QCD Master Class 2017

18-24 June 2017
Saint-Jacut-de-la-Mer
Europe/Paris timezone

Theory of Quarkonium Production

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Lecture five/six

Theory Center

Jefferson Lab
EXPLORING THE NATURE OF MATTER

The plan for my eight lectures

- **The Goal:**

  To understand the theory of heavy quarkonium production, and strong interaction dynamics in terms of QCD

- **The Plan (approximately):**

  Inclusive production of a single heavy quarkonium

  The November Revolution

  Theoretical models and approximations

  Surprises and anomalies

  QCD factorization at the leading and next-to-leading power

  Five lectures

  Heavy quarkonium associate and in medium production

  Quarkonium associate production

  Quarkonium production in a jet

  Quarkonium production in cold/hot medium

  Three lectures

Factorization for heavy quarkonium production

Kang, Qiu and Sterman, 2009

❑ Factorized cross section:

$$
\begin{aligned}
E\frac{d\sigma_{AB\to \mathrm{J}/\psi}}{d^3P} &= \sum_{a,b,c} \phi_{a/A}\otimes\phi_{b/B}\otimes H^{(2)}_{ab\to c}\otimes D_{c\to \mathrm{J}/\psi} \\
&+ \sum_{a,b} \phi_{a/A}\otimes\phi_{b/B}\otimes H^{(4)}_{ab\to Q\bar{Q}}\otimes \mathcal{D}^{(4)}_{Q\bar{Q}\to \mathrm{J}/\psi} \\
&+ \sum_{a,b,c} \phi^{(4)}_{a/A}\otimes\phi_{b/B}\otimes H^{(4a)}_{ab\to c}\otimes D_{c\to \mathrm{J}/\psi} \\
&+ \sum_{a,b,c} \phi_{a/A}\otimes\phi^{(4)}_{b/B}\otimes H^{(4b)}_{ab\to c}\otimes D_{c\to \mathrm{J}/\psi} + \mathcal{O}\left(\frac{1}{P_T^4}\right)
\end{aligned}
$$

❑ Expect the first two terms to dominate:

- ✧ $\mathrm{H}^{(4)}$ are IR safe and free of large logarithms
- ✧ $\mathrm{D}^{(4)}$ are fragmentation functions of 4-quark operators

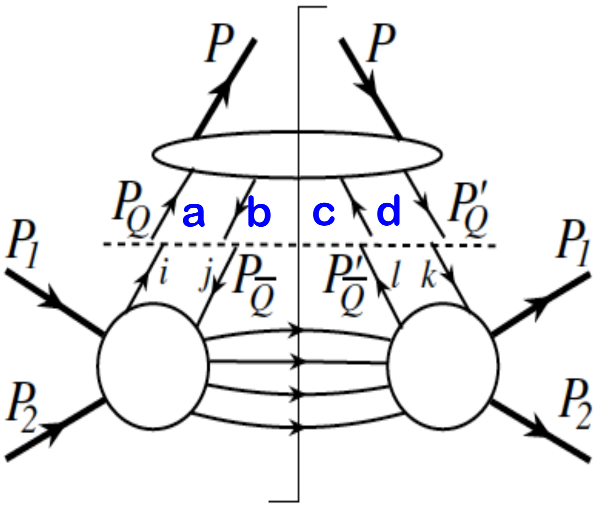
Qiu, 1990

❑ New perturbative inputs:

Calculation of $\mathrm{H}^{(4)}$ and evolution of $\mathrm{D}^{(4)}$

Heavy quark pair fragmentation functions

❑ Cut vertex = Momentum * Color * Spin:



✧ **Momentum:**

$$\frac{d^4p_c}{(2\pi)^4}\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\, z^4\,\delta(z-\frac{P^+}{p_c^+})\delta(u-\frac{P_Q^+}{p_c^+})\delta(v-\frac{P_Q'^+}{p_c^+})$$

$$P_Q=\frac{p_c}{2}+q_1,\, P_{\bar{Q}}=\frac{p_c}{2}-q_1;\, P'_Q=\frac{p_c}{2}+q_2,\, P'_{\bar{Q}}=\frac{p_c}{2}-q_2$$

✧ **Color:**

$$\mathcal{C}^{(1)}_{ab,cd}=\frac{1}{N_c^2}\delta_{ab}\,\delta_{cd}$$

$$\mathcal{C}^{(8)}_{ab,cd}=\frac{4}{N_c^2-1}\sum_B(t^B)_{ab}\,(t^B)_{cd}$$

✧ **Spin:**

$$\mathcal{P}^{(v)}_{ij,lk}=\left[\frac{1}{4P^+}\gamma\cdot n\right]_{ij}\left[\frac{1}{4P^+}\gamma\cdot n\right]_{lk}$$ **Vector**

$$\mathcal{P}^{(a)}_{ij,lk}=\left[\frac{1}{4P^+}\gamma\cdot n\gamma_5\right]_{ij}\left[\frac{1}{4P^+}\gamma\cdot n\gamma_5\right]_{lk}$$ **Axial vector**

$$\mathcal{P}^{(t)}_{ij,lk}=\frac{1}{2}(-g_\perp^{\alpha\beta})\left[\frac{1}{4P^+}\gamma\cdot n\gamma_\alpha\right]_{ij}\left[\frac{1}{4P^+}\gamma\cdot n\gamma_\beta\right]_{lk}$$ **Tensor**

Corresponding projection operators define the hard part

Evolution of fragmentation functions

Kang, Qiu and Sterman, 2011

- **Independence of the factorization scale:**

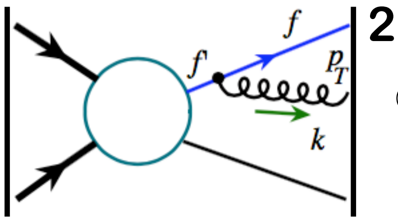
$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T)=0$$

✧ at Leading power in $1/P_T$:

$$\left[\frac{\partial}{\partial\ln\mu^2}D_{f\to H}(z,\mu^2;m_Q)\right]\otimes d\hat{\sigma}_{A+B\to f(p_c)+X}(p_c=p/z,\mu^2)$$
$$+D_{f\to H}(z,\mu^2;m_Q)\otimes\left[\frac{\partial}{\partial\ln\mu^2}d\hat{\sigma}_{A+B\to f(p_c)+X}(p_c=p/z,\mu^2)\right]=0,$$

✧ μ^2 - dependence of $\hat{\sigma}$:

$$\hat{\sigma}_{AB\to f}(p_T,\mu^2)\propto$$



$$\propto\hat{\sigma}_{AB\to f'}\otimes\gamma_{f'\to f}\int_{\mu^2}^{\mathcal{O}(p_T^2)}\frac{dk_T^2}{k_T^2}$$

$$\Rightarrow\left[\frac{\partial}{\partial\ln\mu^2}D_{f\to H}\right]\otimes d\hat{\sigma}_{AB\to f}+D_{f\to H}\otimes[-\gamma_{f'\to f}\otimes d\hat{\sigma}_{AB\to f'}]=0$$

$$\Rightarrow\left[\frac{\partial}{\partial\ln\mu^2}D_{f\to H}-D_{f'\to H}\otimes\gamma_{f\to f'}\right]\otimes d\hat{\sigma}_{AB\to f}=0$$

$$\Rightarrow\boxed{\frac{\partial}{\partial\ln\mu^2}D_{f\to H}=\gamma_{f\to f'}\otimes D_{f'\to H}}$$

DGALP evolution

Evolution of fragmentation functions

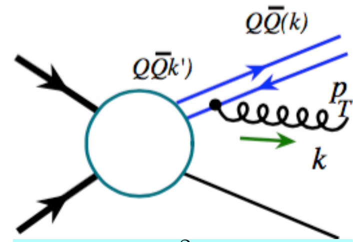
Kang, Qiu and Sterman, 2011

❑ Independence of the factorization scale:

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T)=0$$

✧ **at the Next-to-Leading power in $1/P_T$:**

$$0 = D'_{f\to H}\otimes\hat{\sigma}_{A+B\to f(p_c)+X} + \mathcal{D}'_{[Q\bar{Q}(\kappa)]\to H}\otimes d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)](p_c)+X}$$
$$+ D_{f\to H}\otimes\hat{\sigma}'_{A+B\to f(p_c)+X} + \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}\otimes d\hat{\sigma}'_{A+B\to[Q\bar{Q}(\kappa)](p_c)+X}$$

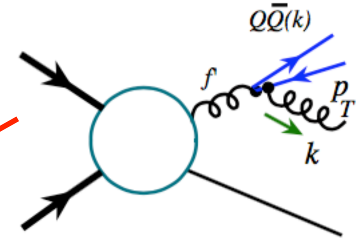


$$\propto\int_{\mu^2}^{\mathcal{O}(p_T^2)}\frac{dk_T^2}{k_T^2}$$

✧ μ^2 **- dependence of $\hat{\sigma}$:**

$$d\hat{\sigma}'_{A+B\to f+X} = -\gamma_{f'\to f}\otimes d\hat{\sigma}_{A+B\to f'+X}$$

$$d\hat{\sigma}'_{A+B\to[Q\bar{Q}(\kappa)](p_c)+X} = -\Gamma_{[Q\bar{Q}(\kappa')]\to[Q\bar{Q}(\kappa)]}\otimes d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa')]+X}$$
$$-\frac{1}{\mu^2}\gamma_{f'\to[Q\bar{Q}(\kappa)]}\otimes d\hat{\sigma}_{A+B\to f'+X}$$



$$\propto\int_{\mu^2}^{\mathcal{O}(p_T^2)}\frac{dk_T^2}{(k_T^2)^2}$$

$$0=\left[D'_{f\to H}-D_{f'\to H}\otimes\gamma_{f'\to f}-\frac{1}{\mu^2}\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}\otimes\gamma_{f\to[Q\bar{Q}(\kappa)]}\right]\otimes d\hat{\sigma}_{AB\to f}$$
$$+\left[\mathcal{D}'_{[Q\bar{Q}(\kappa)]\to H}-\mathcal{D}_{[Q\bar{Q}(\kappa')]\to H}\otimes\Gamma_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(\kappa')]}\right]\otimes d\hat{\sigma}_{AB\to[Q\bar{Q}(\kappa)]}$$

Evolution of fragmentation functions

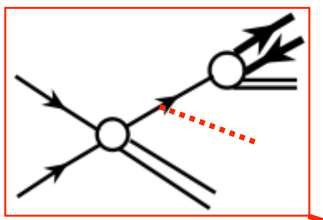
Kang, Qiu and Sterman, 2011

❑ Independence of the factorization scale:

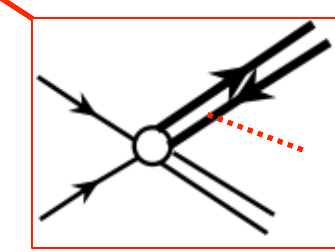
$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$$

✧ **New evolution equations at the Next-to-Leading power in $1/P_T$:**

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes\,\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$



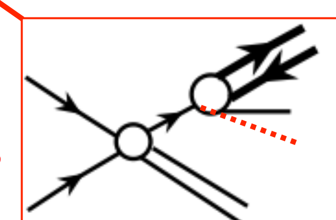
Form a close set of evolution equations



$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

NOTE (scheme dependence):

Impact the polarization!



Use dimensional regularization, drop power divergence!
The pair is effectively only produced at p_T or the input scale?

