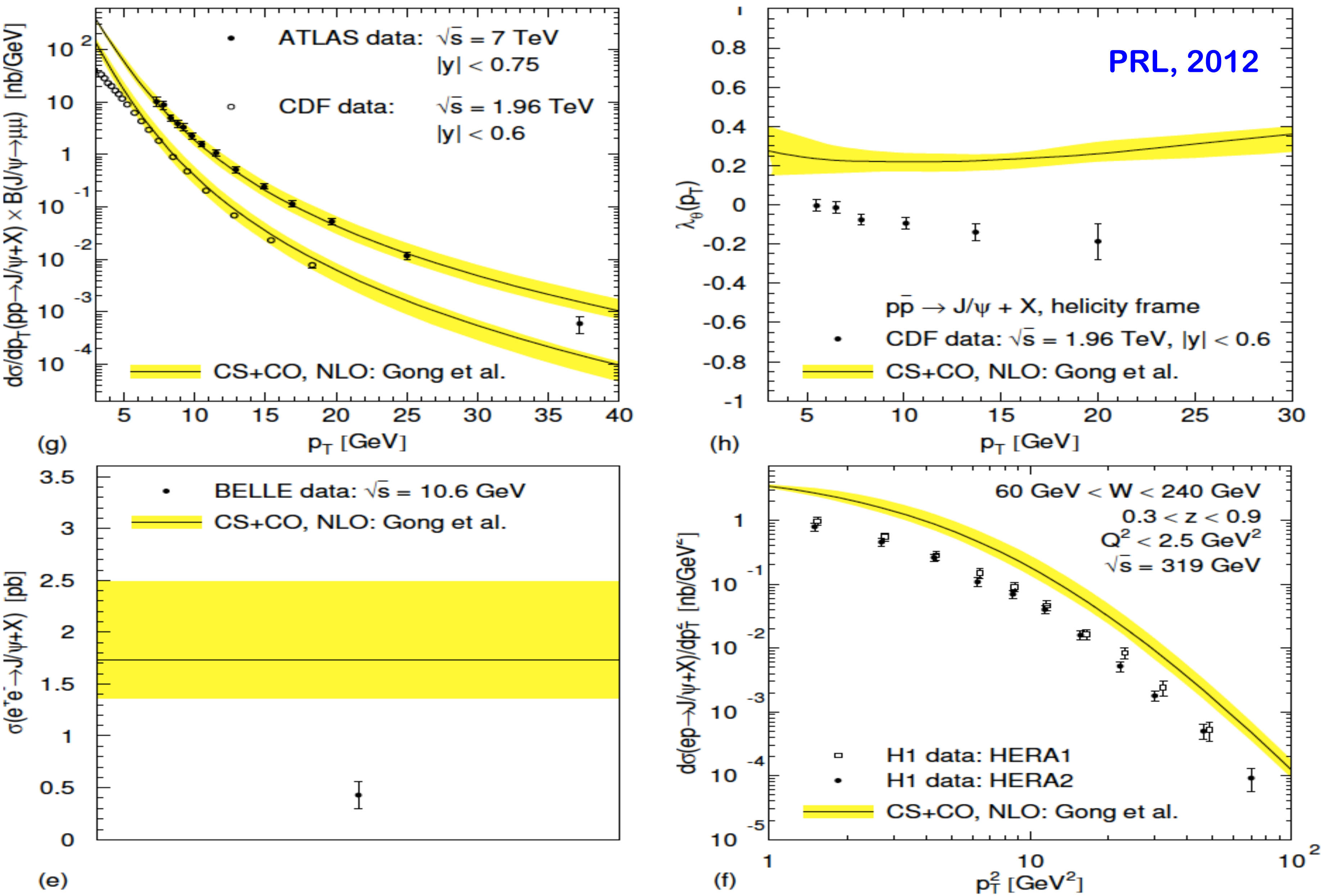
Artoisenet, et al. 2008

Lansberg, 2009

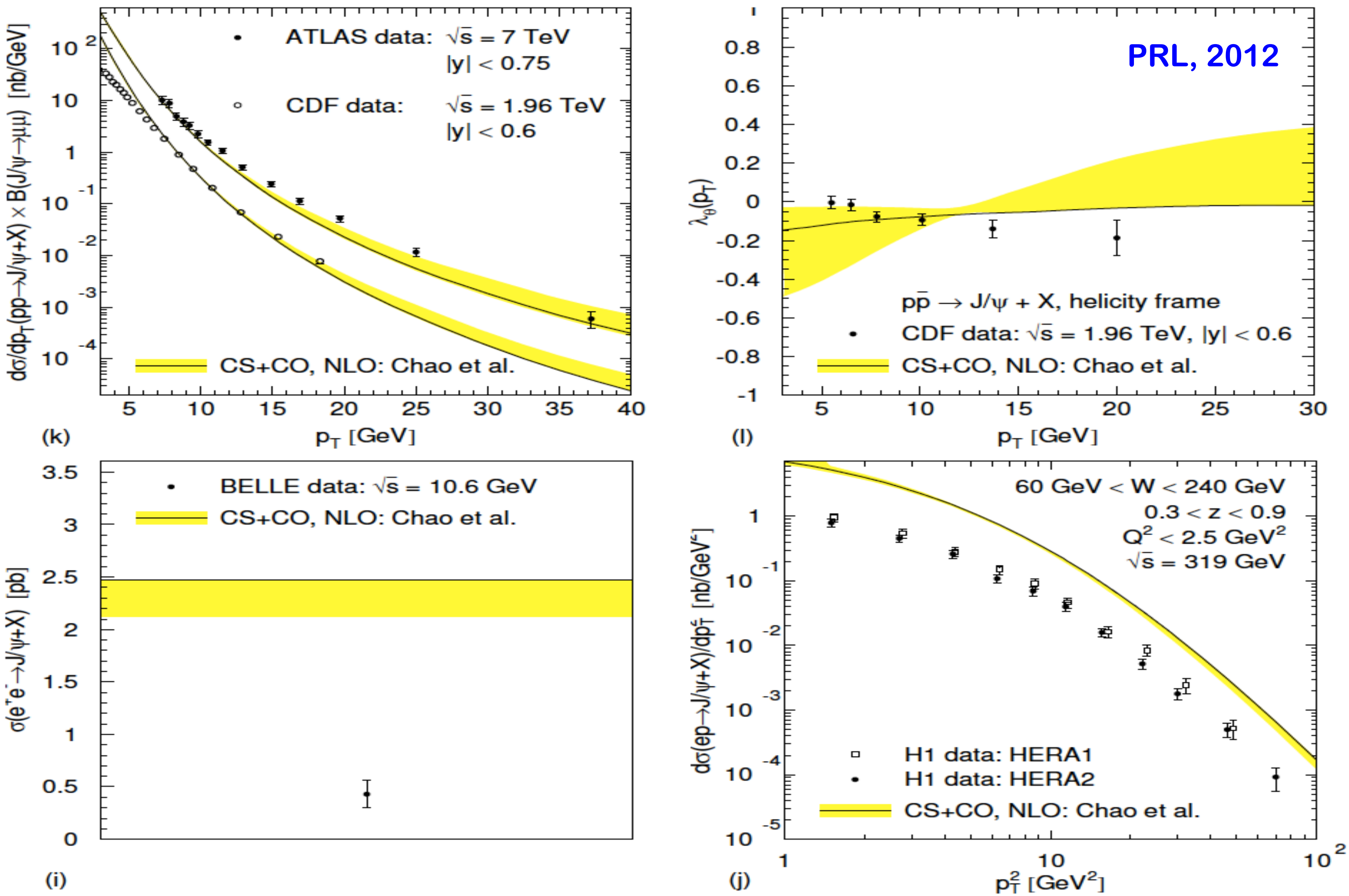
NLO theory fits – Butenschoen et al.



NLO theory fits – Gong et al.



NLO theory fits – Chao et al.



Questions

- Why the high order correction in CSM is so large? How many orders should we calculate?

- Why the CSM predicts the longitudinally polarized J/psi?

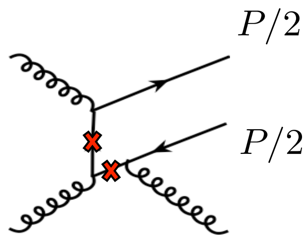
- Why NRQCD model predicts wrong polarization and “wrong rate” for association production?

