Direct CPV in Three-Body Charmless $B$ Decays:

Prospects for a model-independent interpretation of LHCb data

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Just some thoughts based on collaborations with S. Kränkl, T. Mannel, A. Khodjamirian, S. Cheng, T. Huber and K. Vos
Direct CP Violation

\[ \mathcal{A}(\bar{B} \to f) \equiv \mathcal{A}_f = \lambda_u \left( T^u_f - P_f \right) + \lambda_c \left( T^c_f - P_f \right) \]

\[ \lambda_p = V_{pb} V^*_{p\{d,s\}} \]

\[ T^p_f = \sum_{1,2} C_i^p \langle f | Q_i^p | \bar{B} \rangle \quad (\text{current-current operators}) \]

\[ P_f = \sum_{3,...,6} C_i \langle f | Q_i^p | \bar{B} \rangle \quad (\text{penguin operators}) \]

\[ \lambda_p = V_{pb} V^*_{p\{d,s\}} \]

- In the SM, \( C_i \) contain no phases.

- We write \( \mathcal{A}^p = |\mathcal{A}^p| e^{i\delta_p} \). Then:

\[ \mathcal{A}_{\text{CP}} \equiv \frac{|\mathcal{A}_f| - |\bar{\mathcal{A}}_f|}{|\mathcal{A}_f| + |\bar{\mathcal{A}}_f|} \propto \frac{\lambda_u \mathcal{A}^u}{\lambda_c \mathcal{A}^c} \cdot \sin \gamma \cdot \sin(\delta_c - \delta_u) \]

- Look for relative strong phases in interfering amplitudes
Two-body decays

To leading power in the heavy-quark expansion

$$\langle M_1 M_2 | O_i | B \rangle = F^{BM_1} \int du \ T^I_i(u) \phi_{M_2}(u) + \int d\omega \ du \ dv \ T^{II}_i(\omega, u, v) \phi_B(\omega) \phi_{M_1}(u) \phi_{M_2}(v)$$

- Vertex corrections: $T^I_i(u) = 1 + \mathcal{O}(\alpha_s/\pi)$
- Spectator scattering: $T^{II}_i(\omega, u, v) = \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2/\pi)$

$A_{CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$  -  But ... $\alpha_s(m_b)/\pi \sim \Lambda/m_b$ !!
Three-body decays

- Richer dynamics
- May have non-perturbative strong phases not suppressed by $\Lambda/m_b$

We do not want just a model that fits well. Instead we want to know if CKM+QCD is compatible with the data.
Three-body decays – kinematics

\[ \bar{B} \rightarrow M_a(p_a)M_b(p_b)M_c(p_c) \]

Two independent invariants, e.g.

\[ s_{ab} = \frac{(p_a + p_b)^2}{m_B^2} \quad \text{and} \quad s_{ac} = \frac{(p_a + p_c)^2}{m_B^2} \]
Three collinear directions $n_1$, $n_2$, $n_3$, disconnected at the leading power.

\[
\langle \pi^- \pi^+ \pi^- | \mathcal{O}_i | \bar{B} \rangle = F^{B \to \pi} \int du \, dv \, T^I_i(u, v) \, \phi_\pi(u) \, \phi_\pi(v) \\
+ \int d\omega \, du \, dv \, dy \, T^{II}_i(\omega, u, v, y) \, \phi_B(\omega) \, \phi_\pi(u) \, \phi_\pi(v) \, \phi_\pi(y)
\]

- Power $(1/m_b^2)$ & $\alpha_s$ suppressed with respect to two-body.
- At leading order/power/twist all convolutions are finite $\rightarrow$ factorization ✓
- Some pieces proven at NLO: Factorization of $B \to \pi\pi$ form factors [Böer, Feldmann, van Dyk '16] and $2\pi$ LCDAs [Diehl, Feldmann, Kroll, Vogt '99]

- $A_{CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ – Like two-body!
- But this region might not exist for $m_B = 5$ GeV