Predictive power

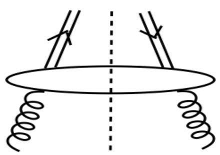
- Calculation of short-distance hard parts in pQCD:

  Power series in α_s, without large logarithms

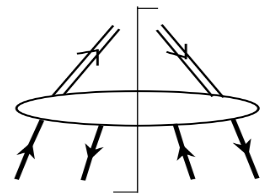
- Calculation of evolution kernels in pQCD:

  Power series in α_s, scheme in choosing factorization scale μ
  Could affect the term with mixing powers

- Universality of input fragmentation functions at μ_0:



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

- Physics of $\mu_0 \sim 2m_Q$ – a parameter:

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

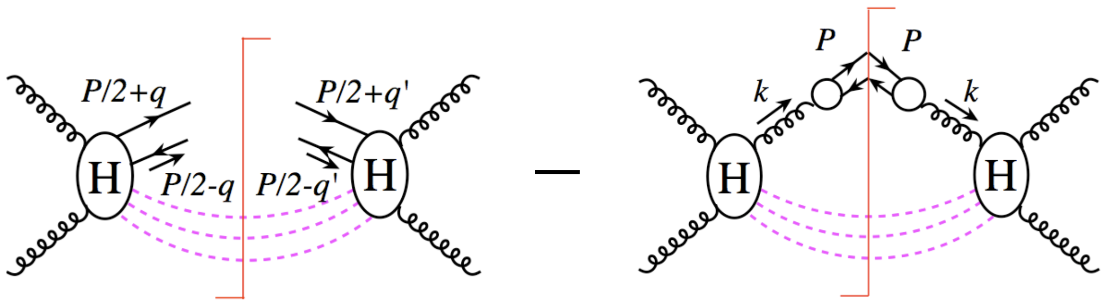
  Different quarkonium states require different input distributions!

"Direct" production of heavy quark pairs

❑ Calculation of $H^{(4)}$:

Apply the factorized formula to the production of $Q\bar{Q}(P)$ state

$$H^{(4)}_{ab\to Q\bar{Q}} = E\frac{d\sigma_{AB\to Q\bar{Q}(P)}}{d^3P} - \sum_{a,b,c}\phi_{a/A}\otimes\phi_{b/B}\otimes H^{(2)}_{ab\to c}\otimes D_{c\to Q\bar{Q}} - \ldots$$



$H^{(4)}$ are free of large logarithms

– absorbed into the PDFs and/or fragmentation functions

❑ All partonic hard parts are evaluated at P_T:

Smooth transition from high P_T to $P_T \sim M_H$

Need "new" non-perturbative fragmentation functions

Short-distance hard part– an example

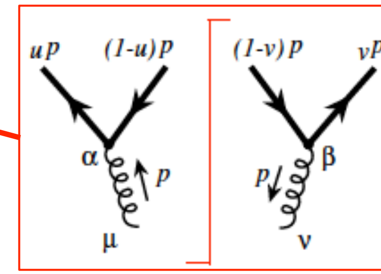
PRD91, 014030 (2015)

❑ Even tree-level needs subtraction – H(4) at α_s^3:

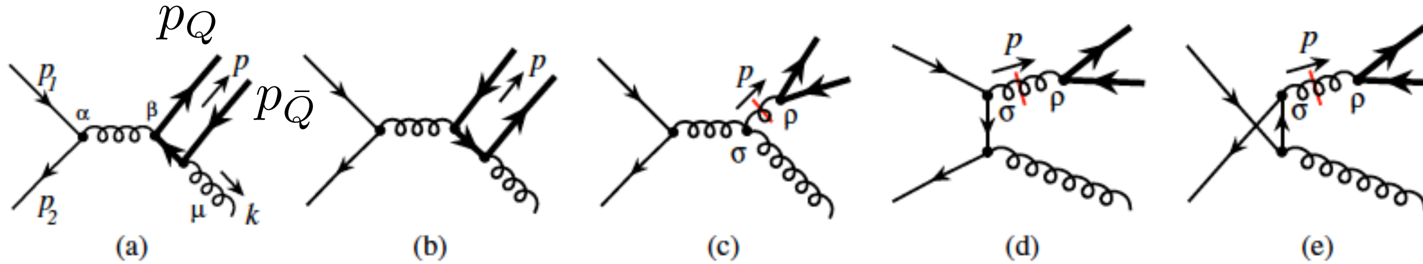
$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

$$\Longrightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

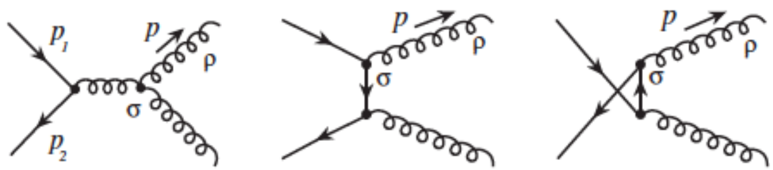
$$\frac{\alpha_s^3(\mu)}{p_T^6} \qquad \frac{\alpha_s^2(\mu)}{p_T^4}$$



$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$:



$\sigma^{(2)}_{q\bar{q}\to g}$:



$$E_p \frac{d\hat{\sigma}^{(3)}_{q+\bar{q}\to[Q\bar{Q}(\kappa)](p)}}{d^3p} \equiv \left[\frac{4\pi\alpha_s^3}{\hat{s}}\right] \frac{1}{\bar{u}u\bar{v}v} H_{q\bar{q}\to[Q\bar{Q}(\kappa)]}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u})$$

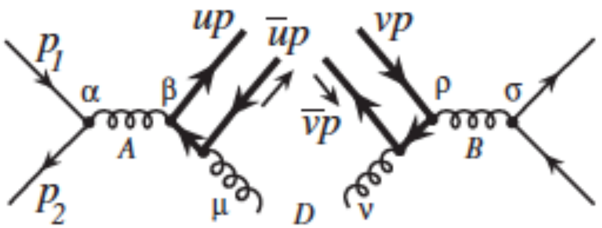
$$\Longrightarrow \quad H_{q\bar{q}\to[Q\bar{Q}(\kappa)]}(\hat{s},\hat{t},\hat{u}) = |\overline{\mathcal{M}}_{q\bar{q}\to[Q\bar{Q}(\kappa)]}|^2_c \left[\frac{\bar{u}u\bar{v}v}{g_s^6}\right]$$

Short-distance hard part – an example

❑ **Simplest partonic channel** $q+\bar{q}\to[Q\bar{Q}(a1)]$:

Diagrams (a) and (b) contribute

✧ **Color factor:**



Other 3 terms have the same color factor

$$C^{[1]}=\left(\frac{1}{N_c}\right)^2\sum_{A,B,D}\mathrm{Tr}[t^At^B]\frac{1}{\sqrt{N_c}}\mathrm{Tr}[t^At^D]\frac{1}{\sqrt{N_c}}\mathrm{Tr}[t^Bt^D]=\frac{N_c^2-1}{8N_c^3}$$

✧ **Partonic matrix element squares:**

$$|\overline{\mathcal{M}}^{aa^\dagger}_{q\bar{q}\to[Q\bar{Q}(a1)]}|^2=C^{[1]}g_s^6\left(\frac{1}{2}\right)^2\mathrm{Tr}[\gamma\cdot p_1\gamma^\sigma\gamma\cdot p_2\gamma^\alpha]\frac{(-g_{\alpha\beta})}{(p_1+p_2)^2}\frac{(-g_{\sigma\rho})}{(p_1+p_2)^2}$$

$$\times\mathrm{Tr}\left[\gamma\cdot p\gamma_5\gamma^\beta\frac{\gamma\cdot(up-p_1-p_2)}{(up-p_1-p_2)^2}\gamma^\mu\right]$$

$$\times\mathrm{Tr}\left[\gamma\cdot p\gamma_5\gamma^\nu\frac{\gamma\cdot(vp-p_1-p_2)}{(vp-p_1-p_2)^2}\gamma^\rho\right](-g_{\mu\nu})=C^{[1]}\left[\frac{g_s^6}{\bar{u}\,\bar{v}}\right]\frac{4}{\hat{s}}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

$$|\overline{\mathcal{M}}^{ab^\dagger}_{q\bar{q}\to[Q\bar{Q}(a1)]}|^2=C^{[1]}\left[\frac{g_s^6}{\bar{u}v}\right]\frac{4}{\hat{s}}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right],\quad|\overline{\mathcal{M}}^{ba^\dagger}_{q\bar{q}\to[Q\bar{Q}(a1)]}|^2=C^{[1]}\left[\frac{g_s^6}{u\bar{v}}\right]\frac{4}{\hat{s}}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right],$$

$$|\overline{\mathcal{M}}^{bb^\dagger}_{q\bar{q}\to[Q\bar{Q}(a1)]}|^2=C^{[1]}\left[\frac{g_s^6}{uv}\right]\frac{4}{\hat{s}}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

Short-distance hard part – an example

- **Simplest partonic channel** $q+\bar{q}\to[Q\bar{Q}(a1)]$:

Diagrams (a) and (b) contribute

- ✧ **Partonic hard part:**

$$|\overline{\mathcal{M}}_{q\bar{q}\to[Q\bar{Q}(a1)]}|^2=\left[\frac{g_s^6}{\bar{u}u\bar{v}v}\right]4C^{[1]}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^3}\right]$$

used $\frac{1}{\bar{u}\,\bar{v}}+\frac{1}{\bar{u}v}+\frac{1}{u\bar{v}}+\frac{1}{uv}=\frac{1}{\bar{u}u\bar{v}v}$

$$\longrightarrow\quad H_{q\bar{q}\to[Q\bar{Q}(a1)]}(\hat{s},\hat{t},\hat{u})=4\left[\frac{N_c^2-1}{8N_c^3}\right]\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^3}\right]$$

- **Similarly, for the octet channel** $q+\bar{q}\to[Q\bar{Q}(a8)]$:
  - ✧ **Difference in color factors:**

$$C^{[8]}_{aa^\dagger}=\left(\frac{1}{N_c}\right)^2\sum_{A,B,D}\mathrm{Tr}[t^At^B]\sqrt{2}\mathrm{Tr}[t^Ct^At^D]\sqrt{2}\mathrm{Tr}[t^Ct^Dt^B]=\left[\frac{N_c^2-1}{8N_c^3}\right](N_c^2-2)=C^{[8]}_{bb^\dagger}$$

$$C^{[8]}_{ab^\dagger}=-\left[\frac{N_c^2-1}{4N_c^3}\right]=C^{[8]}_{ba^\dagger}$$

See PRD91, 014030 (2015) for all other channels!