Why high orders in CSM are so large?

Kang, Qiu and Sterman, 2011

- **LO in α_s but higher power in $1/p_T$:**

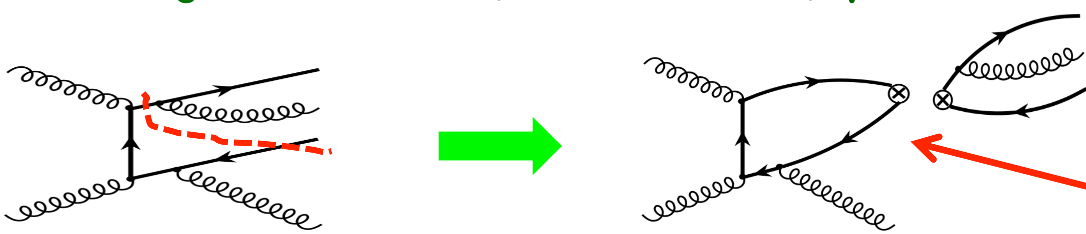
LO in α_s:



$$\hat{\sigma}^{\rm LO} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

CSM and NRQCD spin-1 projection NNLP in $1/p_T$!

- **NLO in α_s but lower power in $1/p_T$:**

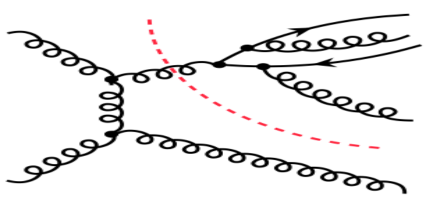


Relativistic projection to all "spin states"

$$\hat{\sigma}^{\rm NLO} \rightarrow \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)$$

$$\mu_0 \gtrsim 2m_Q$$

- **NNLO in α_s but leading power in $1/p_T$:**



$$\hat{\sigma}^{\rm NNLP} \rightarrow \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu)\log^m(\mu^2/\mu_0^2)$$

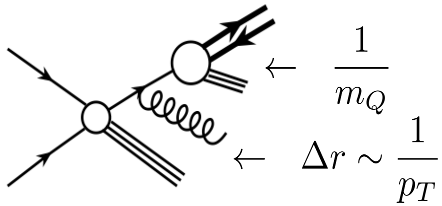
Leading order in α_s-expansion =\= leading power in $1/p_T$-expansion!

PQCD power counting

Kang, Qiu and Sterman, 2011

- IF $p_T >> m_Q$, the pair produced

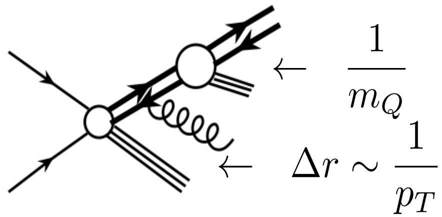
  - ✧ at $1/m_Q$:



$$\frac{1}{p_T^4}\sum_n \left[\log\left(\frac{p_T^2}{\mu_0^2}\right)\right]^n$$

Only final-state fragmentation

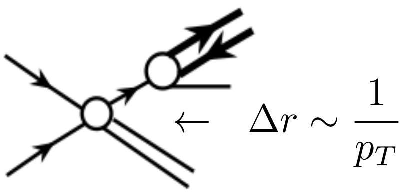
  - ✧ at $1/P_T$:



$$\frac{1}{p_T^6}\sum_n \left[\log\left(\frac{p_T^2}{\mu_0^2}\right)\right]^n$$

Short-distance Production

  - ✧ between: $[\,1/m_Q\,,\,1/P_T]$

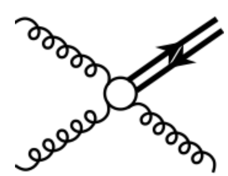


$$\frac{1}{p_T^4}$$

Modified evolution + pair production

- Role of relativity, color and spin projection:

  - ✧ Color can be perturbatively resolved between m_Q and P_T
  - ✧ Factorize into a singlet or octet pair
  - ✧ Relativity and spin affects the p_T-dependence



$$\left\{ \begin{array}{l} \frac{1}{p_T^8} \\ \frac{1}{p_T^6} \end{array} \right.$$

