Lattice prospects for CP violation and multi-hadron decays

Maxwell T. Hansen
Institut für Kernphysik and Helmholtz-Institut Mainz
Johannes Gutenberg Universität
Mainz, Germany

October 13th, 2016

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JG|U

Introduction

CP violating processes are a promising tool in searching for new physics beyond the Standard Model (BSM)

This requires understanding the SM prediction for the CP violating process

experiment = (SM) (perturbative QCD) **(non-perturbative QCD)**
+ (BSM) **(non-perturbative QCD)**

Lattice QCD (LQCD) is a powerful tool for extracting non-perturbative QCD predictions

Here I focus on prospects for multi-hadron decays

$D \to \pi\pi,\ K\overline{K}$ $\quad B \to K^*(\to K\pi)\ell\ell$ $\quad \Lambda_b \to J/\psi\, p\, \pi^-$

Lattice QCD in a nutshell

In LQCD we evaluate the Feynman path-integral numerically

$$\textbf{observable} = \int \mathcal{D}\phi \; e^{iS} \left[\begin{array}{c} \textbf{quantum fields} \\ \textbf{of the observable} \end{array} \right]$$

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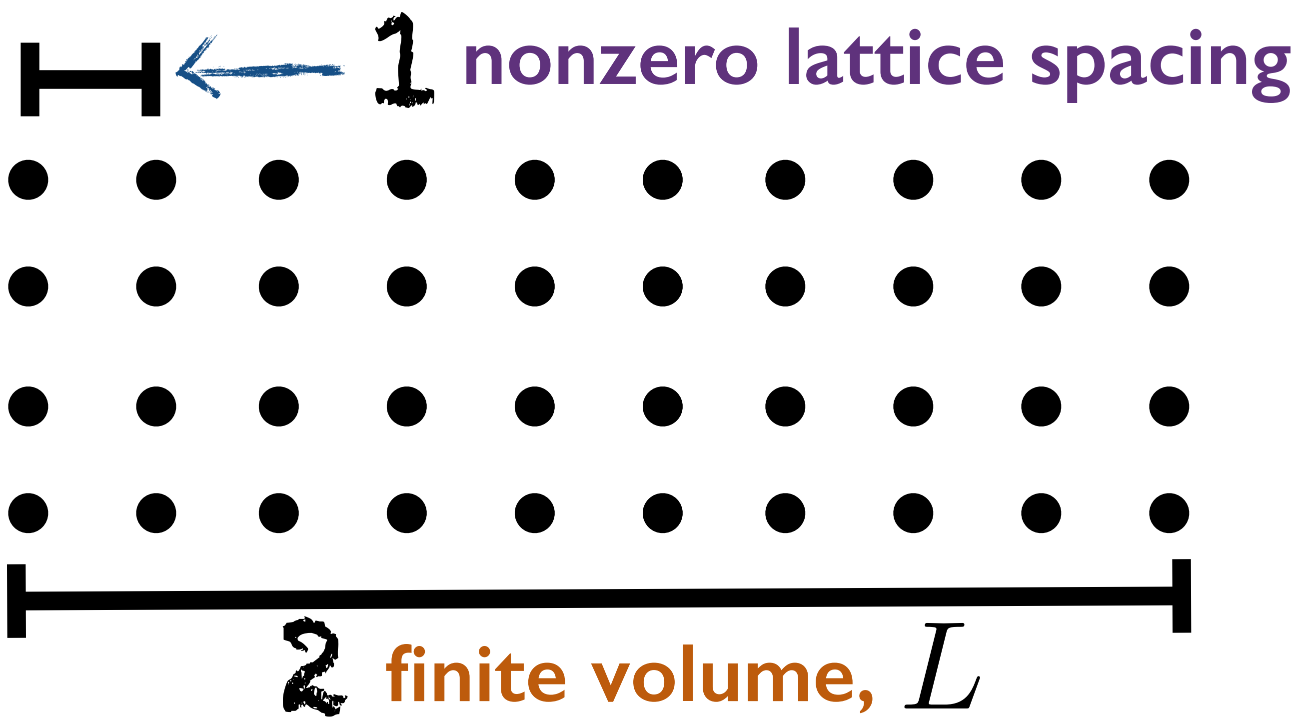

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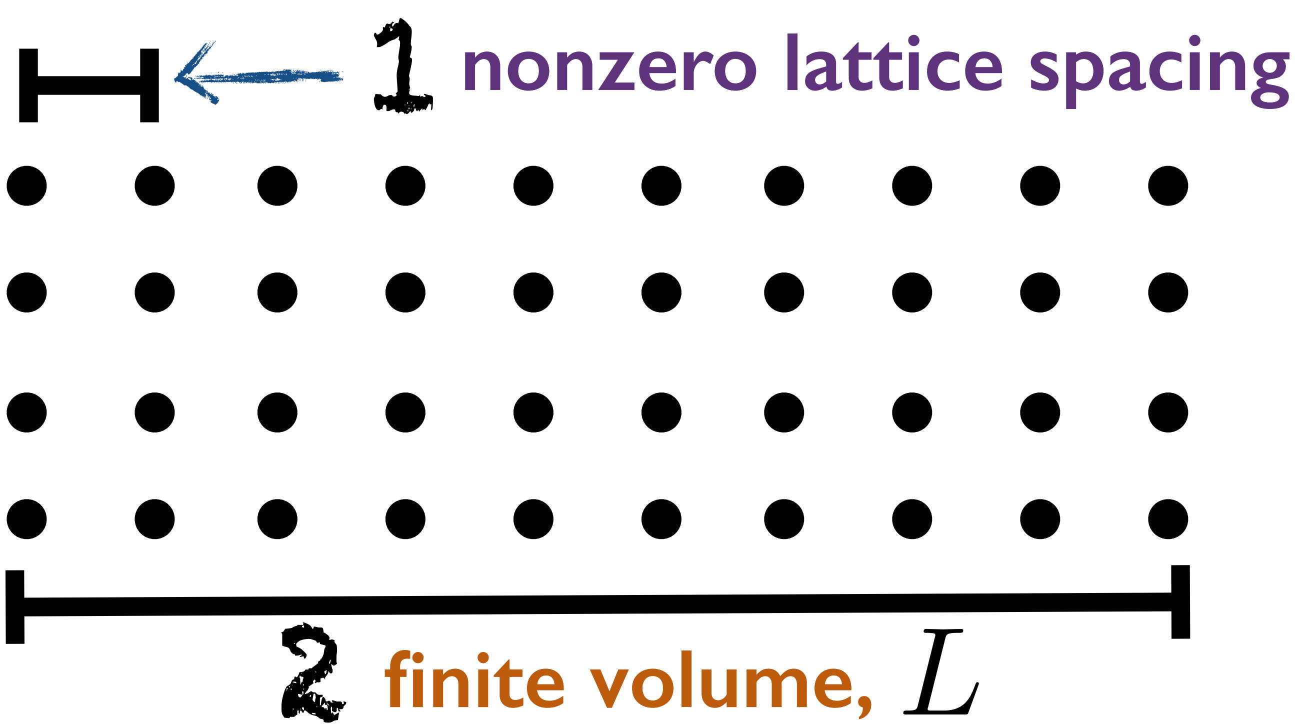

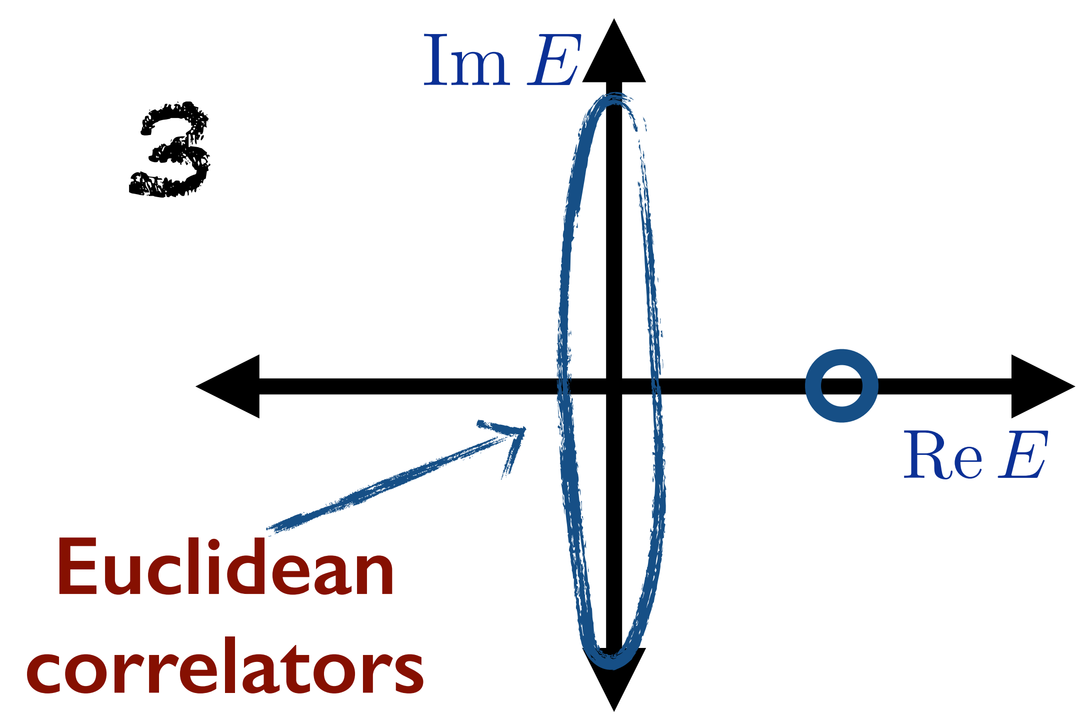

4 unphysical quark content $\quad M_{\pi,\text{lattice}} > M_{\pi,\text{our universe}}$

Calculations at the physical pion mass do now exist

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$$\left(\begin{matrix}\textbf{observable?}\\ \text{discretized, finite volume,}\\ \text{Euclidean, heavy pions}\end{matrix}\right) = \int \prod_i^N d\phi_i \; e^{-S} \left[\begin{matrix}\text{quantum fields}\\ \text{of the observable}\end{matrix}\right]$$

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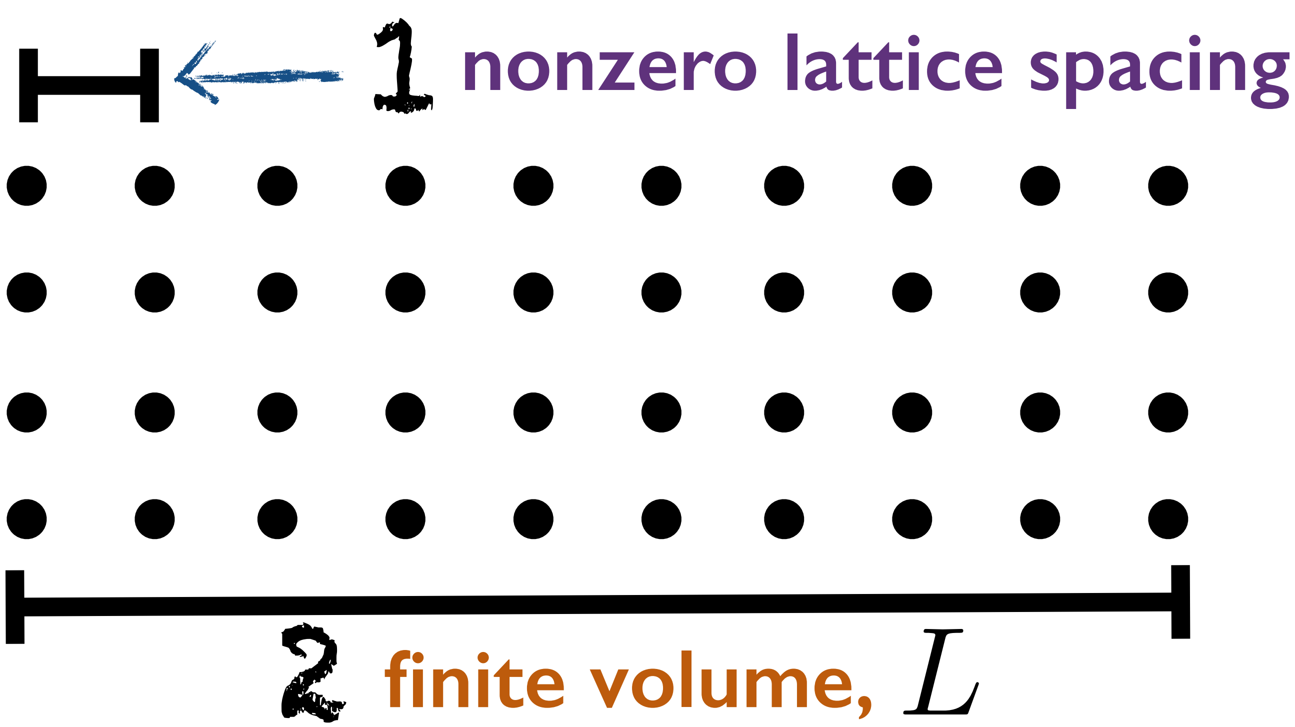