  - ✧ **Partonic hard part:**

using $\frac{1}{\bar{u}\,\bar{v}}C^{[8]}_{aa^\dagger}+\frac{1}{\bar{u}\,v}C^{[8]}_{ab^\dagger}+\frac{1}{u\bar{v}}C^{[8]}_{ba^\dagger}+\frac{1}{uv}C^{[8]}_{bb^\dagger}=\frac{1}{\bar{u}u\bar{v}v}\left[\frac{1}{2}(1+\zeta_1\zeta_2)\mathcal{C}_1+\mathcal{C}_2\right]$

$$\longrightarrow\quad H_{q\bar{q}\to[Q\bar{Q}(a8)]}(\hat{s},\hat{t},\hat{u})=2\left[\frac{N_c^2-1}{8N_c}\right]\left[1+\zeta_1\zeta_2-\frac{4}{N_c^2}\right]\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^3}\right]$$

Evolution equations and kernels

PRD90, 034006 (2914)

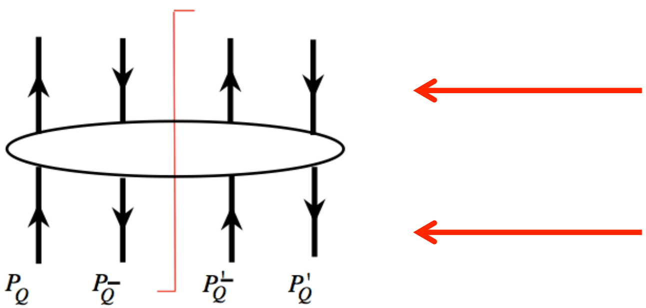
❑ Evolution equations:

$$\frac{\partial}{\partial \ln \mu^2} \mathcal{D}_{Q\bar{Q}[\kappa]\to J/\psi}(z_h, \zeta_1, \zeta_2, \mu^2)$$

$$= \frac{\alpha_s}{2\pi} \int_{z_h}^{1} \frac{dz}{z} \int_{-1}^{1} d\zeta_1' \int_{-1}^{1} d\zeta_2' P_{\kappa\to\kappa'}(\zeta_1, \zeta_2, \zeta_1', \zeta_2', z) \mathcal{D}_{Q\bar{Q}[\kappa']\to J/\psi}(z_h/z, \zeta_1', \zeta_2', \mu^2)$$

$$\kappa, \kappa' = v, a, t$$

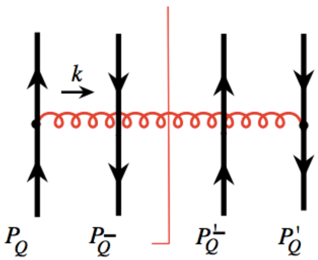
❑ Evolution kernels:



Projection operators

Cut vertex

❑ Lowest order Feynman diagrams:



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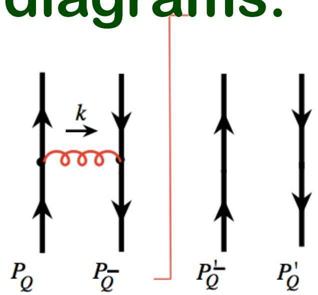

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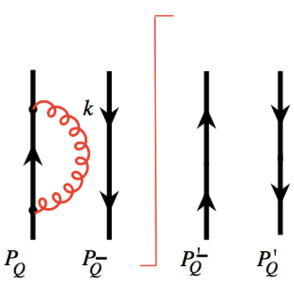


+ c.c.

Real

Virtual

IR divergence canceled between the real and virtual diagrams!

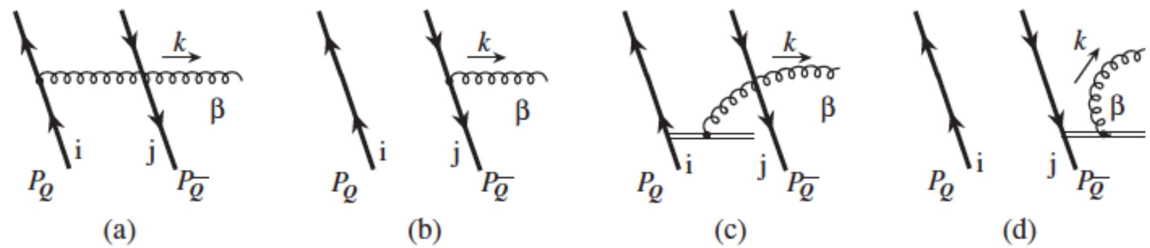
Calculation of evolution kernels

Fragmentation function of quark-antiquark pair

- **Evolution kernels:**

$$\Gamma^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(\kappa')]}(z/z',u,v;u',v') = \frac{\partial}{\partial \ln \mu^2}\mathcal{D}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(\kappa')]}(z/z',u,v;u',v';\mu^2)$$

- **Real diagrams:**



$$\mathcal{D}^{(1,R-aa^\dagger)}_{[Q\bar{Q}(v8)]\to[Q\bar{Q}(v8)]}(z,u,v;u',v';\mu^2) = g_s^2 C_a \int \frac{d^4p_c}{(2\pi)^4}\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\theta(\mu^2 - p_{c\perp}^2)$$
$$\times (2\pi)^4\delta^4(p_c - p^+ - k) z^2 \delta\left(z - \frac{p^+}{p_c^+}\right)\delta\left(u - \frac{1}{2} - \frac{q_1^+}{p_c^+}\right)\delta\left(v - \frac{1}{2} - \frac{q_2^+}{p_c^+}\right)$$
$$\times (2\pi)^4\delta^4\left(\frac{p_c}{2} - q_1 - \bar{u}'p^+\right)(2\pi)^4\delta^4\left(\frac{p_c}{2} - q_2 - \bar{v}'p^+\right)(2\pi)\delta(k^2)\mathcal{P}_{\alpha\beta}(k)$$
$$\times \frac{1}{4p_c^+}\mathrm{Tr}[\gamma\cdot n\gamma\cdot p\gamma^\beta\gamma\cdot(p_c/2+q_1)]\frac{1}{(p_c/2+q_1)^2 + i\varepsilon}$$
$$\times \frac{1}{4p_c^+}\mathrm{Tr}[\gamma\cdot n\gamma\cdot(p_c/2+q_2)\gamma^\alpha\gamma\cdot p]\frac{1}{(p_c/2+q_2)^2 - i\varepsilon},$$
$$= \int^{\mu^2}\frac{dk_\perp^2}{k_\perp^2}\left(\frac{\alpha_s}{2\pi}\right)\frac{N_c^2-2}{2N_c}\left[\frac{u}{u'}+z\right]\left[\frac{v}{v'}+z\right]\delta(\bar{u}-z\bar{u}')\delta(\bar{v}-z\bar{v}')\frac{z}{2}\int_0^{p^+/z}\frac{dk^+}{k^+}\delta\left(1-z+\frac{k^+}{p^+/z}\right)$$

Calculation of evolution kernels

❑ **Total real contribution:**

$$\mathcal{D}^{(1,R)}_{[Q\bar{Q}(v8)]\to[Q\bar{Q}(v8)]}(z,u,v;u',v';\mu^2) = \int^{\mu^2}\frac{dk_\perp^2}{k_\perp^2}\left(\frac{\alpha_s}{2\pi}\right)\left(\frac{1}{2N_c}\right)S_+\Delta_-^{[8]}\frac{z}{2}\int_0^{p^+/z}\frac{dk^+}{k^+}\delta\left(1-z+\frac{k^+}{p^+/z}\right)$$

with $S_\pm = \left[\frac{u}{u'}\pm\frac{\bar{u}}{\bar{u}'}\right]\left[\frac{v}{v'}\pm\frac{\bar{v}}{\bar{v}'}\right]$,

$$\Delta_\pm^{[8]} = \{(N_c^2-2)[\delta(u-zu')\delta(v-zv')+\delta(\bar{u}-z\bar{u}')\delta(\bar{v}-z\bar{v}')]$$
$$\mp 2[\delta(u-zu')\delta(\bar{v}-z\bar{v}')+\delta(\bar{u}-z\bar{u}')\delta(v-zv')]\}$$

❑ **Kernel for v8 to v8 transition after adding virtual contribution:**

$$\Gamma^{(1)}_{[Q\bar{Q}(v8)]\to[Q\bar{Q}(v8)]}(z,u,v;u',v') = \left(\frac{\alpha_s}{2\pi}\right)\left\{\left[\frac{1}{2N_c}\right]\left\{\frac{1}{2}\frac{z}{(1-z)_+}S_+\Delta_-^{[8]}-\delta(z-1)\right.\right.$$
$$\times\left[\delta(v-v')\left(\frac{\theta(\bar{u}'-\bar{u})}{(\bar{u}'-\bar{u})_+}\left(\frac{\bar{u}}{\bar{u}'}\right)(\bar{u}'+u)+\frac{\theta(u'-u)}{(u'-u)_+}\left(\frac{u}{u'}\right)(u'+\bar{u})\right)\right.$$
$$+\delta(u-u')\left(\frac{\theta(\bar{v}'-\bar{v})}{(\bar{v}'-\bar{v})_+}\left(\frac{\bar{v}}{\bar{v}'}\right)(\bar{v}'+v)+\frac{\theta(v'-v)}{(v'-v)_+}\left(\frac{v}{v'}\right)(v'+\bar{v})\right)$$
$$\left.\left.+3\delta(v-v')\delta(u-u')\right]\right\}$$
$$-\left[\frac{1}{2}\right]\delta(z-1)[\delta(v-v')[4u(1-u)]+\delta(u-u')[4v(1-v)]]$$
$$\left.+\left[\frac{N_c}{2}\right]\delta(z-1)\delta(u-u')\delta(v-v')[3-\ln(u\bar{u}v\bar{v})]\right\},$$

Infrared safe!
evolution in pair's total momentum, z-dependence, as well as pair's relative momentum fractions, u, v, …

Evolution equations and kernels

- Evolution equation – matrix form:

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} \mathcal{D}_{Q\bar{Q}[v8]} \\ \mathcal{D}_{Q\bar{Q}[v1]} \\ \mathcal{D}_{Q\bar{Q}[a8]} \\ \mathcal{D}_{Q\bar{Q}[a1]} \\ \mathcal{D}_{Q\bar{Q}[t8]} \\ \mathcal{D}_{Q\bar{Q}[t1]} \end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix} \mathcal{K}_1 & \mathcal{T}_1 & \mathcal{K}_2 & \mathcal{T}_2 & 0 & 0 \\ \mathcal{R}_1 & \mathcal{S}_1 & \mathcal{R}_2 & 0 & 0 & 0 \\ \mathcal{K}_2 & \mathcal{T}_2 & \mathcal{K}_1 & \mathcal{T}_1 & 0 & 0 \\ \mathcal{R}_2 & 0 & \mathcal{R}_1 & \mathcal{S}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{K}_1' & \mathcal{T}_1' \\ 0 & 0 & 0 & 0 & \mathcal{R}_1' & \mathcal{S}_1' \end{pmatrix} \otimes \begin{pmatrix} \mathcal{D}_{Q\bar{Q}[v8]} \\ \mathcal{D}_{Q\bar{Q}[v1]} \\ \mathcal{D}_{Q\bar{Q}[a8]} \\ \mathcal{D}_{Q\bar{Q}[a1]} \\ \mathcal{D}_{Q\bar{Q}[t8]} \\ \mathcal{D}_{Q\bar{Q}[t1]} \end{pmatrix}$$

- Evolution kernels – singlet case:

$$\begin{aligned} \mathcal{S} = P_{v1\to v1} = P_{a1\to a1} = C_F\delta(1-z)\Big\{ & 3\delta(u-u')\delta(v-v') \\ & + \delta(v-v')\left[\frac{\theta(\bar{u}'-\bar{u})}{(\bar{u}'-\bar{u})_+}\frac{\bar{u}}{\bar{u}'}(\bar{u}'+u) + \frac{\theta(u'-u)}{(u'-u)_+}\frac{u}{u'}(u'+\bar{u})\right] \\ & + \delta(u-u')\left[\frac{\theta(\bar{v}'-\bar{v})}{(\bar{v}'-\bar{v})_+}\frac{\bar{v}}{\bar{v}'}(\bar{v}'+v) + \frac{\theta(v'-v)}{(v'-v)_+}\frac{v}{v'}(v'+\bar{v})\right]\Big\} \end{aligned}$$

The $\delta(1-z)$ is a unique feature of singlet-to-singlet kernel, which the relative momentum between quark and antiquark evolves

NRQCD for input distributions

- Input distributions are universal, non-perturbative:

  Should, in principle, be extracted from experimental data

- Apply NRQCD to the input distributions:
  - ✧ Single parton fragmentation – valid to 2-loop: Nayak, Qiu and Sterman, 2005

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\mathrm{NRQCD}}$$

Braaten, Yuan, 1994
Ma, 1995, …
Braaten, Chen, 1997
Braaten, Lee, 2000,
Ma, Qiu, Zhang, 2013
…

Complete LO+NLO for S, P states & NNLO for singlet S state

Dominated by transverse polarization

  - ✧ heavy quark pair fragmentation – no proof:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi}(z,\zeta,\zeta',\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle_{\mathrm{NRQCD}}$$

Kang, Qiu and Sterman, 2011, 2014

Ma, Qiu, Zhang, 2014

Dominated by longitudinal polarization

- If it is valid – No proof of such factorization yet!