Non-relativistic projection

Relativistic projection

New factorization formalism

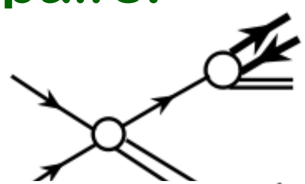
❑ Factorization formalism:

Kang, Qiu and Sterman, 2010

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\to i+X}(p_T/z,\mu)\otimes D_{i\to H}(z,m_Q,\mu)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+|B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}(\kappa)]}=p_T/z,\mu)$$
$$\otimes D_{[Q\bar{Q}(\kappa)]\to H}(z,m_Q,\mu)$$
$$+\mathcal{O}(m_Q^4/p_T^4)$$

❑ Production of the pairs:

✧ at $1/m_Q$:

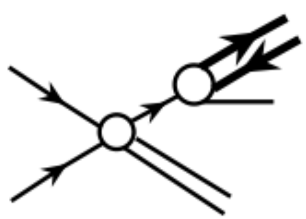


$$D_{i\to H}(z,m_Q,\mu_0)$$

✧ at $1/P_T$:

$$d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]}(\kappa),\mu)$$

✧ between: $[\,1/m_Q\,,\,1/P_T]$

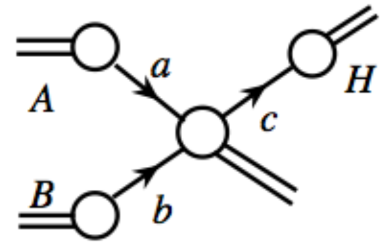


$$\frac{d}{d\ln(\mu)}D_{i\to H}(z,m_Q,\mu)=\ldots$$
$$+\frac{m_Q^2}{\mu^2}\Gamma(z)\otimes D_{[Q\bar{Q}(\kappa)\to H}(\{z_i\},m_Q,\mu)$$

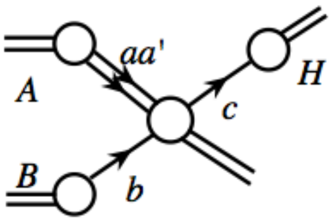
Why such power correction important?

- **Leading power in hadronic collisions:**

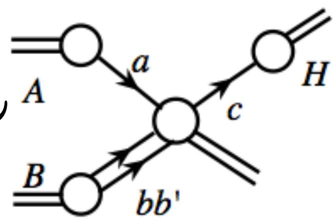
$$d\sigma_{AB\to H} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to cX} \otimes D_{c\to H}$$



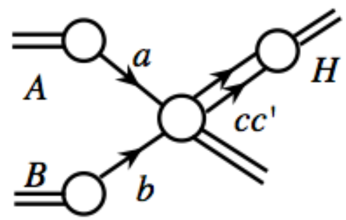
- **1st power corrections in hadronic collisions:**



$\sim$



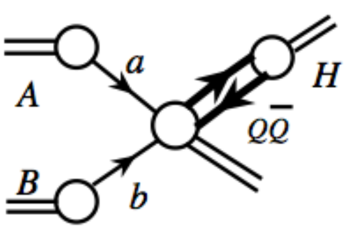
$\sim$



$$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$$

or $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes \mathcal{D}_{[ff]\to H}$