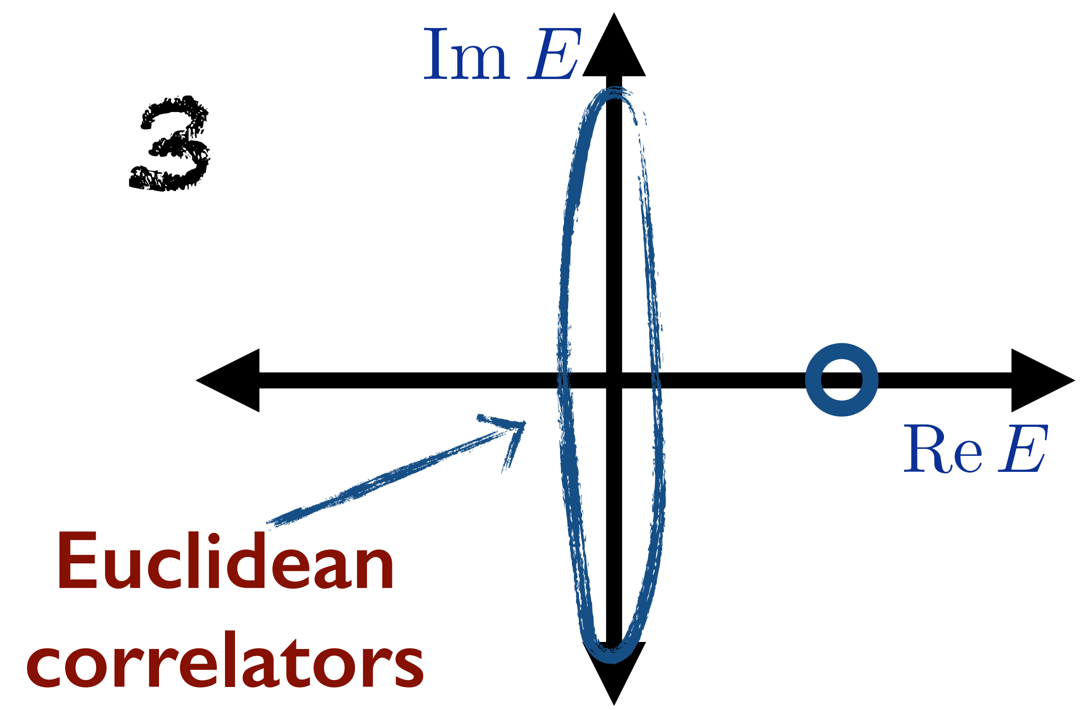

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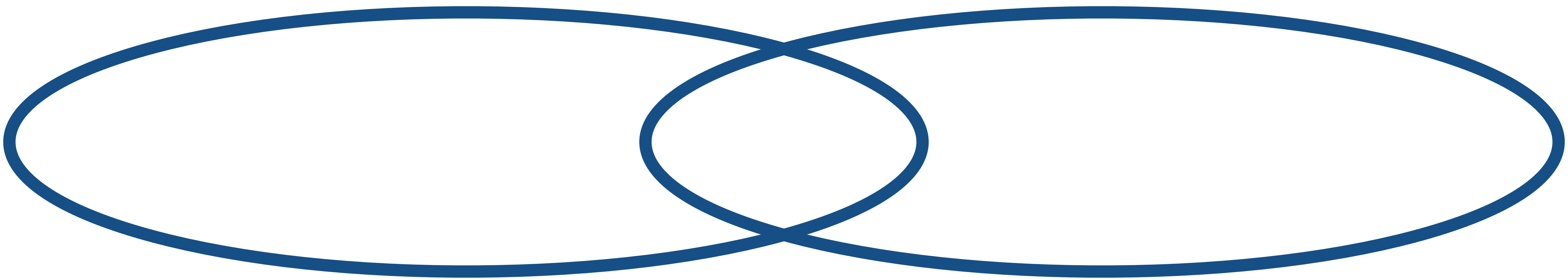
Extracting physical predictions…

Two basic approaches to handle these modifications

Perform multiple calculations and extrapolate

Use theoretical methods to understand the modification



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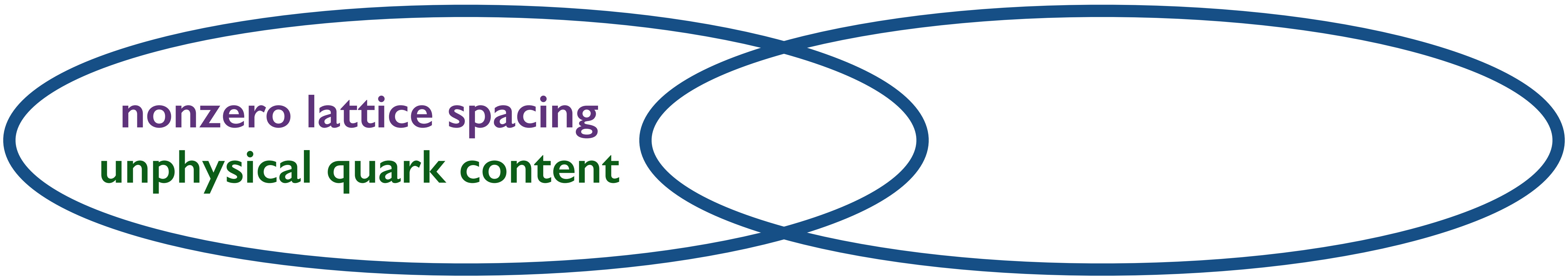

Modern calculations often have reliable chiral-continuum extrapolations (see e.g. FLAG)

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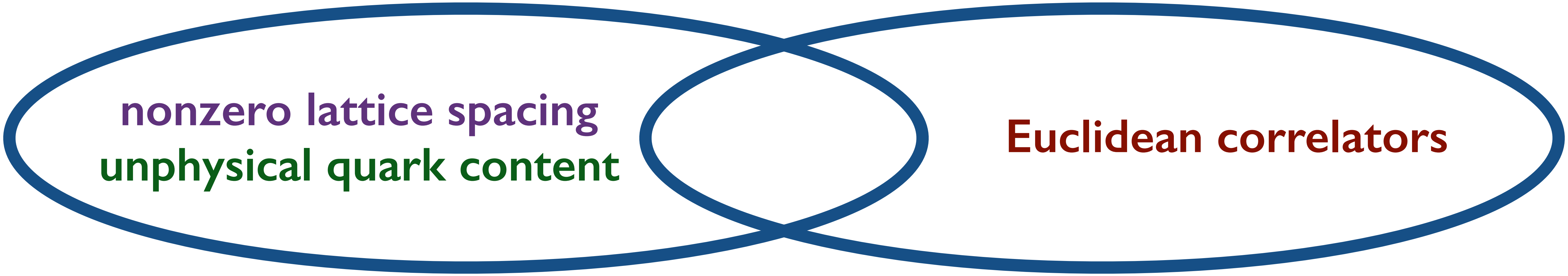

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For decay constants and form factors one should extrapolate to infinite-volume…

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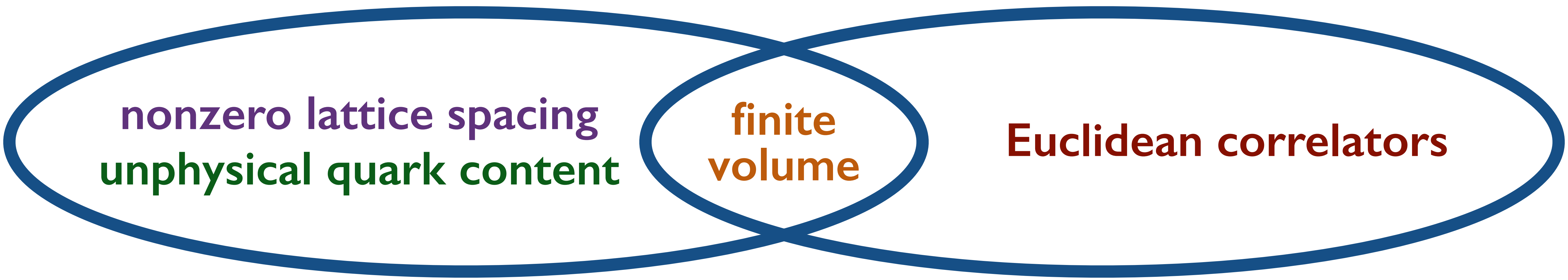

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The role of finite volume is also observable dependent:

For decay constants and form factors one should extrapolate to infinite-volume…

To extract multi-hadron decay and scattering amplitudes we do not take the infinite-volume limit

$$D \to \pi\pi, \ K\overline{K} \qquad B \to K^*(\to K\pi)\ell\ell \qquad \Lambda_b \to J/\psi\, p\, \pi^-$$

This is the focus of this talk!

Multi-hadron processes from LQCD…

In a LQCD calculation it is possible to access

$$H_{\rm QCD}|n, \text{“}\pi\pi\text{”}, L\rangle = |n, \text{“}\pi\pi\text{”}, L\rangle E_n(L)$$

$$\langle n, \text{“}\pi\pi\text{”}, L|\mathcal{H}_W|\text{“}D\text{”}, L\rangle$$

finite-volume energies and matrix elements

(labels in quotes indicate quantum numbers)

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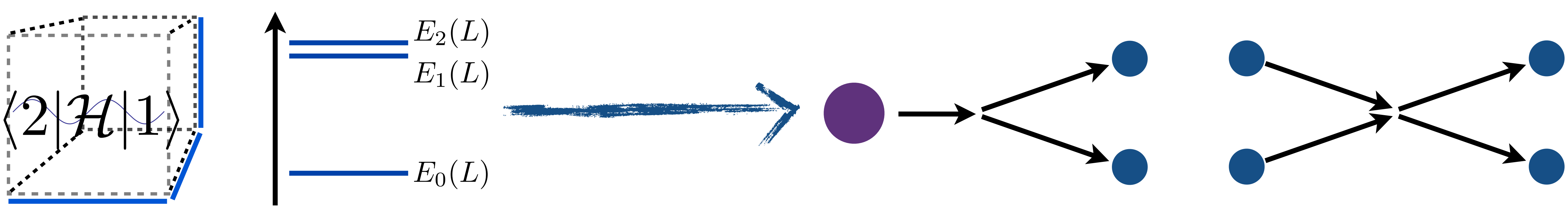
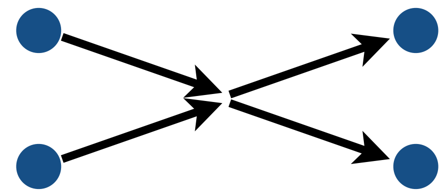
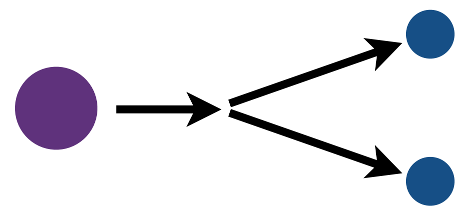
Lüscher (1991) + Lellouch and Lüscher (2001) derived relations between such finite-volume quantities and infinite-volume experimental observables



Neglect contributions scaling as $e^{-M_\pi L}$**.**

Status of multi-hadron matrix elements in LQCD…

physical system		Method to get it from LQCD
$\pi\pi \to \pi\pi,\ \sqrt{s} < 4M_\pi$ ($\mathbf{P} \neq 0$ in finite-volume frame)*		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)*
$K \to \pi\pi$ (relies on $M_K < 4M_\pi$) ($\mathbf{P} \neq 0$ in finite-volume frame)*		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005)*, Christ, Kim and Yamazaki (2005)*