  Replace unknown functions by a few unknown matrix elements!

Ma, Qiu, Zhang, PRD89, 094029 (2014) – S-Wave; 094030 (2014) – P-Wave

NRQCD for input distributions

Ma, Qiu, Zhang, 2014

❑ Heavy quark pair FFs in NRQCD:

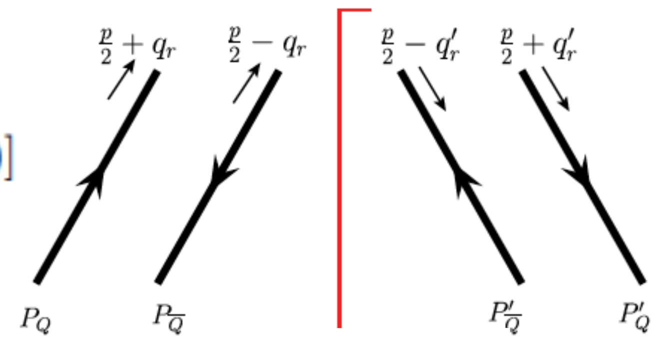
$$D_{f\to H}(z;m_Q,\mu_0)=\sum_{[Q\bar{Q}(n)]}\hat{d}_{f\to[Q\bar{Q}(n)]}(z;m_Q,\mu_0,\mu_\Lambda)\times\langle\mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle$$

✧ Apply the factorized formula to $[Q\bar{Q}(\kappa)]$ states, instead of the "H"

$$\mathcal{D}^{\rm LO}_{[Q\bar{Q}(s^{[b]})]\to[Q\bar{Q}(i^{[b']})]}(z,\zeta_1,\zeta_2;m_Q,\mu_0)=\hat{d}^{\rm LO}_{[Q\bar{Q}(s^{[b]})]\to[Q\bar{Q}(i^{[b']})]}(z,\zeta_1,\zeta_2;m_Q,\mu_0)$$

✧ Consider :

$$[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]$$



NRQCD states:
Operators for LDMEs

PQCD states:
Operators for FFs

$$\hat{d}^{\rm LO}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2;m_Q,\mu_0)=\frac{1}{N_c^2-1}\,\delta(1-z)\mathcal{A}(\zeta_1)\mathcal{A}^\dagger(\zeta_2)$$

$$=\frac{1}{N_c^2-1}\frac{1}{2m_Q}\delta(1-z)\delta(\zeta_1)\delta(\zeta_2)$$

where

$$\mathcal{A}(\zeta_1)=\mathrm{Tr}_c[\sqrt{2}t^{(F)}_{c_{\rm in}}\sqrt{2}t^{(F)}_{c_{\rm out}}]\mathrm{Tr}_\gamma\left[\frac{\gamma\cdot\hat{n}\gamma_5-\gamma_5\gamma\cdot\hat{n}}{8p\cdot\hat{n}}\frac{1}{\sqrt{8m_Q^3}}\times\left(\frac{\not{p}}{2}-m_Q\right)\gamma_5\left(\frac{\not{p}}{2}+m_Q\right)\right]2\delta(\zeta_1)$$

$$=-\frac{1}{\sqrt{2m_Q}}\delta_{c_i,c_f}\delta(\zeta_1)$$

NRQCD for input distributions

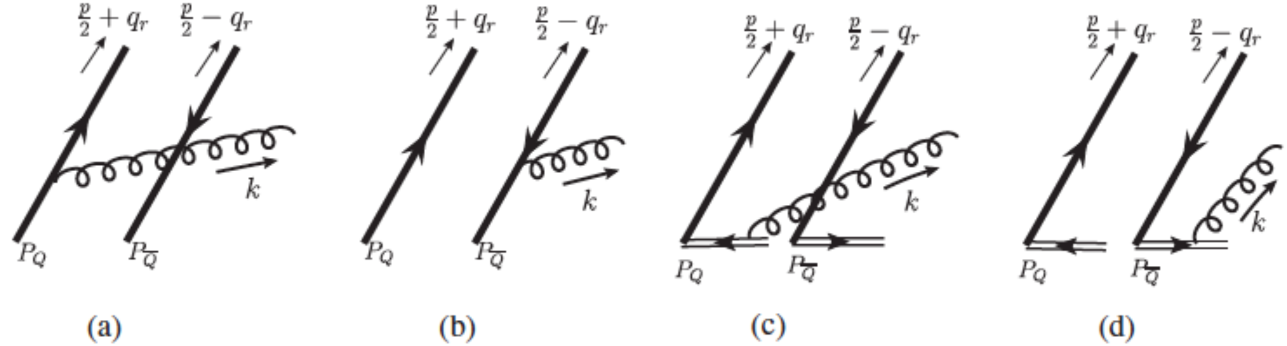
Ma, Qiu, Zhang, 2014

❑ Heavy quark pair FFs in NRQCD:

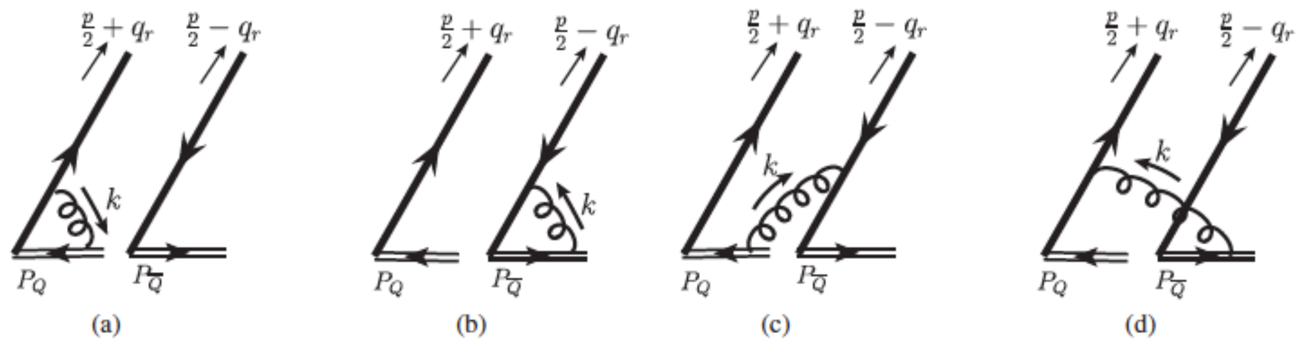
$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \Big\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \Big\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

❑ NLO contribution:

Real diagrams:



Virtual diagrams:



Plus 8 more diagrams

NRQCD for input distributions

Ma, Qiu, Zhang, 2014

❑ Heavy quark pair FFs in NRQCD:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \Big\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \Big\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

❑ NLO contribution:

$$\begin{aligned}
&\mathcal{D}^{\rm NLO,real}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2;m_Q,\mu_0) \\
&= \frac{\alpha_s}{4\pi m_Q N_c(N_c^2-1)} \left(\frac{4\pi\mu^2}{m_Q^2}\right)^{\epsilon} \Gamma(1+\epsilon) \\
&\times \Big\{ -N_c^2 \delta(\zeta_1)\delta(\zeta_2)\delta(1-z) \left(\frac{1}{\epsilon_{\rm UV}\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm IR}}\right) + \frac{1}{\epsilon_{\rm UV}} \frac{z}{(1-z)_+} \frac{\Delta_-^{[8]}}{4} \\
&+ 2(\ln 2) N_c^2 \delta(\zeta_1)\delta(\zeta_2)\delta(1-z) \frac{1}{\epsilon_{\rm UV}} - 2[(\ln 2)^2 + \ln 2] N_c^2 \delta(\zeta_1)\delta(\zeta_2)\delta(1-z) \\
&- (2\ln 2) \frac{z}{(1-z)_+} \frac{\Delta_-^{[8]}}{4} + \frac{\Delta_-^{[8]}}{4} \left[-\frac{z}{(1-z)_+} - 2z \left(\frac{\ln(1-z)}{1-z}\right)_+ \right] \Big\},
\end{aligned}$$

NRQCD for input distributions

Ma, Qiu, Zhang, 2014

❑ Heavy quark pair FFs in NRQCD:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) \right.$$
$$\left. + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \right\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

❑ NLO contribution:

$$\mathcal{D}^{\rm NLO,virtual}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}(^1S_0^{[8]})]}(z,\zeta_1,\zeta_2;m_Q,\mu_0)$$
$$= \frac{\alpha_s}{8\pi m_Q}\frac{1}{2N_c(N_c^2-1)}\delta(1-z)\delta(\zeta_2)\left(\frac{4\pi\mu^2}{m_Q^2}\right)^\epsilon \Gamma(1+\epsilon)$$
$$\times \left\{ 2N_c^2\delta(\zeta_1)\left[\frac{1}{\epsilon_{\rm UV}\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm IR}} + 2\right] + \frac{1}{\epsilon_{\rm UV}}\left[3(N_c^2-1)\delta(\zeta_1) - 2\left(\frac{1}{\zeta_1}\right)_{1+}\right.\right.$$
$$\left. +(\zeta_1+1)_{0+}\right] + 2\left[\left(\frac{1}{\zeta_1}\right)_{1+} - \left(\frac{1}{\zeta_1^2}\right)_{2+} + \left(\frac{\ln(\zeta_1^2)}{\zeta_1}\right)_{1+}\right]$$
$$\left. -((\zeta_1+1)\ln(\zeta_1^2))_{0+} - (\zeta_1-1)_{0+} \right\} + (\zeta_1 \longleftrightarrow \zeta_2).$$

NRQCD for input distributions

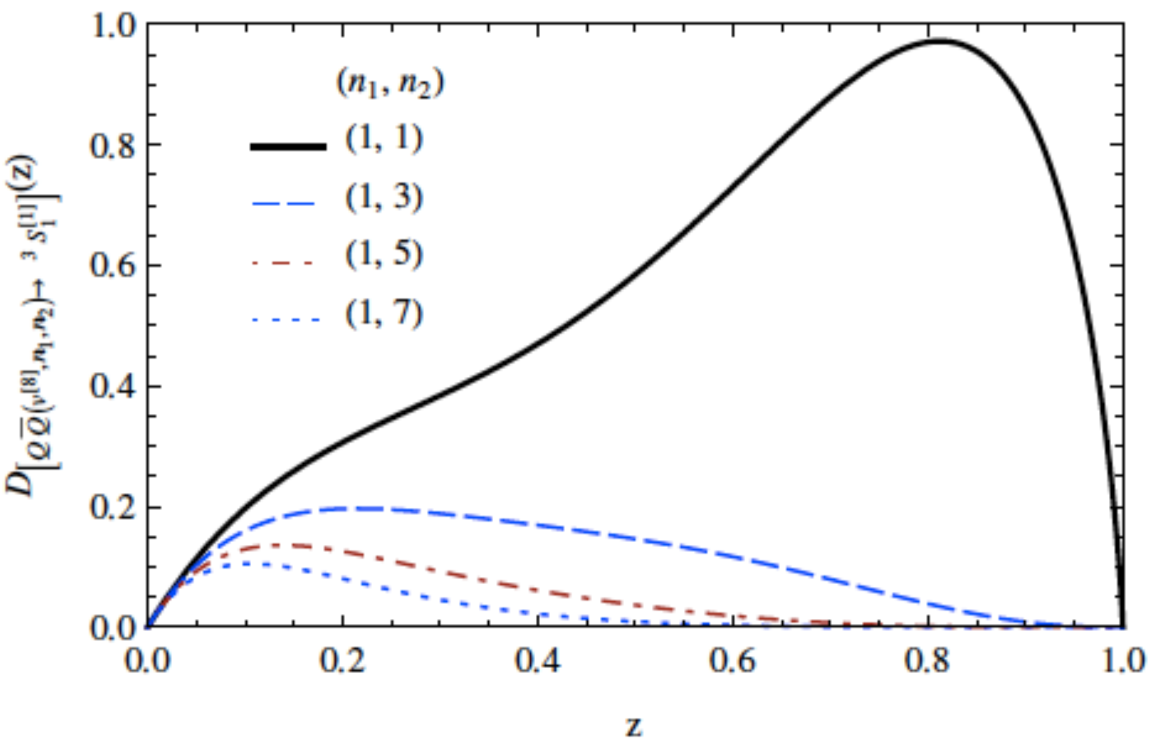
Ma, Qiu, Zhang, 2014

- **NLO contribution:**

Real + virtual + UV counter-term (renormalization)