- **Dominated 1st power corrections:**



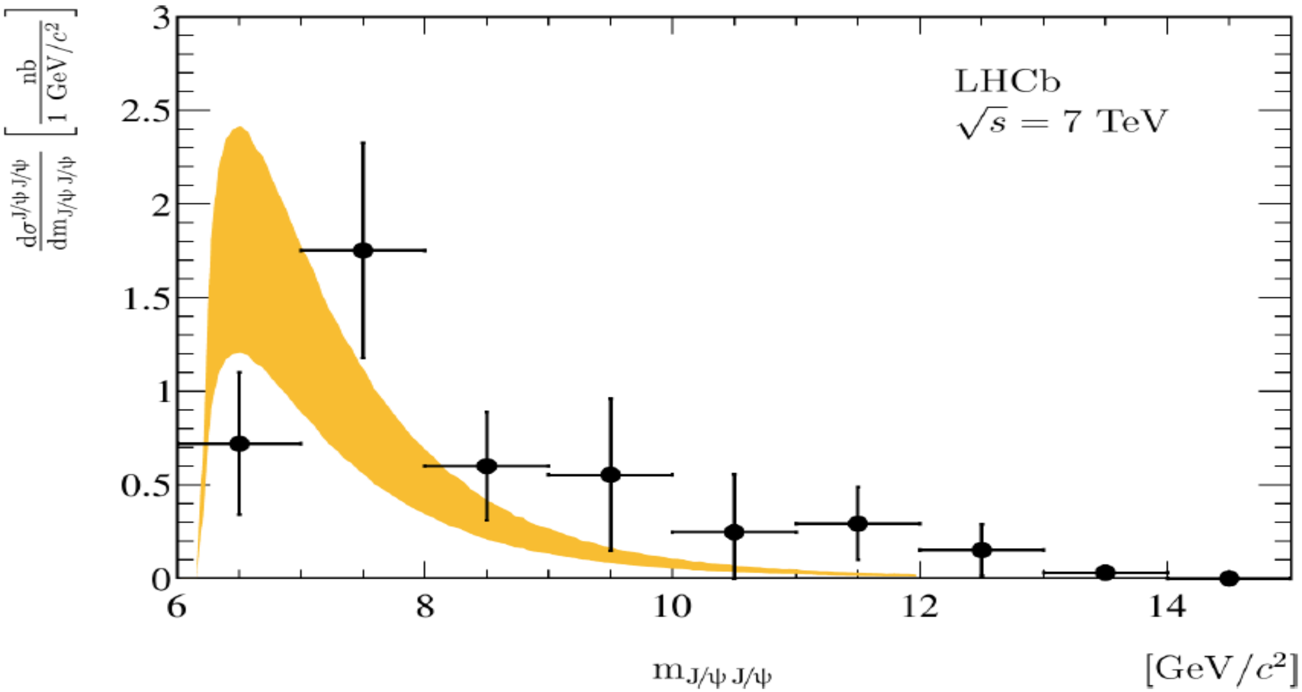
$$\sim \mathcal{O}\left(\frac{(2m_Q)^2}{P_T^2}\right) \otimes D^{(2)}_{[Q\bar{Q}]\to H}$$

Key: competition between $P_T^2 \gg (2m_Q)^2$ **and** $D^{(2)}_{[Q\bar{Q}]\to H} \gg D_{c\to H}$

Double J/ψ production at LHC

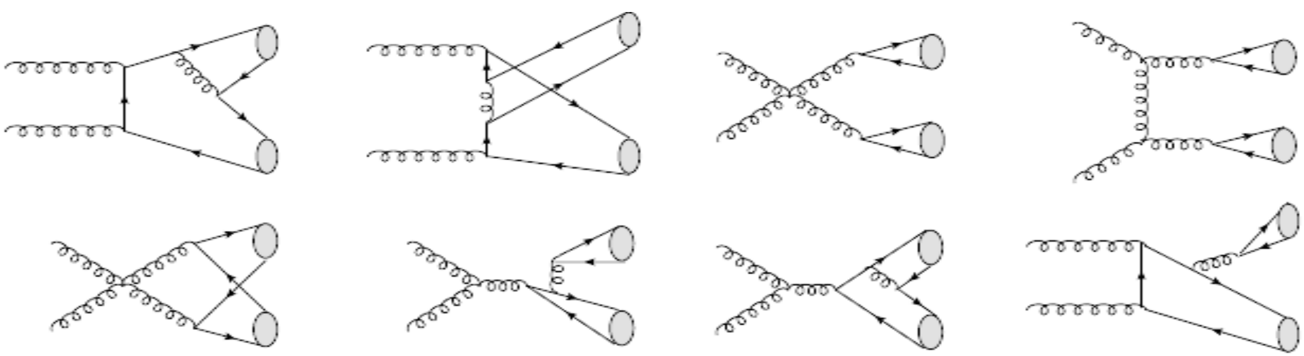
M. Frosini, QWG2011

❑ LHC data:



❑ Theory:

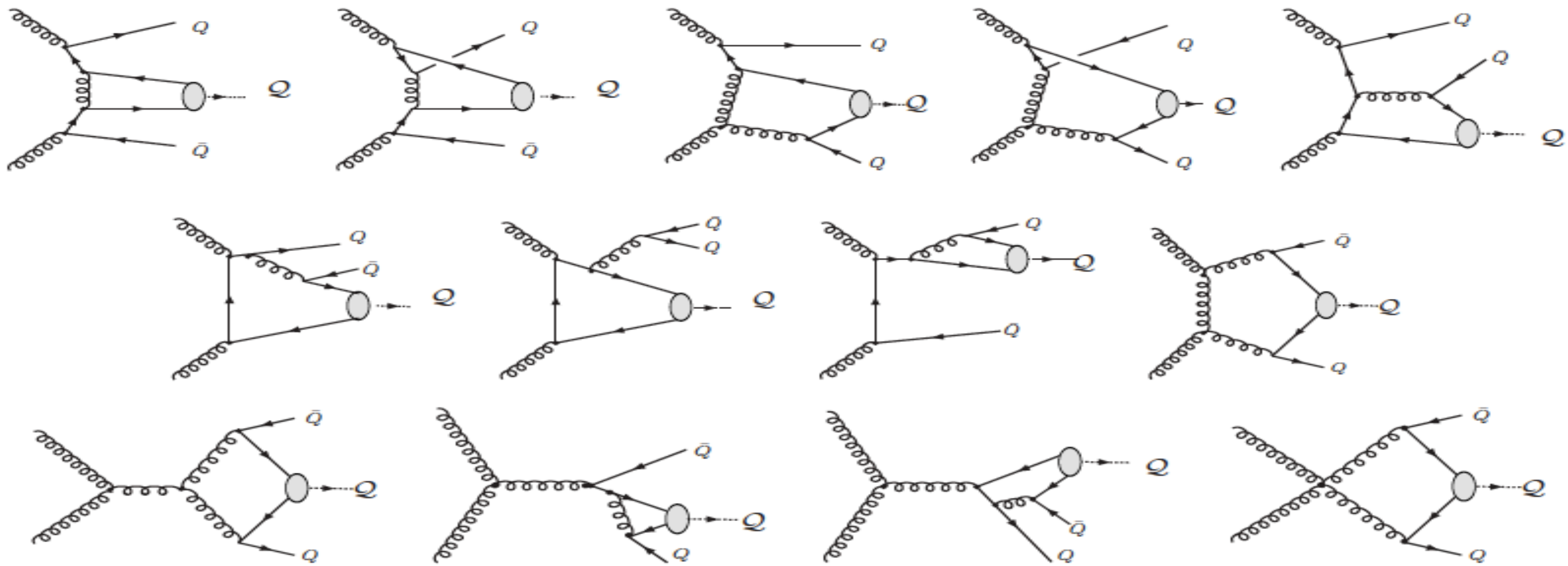
A.V. Berezhnoy, et al, 2011
C.F. Qiao, 2009, 2010



Associate production as an example

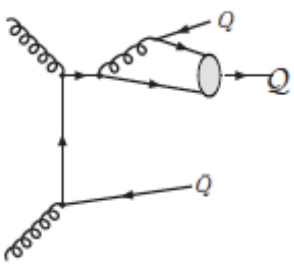
- **Complete set of diagrams:**

Artoisenet, Lansburg, Maltoni (2007)

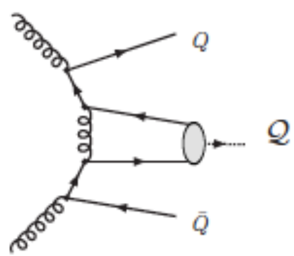


❑ **Contribution to inclusive J/ψ is NOT perturbatively stable!**

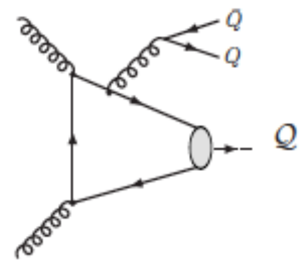
when $p_T >> m_Q$

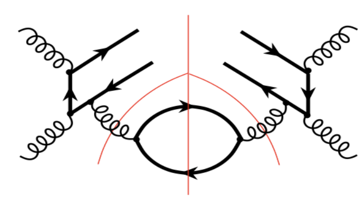


Q-fragmentation



Logs in PDFs





Need interference (virtual) diagrams

Backup slides