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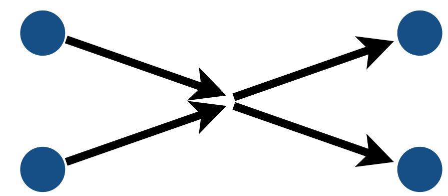
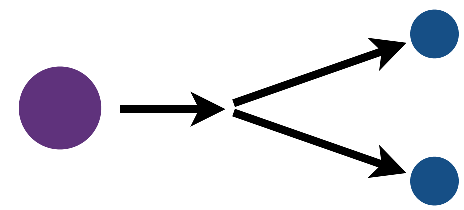
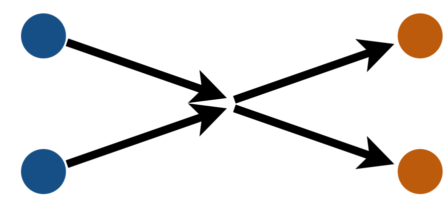
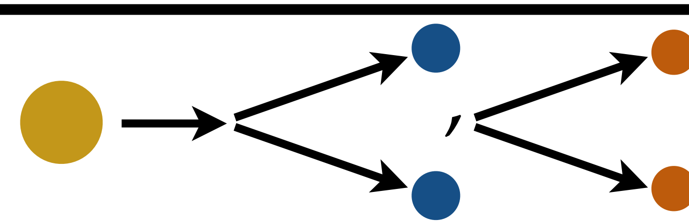
$\pi\pi \to \pi\pi,\ \sqrt{s} < 4M_\pi$ $(\mathbf{P} \neq 0$ in finite-volume frame)*		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)*
$K \to \pi\pi$ (relies on $M_K < 4M_\pi$) $(\mathbf{P} \neq 0$ in finite-volume frame)*		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005)*, Christ, Kim and Yamazaki (2005)*
$\pi\pi \to K\overline{K},\ \sqrt{s} < 4M_\pi$ (not possible for physical masses)		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
$D \to \pi\pi,\ K\overline{K}$ (ignores four-particle states)		MTH and Sharpe (2012)

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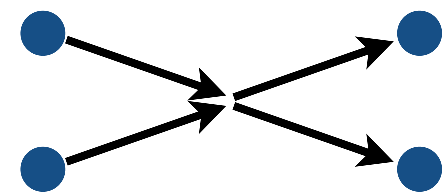
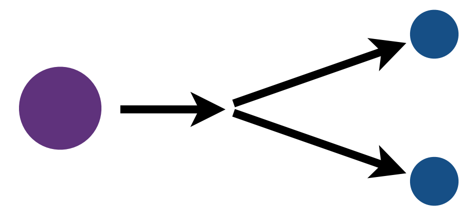
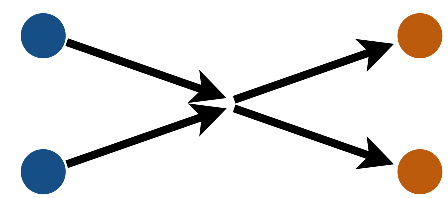
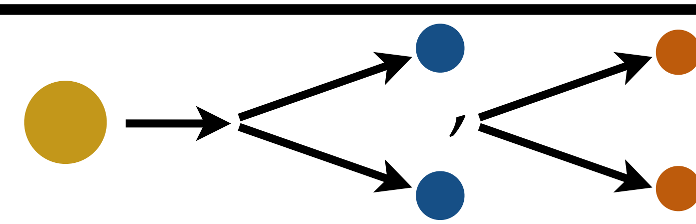
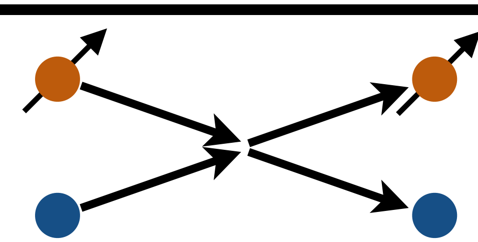
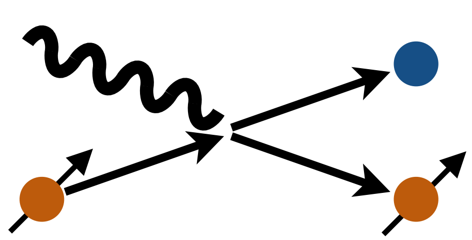
$\pi\pi \to \pi\pi, \ \sqrt{s} < 4M_\pi$ ($\mathbf{P} \neq 0$ in finite-volume frame)*		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)*
$K \to \pi\pi$ (relies on $M_K < 4M_\pi$) ($\mathbf{P} \neq 0$ in finite-volume frame)*		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005)*, Christ, Kim and Yamazaki (2005)*
$\pi\pi \to K\overline{K}, \ \sqrt{s} < 4M_\pi$ (not possible for physical masses)		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
$D \to \pi\pi, \ K\overline{K}$ (ignores four-particle states)		MTH and Sharpe (2012)
$NN \to NN, \ N\pi \to N\pi$ (energies below three-particle production)		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
$\gamma^* \to \pi\pi, \ \pi\gamma^* \to \pi\pi,$ $N\gamma^* \to N\pi$ $B \to K^*(\to K\pi)\ell\ell$ (energies below three-particle production)		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

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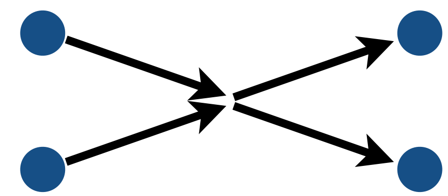
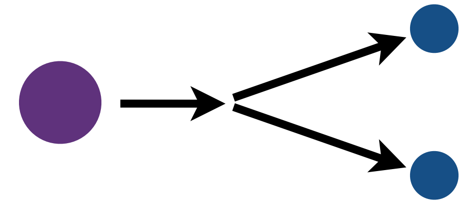
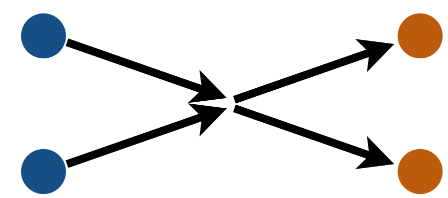
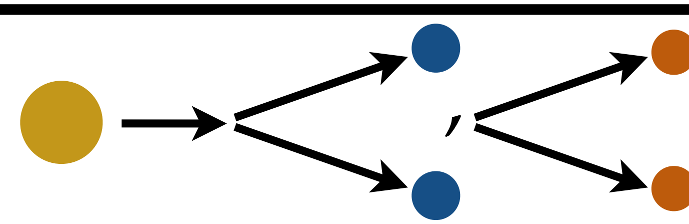
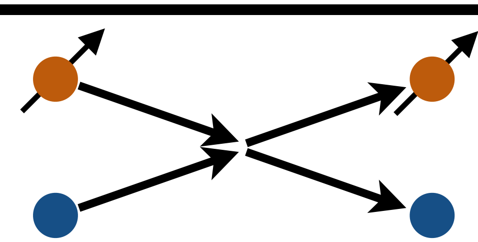
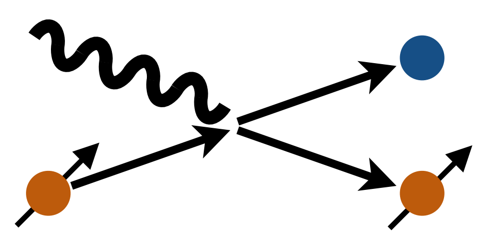
elastic scattering of identical scalars		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)
decay into identical scalars (no other open decay channels)		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005), Christ, Kim and Yamazaki (2005)
non-identical scalars, multiple coupled channels*		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
decay into multiple, coupled two-particle channels*		MTH and Sharpe (2012)
scattering of particles with intrinsic spin*		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
particle production mediated by a generic local current*		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

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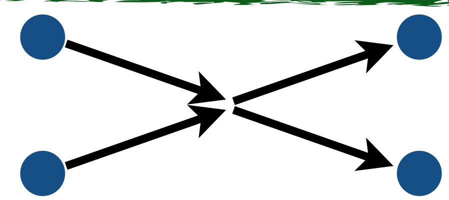
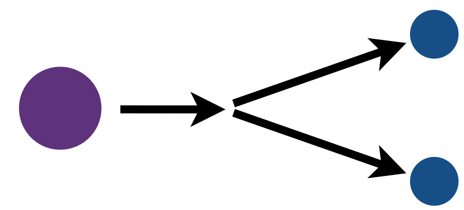
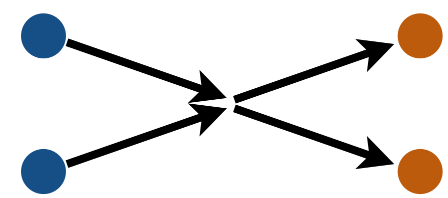
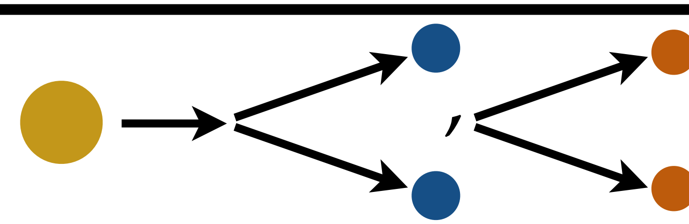
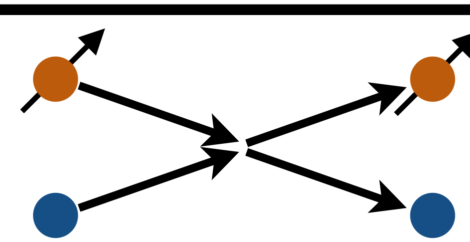
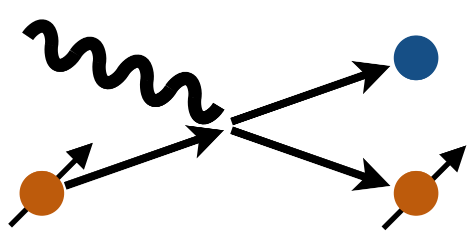
elastic scattering of identical scalars		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)
decay into identical scalars (no other open decay channels)		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005), Christ, Kim and Yamazaki (2005)
non-identical scalars, multiple coupled channels*		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
decay into multiple, coupled two-particle channels*		MTH and Sharpe (2012)
scattering of particles with intrinsic spin*		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
particle production mediated by a generic local current*		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

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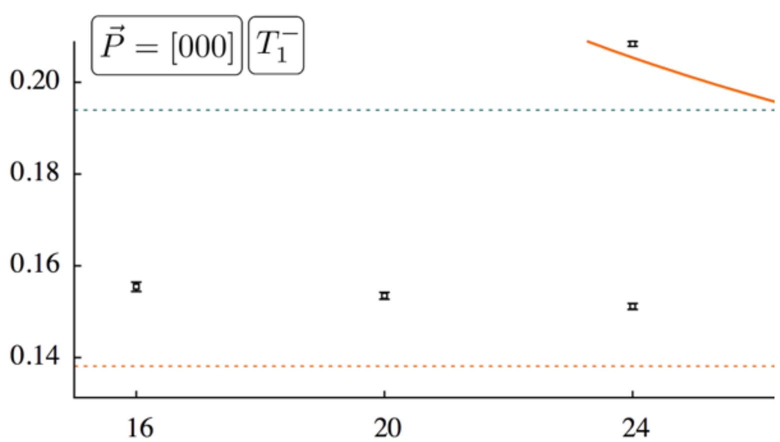
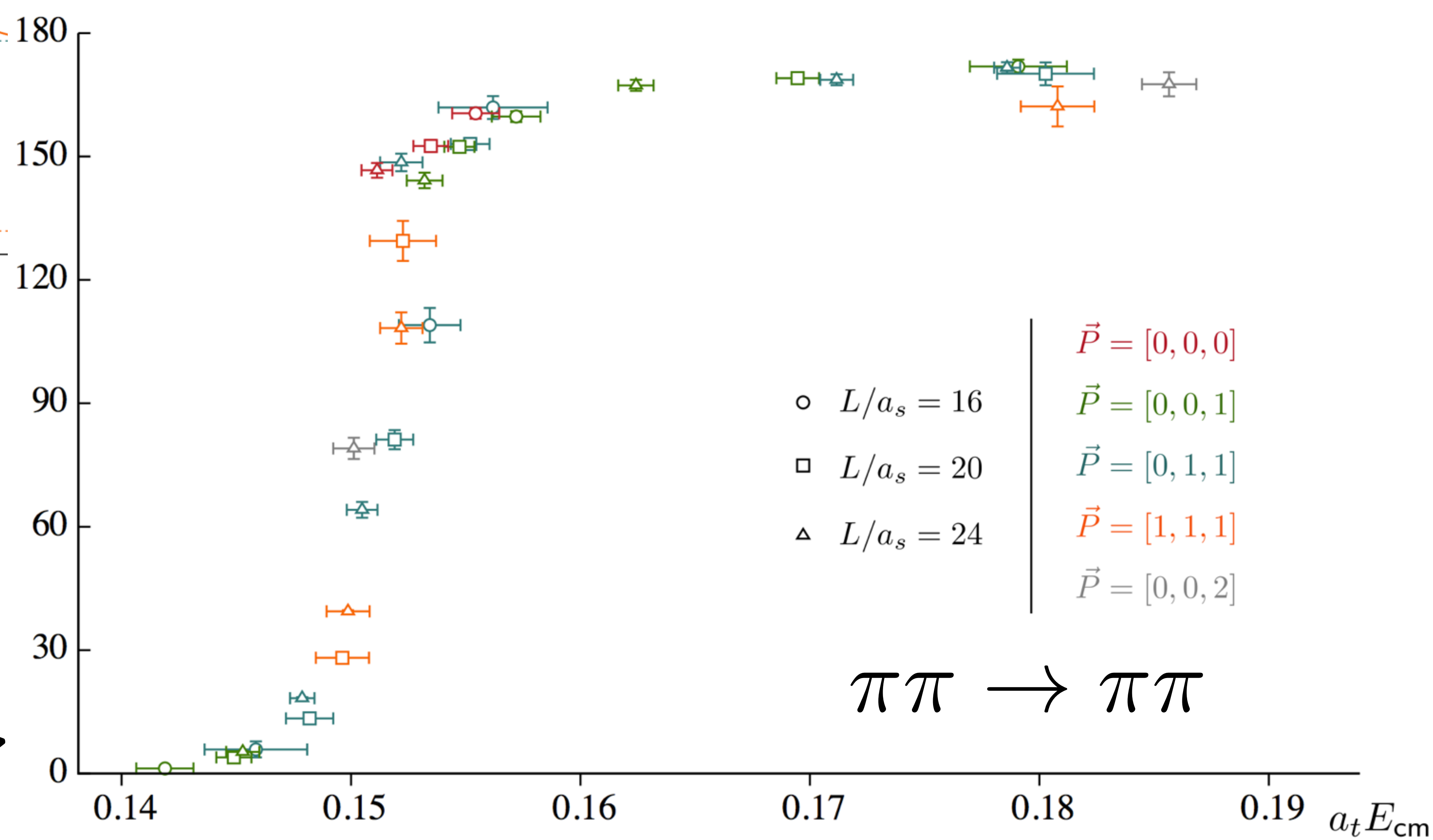
Example: isospin scattering in the ρ channel…