- **Moment of the FFs:**

$$\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$$



Factorized cross section

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- **Connection to NRQCD factorization:**

$$\sigma^H(p_T, m_Q) = \sum_{[Q\bar{Q}(n)]} \hat{\sigma}_{[Q\bar{Q}(n)]}(p_T, m_Q, \Lambda)\langle 0|\mathcal{O}^H_{[Q\bar{Q}(n)]}(\Lambda)|0\rangle$$

$$D_{H/f}(z, m_Q) \equiv \sum_{[Q\bar{Q}(n)]} \hat{d}_{f\to[Q\bar{Q}(n)]}(z, m_Q, \Lambda)\langle 0|\mathcal{O}^H_{[Q\bar{Q}(n)]}(\Lambda)|0\rangle$$

$$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q) \equiv \sum_{[Q\bar{Q}(n)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta,\zeta',m_Q,\Lambda)\langle 0|\mathcal{O}^H_{[Q\bar{Q}(n)]}(\Lambda)|0\rangle$$

For every NRQCD state $[Q\bar{Q}(n)]$:

$$\hat{\sigma}_{[Q\bar{Q}(n)]}(p_T, m_Q, \Lambda) = \sum_f \hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes \hat{d}_{f\to[Q\bar{Q}(n)]}(z,\mu,m_Q)$$

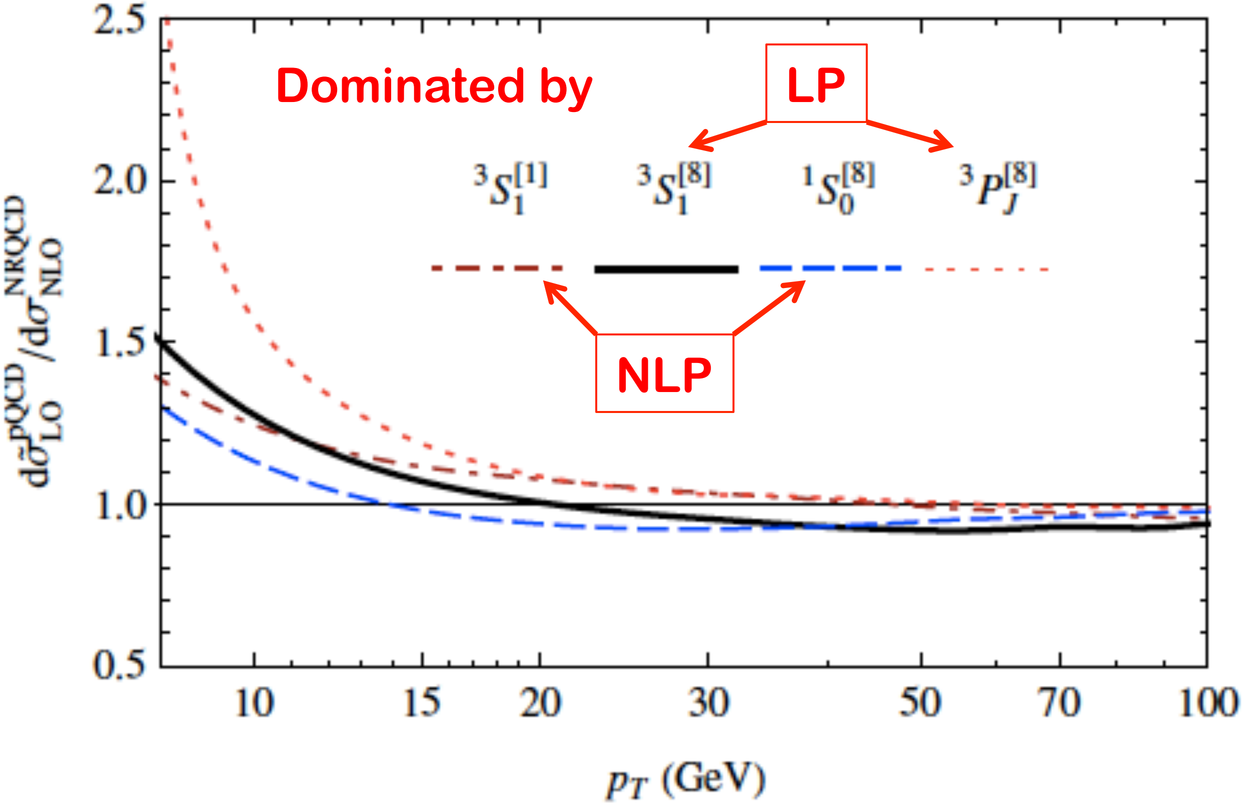
$$+ \sum_{[Q\bar{Q}(\kappa)]} \hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta,\zeta',\mu,m_Q)$$

Factorized cross section

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- Channel-by-channel comparison:



independent of NRQCD matrix elements

LO analytical results reproduce NLO NRQCD calculations (numerical)

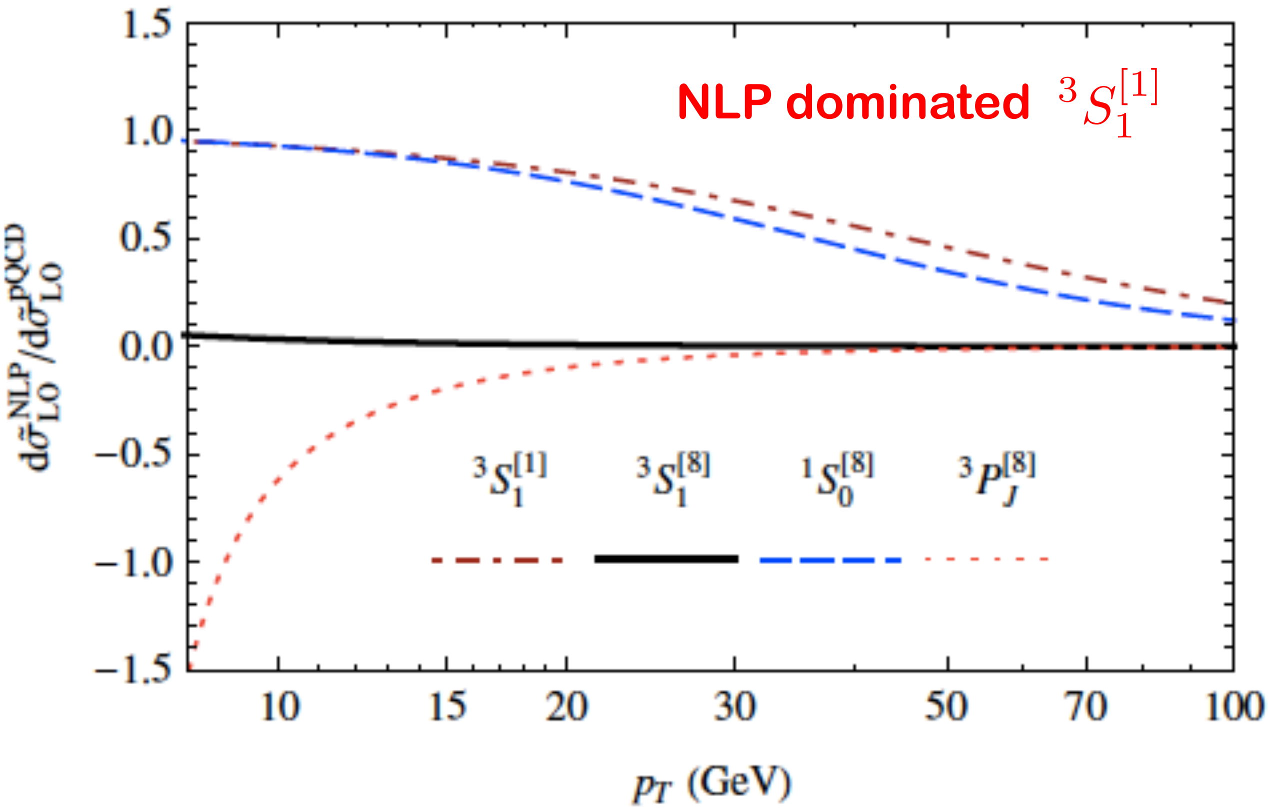
PRL, 2014

Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- LP vs. NLP (both LO):



NLP dominated $^1S_0^{[8]}$ for wide pT

LP dominated $^3S_1^{[8]}$ and $^3P_J^{[8]}$

PT distribution is consistent with distribution of $^1S_0^{[8]}$

PRL, 2014

QCD factorization vs NRQCD factorization

- **QCD factorization – not always true:**
  - ✧ Expand physical cross section in powers of $1/p_T$
  - ✧ Expand the coefficient of each term in powers of α_s
  - ✧ Factorization is valid for all powers of α_s of the 1st two terms in $1/p_T$

- **NRQCD factorization – conjectured:**
  - ✧ Expand physical cross section in powers of relative velocity of HQ
  - ✧ Expand the coefficient of each term in powers of α_s
  - ✧ Verified to NNLO in α_s for the leading power term in the *v*-expansion

- **Connection:**

If NRQCD factorization for fragmentation functions is valid,

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P, m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

$$+ E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P, m_Q \neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

Mass effect + connection to lower p_T region

Heavy quarkonium polarization

Ma et al. 2014

- Polarization = input fragmentation functions:
  - Partonic hard parts and evolution kernels are perturbative
  - Insensitive to the properties of produced heavy quarkonia
- Projection operators – polarization tensors:

$$\mathcal{P}^{\mu\nu}(p) \equiv \sum_{\lambda=0,\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}$$

Unpolarized quarkonium

$$\mathcal{P}_T^{\mu\nu}(p) \equiv \frac{1}{2}\sum_{\lambda=\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = \frac{1}{2}\left[-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n}\right]$$

Transversely polarized quarkonium

$$\mathcal{P}_L^{\mu\nu}(p) \equiv \mathcal{P}^{\mu\nu}(p) - 2\mathcal{P}_T^{\mu\nu}(p) = \frac{1}{p^2}\left[p^\mu - \frac{p^2}{2p\cdot n}n^\mu\right]\left[p^\nu - \frac{p^2}{2p\cdot n}n^\nu\right]$$

