Emittance and cooling measurement

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Emittance definition

The RMS normalised emittance is expressed as

$$\epsilon_n = \frac{1}{m}\sqrt[4]{D} \tag{1}$$

with D the determinant of the covariance matrix defined by

$$D = \det \begin{bmatrix} V_{xx} & V_{xp_x} & V_{xy} & V_{xp_y} \\ V_{p_x x} & V_{p_x p_x} & V_{p_x y} & V_{p_x p_y} \\ V_{yx} & V_{yp_x} & V_{yy} & V_{yp_y} \\ V_{p_y x} & V_{p_y p_x} & V_{p_y y} & V_{p_y p_y} \end{bmatrix} = \sum_\beta V_{\alpha\beta} C_{\alpha\beta}, \, \forall\alpha \tag{2}$$

with $V_{\alpha\beta}$ the covariance of α and β defined as

$$V_{\alpha\beta} = \frac{1}{N}\sum_{i=1}^{N}(\alpha_i - \langle\alpha\rangle)(\beta_i - \langle\beta\rangle) = \langle\alpha\beta\rangle - \langle\alpha\rangle\langle\beta\rangle, \tag{3}$$

and $C_{\alpha\beta}$ the (α, β)-cofactor of the covariance matrix.

Emittance error propagation

The covariances error correlation can be expressed as a rank-4 tensor,

$$\Sigma^V = A\Sigma A^T, \tag{4}$$

with $\Sigma_{i\alpha\beta j} = \delta_{ij}\delta_{\alpha\beta}\sigma^2_{\alpha_i}$ and A the derivative tensor:

$$A_{\alpha\beta\eta k} = \frac{\partial V_{\alpha\beta}}{\partial \eta_k} = \frac{1}{N}\left[\delta_{\eta\alpha}\left(\beta_k - \langle\beta\rangle\right) + \delta_{\eta\beta}\left(\alpha_k - \langle\alpha\rangle\right)\right]. \tag{5}$$

Inputting equation 5 into equation 4 yields

$$\begin{aligned}
\Sigma_{\alpha\beta\kappa\lambda} \quad = \quad & \frac{1}{N^2}\sum_{i=1}^{N}\left[\delta_{\alpha\kappa}\sigma^2_{\alpha_i}\left(\beta_i - \langle\beta\rangle\right)\left(\lambda_i - \langle\lambda\rangle\right)\right.\\
& +\delta_{\alpha\lambda}\sigma^2_{\alpha_i}\left(\beta_i - \langle\beta\rangle\right)\left(\kappa_i - \langle\kappa\rangle\right)\\
& +\delta_{\beta\kappa}\sigma^2_{\beta_i}\left(\alpha_i - \langle\alpha\rangle\right)\left(\lambda_i - \langle\lambda\rangle\right)\\
& \left.+\delta_{\beta\lambda}\sigma^2_{\beta_i}\left(\alpha_i - \langle\alpha\rangle\right)\left(\kappa_i - \langle\kappa\rangle\right)\right]
\end{aligned} \tag{6}$$

Emittance error propagation (2)

This error tensor propagates into the determinant error through

$$\begin{aligned}\sigma_D^2 &= \sum_{\alpha\beta\kappa\lambda} \frac{\partial D}{\partial V_{\alpha\beta}} \Sigma^V_{\alpha\beta\kappa\lambda} \frac{\partial D}{\partial V_{\kappa\lambda}} \\ &= \frac{4}{N^2} \sum_{i=1}^{N} \sum_{\alpha\beta} \left[\left(C^T \hat{\sigma}^i C \right)_{\alpha\beta} \left(\alpha_i - \langle\alpha\rangle \right) \left(\beta_i - \langle\beta\rangle \right) \right] \end{aligned} \quad (7)$$

with $\hat{\sigma}^i_{\alpha\beta} = \delta_{\alpha\beta}\sigma^2_{\alpha_i}$, the diagonal matrix that contains the errors. This eventually yields a measurement error on the emittance of

$$\sigma_{\epsilon_n} = \left| \frac{\partial \epsilon_n}{\partial D} \right| \sigma_D = \frac{D^{-3/4}}{4m} \sigma_D \quad (8)$$