$$\cot \delta_{\ell=1}(E_n^*) + \cot \phi(E_n, \vec{P}, L) = 0$$

scattering phase — **known geometric function**

Lüscher (1986, 1991), Rummukainen and Gottlieb (1995)





Dudek, Edwards, Thomas in *Phys.Rev*. D87 (2013) 034505

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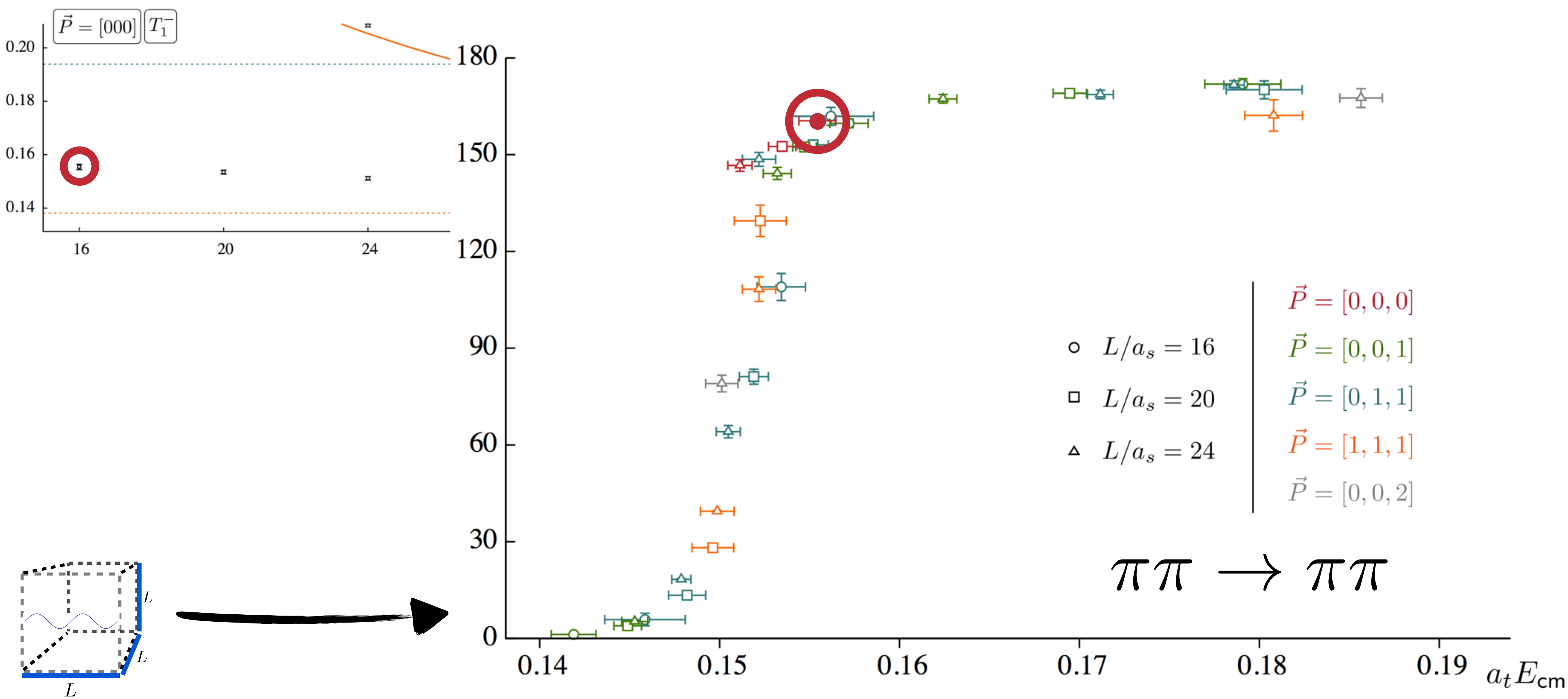

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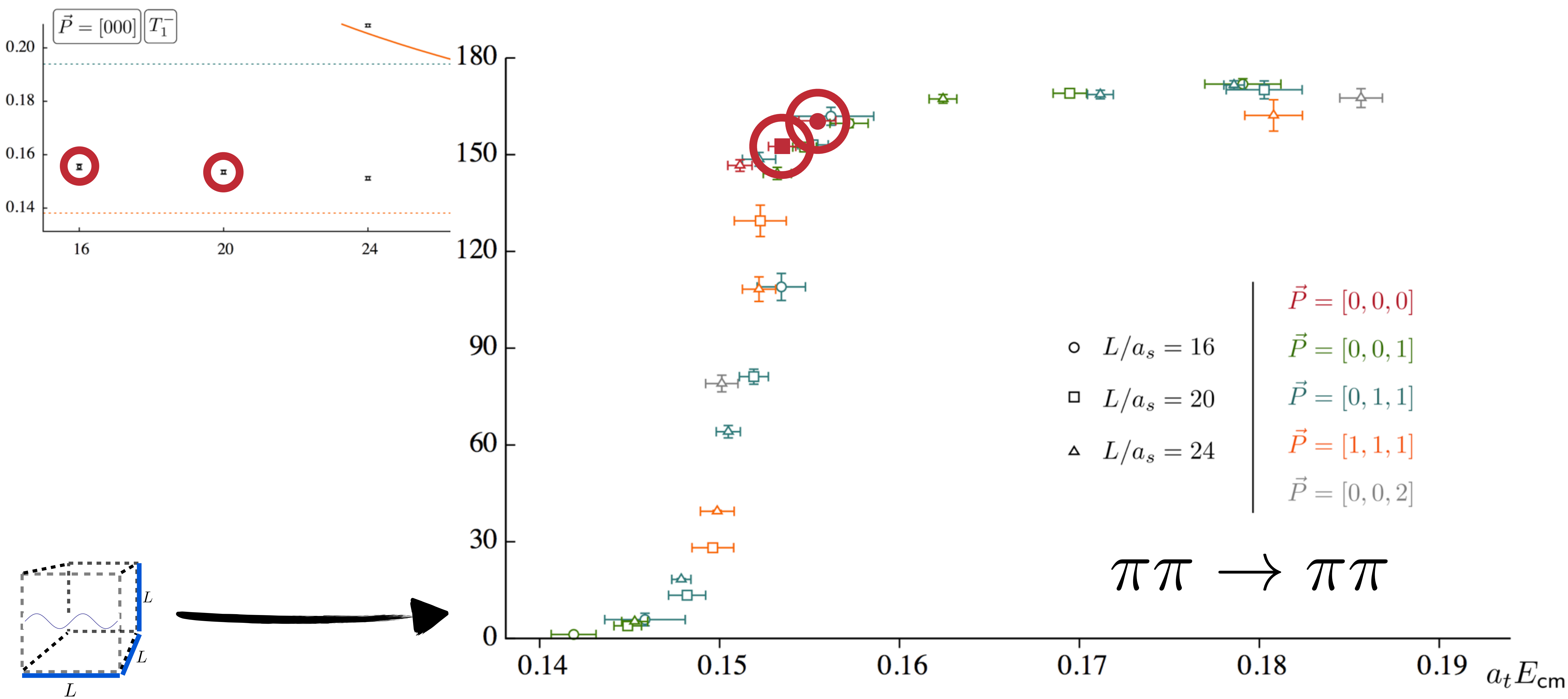

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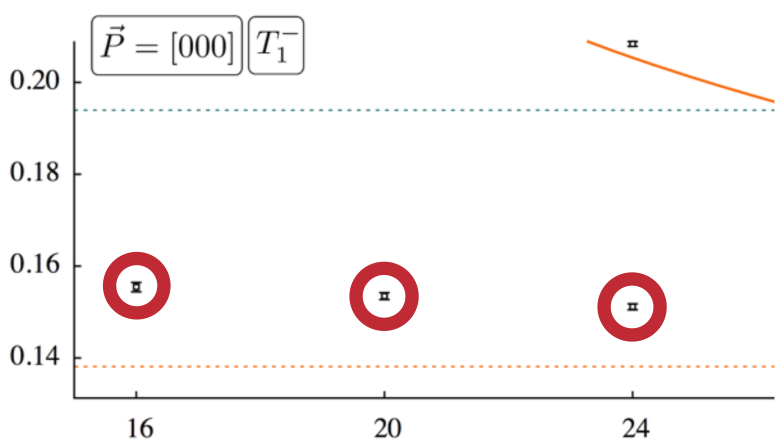

