Multiple Scattering Analysis

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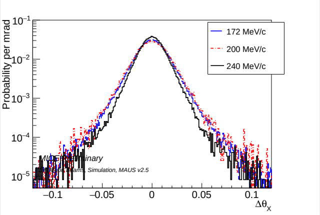
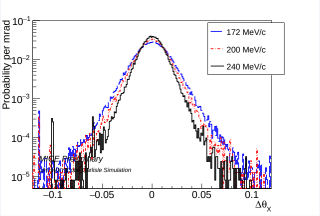
Motivation

- Emittance (especially at equilibrium) depends on multiple Coulomb scattering (MCS).
- We depend on low Z absorbers but MCS in low Z materials not well implemented in simulation
- We want to
  - measure MCS in MICE absorbers (LiH in this case)
  - compare data to simulation to validate model (or propose something better)
- Will compare data to GEANT default scattering and to implementation by Carlisle and Cobb (CC)

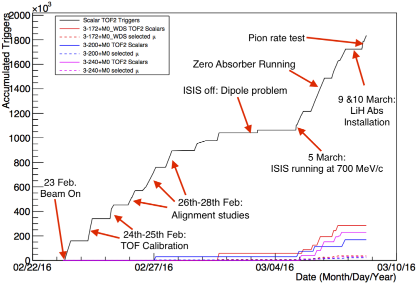
GEANT

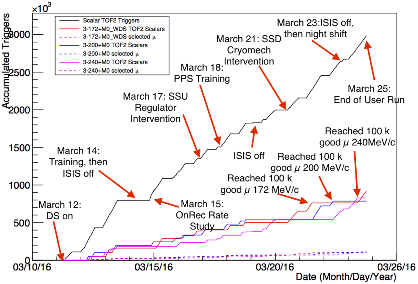


Carlisle-Cobb



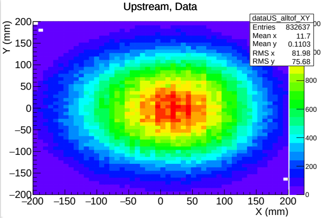
Data Collection



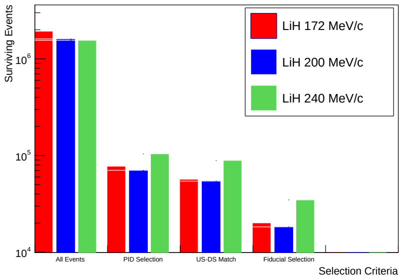


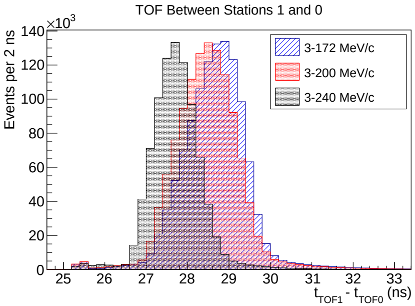
Collected Data

State	TOF1	TOF2
Empty 172 MeV/c, Muon	624577	94722
Empty 200 MeV/c, Muon	384909	56314
Empty 240 MeV/c, Muon	314739	62546
LiH 172 MeV/c, Muon	1282488	174405
LiH 200 MeV/c, Muon	1223560	177460
LiH 240 MeV/c, Muon	1239827	232982