Other quantities of interest

→ Optical beta function in the two projections:

$$\beta_x = \frac{V_{xx}}{\det(\epsilon_x^{2D})} \qquad \beta_y = \frac{V_{yy}}{\det(\epsilon_y^{2D})}$$
$$\text{with} \qquad \epsilon_q^{2D} = \begin{bmatrix} V_{qq} & V_{qq'} \\ V_{q'q} & V_{q'q'} \end{bmatrix}, \quad q' = p_q/p_z \tag{9}$$

→ Mean total momentum:

$$|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} \tag{10}$$

→ Transmission in the cooling channel

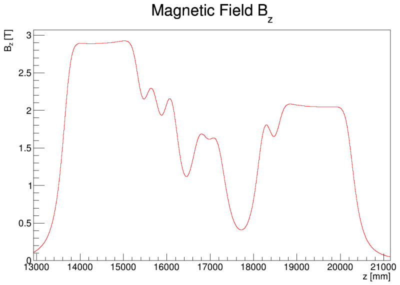
$$T_i = \frac{N_i}{N_0} \tag{11}$$

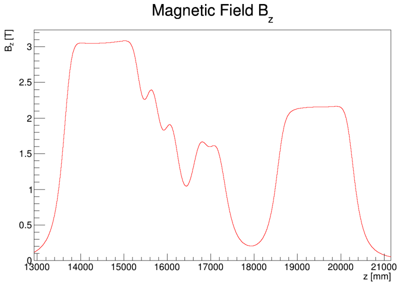
8 configurations under investigation

- Two solenoid modes 200 MeV/c magnet settings (from A. Liu):

	ECE_U [%]	$M2_U$	$M1_U$	FC	$M1_D$	$M2_D$	ECE_D [%]
w/ $M2_D$	0.72	219.8	162.7	55.9	0	205.66	0.51
w/o $M2_D$	0.76	236.8	135.2	56	0	0	0.54

- 3 mm and 6 mm input normalised emittance
- With or without absorber (65 mm of LiH in this study)





Geometries

In first approximation, a simplified geometry was used

→ Two trackers in, 5 stations/tracker, 3 planes/station, full geometry

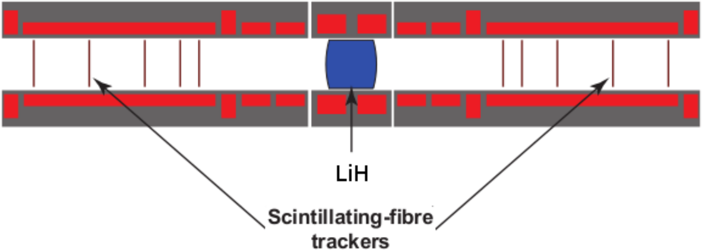
→ A simple 65 mm-thick, 225 mm in radius cylinder of LiH (or not)

→ Field maps generated in MAUS from the cooling channel currents

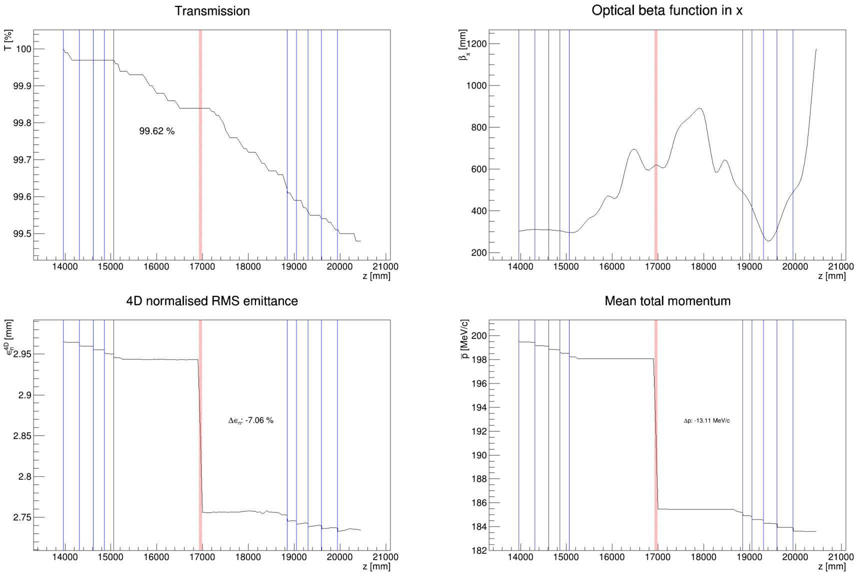
→ Fixed emittance input beam at 13800 m (just before TKUS5)