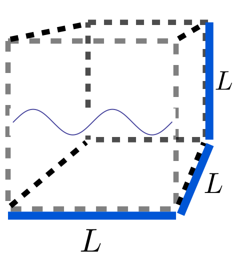

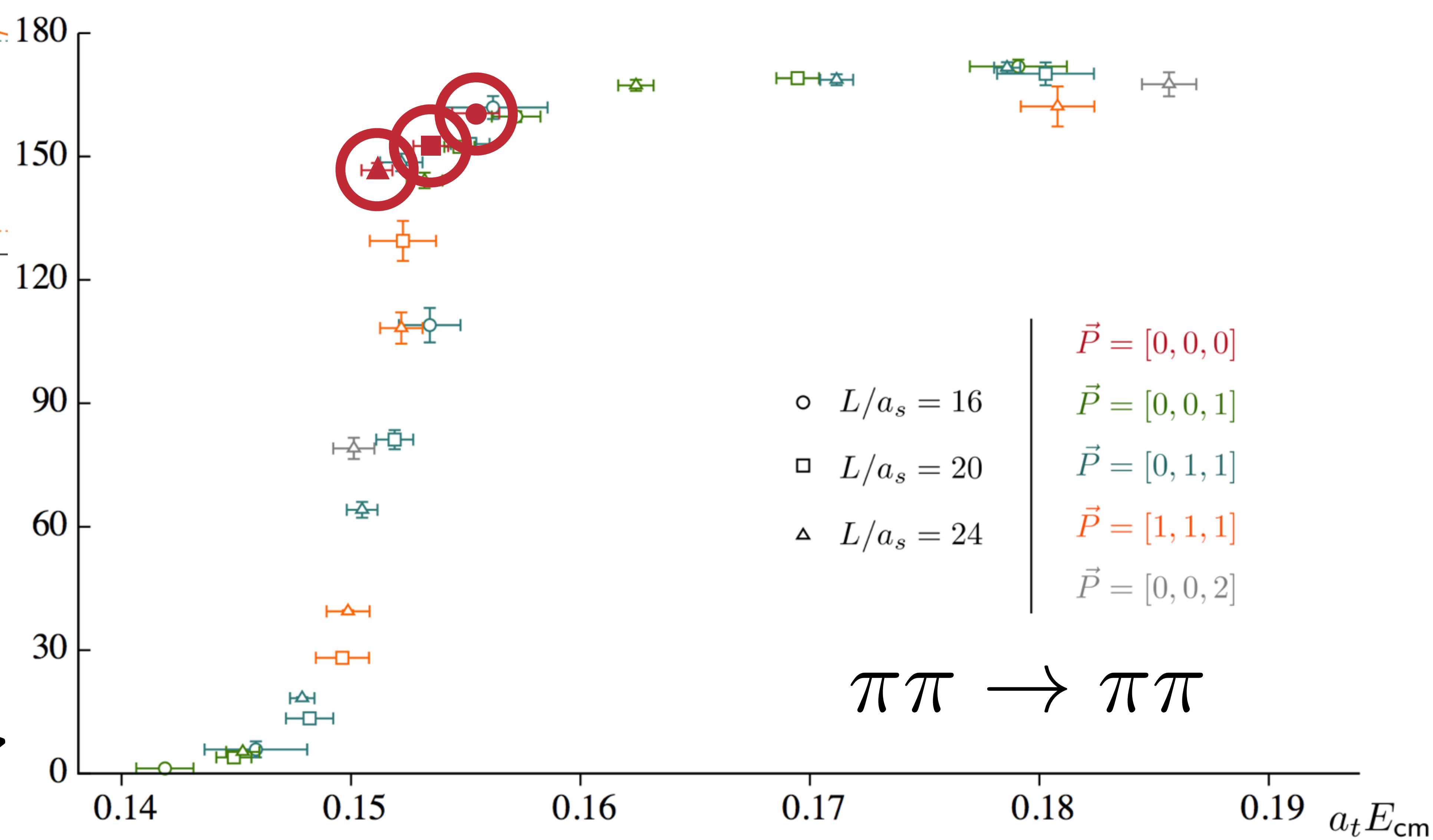

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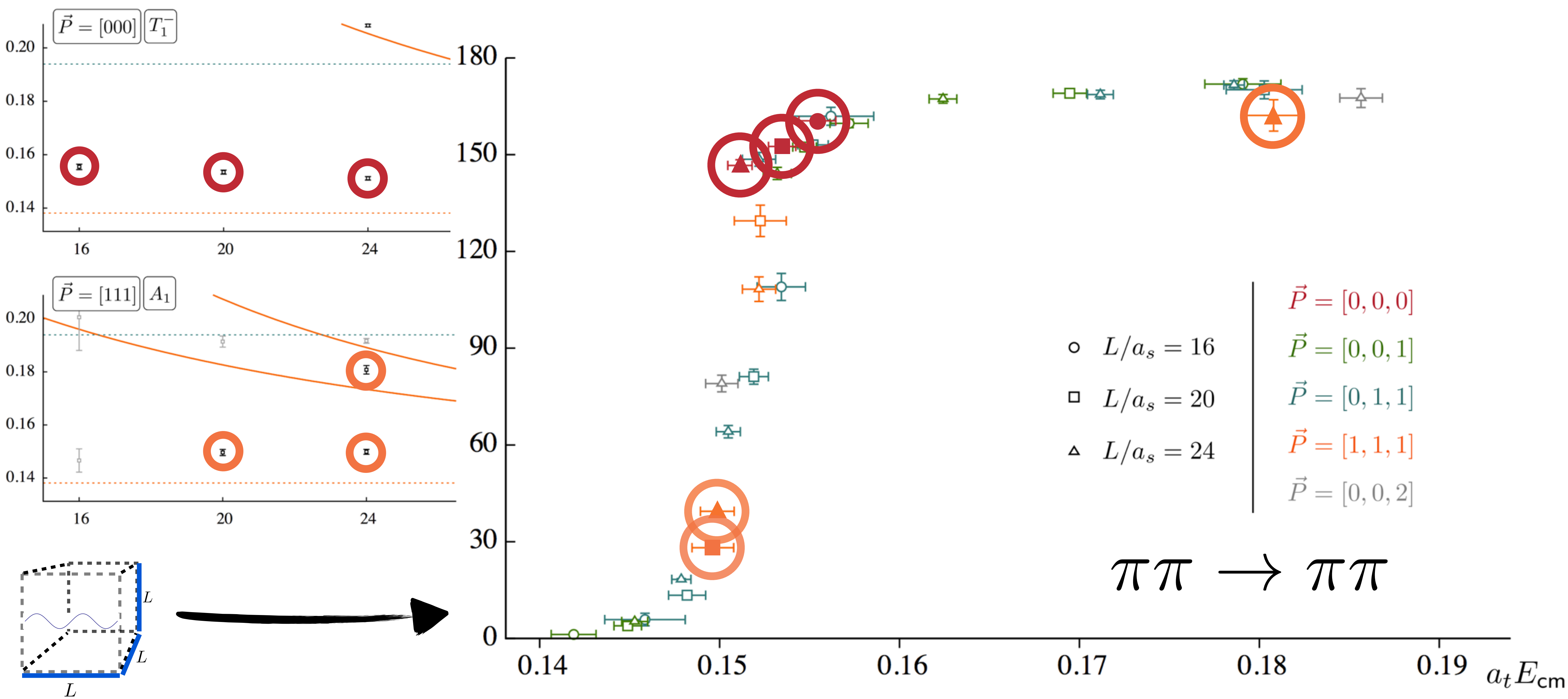

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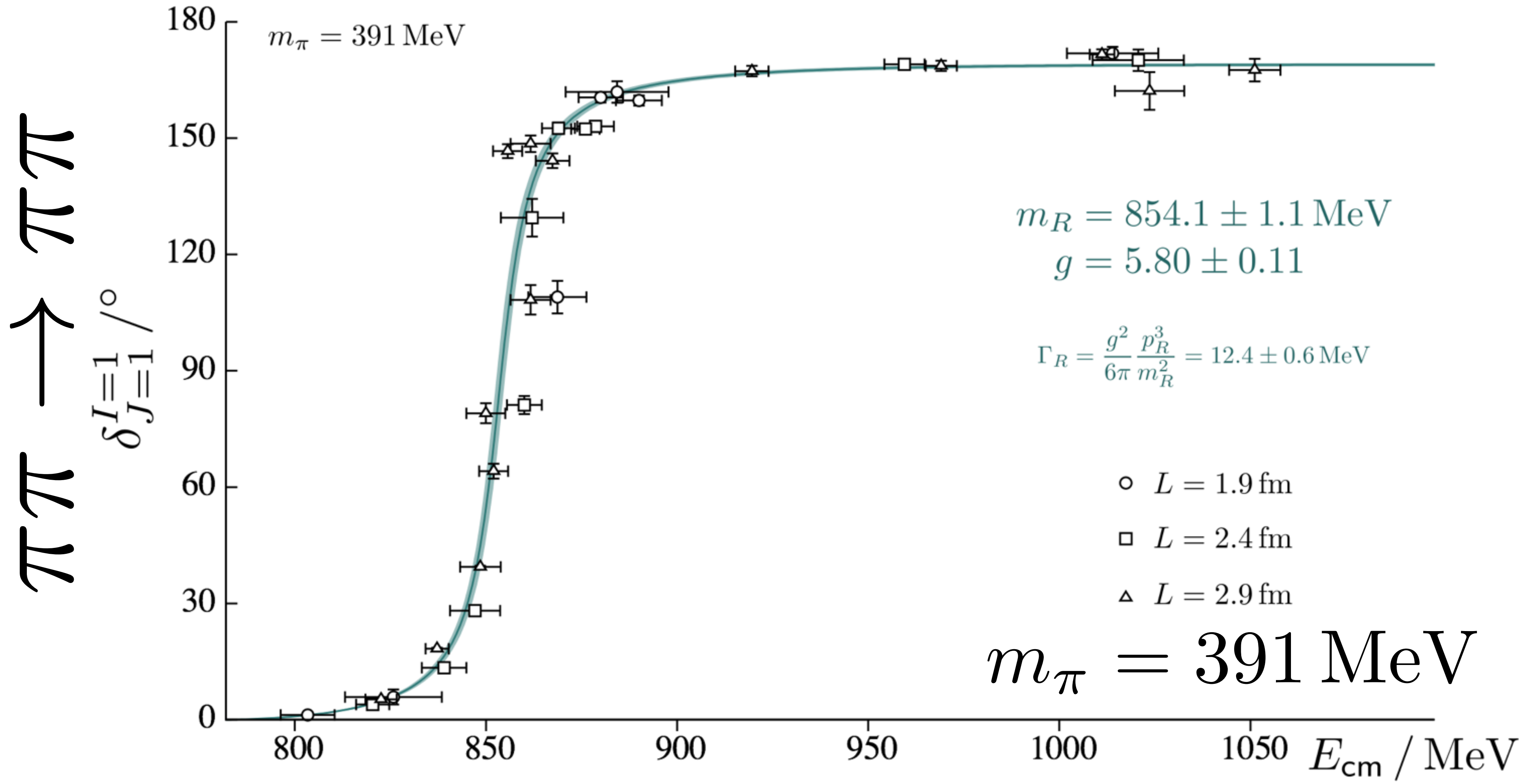

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$$\cot\delta_{\ell=1}(E_n^*) + \cot\phi(E_n, \vec{P}, L) = 0$$

Caveats

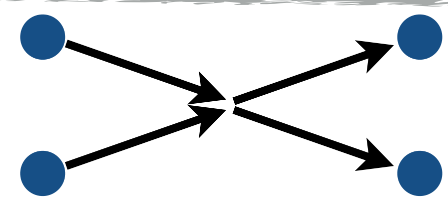
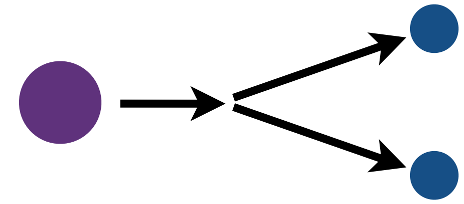
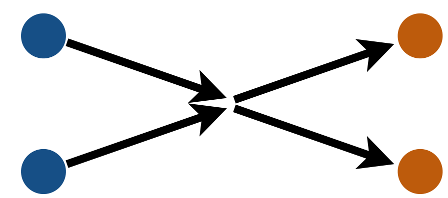
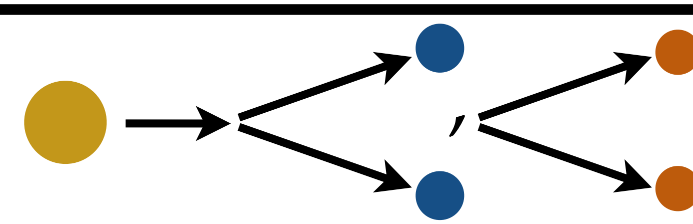
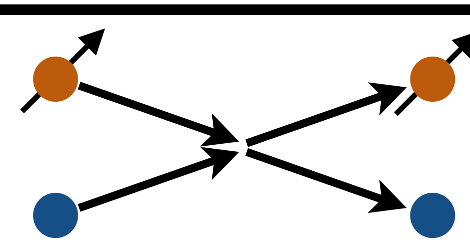
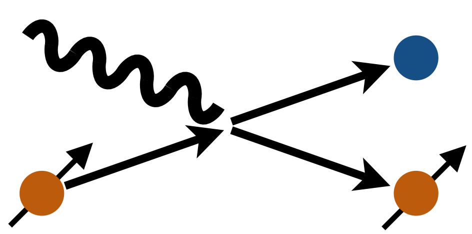
Neglects contributions scaling as $e^{-M_\pi L}$

Full result is a determinant of matrices in the partial-wave basis

Tower of partial waves contribute to each given finite-volume energy…

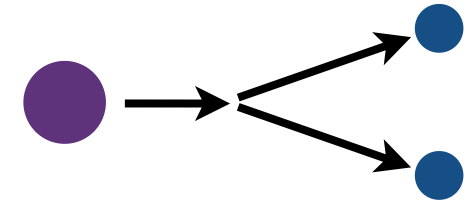
Must truncate to solve… can estimate uncertainty by varying truncation

Status of multi-hadron matrix elements in LQCD…

physical system		Method to get it from LQCD
elastic scattering of identical scalars		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)
decay into identical scalars (no other open decay channels)		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005), Christ, Kim and Yamazaki (2005)
non-identical scalars, multiple coupled channels*		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
decay into multiple, coupled two-particle channels*		MTH and Sharpe (2012)
scattering of particles with intrinsic spin*		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
particle production mediated by a generic local current*		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

***(assumes no three or four-particle channels open)**

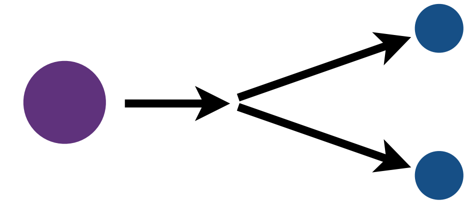
$K \to \pi\pi$ from LQCD



$$|\langle \pi\pi, L|\widetilde{\mathcal{H}}_W|K, L\rangle|^2 = \mathcal{B}[\delta_{\pi\pi}]\ |\langle \pi\pi, \mathrm{out}|\mathcal{H}_W|K\rangle|^2$$

$$\mathcal{B}[\delta_{\pi\pi}] = \frac{p}{32\pi M_K^2}\left[\frac{\partial}{\partial E}\left(\phi + \delta_{\pi\pi}\right)\right]^{-1}_{E=M_K}$$

Lellouch and Lüscher (2001)

$K \to \pi\pi$ from LQCD 

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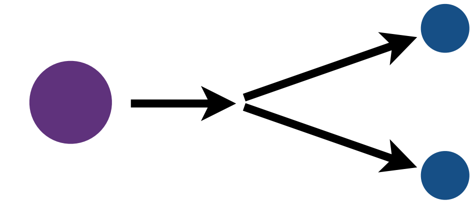
Lellouch and Lüscher (2001)

To convert finite-volume LQCD matrix elements to physically observable decay amplitudes one uses the Lellouch-Lüscher conversion factor $\mathcal{B}[\delta_{\pi\pi}]$.