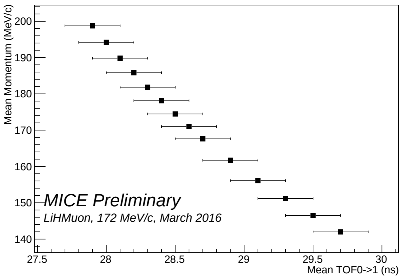
Track Selection

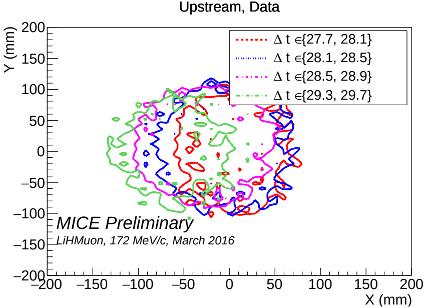


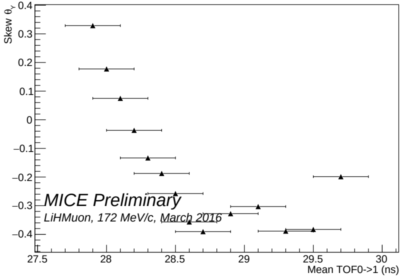


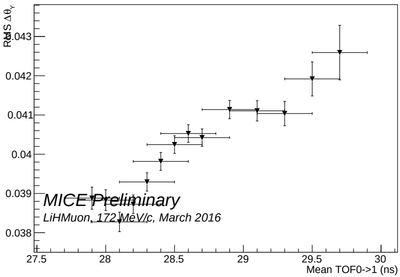
- TOF (PID) selection is vital for beam momentum and position selection
- Assumed a selection of
  - $\Delta t_{01} \in \{27.9, 28.1\}$ for 172 MeV/c beams
  - $\Delta t_{01} \in \{27.7, 27.9\}$ for 200 MeV/c beams
  - $\Delta t_{01} \in \{27.2, 27.4\}$ for 240 MeV/c beams
  - Considering 4 ns selection.
- Require a US track. If a DS track not extant, statistics are set to overflow values.
- Require projection of US tracks to appear within central 150 mm radius of DS plane 1.

