Multiple Scattering Analysis

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Motivation

- Cooling performance in the channel given by

$$\frac{d\epsilon_n}{dz} = -\frac{\epsilon}{p_\mu \beta}\langle\frac{dE_\mu}{dz}\rangle + \frac{\beta_\perp p_\mu}{2m}\frac{d\theta_0^2}{dz}$$

- RMS scattering width recommended by the Particle Data Group is

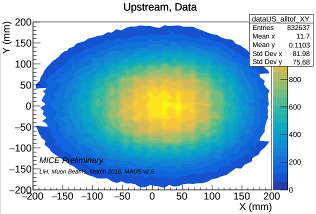
$$\theta_0 = \frac{13.6 \text{ MeV/c}}{p_\mu \beta}\sqrt{\frac{x}{X_0}}\left(1 + 0.0038 \ln\frac{x}{X_0}\right)$$

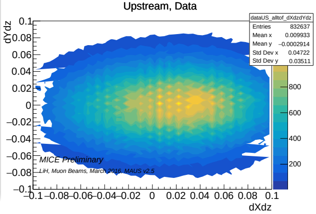
- Theory developed by Rossi and Greisen, Moliere, and Bethe
- GEANT4 uses a Legendre polynomial expansion implemented by Urban.
  - Theories do not agree with measurements of muon scattering in low Z materials (see MuScat `http://arxiv.org/pdf/hep-ex/0512005.pdf`)

Data Collection

Collected Data

State	TOF1	TOF2
Xe 240 MeV/c, Pion	883118	75879
He 240 MeV/c, Pion	185983	16155
Empty 172 MeV/c, Muon	624577	94722
Empty 200 MeV/c, Muon	384909	56314
Empty 240 MeV/c, Muon	314739	62546
LiH 172 MeV/c, Muon	1282488	174405
LiH 200 MeV/c, Muon	1223560	177460
LiH 240 MeV/c, Muon	1239827	232982





- Data recently re-analyzed with MAUS v2.5.0
- Collection of 100000 good muons targeted

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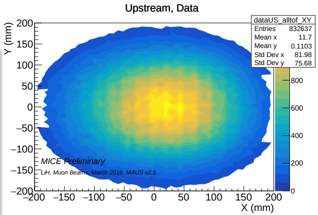

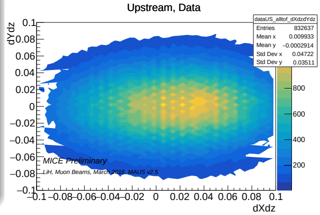

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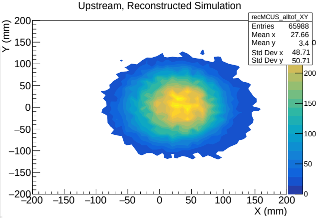
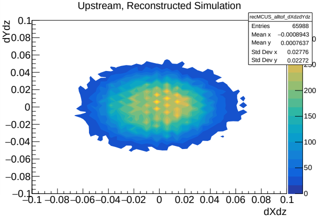
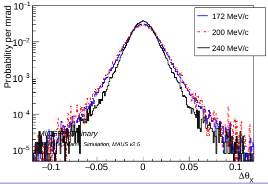
Simulation

Generated Simulations

State	TOF1	TOF2
Xe 240 MeV/c, Pion	23436	4027
He 240 MeV/c, Pion	23142	3978
Empty 172 MeV/c, Muon	771720	127245
Empty 200 MeV/c, Muon	370079	51822
Empty 240 MeV/c, Muon	1204155	261244
LiH 172 MeV/c, Muon	718185	108777
LiH 200 MeV/c, Muon	364587	45638
LiH 240 MeV/c, Muon	1266073	236582