$K \to \pi\pi$ from LQCD 

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Lellouch and Lüscher (2001)

To convert finite-volume LQCD matrix elements to physically observable decay amplitudes one uses the Lellouch-Lüscher conversion factor $\mathcal{B}[\delta_{\pi\pi}]$.

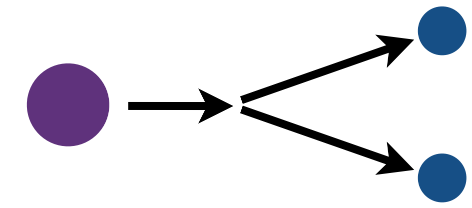
(1). **Determine finite-volume energies**

(2). **Use these to determine the (derivative of the) scattering phase**

(3). **Calculate the finite-volume matrix element**

(4). **Combine Lellouch-Lüscher factor and finite-volume matrix element to deduce decay rate**

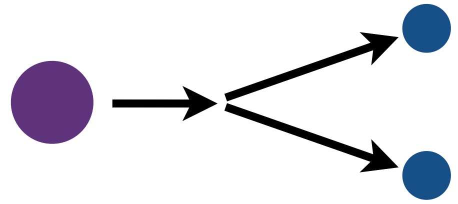
$K \to \pi\pi$ from LQCD



A full error budget LQCD calculation of this decay is being pursued by the RBC/UKQCD collaboration

(I=2:1502.00263, I=0: 1505.07863)

$K \to \pi\pi$ from LQCD



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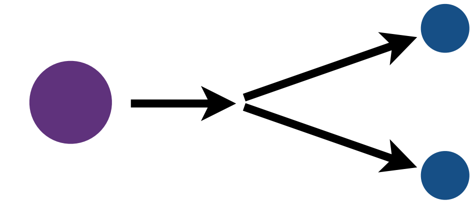
Isospin-two decay

Find significant cancellation between two dominant contributions (insight on $\Delta I = 1/2$ rule)

$K \to \pi\pi$ from LQCD



A full error budget LQCD calculation of this decay is being pursued by the RBC/UKQCD collaboration

(I=2:1502.00263, I=0: 1505.07863)

Isospin-two decay

Find significant cancellation between two dominant contributions (insight on $\Delta I = 1/2$ rule)

More difficult isospin-zero decay

$$\mathrm{Re}[A_0] = 4.66(1.00)(1.26) \times 10^{-7}\,\mathrm{GeV}$$

$$\mathrm{Re}[A_0]_{\mathrm{expt}} = 3.3201(18) \times 10^{-7}\,\mathrm{GeV}$$

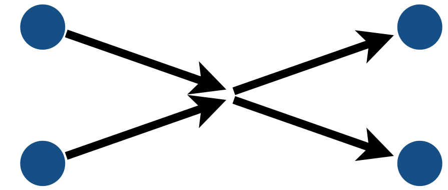
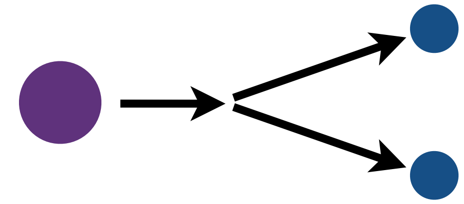
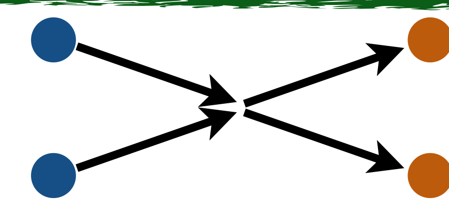
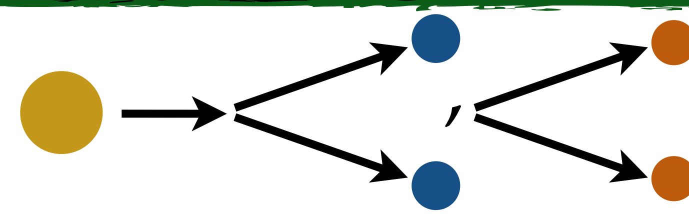
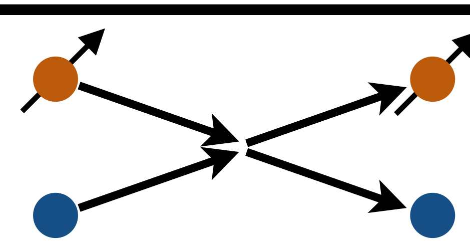
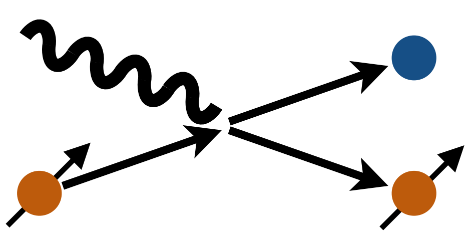
Direct CP violating ratio

$$\mathrm{Im}[A_0] = -1.90(1.23)(1.08) \times 10^{-11}\,\mathrm{GeV} \longrightarrow \mathrm{Re}[\varepsilon'/\varepsilon] = 1.38(5.15)(4.59) \times 10^{-4}$$

$$\mathrm{Re}[\varepsilon'/\varepsilon]_{\mathrm{expt}} = 16.6(2.3) \times 10^{-4}$$

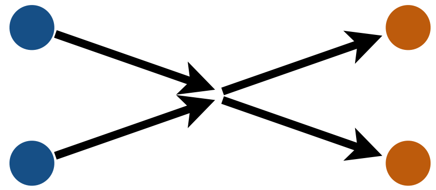
Only LQCD study of a multi-hadron decay so far

Status of multi-hadron matrix elements in LQCD…

physical system	Method to get it from LQCD	
elastic scattering of identical scalars		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)
decay into identical scalars (no other open decay channels)		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005), Christ, Kim and Yamazaki (2005)
non-identical scalars, multiple coupled channels*		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
decay into multiple, coupled two-particle channels*		MTH and Sharpe (2012)
scattering of particles with intrinsic spin*		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
particle production mediated by a generic local current*		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

***(assumes no three or four-particle channels open)**

Multiple two-particle channels

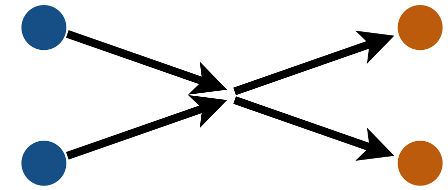


Must now include a channel index

MTH and Sharpe/Briceño and Davoudi

$$\det\left[\begin{pmatrix}\mathcal{M}_{a\to a} & \mathcal{M}_{a\to b}\\ \mathcal{M}_{b\to a} & \mathcal{M}_{b\to b}\end{pmatrix}^{-1}+\begin{pmatrix}F_a & 0\\ 0 & F_b\end{pmatrix}\right]=0$$

Multiple two-particle channels



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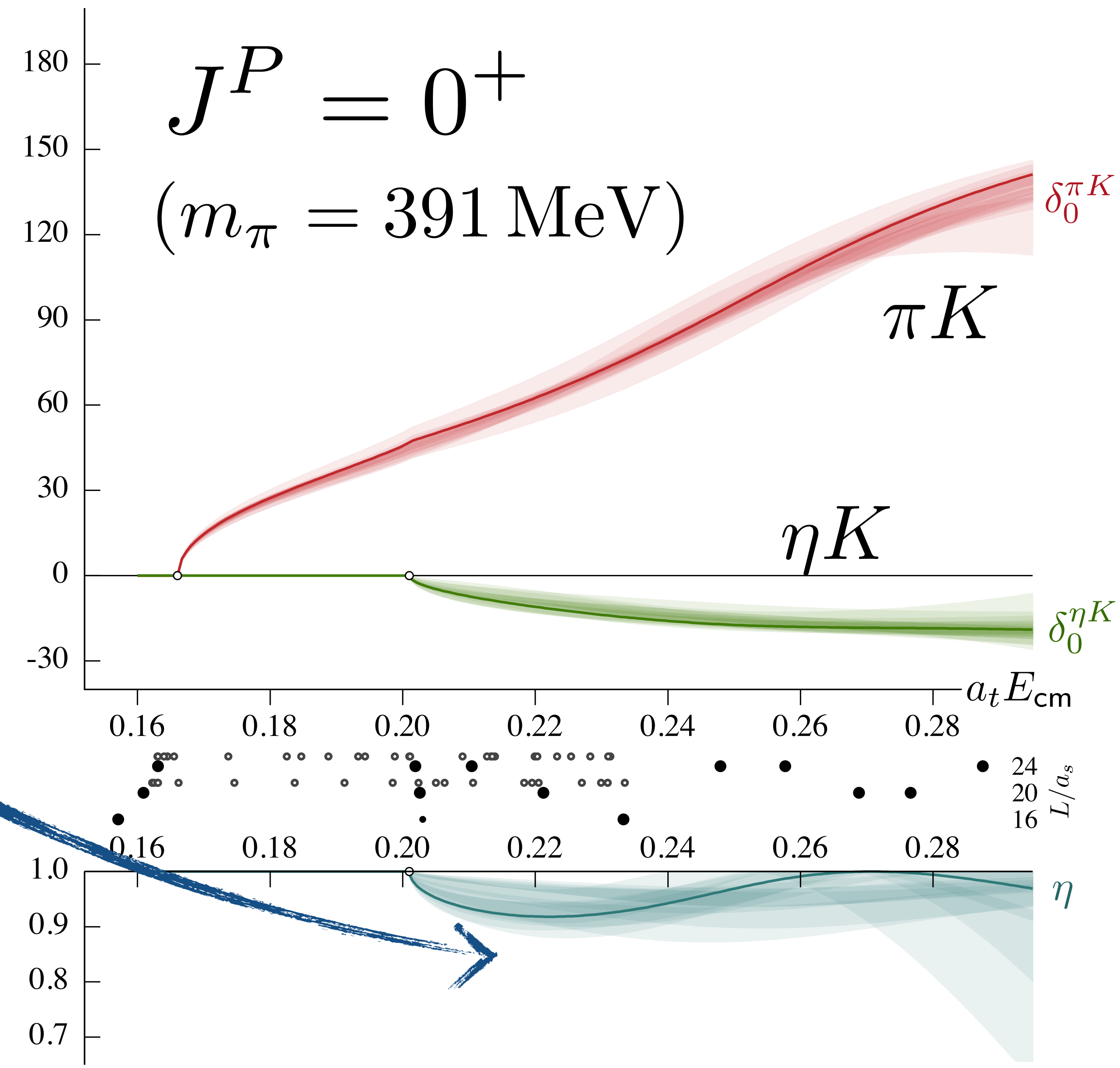
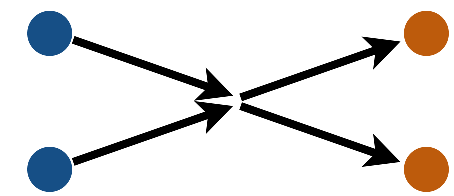
First used in HadSpec study of
$\pi K,\ \eta K$