Scans over TOF Selections: 172 MeV/c Data Set

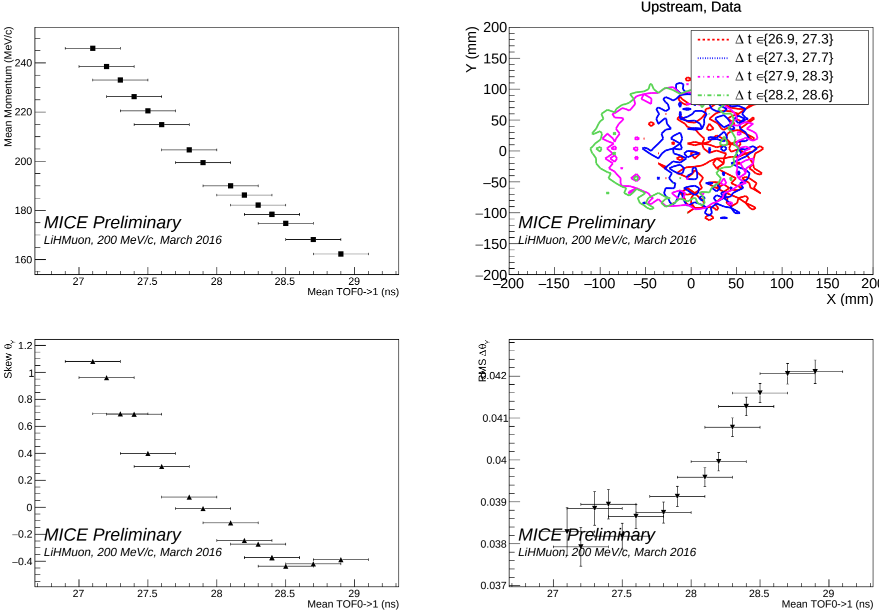




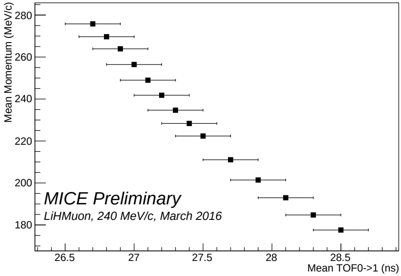


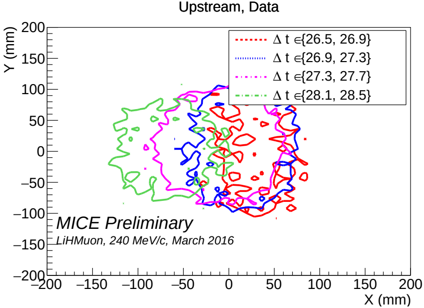


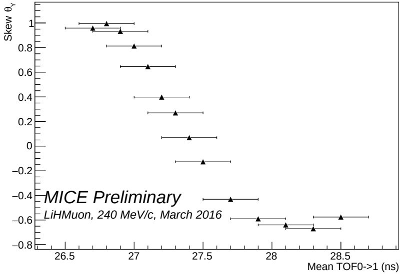
Scans over TOF Selections: 200 MeV/c Data Set

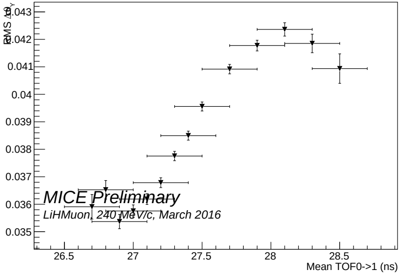


Scans over TOF Selections: 240 MeV/c Data Set





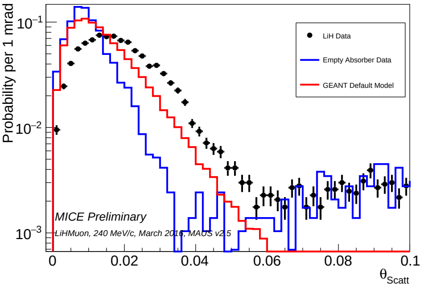


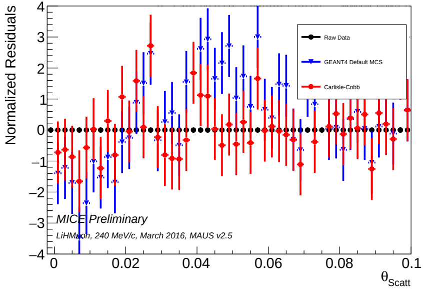


Comments Regarding the TOF Scans

- Require that the mean momentum of the data matches the mean momentum of the model
  - Not a problem for tests against GEANT but CC model is fixed.
  - Motivates narrow TOF selection.
- Would like the TOF selection to also place the beam near the centre of the tracker with small mean off-axis gradient
  - This happens for 200 MeV/c and 240 MeV/c beams but not for 172 MeV/c beam.
- Scans show dependence of the RMS on the TOF selection
  - Suggests the expected $1/\sqrt{p}$ behaviour
  - Needs to be evaluated properly

Analysis — Convolution





- Convolve empty absorber data with different MCS implementations
- Compare with data.
- Method suggested by John Cobb.

χ^2 of θ_{Scatt} distributions

p (MeV/c)	GEANT	Carlisle-Cobb
172	121.6	85.3
200	201.2	102.4
240	131.2	48.3

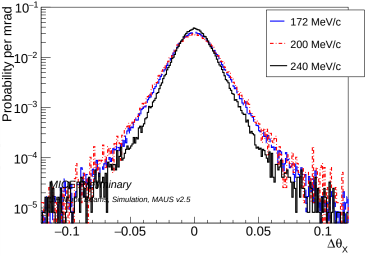
$$\chi^2 = \sum_{n=1}^{100} \frac{(n(\theta_i^{data}) - n(\theta_i^{model})^2}{n(\theta_i^{data}) + n(\theta_i^{model})}$$

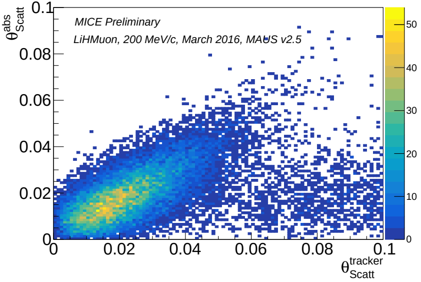
Analysis — Deconvolution

$$n_q(\theta_i^{abs}) = \sum_{j=1}^{n_E} n(\theta_j^{rec}) P_q(\theta_i^{abs}|\theta_j^{rec})$$

$$P_q(\theta_i^{abs}|\theta_j^{rec}) = \frac{P_{MC}(\theta_j^{rec}|\theta_i^{abs}) P_q(\theta_i^{abs})}{\sum_{l=1}^{n_c} P_{MC}(\theta_j^{rec}|\theta_l^{abs}) P_q(\theta_l^{abs})}$$