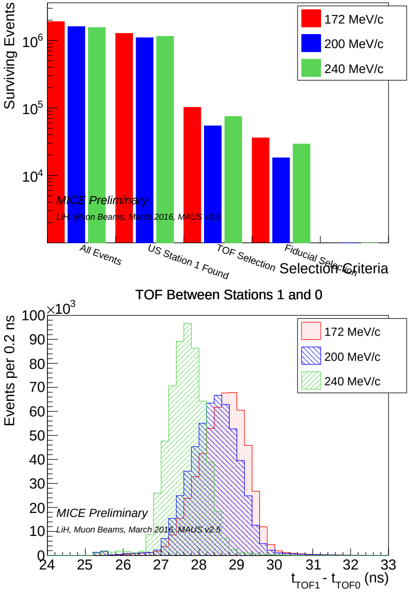






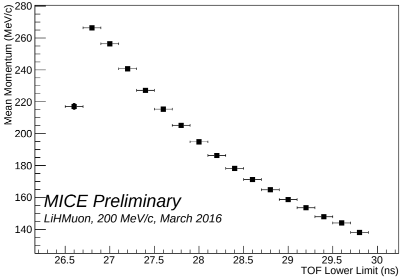
- Grid MC with MAUS v2.5.0
- Muon beams stats comparable to data
- Primarily used to predict scattering in absorber.

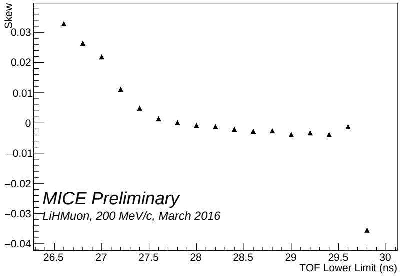
Particle Selection

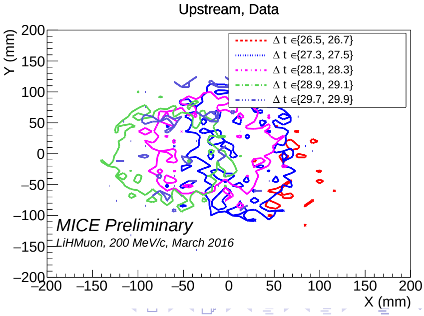


- TOF (PID) selection is vital for beam momentum and position selection
- Assumed a selection of
  - $\Delta t_{01} \in \{27.9, 28.1\}$ for 172 MeV/c beams
  - $\Delta t_{01} \in \{27.7, 27.9\}$ for 200 MeV/c beams
  - $\Delta t_{01} \in \{27.2, 27.4\}$ for 240 MeV/c beams
- Require a US track. If a DS track not extant, statistics are set to overflow values.
- Require projection of US tracks to appear within central 150 mm radius of DS plane 1.

TOF Selection Examined





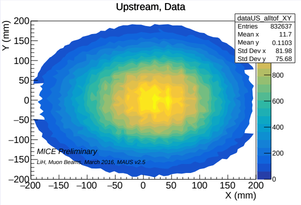


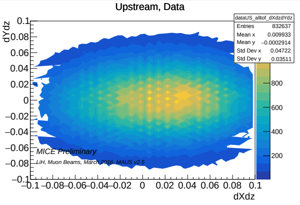
- Selection of TOF limits made by matching measured mean momentum to beam momentum
- Mean momentum of the data must be matched to the model for testing purposes

Effect of Selections on Beam Distributions

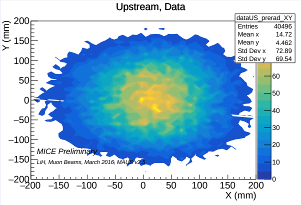
- 200 MeV/c muon beam at TkU Station 1 shown.