Krankl, Mannel, JV '15
• Breakdown of factorization at resonant edges requires new NP functions.
• 3-body decay resembles 2-body, but with new \((\pi\pi)\) “compound object”:

\[
\langle \pi^-_n \pi^+_n \pi^-_n \mid O \mid B \rangle = \langle \pi^-_n \mid \bar{h}_\nu \Gamma \xi_n \mid B \rangle \times \int dz \, T_1(z) \langle \pi^-_n \pi^+_n \mid \bar{\chi}(z) \Gamma \chi (0) \rangle \\
+ \langle \pi^-_n \pi^+_n \mid \bar{h}_\nu \Gamma \xi_n \mid B \rangle \times \int dz \, T_2(z) \langle \pi^-_n \mid \bar{\chi}(zn) \Gamma \chi (0) \rangle \\
= F_{B \to \pi} T_1 \ast \phi_{\pi\pi} + F_{B \to \pi \pi} T_2 \ast \phi_{\pi}
\]

• New non-perturbative input: (Contains NP strong phases!!)
  ▶ Generalized Distribution Amplitudes (GDAs) [Diehl, Polyakov, Gousset, Pire, Grozin...]
  ▶ Generalized Form Factors (GFFs) [Faller, Feldmann, Khodjamirian, Mannel, van Dyk...]

This is **always** an improvement w.r.t. quasi-two-body decays:

\[
A(B^- \rightarrow \pi^- [\pi^+ \pi^-]) = F^{B \rightarrow \pi} T_1 \phi_{\pi \pi} + F^{B \rightarrow \pi \pi} T_2 \phi_{\pi} \\
\rho \text{ dominance + zero-width limit} \\
A(B^- \rightarrow \pi^- \rho) = F^{B \rightarrow \pi} T_1 \phi_{\rho} + F^{B \rightarrow \rho} T_2 \phi_{\pi}
\]

This limit can be checked analytically.

- Factorization is at the same level of theoretical rigour for quasi-two-body and 3-body.
- Any model for \(\phi_{\pi \pi}\) and \(F^{B \rightarrow \pi \pi}\) satisfying axiomatic constraints and compatible with data (e.g. \(e^+ e^- \rightarrow \pi \pi\)) replaces any notion of \(\rho\).
2π GDAs

• Definition: \( s = (k_1 + k_2)^2, \ k_1 = \zeta k_{12}, \ k_2 = (1 - \zeta) k_{12} \)

\[
\phi^q_{\pi\pi}(z, \zeta, s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^-n_-)\not{p} + q(0)|0 \rangle
\]

• Normalization (local correlator):

\[
\int dz \ \phi_{\pi\pi}(z, \zeta, s) = (2\zeta - 1)F_{\pi}(s) \quad \text{(pion vector FF)}
\]

• \( F_{\pi}(s) \): Data \((e^+e^- \rightarrow \pi\pi(\gamma) \ [BaBar])\)
\[ \mathcal{B} \rightarrow \pi \pi \text{ form factors from } B\text{-meson LCSRs} \]

Cheng, Khodjamirian, JV '16?

**Correlation function**

\[
F_\mu(k, q) = i \int d^4x e^{ik \cdot x} \langle 0 | T \{ \bar{d}(x) \gamma_\mu u(x), m_b \bar{u}(0) \gamma_5 b(0) \} | \bar{B}^0(q + k) \rangle
\]

**Unitarity relation**

\[
2 \text{Im} F_\mu(k, q) = m_b \int d\tau 2\pi \left[ \langle 0 | \bar{d} \gamma_\mu \pi(k_1) \pi(k_2) | \langle \pi(k_1) \pi(k_2) | \bar{u} \gamma_5 b | \bar{B}^0(q + k) \rangle \right]
\]

\[
= q_\mu \frac{s \sqrt{q^2} \beta_\pi(s)^2}{4 \sqrt{6\pi} \sqrt{\lambda}} \ F^*_\pi(s) \ F^{(\ell=1)}_t(s, q^2) + \cdots
\]

**Corollary:** \( F^*_\pi(s) \ F^{(\ell=1)}_t(s, q^2) \) is real for all \( s < 16 m^2_\pi \) \( \Rightarrow \)

**Phase** \( (F^B_{\rightarrow \pi \pi}) = \text{Phase(pion form factor)} \)

Important for CP violation!!!