→ No momentum spread in the beam

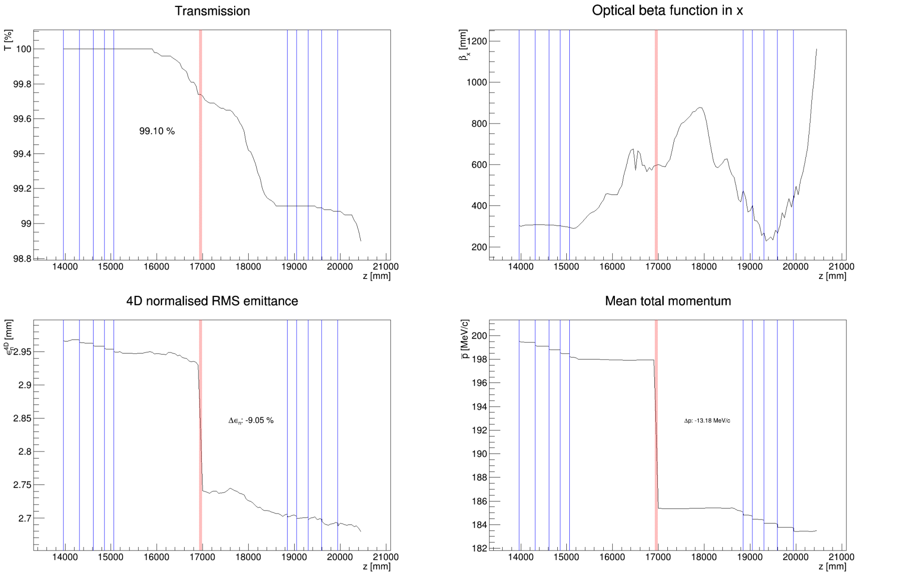
The simulations were also run with the full MAUS geometry and the same input beam, it did not have any significant effect on the measurements.



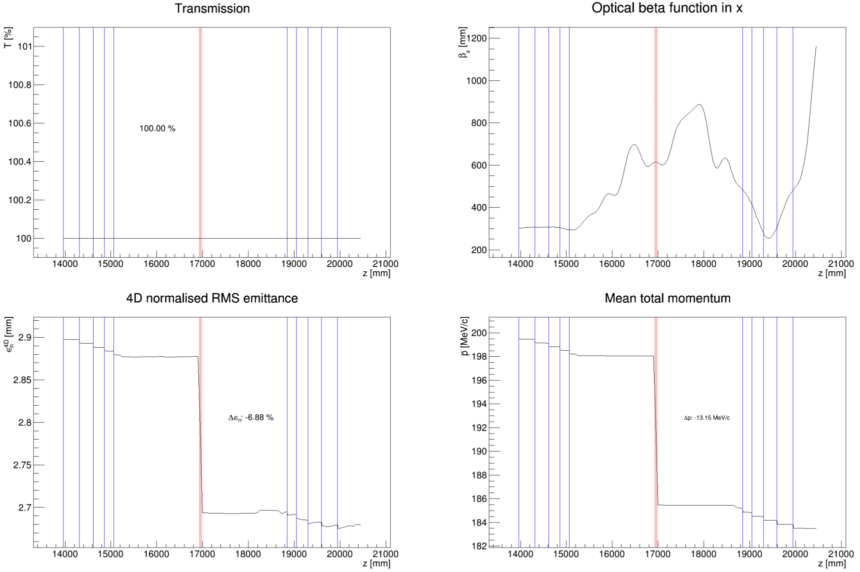
3mm, M2-on, LiH (no fiducial)