$$\mathcal{M}(\pi K\to\eta K)\sim\sqrt{1-\eta^2}$$



Wilson, Dudek, Edwards, Thomas,
Phys. Rev. D 91, 054008 (2015)
arXiv: 1411.2004

Multiple two-particle channels



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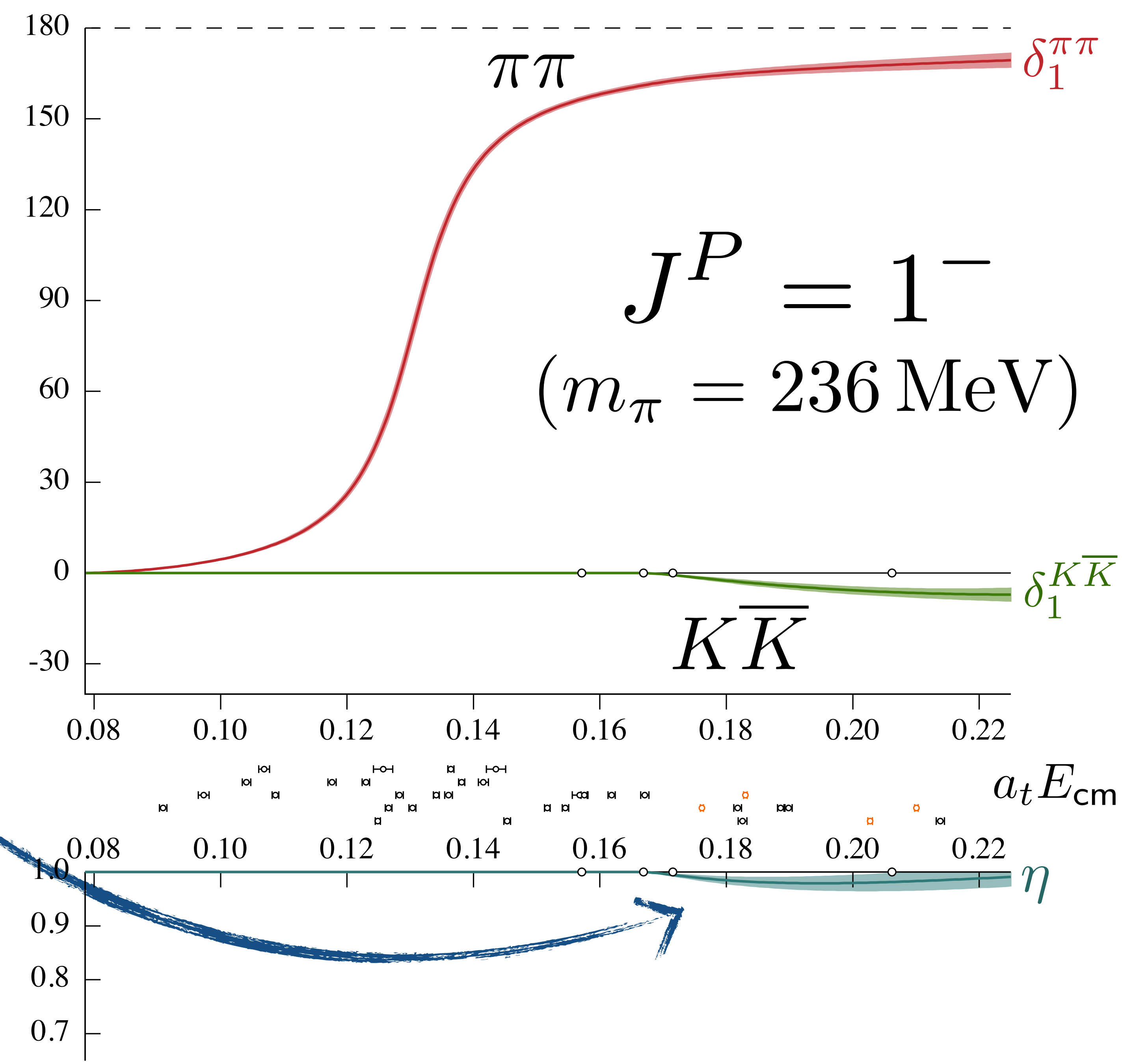
As well as JLab rho study with

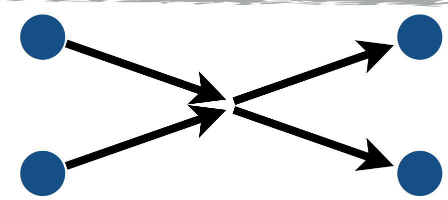
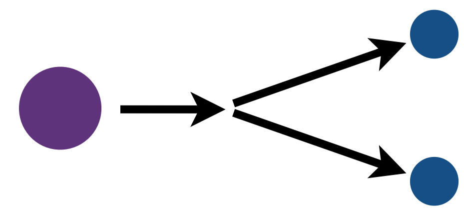
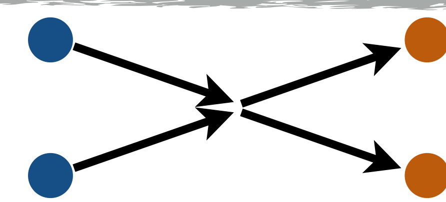
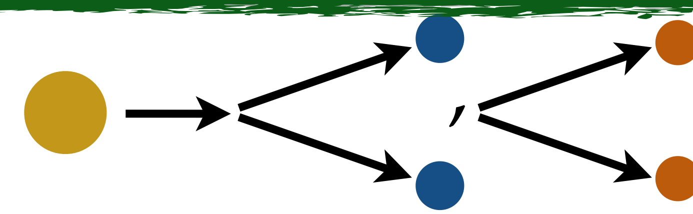
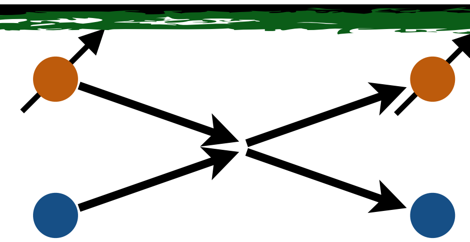
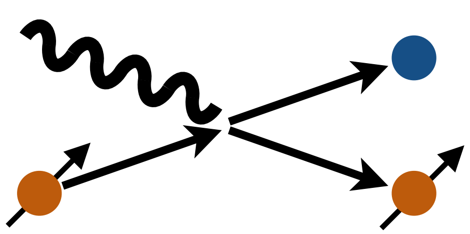
$$\pi\pi,\ \ K\overline{K}$$

$$\mathcal{M}(\pi\pi\to K\overline{K})\sim\sqrt{1-\eta^2}$$



Wilson, Briceño, Dudek, Edwards, Thomas,
Phys. Rev. D 92, 094502 (2015)
arXiv:1507:02599

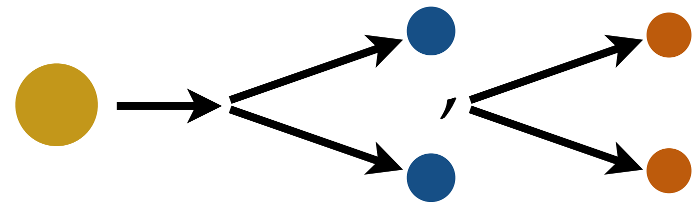
Status of multi-hadron matrix elements in LQCD…

physical system		Method to get it from LQCD
elastic scattering of identical scalars		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)
decay into identical scalars (no other open decay channels)		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005), Christ, Kim and Yamazaki (2005)
non-identical scalars, multiple coupled channels*		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
decay into multiple, coupled two-particle channels*		MTH and Sharpe (2012)
scattering of particles with intrinsic spin*		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
particle production mediated by a generic local current*		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

*(assumes no three or four-particle channels open)

D decays from LQCD?...

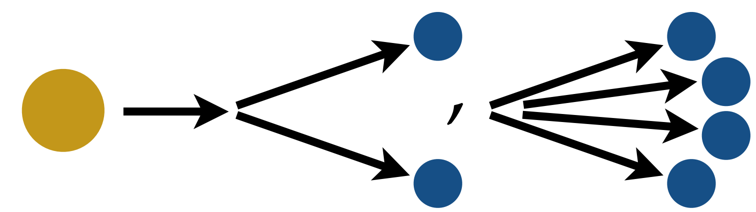
$D \to \pi\pi,\ K\overline{K}$
(ignores four-particle states)



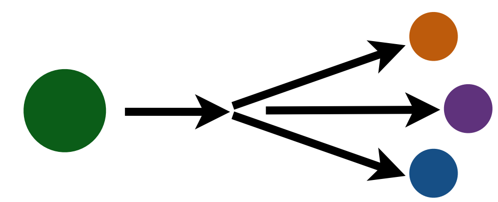
MTH and Sharpe (2012)

LQCD formalism has not yet been developed for

$D \to \pi\pi,\ K\overline{K},\ \pi\pi\pi\pi$



$B^\pm \to K^\pm K^+ K^-,\ \Lambda_b \to J/\psi\, p\, \pi^-$



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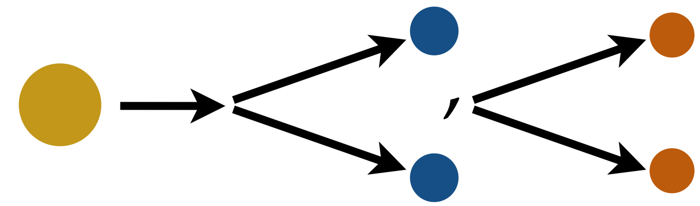

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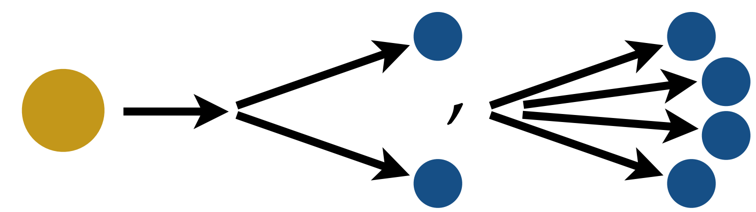
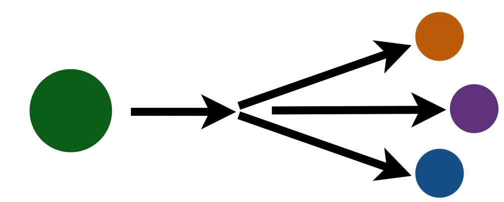


$B^{\pm} \to K^{\pm}K^{+}K^{-},\ \Lambda_b \to J/\psi\, p\, \pi^{-}$



Three-particle scattering formalism has been developed for pions and extensions to all systems are underway

(MTH and Sharpe, arXiv:1408.5933 and 1504.04248)

D decays from LQCD?…

$$D \to \pi\pi,\ K\overline{K}$$

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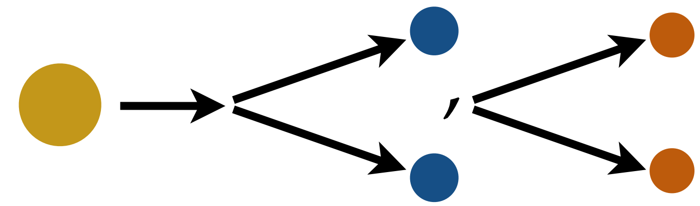
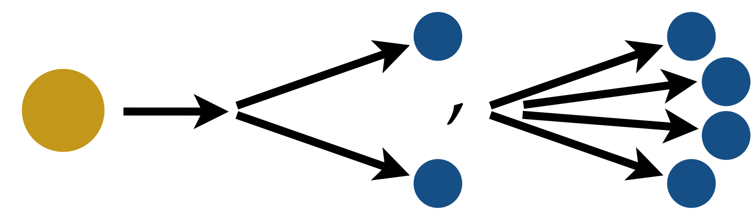


MTH and Sharpe (2012)

LQCD formalism has not yet been developed for

$$D \to \pi\pi,\ K\overline{K},\ \pi\pi\pi\pi$$



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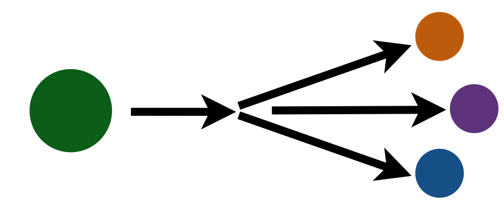

Three-particle scattering formalism has been developed for pions and extensions to all systems are underway

(MTH and Sharpe, arXiv:1408.5933 and 1504.04248)

Even if we just want two-particle decays…
These can only be studied rigorously in LQCD by including the effects of all open thresholds

This is the central limitation of all current formalism

The finite-volume mixes all open channels

D decays from LQCD?...

If we ignore four (and higher) particle states then

$$|\langle n, L|\widetilde{\mathcal{H}}_W|D, L\rangle| = |b_{\pi\pi}\langle \pi\pi, \mathrm{out}|\mathcal{H}_W|D\rangle + b_{K\overline{K}}\langle K\overline{K}, \mathrm{out}|\mathcal{H}_W|D\rangle + \cdots|$$

MTH and Sharpe, 1204.0826

Like the original Lellouch-Lüscher factor $b_{\pi\pi}$ and $b_{K\overline{K}}$ depend on derivatives of QCD scattering amplitudes

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MTH and Sharpe, 1204.0826

Like the original Lellouch-Lüscher factor $b_{\pi\pi}$ and $b_{K\overline{K}}$ depend on derivatives of QCD scattering amplitudes

(1). Determine finite-volume energies

(2). Use these to determine the (derivatives of) all scattering parameters in the coupled-channel sector

(3). Calculate multiple finite-volume matrix elements

(4). Deduce multiple, linearly independent relations between finite- and infinite-volume matrix elements

(5). Solve for the infinite-volume decay amplitudes

D decays from LQCD?...