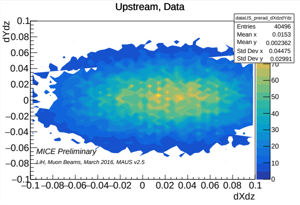
All TkU Events



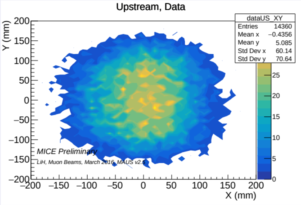


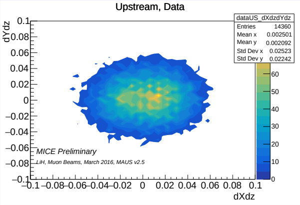
After TOF Selection





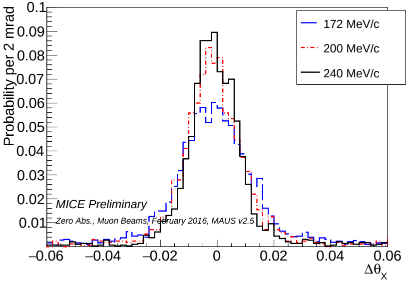
After Fid. Selection

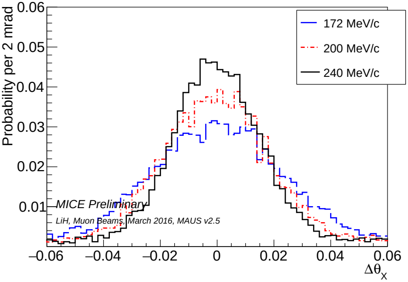




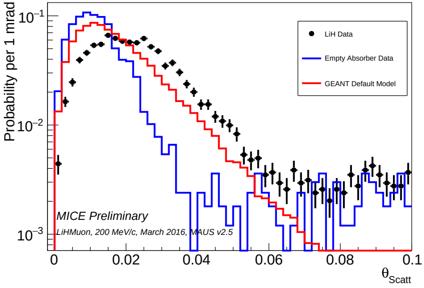
Angle Distributions

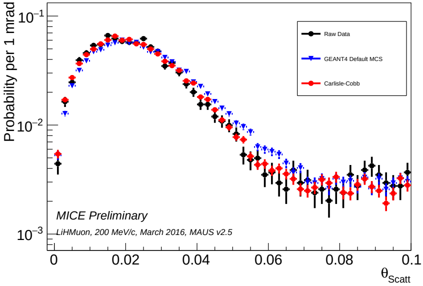
- Particles follow straight paths through spectrometers.
- Scatter off of materials along its path.
  - Need to identify scattering from absorber material only.
- Use TOFs to measure momentum.
  - Consider scattering as a function of momentum.
- Define scattering angles for trajectories
  - $\Delta\theta_x = \text{atan}(\frac{dy}{dz})_{US} - \text{atan}(\frac{dy}{dz})_{DS}$
  - $\Delta\theta_y = \text{atan}(\frac{dx}{dz})_{US} - \text{atan}(\frac{dx}{dz})_{DS}$
  - $\theta_{scatt} = \text{acos}\left(\frac{\mathbf{p}_u \cdot \mathbf{p}_d}{|\mathbf{p}_u| p_d|}\right)$



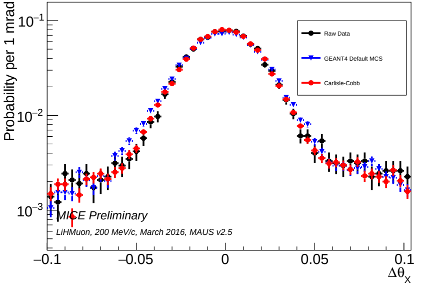


Convolution

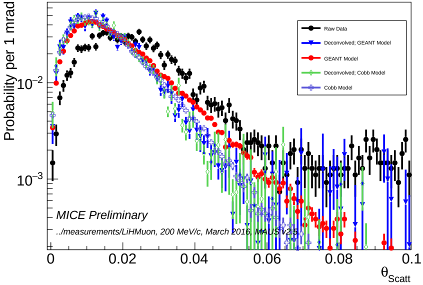


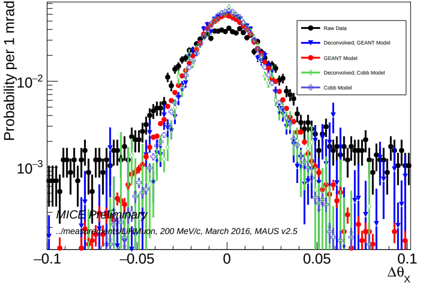


- Add model of scattering in LiH to Empty absorber simulation on the event by event level
- The model is sampled 10 times to increase statistics.
- χ^2 calculated from the data and the model convolution.
  - Ideal for model testing