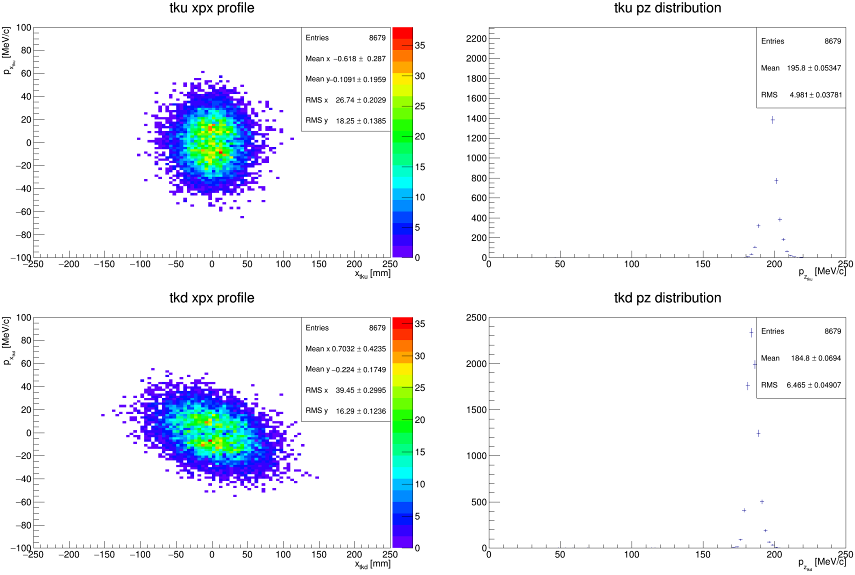
3mm, M2-on, LiH (150 mm fiducial)



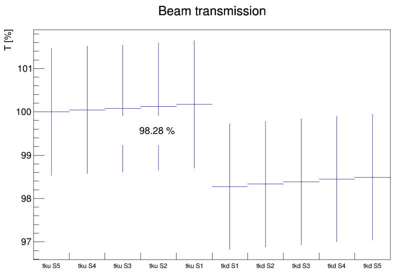
3mm, M2-on, LiH (150 mm fiducial+through)

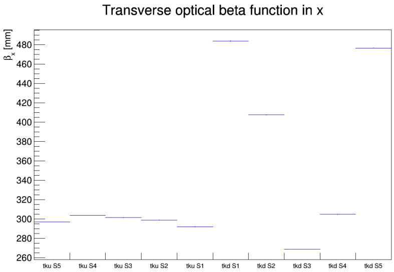


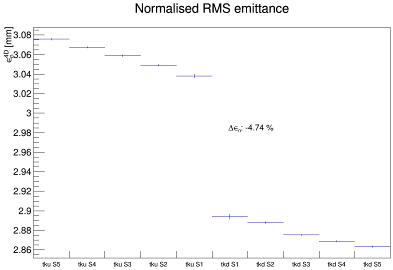
Reconstruction

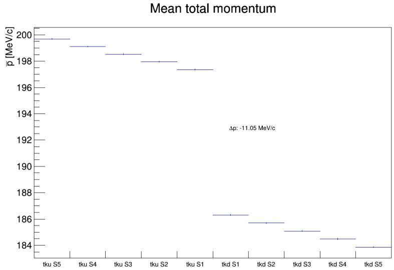


3mm, M2-on, LiH (reconstructed)