- iterate q: $P_q(\theta_i^{abs}) = \frac{n_{q-1}(\theta_i^{abs})}{\sum_{i=1}^{n_E} n_{q-1}(\theta_i^{abs})}$





- Developed using LiH data and MC.
- Confirming insensitivity to particulars of MC.
- Compare deconvolved distribution to different MCS implementations.

Systematics

Absorber thickness or density – $\rho_{LiH} = 0.65 \pm 0.16$ g/cm^3

- 24% uncertainty
- Scattering width goes as $\sqrt{x} = \sqrt{\Delta z \rho}$
- Modelled by exaggerating convolved distribution width by $\pm$25% and divide by 2.
- Applied to the analysis of all three data sets.

TOF selection — TOF resolution given as 60 ps

- Offset the TOF selection by $\pm$ 800 ps to exaggerate uncertainty.
- Scans show that the RMS is mostly linear wrt TOF selection.

Other Systematics

Alignment — uncertainty of ≈ 0.08 mrad

- Expect to produce a bias in the scattering distribution
- Should not affect the distribution width.

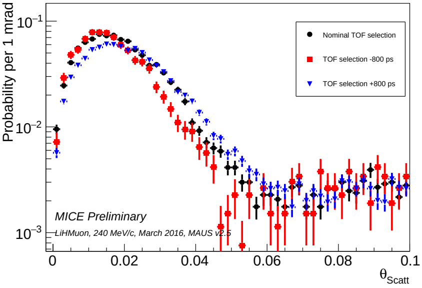
Alignment deviations between reconstruction and collection

	Δx	Δy	$\Delta\theta_x$	$\Delta\theta_y$
TKU	0.51 ± 0.25	-1.20 ± 0.27	0.085 ± 0.088	-0.185 ± 0.084
TKD	-0.03 ± 0.25	-0.02 ± 0.25	0.055 ± 0.081	-0.333 ± 0.081

Particle identification

- 0.5% of all muons decay according to simulation.
- No decay electrons selected (but statistics are extremely low).
- No pion background expected.

Systematics — TOF selection



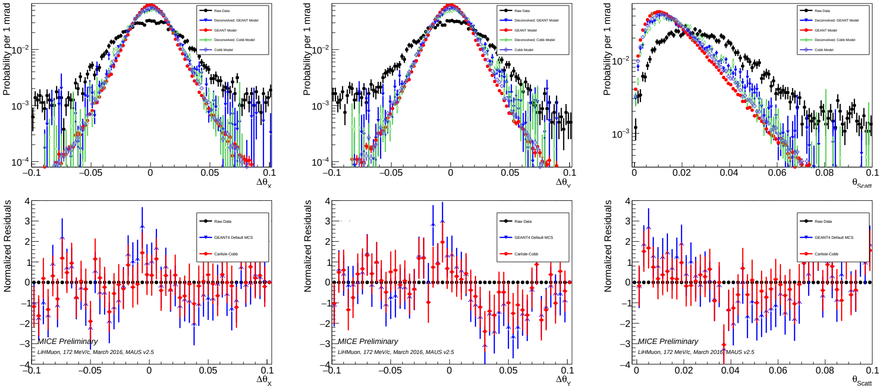
- Offset the TOF selection by $\pm$ 800 ps to exaggerate uncertainty.
- Scans show that the RMS is mostly linear wrt TOF selection.
- Scale factor is 0.06 ns/0.8 ns.