Longitudinally polarized quarkonium

for produced the quarknium moving in +z direction with

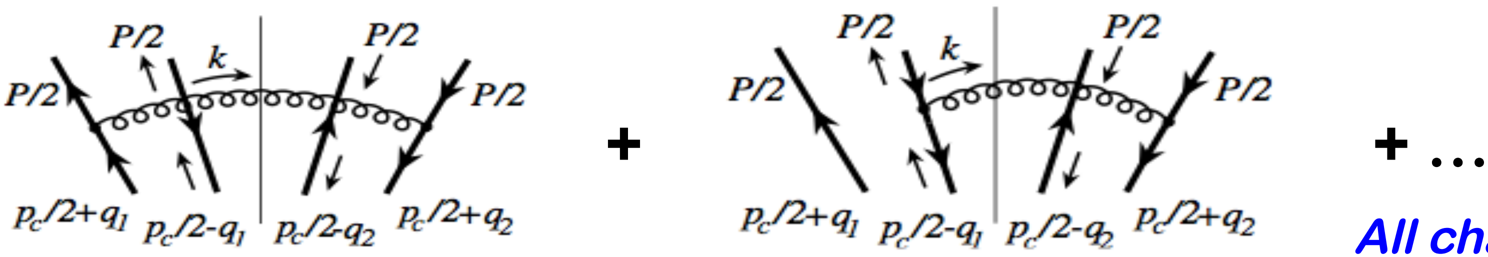
$$p^\mu = (p^+, p^-, p_\perp) = p^+(1, 0, \mathbf{0}_\perp) \qquad p^2 = n^2 = 0$$

$$n^\mu = (n^+, n^-, n_\perp) = (0, 1, \mathbf{0}_\perp) \qquad p\cdot n = p^+$$

Polarized fragmentation functions

Kang, Ma, Qiu and Sterman, 2014
Zhang, Ph.D. Thesis, 2014

- **Color singlet as an example:**



All channels are calculated

✧ **A axial vector pair to a singlet NRQCD pair:**

$$\mathcal{D}^{L,CR}_{[Q\bar{Q}(a8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_+(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^{T,CR}_{[Q\bar{Q}(a8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_+(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

✧ **A vector pair to a singlet NRQCD pair:**

$$\mathcal{D}^{L,CR}_{[Q\bar{Q}(v8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_-(u,v)\times\frac{\alpha_s}{2\pi}\frac{z}{1-z}\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^{T,CR}_{[Q\bar{Q}(v8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_-(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

where

$$\Delta_+(u,v) = \frac{1}{4}\left[\delta\left(u-\frac{z}{2}\right)+\delta\left(\bar{u}-\frac{z}{2}\right)\right]\left[\delta\left(v-\frac{z}{2}\right)+\delta\left(\bar{v}-\frac{z}{2}\right)\right]$$

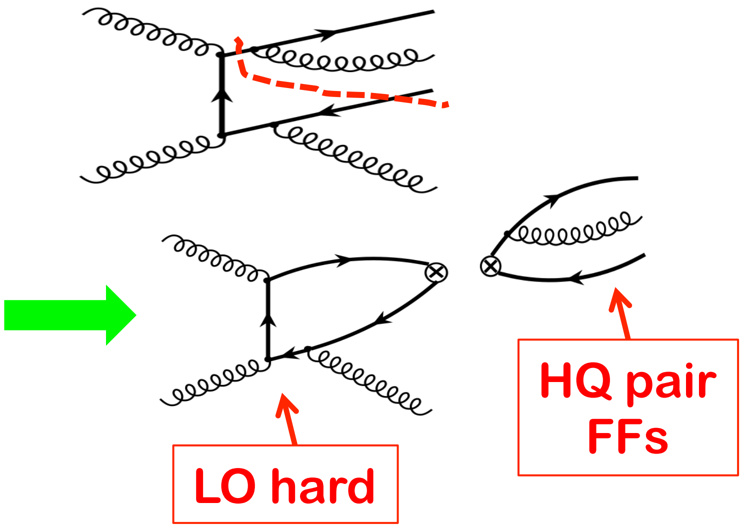
$$\Delta_-(u,v) = \frac{1}{4}\left[\delta\left(u-\frac{z}{2}\right)-\delta\left(\bar{u}-\frac{z}{2}\right)\right]\left[\delta\left(v-\frac{z}{2}\right)-\delta\left(\bar{v}-\frac{z}{2}\right)\right]$$

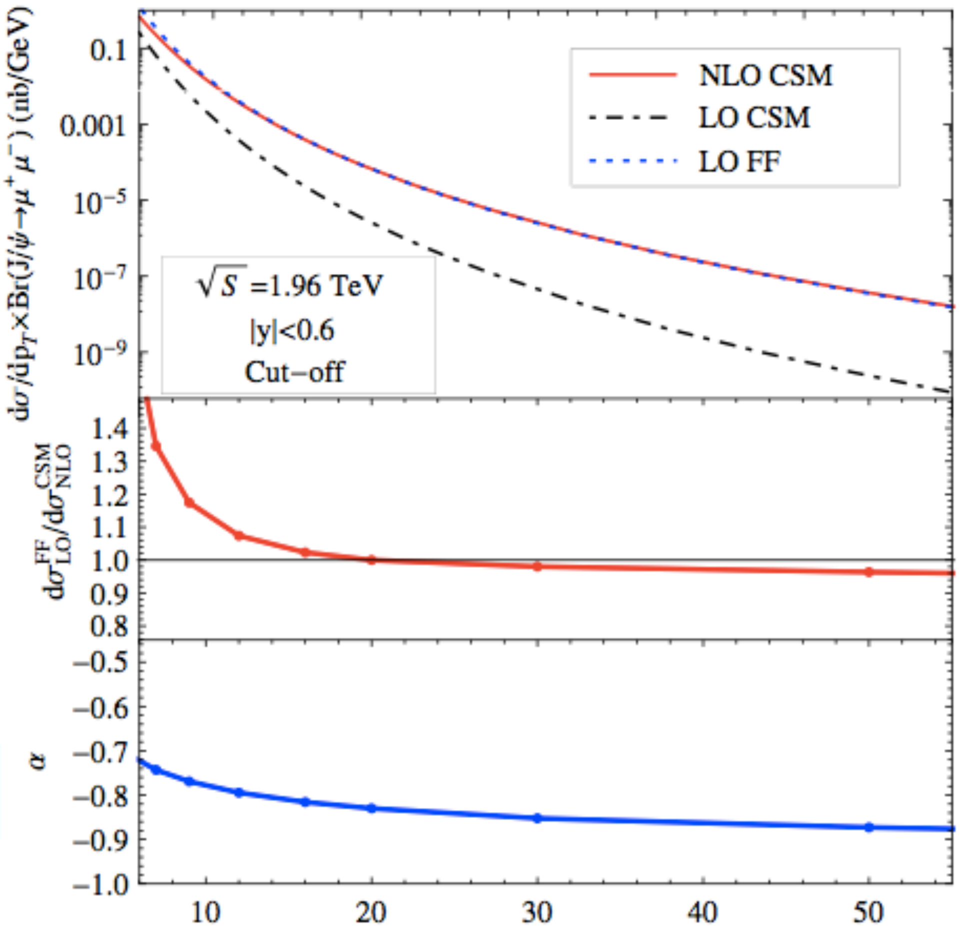
$$r(z) \equiv \frac{z^2\mu^2}{4m_c^2(1-z)^2}$$

Production and polarization

Kang, Ma, Qiu and Sterman, 2014

❑ Color singlet as an example:





$$\sigma^{(NLO)}_{NRQCD} \propto \Big[d\hat{\sigma}^{A(LO)}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(LO)}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S(LO)}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(LO)}_{[Q\bar{Q}(a8)]\to J/\psi} \Big]$$

Reproduce NLO CSM for p_T > 10 GeV!

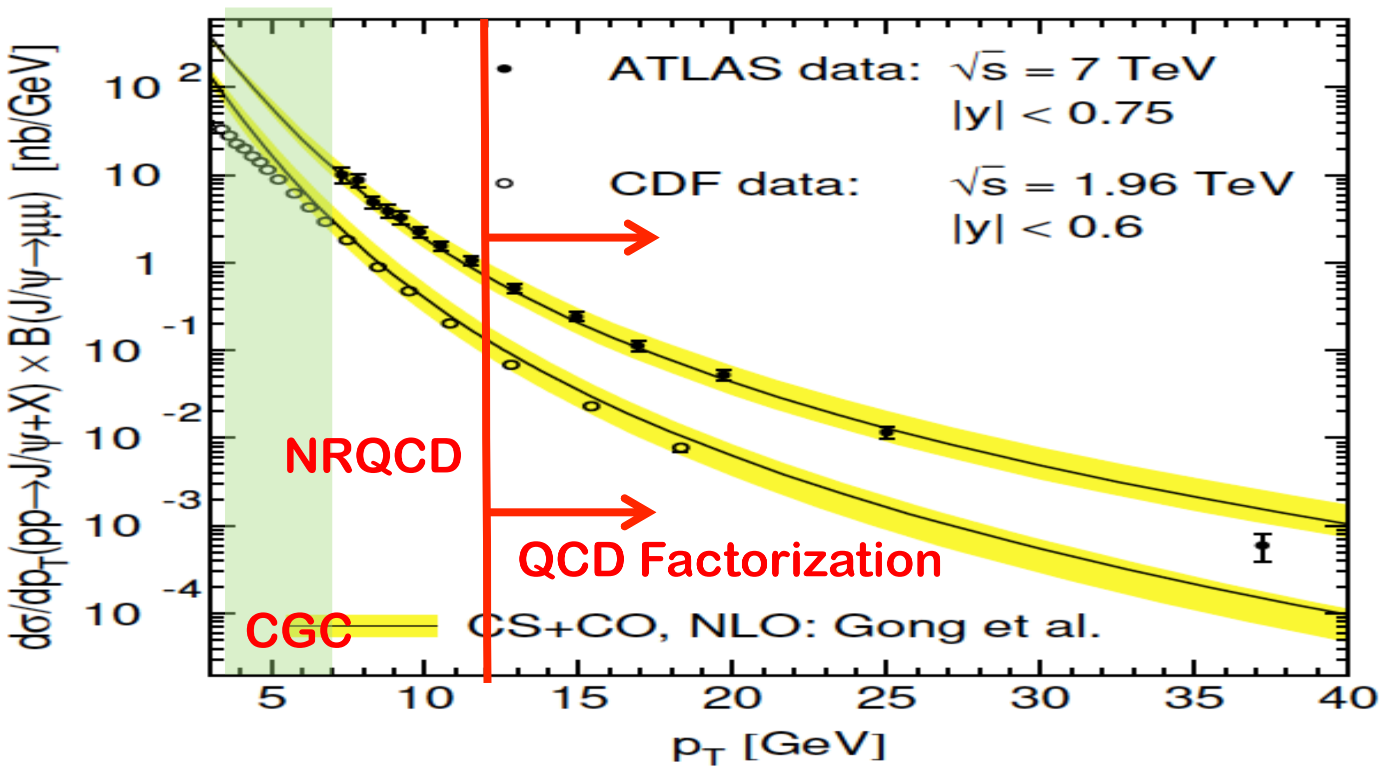
Cross section + polarization

QCD Factorization = better controlled HO corrections!

Can we understand the full p_T spectrum?