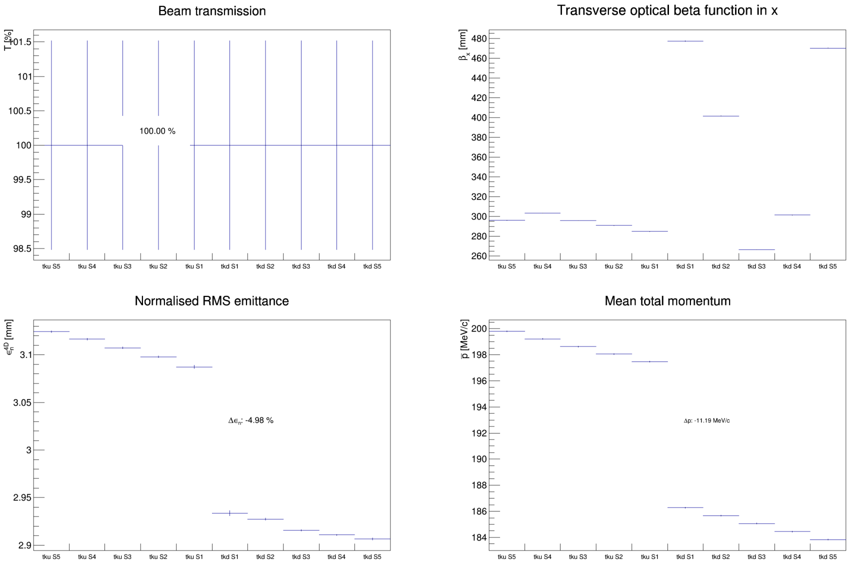








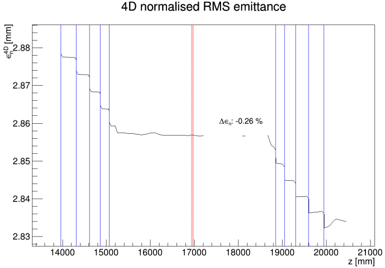
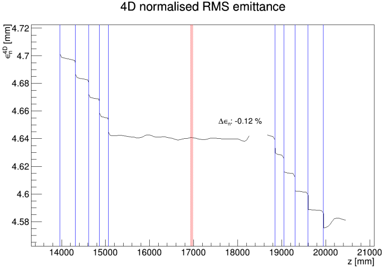
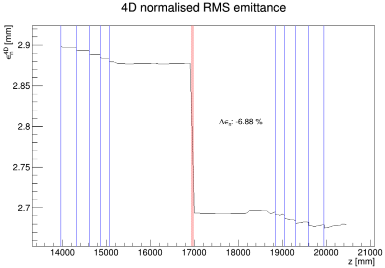
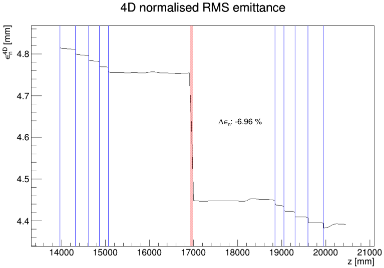
3mm, M2-on, LiH (reconstructed+through)



Summary of all M2$_D$ **on** configurations

3 mm, LiH	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	-7.07%	-9.05%	**-6.88%**	-4.74%	-4.98%
Δp [MeV/c]	-13.11	-13.18	**-13.15**	-11.06	-11.19
Trans.	99.62	99.10	**100**	98.28	100
6 mm, LiH	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	-1.58%	-29.10%	**-6.96%**	-5.71%	-6.01%
Δp [MeV/c]	-12.80	-12.78	**-12.78**	-12.88	-12.86
Trans. [%]	99.64	88.45	**100**	103.9	100
3 mm, $\emptyset$	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	+0.17%	-2.64%	**-0.26%**	+2.06%	+1.80%
Δp [MeV/c]	-0.60	-0.49	**-0.48**	+1.47	+1.3
Trans.	99.66	98.85	**100**	98.07	100
6 mm, $\emptyset$	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	+6.77%	-23.82%	**-0.12%**	+0.58%	+0.39%
Δp [MeV/c]	-0.20	-0.19	**-0.20**	-0.32	-0.30
Trans.	99.66	86.57	**100**	103.23	100

Emittance reduction in the M2_D **on** configurations

	3 mm	**6 mm**
$\emptyset$		
LiH		

Summary of all $M2_D$ **off** configurations

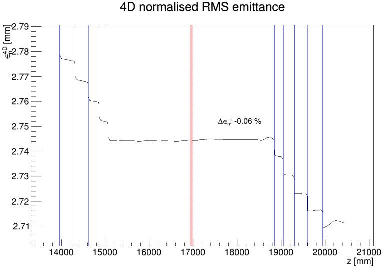
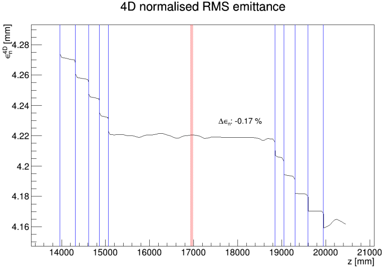
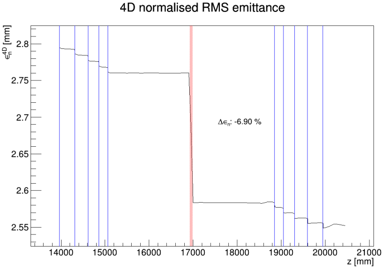
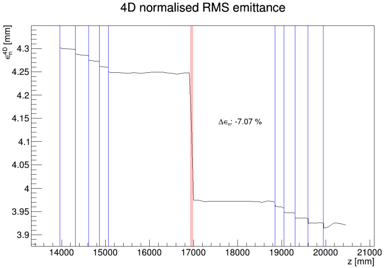
3 mm, LiH	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	-6.81%	-12.20%	**-6.90%**	-6.28%	-5.85%
Δp [MeV/c]	-12.75	-12.74	**-12.74**	-12.57	-12.66
Trans.	99.65	97.60	**100**	103.50	100