Systematic diff. and uncertainties

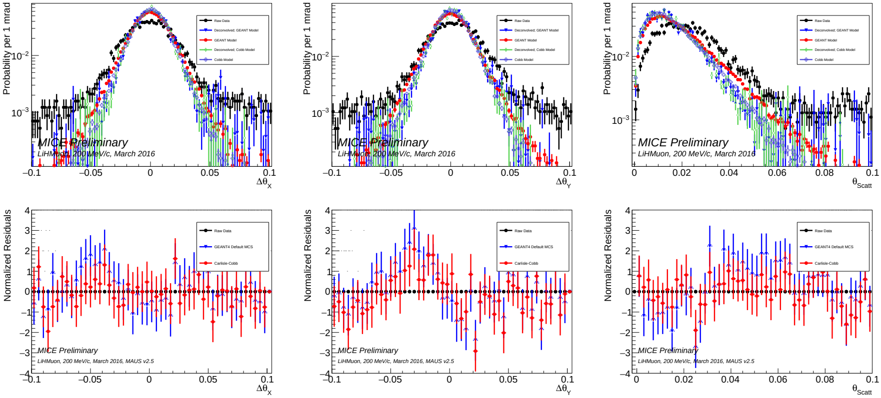
p (MeV/c)	Angle	Raw $\sigma_{high} - \sigma_{low}$	Sys. (mrad)
172	$\Delta\theta_X$	4.0	0.3
	$\Delta\theta_Y$	3.13	0.234
200	$\Delta\theta_X$	3.77	0.283
	$\Delta\theta_Y$	3.96	0.297
240	$\Delta\theta_X$	3.58	0.269
	$\Delta\theta_Y$	3.9	0.292
p		$\Delta\langle\theta_{Scatt}\rangle$	Sys (mrad)
172		3.06	0.23
200		3.66	0.275
240		5.2	0.39

- Sensitivities of widths evaluated for each variable for each set
- Uncertainties are $\approx$ 0.2 mrad (with exceptions).
- A bin by bin systematic added to χ^2 used for model testing.