- **Expectation:**

Kang, Ma, Qiu and Sterman, 2014



- **Matching pQCD and NRQCD:**

Mass effect + expanded P_T region ($P_T \gtrsim m_Q$)

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P, m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

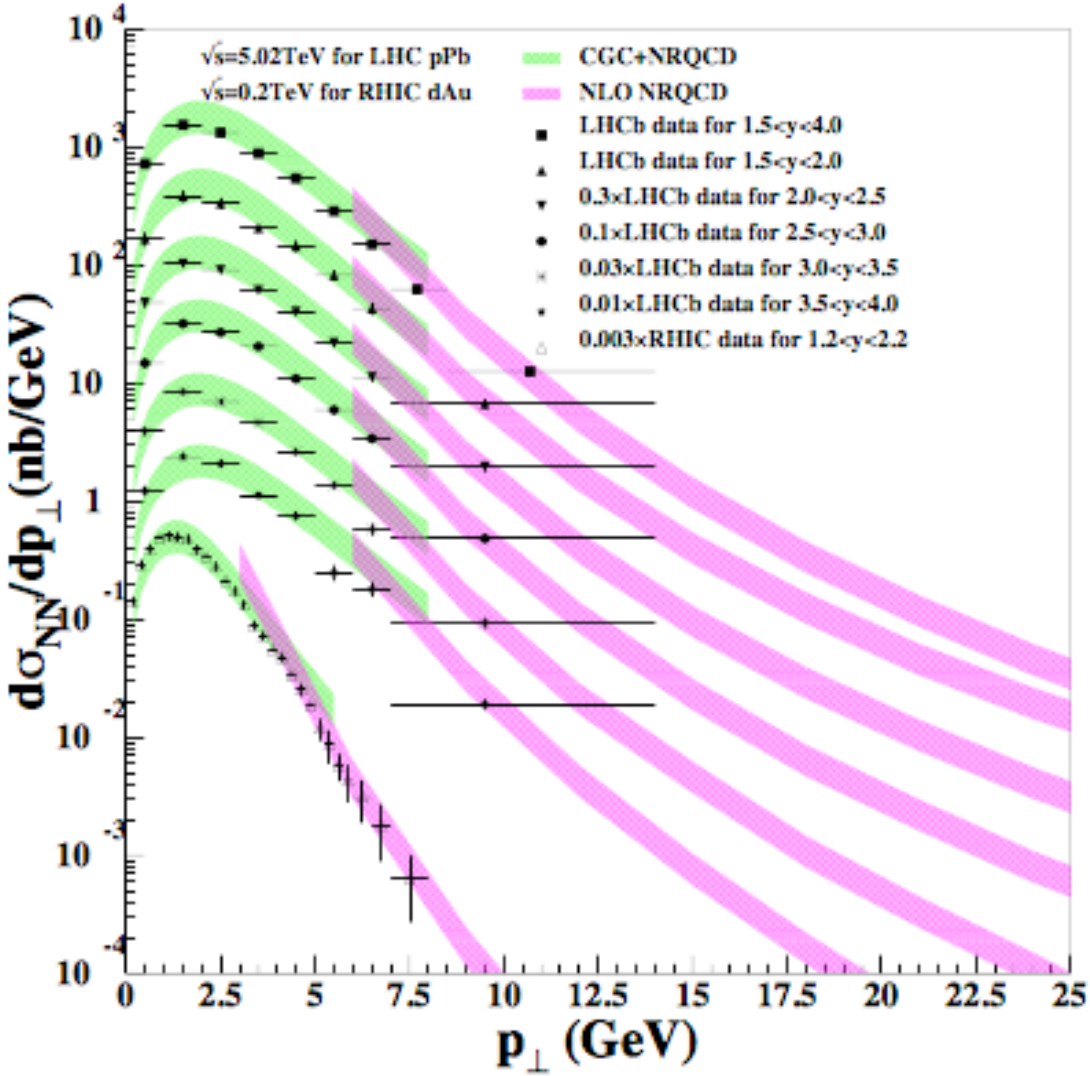
$$+ E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P, m_Q \neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

What can we say for the region where $p_T < m_Q$?

Forward quarkonium production in p(d)+A

Ma et al. Phys.Rev. D92 (2015) 071901

- CGC for low p_T region:



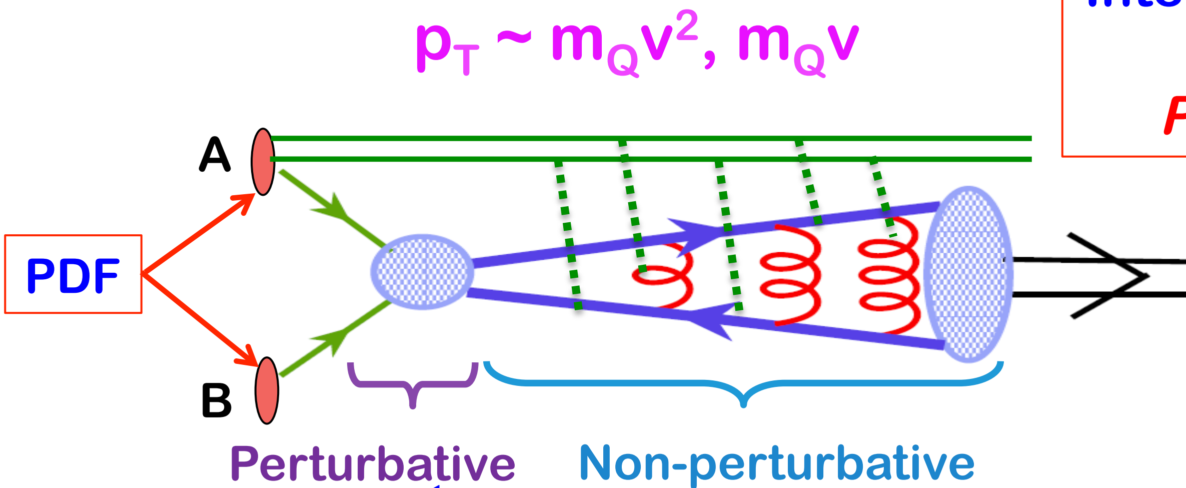
✧ Two free fitted parameters:
  transverse overlap area,
  saturation scale at initial rapidities
seem reasonable

✧ Matching to NLO NRQCD calculation,
  modulo small shadowing effect,
seem to be smooth

✧ Better agreement with data than previous CGC calculations

Production at low p_T (< M_Q)

- Spectator interaction – always there:



$$\Delta r \leq \frac{1}{2m_Q}$$

Interfere with the formation of the quarkonium
Process dependence

- The bad:

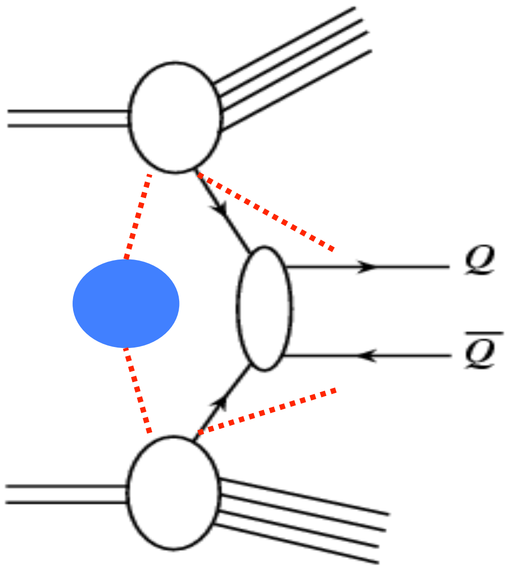
Process dependence – Break factorization – Alter p_T distribution, …

- What if the gluon shower is so strong, playing the dominant role in determining the observed p_T spectrum when $p_T < m_Q$?

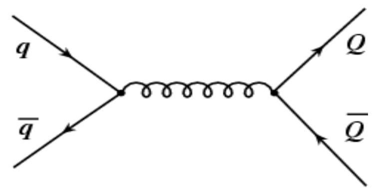
Still have predictive power, if the breaking effect is not strong enough!

Production at low p_T (< M_Q)

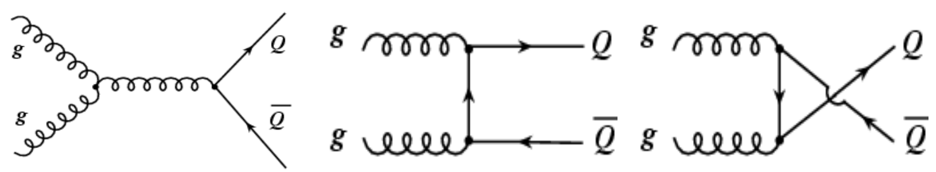
- Gluon shower – Sudakov resummation dominated?



✧ Quark-antiquark channel:



✧ Gluon-gluon channel:



- Assumption:

Leading double logarithms from the gluon shower are from initial-state active partons

⟹ Mimic the Drell-Yan type radiation pattern,
Resum the leading soft radiation into Sudakov form factor

Upsilon production at hadron colliders

❑ CSS formalism (the b-space approach to low P_T region):

Use Drell-Yan as an example:

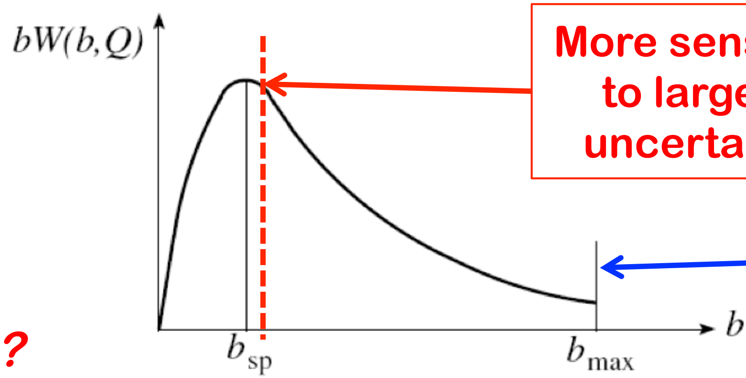
$$\frac{d\sigma_{AB}^{DY}}{dQ^2 dq_T^2}(Q, q_T, x_A, x_B) = \hat{H}_{f\bar{f}}(Q) \otimes \Phi_{f/A}(x_A, k_{a\perp}) \otimes \Phi_{\bar{f}/B}(x_B, k_{b\perp}) \otimes \mathcal{S}(k_{s\perp}) + Y(Q, q_T)$$

$$= \frac{1}{2\pi}\int_0^\infty db\, J_0(bq_T)\, b\widetilde{W}_{AB}(b, Q; x_A, x_B) + \left[\frac{d\sigma_{AB}^{(\mathrm{Pert})}}{dQ^2 dq_T^2} - \frac{d\sigma_{AB}^{(\mathrm{Asym})}}{dQ^2 dq_T^2}\right]$$

The b-space distribution:

$$\widetilde{W}_{AB}^{\mathrm{Pert}}(b, Q; x_A, x_B) = \hat{H}_{f\bar{f}}(Q)\left[\mathcal{C}_{f/a} \otimes \Phi_{a/A}(x_A, 1/b)\right] \otimes \left[\mathcal{C}_{\bar{f}/b} \otimes \Phi_{b/B}(x_B, 1/b)\right] e^{-\mathcal{S}(b,Q)}$$

Very sensitive to the role of non-perturbative Contribution!



The role of large-b region?

Good predictive power (not sensitive to the large-b region):

if the area under the b-space distribution is dominated by small-b region!