6 mm, LiH	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	0.28%	-43.45%	**-7.07%**	-10.91%	-5.65%
Δp [MeV/c]	-12.78	-12.77	**-12.78**	-12.79	-12.80
Trans. [%]	99.64	80.71	**100**	100.34	100

3 mm, ∅	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	-0.00%	-6.05%	**-0.06%**	-0.47%	+0.52%
Δp [MeV/c]	-0.20	-0.19	**-0.20**	+0.02	-0.06
Trans.	99.68	97.09	**100**	103.40	100

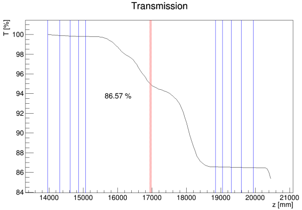
6 mm, ∅	No fid.	Fid.	**Fid.+thru**	Recon.	Recon.+thru
$\Delta\epsilon_n^{4D}$	+7.16%	-37.15%	**-0.17%**	-6.96%	+0.75%
Δp [MeV/c]	-0.19	-0.18	**-0.20**	-0.12	-0.14
Trans.	99.68	78.99	**100**	98.67	100

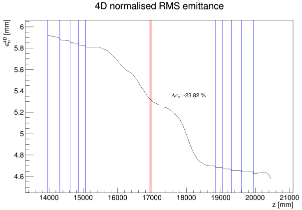
Emittance reduction in the M2$_D$ **off** configurations

	3 mm	**6 mm**
∅		
LiH		

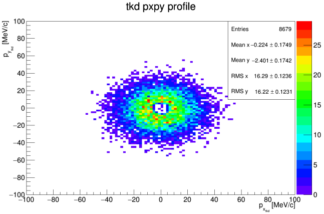
Main sources of bias on the emittance

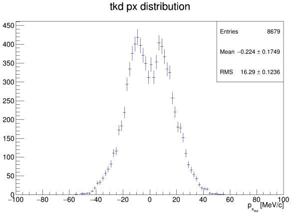
1 **Poor transmission**: scraping gives a seemingly reduced emittance



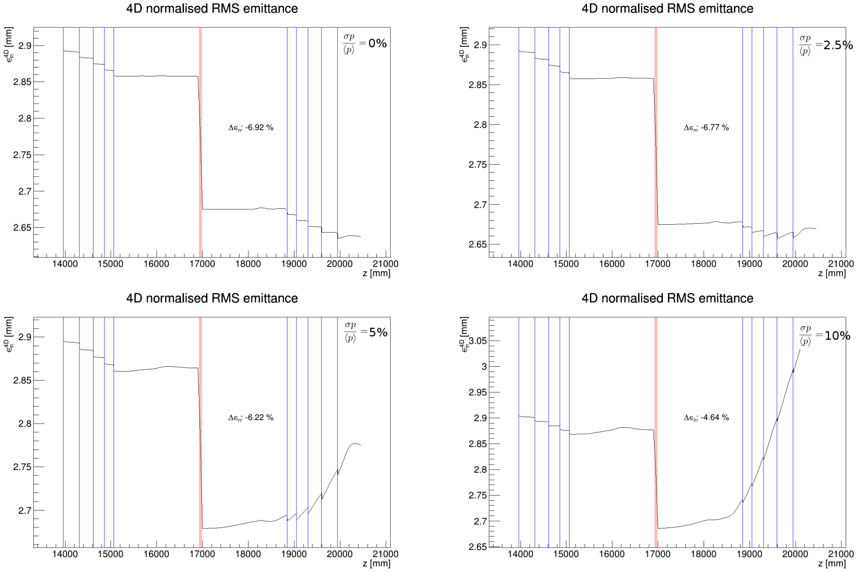


2 **Reconstruction inefficiencies**: The reconstruction produces a seemingly higher emittance due to the poor low p_T efficiency