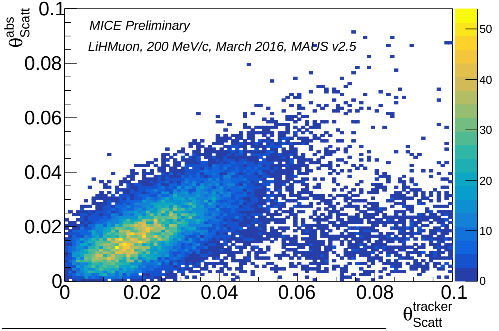


Deconvolution



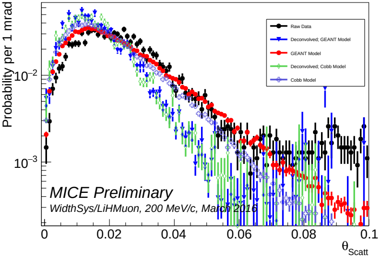


- Want an explicit measurement of the scattering in absorber
- Deconvolve scattering in absorber from measured scattering.
- Used an iterative Bayes deconvolution method[a]



[a] NIM A 362 (1995) 487-498

Independent Width Manipulation



- Model distribution widths exaggerated by factor of 1 $\pm$0.25 in convolution
- Evaluate sensitivity of measurement to variations of material width

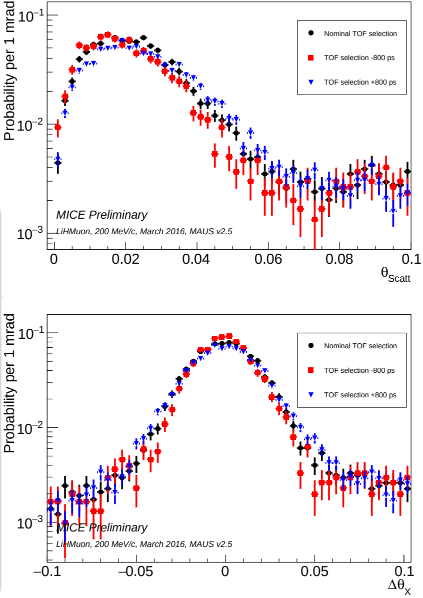
- Errors assume a 24% systematic in absorber thickness (MN 448).
- Uncertainty in LiH absorber thickness expected to be much smaller.
- Density Uncertainty in Xe absorber may be larger.

p	Angle	Raw	
		$\sigma_{high} - \sigma_{low}$	Sys. (mrad)
172	$\Delta\theta_X$	1.28	0.31
	$\Delta\theta_Y$	1.31	0.31
200	$\Delta\theta_X$	1.28	0.31
	$\Delta\theta_Y$	2.26	0.54
240	$\Delta\theta_X$	2.04	0.49
	$\Delta\theta_Y$	1.91	0.46
p		$\Delta\langle\theta_{Scatt}\rangle$	Sys (mrad)
172		3.06	0.74
200		3.66	0.88
240		5.2	1.25

TOF Systematics

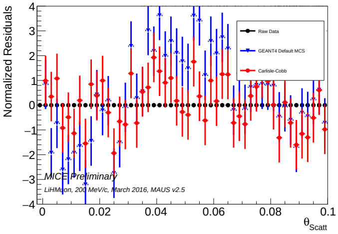
- Defined by making TOF selections offset by ±400 ps.
- Expected to dominate over material systematics.

p	Angle	Raw $\sigma_{high} - \sigma_{low}$	Raw Sys. (mrad)
172	$\Delta\theta_X$	4.0	0.3
	$\Delta\theta_Y$	3.13	0.234
200	$\Delta\theta_X$	3.77	0.283
	$\Delta\theta_Y$	3.96	0.297
240	$\Delta\theta_X$	3.58	0.269
	$\Delta\theta_Y$	3.9	0.292
p		$\Delta\langle\theta_{Scatt}\rangle$	Sys (mrad)
172		3.06	0.23
200		3.66	0.275
240		5.2	0.39



Model Testing

- Widths taken from truncated Gaussian fit ($|\theta| < 0.04$, ndof = 40)
- Define a χ^2 between the absorber full data and the convolution with a model
- Statistical uncertainties for convolution assume Poisson errors (a simplification).