[See also Kang, Kubis, Hanhart, Meissner '13]
\[ B \to \pi \pi \] form factors from \( B \)-meson LCSRs

Cheng, Khodjamirian, JV ’16?

\[ \int_{4m_{\pi}^2}^{s_{\pi}^2} ds \ e^{-s/M^2} \frac{s}{4 \sqrt{6} \pi^2 \sqrt{\lambda}} \left[ \beta_{\pi}(s) \right]^2 F^*_\pi(s) F^{(1)}_t(s, q^2) = f_B m_B^2 m_b \left\{ \int_{s_{\pi}^2}^{\sigma_{\pi}^2} d\sigma \ e^{-s(\sigma, q^2)/M^2} \times \right. \]

\[ \left[ \frac{\sigma}{\sigma} \phi^B(\sigma m_B) - \frac{\sigma}{\sigma} \left[ \phi^B(\sigma m_B) - \phi^B(\sigma m_B) \right] - \frac{1}{\sigma m_B} \phi^B(\sigma m_B) \right] \left[ \frac{\sqrt{s}}{\Gamma(\rho/\pi)} + \frac{s}{(m_{\rho}^2 - s + i \sqrt{2} \Gamma(\rho/\pi))} + A_{0}^{BV} (q^2, \sigma_{\pi}^2, M^2) \right] \]

\[ \rho \text{-dominance} + \text{zero-width limit}: \]

\[ F^*_\pi(s) \simeq \frac{f_{\rho} g_{\rho \pi \pi} m_{\rho} / \sqrt{2}}{m_{\rho}^2 - s + i \sqrt{2} \Gamma(\rho/\pi)} \quad \text{and} \quad F^{(1)}_t(s, q^2) \simeq -\frac{\beta_{\pi}(s) \sqrt{\lambda}}{\sqrt{3} q^2} \frac{m_{\rho} g_{\rho \pi \pi} A_{0}^{B \rho}(q^2)}{m_{\rho}^2 - s + i \sqrt{2} \Gamma(\rho/\pi)} \]

\[ LHS = 2 f_{\rho} m_{\rho} A_{0}^{B \rho}(q^2) \int_{4m_{\pi}^2}^{s_{\pi}^2} ds \ e^{-s/M^2} \left[ \frac{\sqrt{s}}{(m_{\rho}^2 - s)^2 + s \Gamma(\rho/\pi)} \right] \]

\[ \left. \begin{array}{l} \Gamma(\rho/\pi) \to 0 \\ \delta(s - m_{\rho}^2) \end{array} \right\} \quad \text{hep-ph/0611193} \]
Leading order amplitude:

\[ A|_{s+\sim 1} = \frac{G_F}{\sqrt{2}} \left[ 4m_B^2 f_0(s_+)(2\zeta - 1)F_\pi(s+)(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4)F_t(\zeta, s+) \right] \]

- The Wilson coefficients \(a_1, a_2\) have weak phase \(\sim \lambda_u\), and \(a_4\) has weak phase \(\sim \lambda_c\).
- Everything here is LO, so all perturbative strong phases are ignored.
- \(F_\pi(s+)\) and the P-wave contribution to \(F_t(\zeta, s+)\) have the same strong phase.
- S-wave contributions to \(F_t(\zeta, s+)\) can generate a strong phase (S- and P-wave interference).
- The corresponding “scalar-penguin” amplitude (power-suppressed but chirally enhanced) is in this case proportional to the scalar pion form factor. Its interference with the P-wave contribution to the \(F_t\) part may also potentially contribute a large strong phase.
- All these issues are under study.
:: Outlook

▷ Soft corners of Dalitz plot contain interference of crossed resonances: potentially important DCPV. But difficult for QCDF. New ideas?

▷ Central “perturbative” region boring, but it might not exist.

▷ Edges: how large are they? Promising prospects for data-driven understanding of large local asymmetries. Need to improve hadronic input, including vector and scalar (pion) form factors. Study also LCSRs for S-wave $B \to \pi\pi$ form factors.

▷ $B \to \pi\pi$ Form factors: the same approach can be applied to $B \to K\pi$ form factors: Important for $B \to K^*\ell\ell$ !!!!

▷ Distribution amplitudes: $B^- \to D^0(\pi^-\pi^0)$ and $\bar{B}^0 \to D^+(\pi^-\pi^0)$. What can be done? can we measure phases?

![Graphs showing distribution amplitudes](image-url)