If we ignore four (and higher) particle states then

$$|\langle n, L|\widetilde{\mathcal{H}}_W|D, L\rangle| = |b_{\pi\pi}\langle \pi\pi, \mathrm{out}|\mathcal{H}_W|D\rangle + b_{K\overline{K}}\langle K\overline{K}, \mathrm{out}|\mathcal{H}_W|D\rangle + \cdots|$$

MTH and Sharpe, 1204.0826

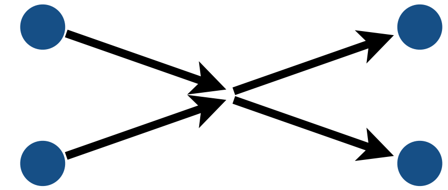
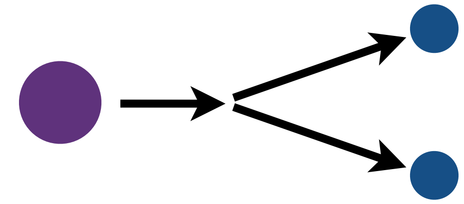
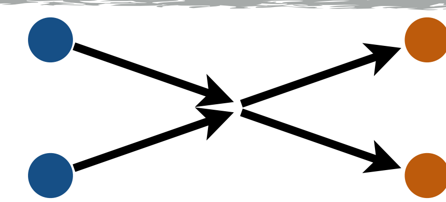
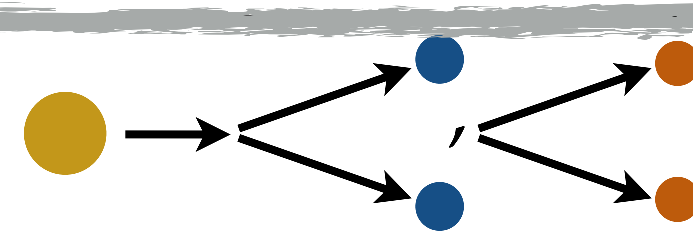
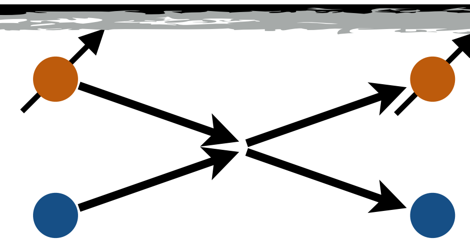
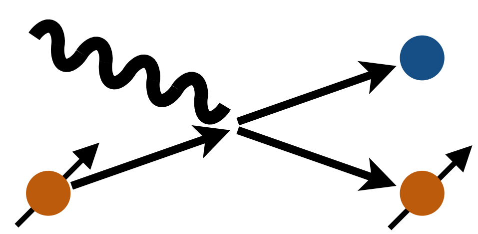
This sounds very challenging!
Probably need tricks to make progress

(Example: Maybe we can find certain energy-volume combinations where one coefficient dominates?)

Turning on four-particle states is the biggest challenge.
We expect this will give rise to additional terms on RHS.
Are they suppressed (in certain cases)?

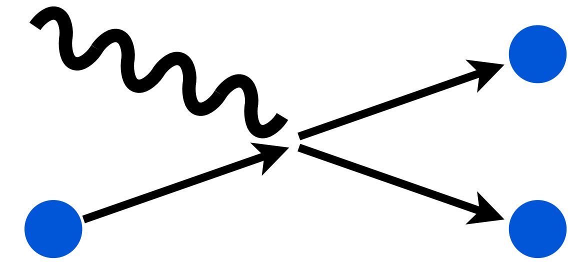
This basic story applies to all heavy multi-hadron decays

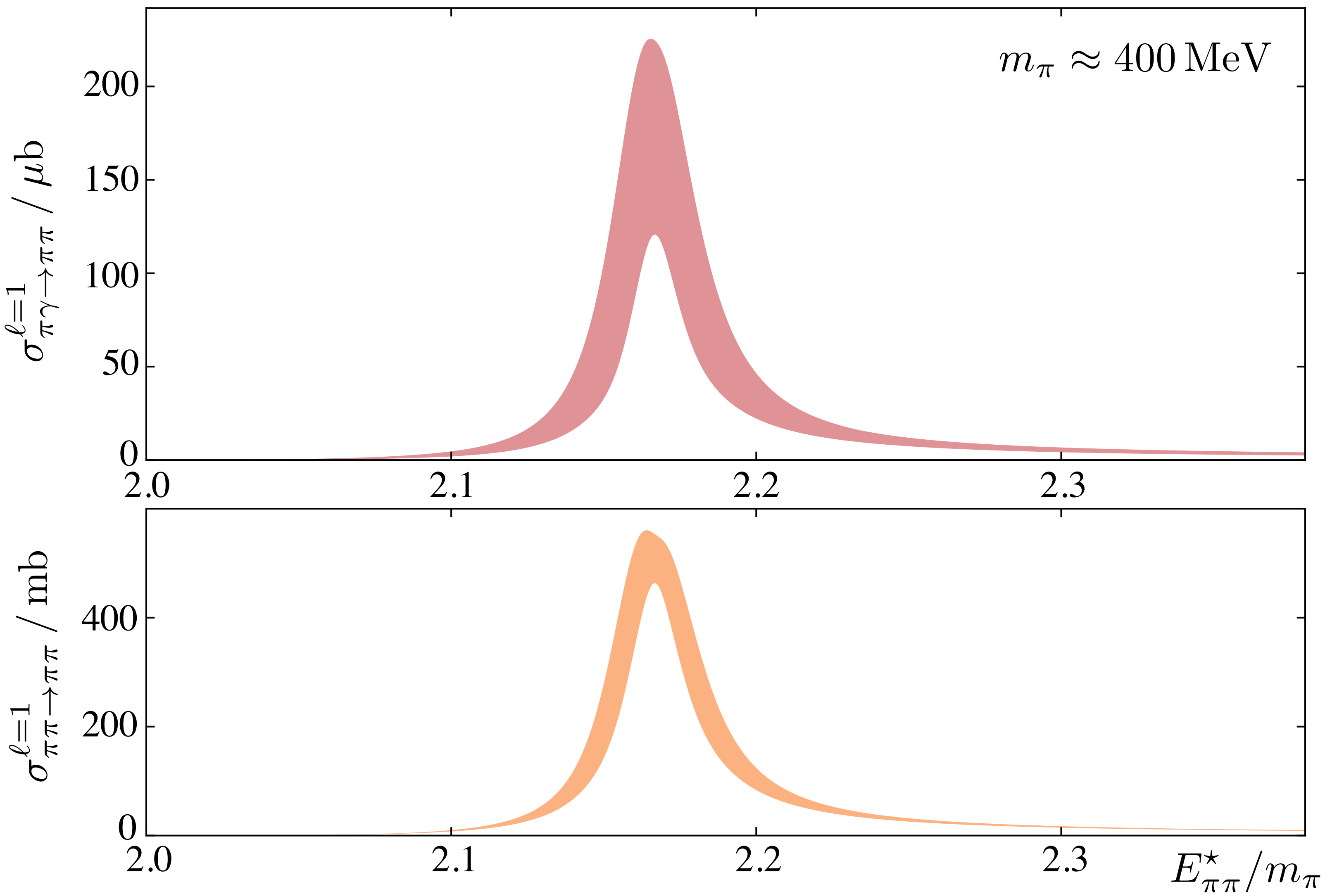
Status of multi-hadron matrix elements in LQCD…

physical system		Method to get it from LQCD
elastic scattering of identical scalars		Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)
decay into identical scalars (no other open decay channels)		Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005), Christ, Kim and Yamazaki (2005)
non-identical scalars, multiple coupled channels*		Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
decay into multiple, coupled two-particle channels*		MTH and Sharpe (2012)
scattering of particles with intrinsic spin*		Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
particle production mediated by a generic local current*		Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

***(assumes no three or four-particle channels open)**

Photoproduction $\langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle \equiv$



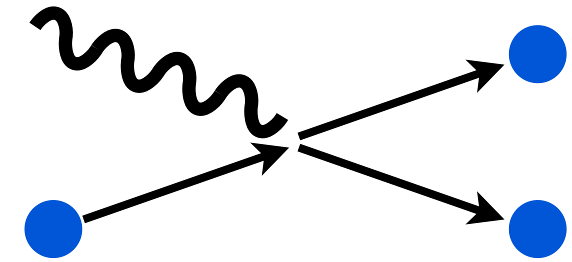


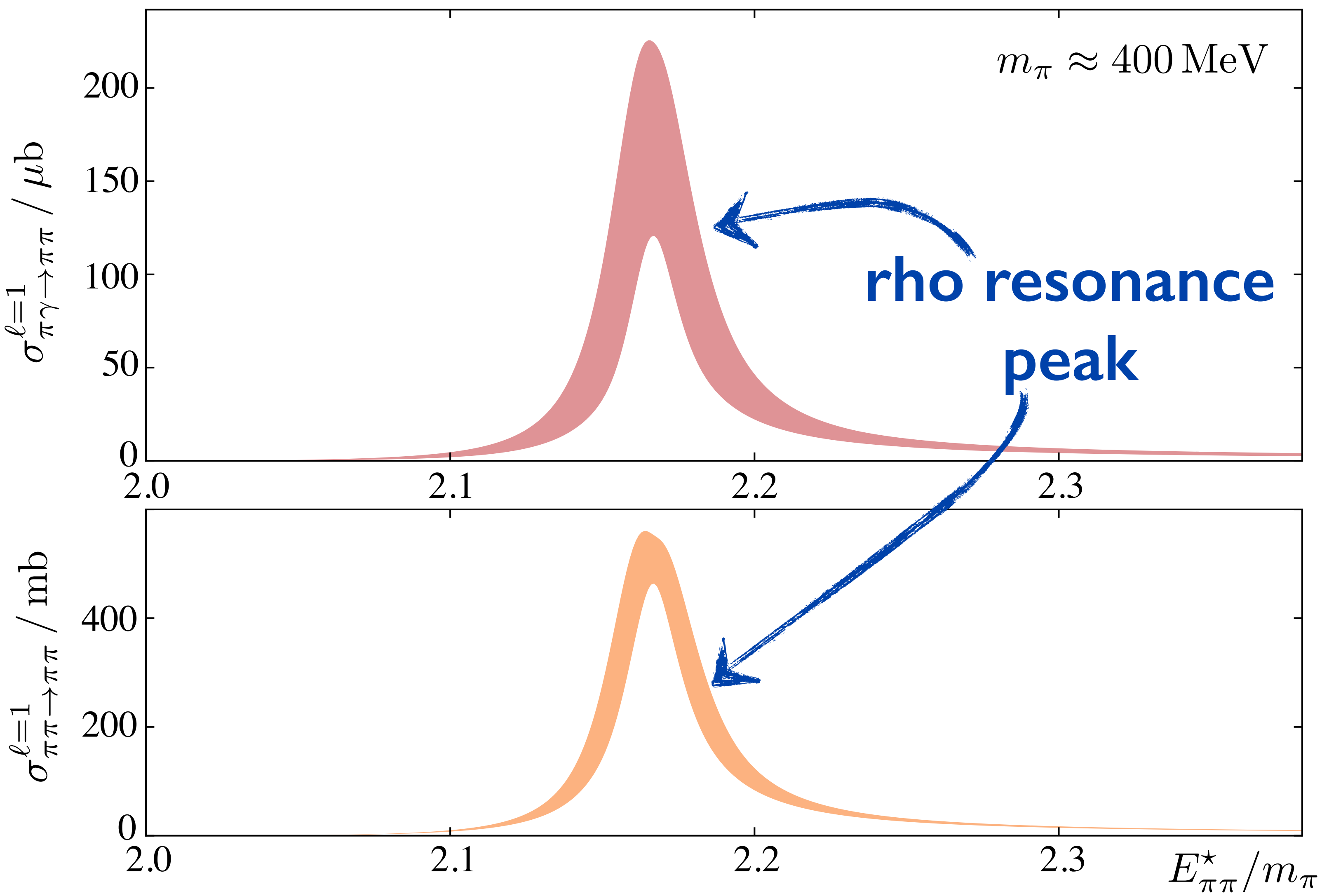
Photoproduction in the rho channel

Briceño, Dudek, Edwards, Schultz, Thomas, Wilson arXiv: 1507.6622

Same technology is needed for

$$B \to K^*(\to K\pi)\ell\ell$$

Photoproduction $\langle \pi\pi, \mathrm{out} | \mathcal{J}_\mu | \pi \rangle \equiv$ 



Photoproduction in the rho channel

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Conclusions

Multi-hadron decays and transitions are very challenging for LQCD

I am very interested to know which multi-hadron matrix elements are most important for using experiment to constrain new physics

Note: The technology discussed here is also relevant for the long-distance contributions to neutral meson mixing

Stay tuned for future LQCD calculations of these difficult quantities