p	Angle	σ_{Data} (mrad)	σ_{G4} (mrad)	χ^2	σ_{CC} (mrad)	χ^2
172	$\Delta\theta_X$	23.9 ± 0.4	21.3 ± 0.1	106.1	22.6 ± 0.1	50.5
	$\Delta\theta_Y$	24.0 ± 0.4	21.2 ± 0.1	154.5	22.2 ± 0.1	77.4
200	$\Delta\theta_X$	18.7 ± 0.3	20.1 ± 0.2	69.0	18.8 ± 0.1	30.3
	$\Delta\theta_Y$	18.5 ± 0.3	20.1 ± 0.2	118.3	18.8 ± 0.1	78.1
240	$\Delta\theta_X$	15.5 ± 0.1	16.5 ± 0.1	121.6	15.9 ± 0.1	85.3
	$\Delta\theta_Y$	15.7 ± 0.1	16.4 ± 0.1	201.2	15.7 ± 0.1	102.4
p		$\langle\theta_{Scatt}\rangle^{meas}_{G4}$ (mrad)	$\langle\theta_{Scatt}\rangle^{true}_{G4}$	χ^2	$\langle\theta_{Scatt}\rangle^{true}_{CC}$	χ^2
172		$12.4 \pm 0.2 \pm 0.3$	11.7 ± 0.1	103.6	12.1 ± 0.1	89.1
200		$10.6 \pm 0.2 \pm 0.5$	11.3 ± 0.1	158.4	11.0 ± 0.1	78.1
240		$10.3 \pm 0.1 \pm 0.3$	10.4 ± 0.1	131.2	10.1 ± 0.1	48.3

Measurements

- Conducted with the Bayesian algorithm.
- Results shown use the GEANT and the Cobb-Carlisle derived distributions to seed the deconvolution
  - independent of the absorber scattering model
- χ^2 shows the difference between the deconvolved distribution and absorber scattering model

p	Angle	σ_{G4}^{meas} (mrad)	σ_{G4}^{true} (mrad)	χ^2	σ_{CC}^{meas} (mrad)	σ_{CC}^{true} (mrad)	χ^2
172	$\Delta\theta_X$	15.0 ± 0.1	13.5 ± 0.03	218.48	15.5 ± 0.1	14.83 ± 0.03	74.6
	$\Delta\theta_Y$	15.3 ± 0.1	13.5 ± 0.03	231.89	15.7 ± 0.1	14.77 ± 0.03	103.8
200	$\Delta\theta_X$	13.1 ± 0.2	14.4 ± 0.05	188.35	12.7 ± 0.2	12.97 ± 0.04	57.1
	$\Delta\theta_Y$	12.9 ± 0.2	14.3 ± 0.05	193.49	12.8 ± 0.2	12.84 ± 0.04	105.7
240	$\Delta\theta_X$	10.9 ± 0.1	11.5 ± 0.03	198.47	10.6 ± 0.1	10.84 ± 0.03	121.4
	$\Delta\theta_Y$	10.9 ± 0.1	11.5 ± 0.03	328.96	10.9 ± 0.1	10.83 ± 0.03	128.8
p		$\langle\theta_{Scatt}\rangle_{G4}^{meas}$ (mrad)	$\langle\theta_{Scatt}\rangle_{G4}^{true}$	χ^2	$\langle\theta_{Scatt}\rangle_{CC}^{meas}$ (mrad)	$\langle\theta_{Scatt}\rangle_{CC}^{true}$	χ^2
172		20.3 ± 0.2	17.5 ± 0.03	351.58	20.4 ± 0.2	18.86 ± 0.03	184.6
200		17.2 ± 0.2	18.8 ± 0.05	203.4	16.3 ± 0.2	16.32 ± 0.04	77.3
240		13.8 ± 0.1	14.8 ± 0.03	325.08	13.5 ± 0.1	13.64 ± 0.03	121.4

Discussion

- Analysis of multiple scattering data is proceeding well.
- Have analysis strategies in place for model testing and absorber scattering measurements.
  - An iterative Bayesian strategy has been used here.
  - A purely data driven strategy using Gold's algorithm under consideration.
- Discussed results have focused on LiH data
- Systematics studies are underway but require further review.
- Results point to a preference for Cobb-Carlisle implementation of scattering.