Upsilon production at hadron colliders

- **Expect good predictive power:**

  Peak of p_T-distribution is around 4 GeV
  >> intrinsic p_T
  >> the Q_s at this energies

  Shower is the dominant source to the observed large p_T

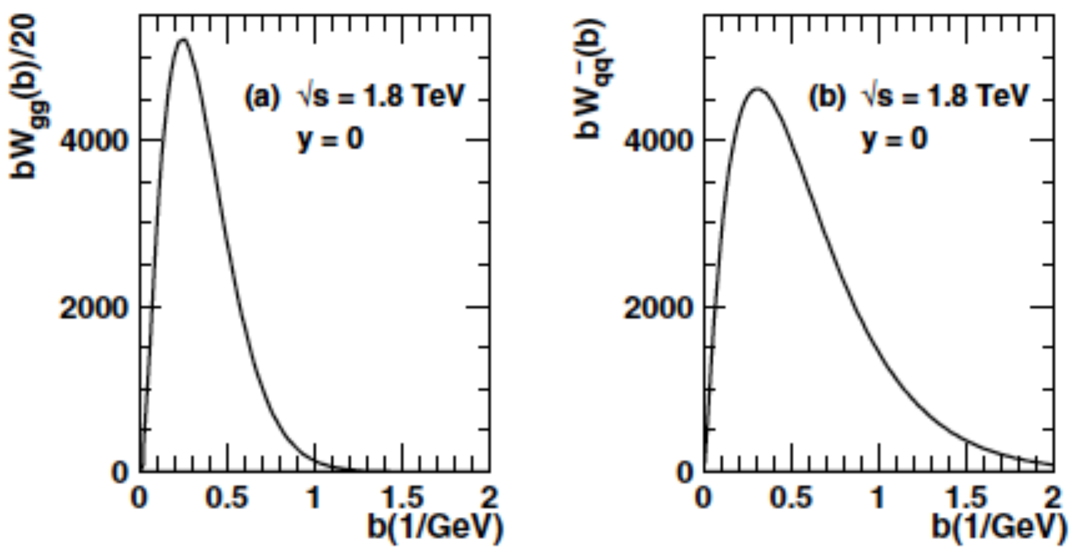
- **Matching procedure to large-b region:**

$$W_{ij}(b,Q,x_A,x_B)=\begin{cases} W_{ij}^{\rm pert}(b,Q,x_A,x_B) & b\le b_{\max} \\ W_{ij}^{\rm pert}(b_{\max},Q,x_A,x_B)F_{ij}^{NP}(b,Q;b_{\max}) & b>b_{\max}\end{cases}$$

$$F_{ij}^{NP}=\exp\left\{-\ln\left(\frac{Q^2b_{\max}^2}{c^2}\right)\{g_1[(b^2)^\alpha-(b_{\max}^2)^\alpha]+g_2(b^2-b_{\max}^2)\}-\bar{g}_2(b^2-b_{\max}^2)\right\}$$

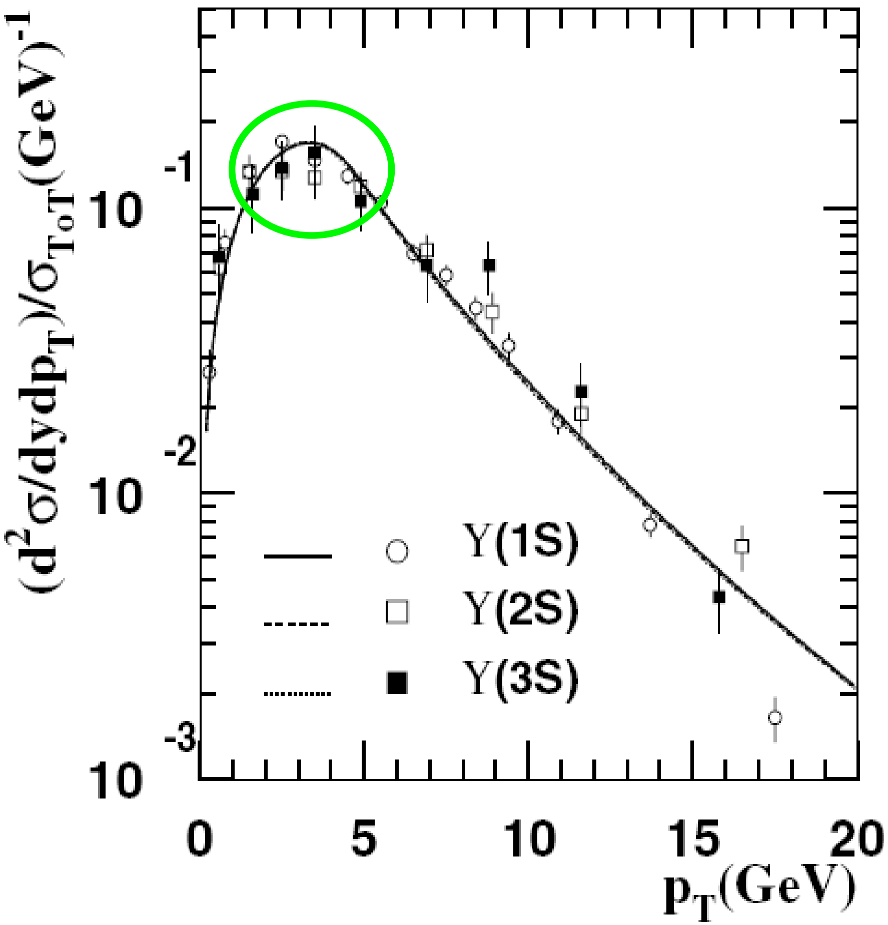
- **b-space distribution for Upsion production at Tevatron energy:**

  All parameters fixed by the derivatives to be continuous at $b = b_{max}$.

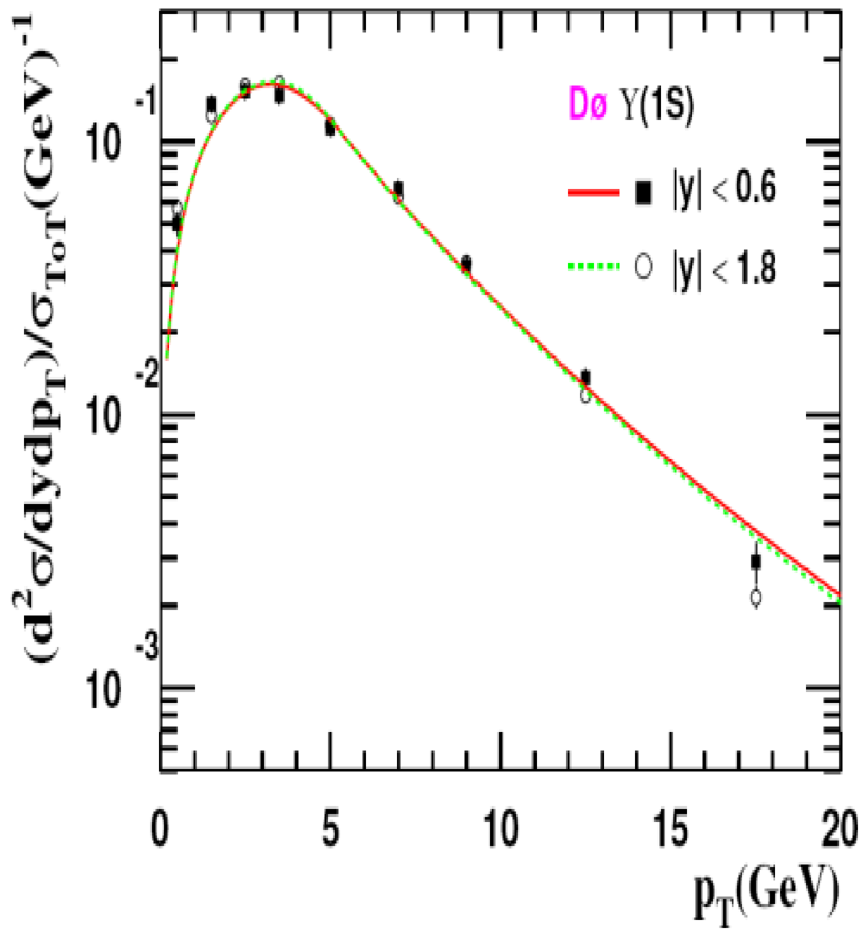


Upsilon production at hadron colliders

CDF Run-I



D0 Run-II



- **Strong gluon shower:**

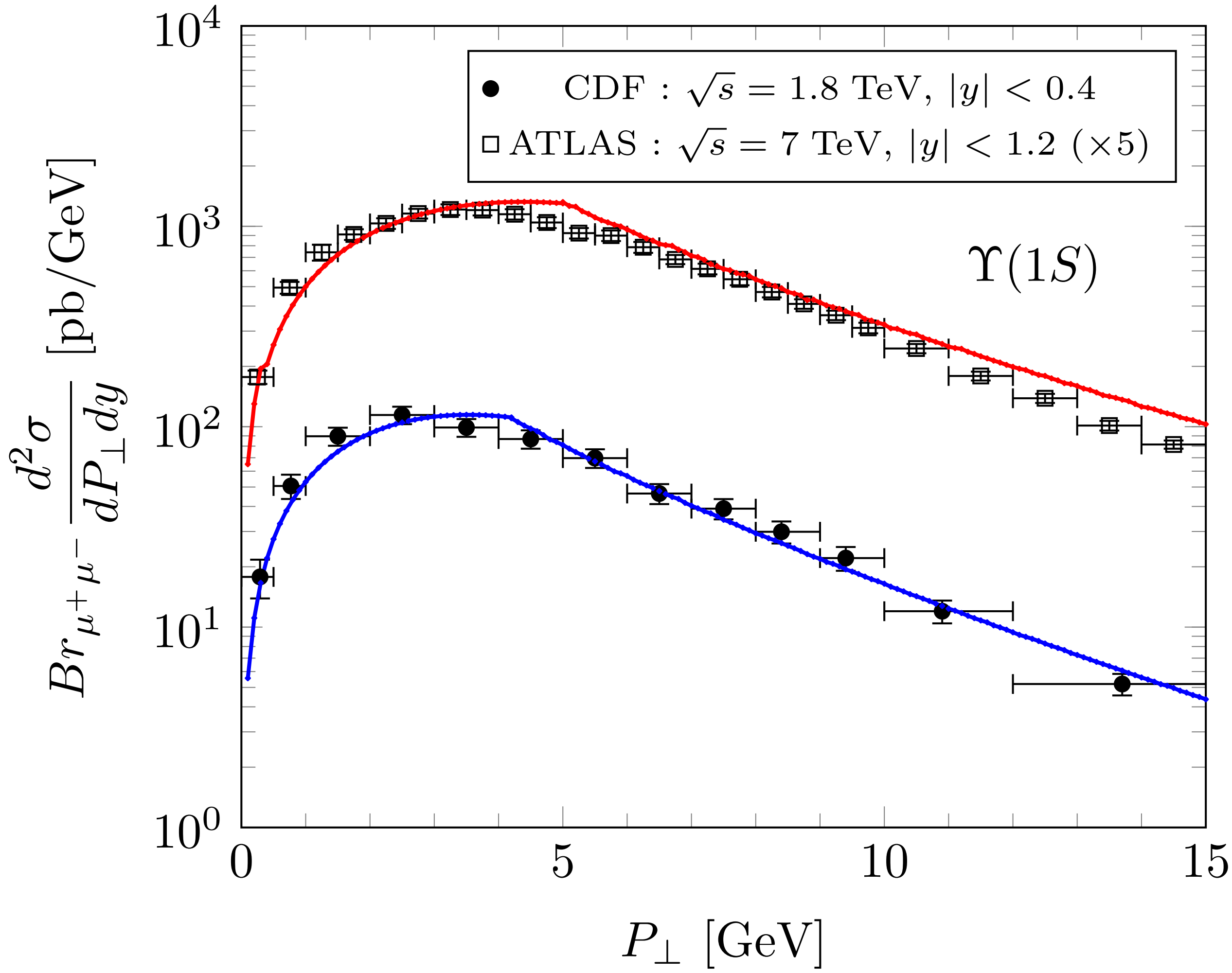
Berger, Qiu, Wang, 2005

Sufficiently large Q (Upsilon mass) + large shower phase space!

Predictive power – Upsilon

Qiu, Watanabe, 2017

❑ Upsilon at the LHC:



No adjustment on any parameter from Tevatron to the LHC!

Backup slides