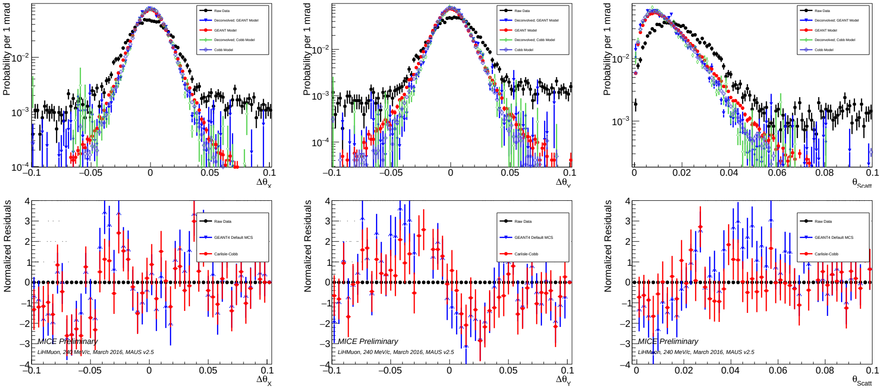
172 MeV/c Results



200 MeV/c Results



240 MeV/c Results



Summary of Results: No Deconvolution

p	Angle	σ_{Data} (mrad)	σ_{G4} (mrad)	χ^2	σ_{CC} (mrad)	χ^2
172	$\Delta\theta_X$	$23.9 \pm 0.4 \pm 0.6$	21.3 ± 0.1	61.4	22.6 ± 0.1	30.1
	$\Delta\theta_Y$	$24.0 \pm 0.4 \pm 0.6$	21.2 ± 0.1	90.7	22.2 ± 0.1	48.2
200	$\Delta\theta_X$	$18.7 \pm 0.3 \pm 0.6$	20.1 ± 0.2	38.7	18.8 ± 0.1	21.9
	$\Delta\theta_Y$	$18.5 \pm 0.3 \pm 0.8$	20.1 ± 0.2	65.2	18.8 ± 0.1	52.1
240	$\Delta\theta_X$	$15.5 \pm 0.1 \pm 0.7$	16.5 ± 0.1	68.4	15.9 ± 0.1	57.8
	$\Delta\theta_Y$	$15.7 \pm 0.1 \pm 0.7$	16.4 ± 0.1	102.7	15.7 ± 0.1	51.0
p		$\langle\theta_{Scatt}\rangle^{meas}_{G4}$ (mrad)	$\langle\theta_{Scatt}\rangle^{true}_{G4}$	χ^2	$\langle\theta_{Scatt}\rangle^{true}_{CC}$	χ^2
172		$12.4 \pm 0.2 \pm 0.3$	11.7 ± 0.1	82.8	12.1 ± 0.1	45.4
200		$10.6 \pm 0.2 \pm 0.5$	11.3 ± 0.1	65.4	11.0 ± 0.1	27.7
240		$10.3 \pm 0.1 \pm 0.3$	10.4 ± 0.1	40.6	10.1 ± 0.1	26.5

- Fits and χ^2 evaluated for bins at angles less than 40 mrad (40 bins).
- Results indicate a preference for Carlisle Cobb model.
- Systematic uncertainties included in χ^2 reported.

Summary of Results: With Deconvolution

p	Angle	σ_{G4}^{meas} (mrad)	σ_{G4}^{true} (mrad)	χ^2	σ_{CC}^{meas} (mrad)	σ_{CC}^{true} (mrad)	χ^2
172	$\Delta\theta_X$	15.0 ± 0.1 ± 0.4	13.52 ± 0.03	206.2	15.5 ± 0.1 ± 0.3	14.83 ± 0.03	134.5
	$\Delta\theta_Y$	15.3 ± 0.1 ± 0.3	13.49 ± 0.03	242.0	15.7 ± 0.1 ± 0.3	14.77 ± 0.03	169.3
200	$\Delta\theta_X$	13.1 ± 0.2 ± 0.3	14.39 ± 0.05	286.3	12.7 ± 0.2 ± 0.2	12.97 ± 0.04	96.9
	$\Delta\theta_Y$	12.9 ± 0.2 ± 0.5	14.32 ± 0.05	335.2	12.8 ± 0.2 ± 0.5	12.84 ± 0.04	130.8
240	$\Delta\theta_X$	10.9 ± 0.1 ± 0.4	11.51 ± 0.03	200.8	10.6 ± 0.1 ± 0.4	10.84 ± 0.03	158.8
	$\Delta\theta_Y$	10.9 ± 0.1 ± 0.5	11.49 ± 0.03	391.0	10.9 ± 0.1 ± 0.5	10.83 ± 0.03	182.4
p		$\langle\theta_{Scatt}\rangle_{G4}^{meas}$ (mrad)	$\langle\theta_{Scatt}\rangle_{G4}^{true}$	χ^2	$\langle\theta_{Scatt}\rangle_{CC}^{meas}$ (mrad)	$\langle\theta_{Scatt}\rangle_{CC}^{true}$	χ^2
172		20.3 ± 0.2 ± 0.5	17.49 ± 0.03	346.0	20.4 ± 0.2 ± 0.5	18.86 ± 0.03	188.7
200		17.2 ± 0.2 ± 0.9	18.81 ± 0.05	461.3	16.3 ± 0.2 ± 0.7	16.32 ± 0.04	149.8
240		13.8 ± 0.1 ± 0.8	14.83 ± 0.03	372.8	13.5 ± 0.1 ± 0.8	13.64 ± 0.03	173.2

- Fits and χ^2 evaluated for bins at angles less than 40 mrad (40 bins).
- Results indicate a preference for Carlisle Cobb model.
- Systematic uncertainties included in χ^2 reported.
- Width systematic is larger than it should be — should be revisited
- Now have alignment corrections in hand
  - Will be adding the corrections soon.
  - Might help correct biases observed in the distributions.