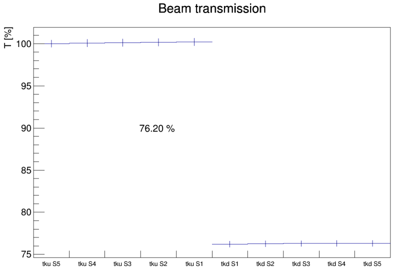


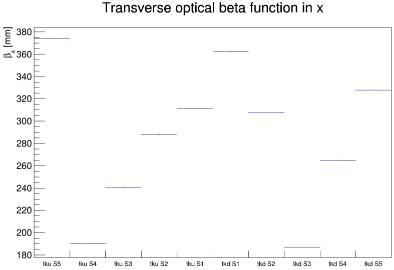


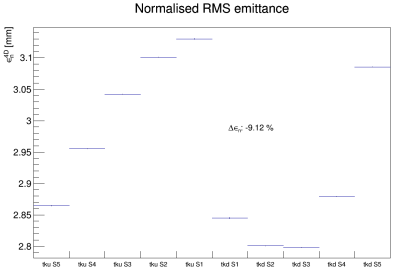
Effect of momentum spread on cooling, first look

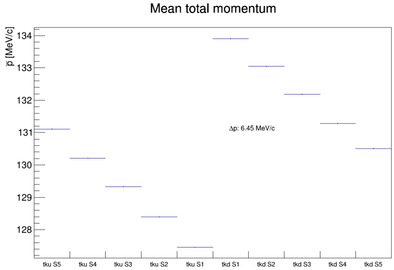


Run 8155, 140MeV/c beam, ECEs 140A, FC 50A

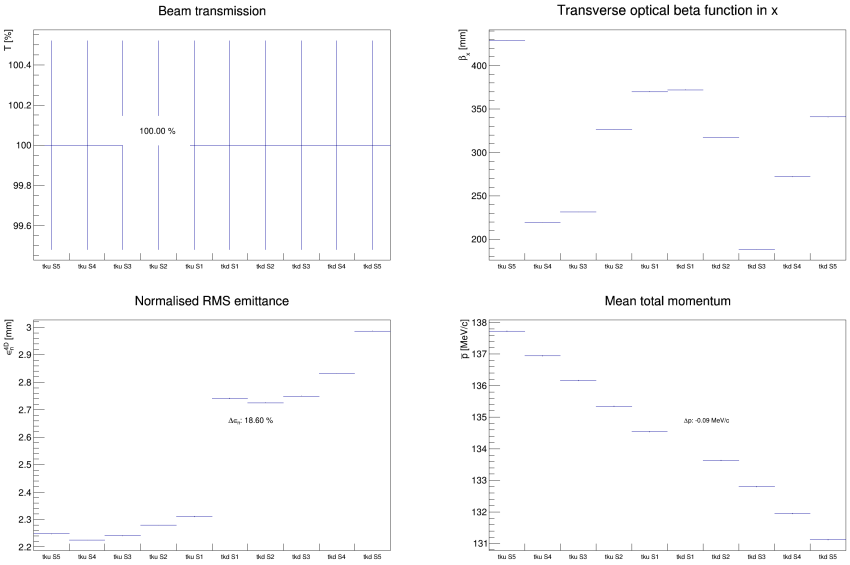








Run 8155, 140MeV/c beam, ECEs 140A, FC 50A (thru)



Conclusions and looking ahead

Observations made so far

→ Having M2 powered provides a higher transmission

3 mm: 98.85% vs 97.09% **6 mm:** 86.57% vs 78.99%

→ Lower transmission means artificial cooling. Selecting the particles that made it through the whole channel gets rid of this bias.

→ With selection, we see the same cooling with or without M2

→ The reconstruction biases the emittance towards higher values:

3 mm: -6.88% vs -4.98% **6 mm:** -7.07% vs -5.65%

To be investigated further:

→ An increase in momentum spread seems to produce emittance growth downstream (caution, plots du jour...)

→ Try to use the G4BL generated beam, more realistic

→ Look into more momentum settings, flip mode

→ Look further into data taken with the cooling channel up (du jour)