CP violation in Charm: theoretical perspective

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"Strong reasons make strong actions."

William Shakespeare, King John (1598), Act III, scene 4, line 182
Introduction

★ Fundamental problem: observation of CP-violation in up-quark sector!
★ Possible sources of CP violation in charm transitions:

★ **CPV in $\Delta c = 1$ decay amplitudes** ("direct" CPV)

$$\Gamma(D \to f) \neq \Gamma(CP[D] \to CP[f])$$

★ **CPV in $D^0 - \bar{D}^0$ mixing matrix ($\Delta c = 2$):**

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle \Rightarrow |D_{CP\pm}\rangle = \frac{1}{\sqrt{2}}(|D^0\rangle \pm |\bar{D}^0\rangle)$$

$$R_m^2 = |\frac{q}{p}|^2 = \left| \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta \Gamma} \right|^2 = 1 + A_m \neq 1$$

★ **CPV in the interference of decays with and without mixing**

$$\lambda_f = \frac{q}{p} \frac{A_f}{A_f} = R_m e^{i(\phi + \delta)} \frac{|A_f|}{A_f}$$

★ One can separate various sources of CPV by customizing observables
CP-violation I: indirect

★ Indirect CP-violation manifests itself in $D\bar{D}$-oscillations

★ “Experimental” mass and lifetime differences of mass eigenstates...

\[ x_D = \frac{M_2 - M_1}{\Gamma_D}, \quad y_D = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D} \]

★ ...can be calculated as real and imaginary parts of a correlation function

\[ y_D = \frac{1}{2M_D \Gamma_D} \text{Im} \langle \overline{D}^0 | i \int d^4x T\{ \mathcal{H}_w[^{\Delta C}=1(x) \mathcal{H}_w[^{\Delta C}=1(0) \} | D^0 \rangle \]

\[ x_D = \frac{1}{2M_D \Gamma_D} \text{Re} \left[ 2\langle \overline{D}^0 | H[^{\Delta C}=2 | D^0 \rangle + \langle \overline{D}^0 | i \int d^4x T\{ \mathcal{H}_w[^{\Delta C}=1(x) \mathcal{H}_w[^{\Delta C}=1(0) \} | D^0 \rangle \right] \]

★ Theoretically, $y_D$ is dominated by long-distance SM-dominated effects
★ CP-violating phases can appear from subleading local SM or NP operators
$y = 0.66^{+0.07}_{-0.10}\%$, $x = 0.37 \pm 0.16$

Note that if $|M_{12}| < |\Gamma_{12}|$:  
\[
x/y = 2 \left| M_{12}/\Gamma_{12} \right| \cos \phi_{12},
\]
\[
A_m = 4 \left| M_{12}/\Gamma_{12} \right| \sin \phi_{12},
\]
\[
\phi = -2 \left| M_{12}/\Gamma_{12} \right|^2 \sin 2\phi_{12}.
\]

CPV is suppressed even if $M_{12}$ is all NP!!!
★ Indirect CP-violation manifests itself in $\bar{D}D$-oscillations
- see time development of a D-system:

$$i \frac{d}{dt} |D(t)\rangle = \left( M - \frac{i}{2} \Gamma \right) |D(t)\rangle$$

$$\langle D^0 | \mathcal{H} | D^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}$$

★ Define "theoretical" mixing parameters

$$y_{12} \equiv |\Gamma_{12}|/\Gamma, \quad x_{12} \equiv 2|M_{12}|/\Gamma, \quad \phi_{12} \equiv \text{arg}(M_{12}/\Gamma_{12})$$

★ Assume that direct CP-violation is absent ($\text{Im} (\Gamma_{12}^* \bar{A}_f / A_f) = 0, |\bar{A}_f / A_f| = 1$)
- can relate $x$, $y$, $\varphi$, $|q/p|$ to $x_{12}$, $y_{12}$ and $\phi_{12}$

"superweak limit"

$$xy = x_{12} y_{12} \cos \phi_{12}, \quad x^2 - y^2 = x_{12}^2 - y_{12}^2,$$

$$(x^2 + y^2)|q/p|^2 = x_{12}^2 + y_{12}^2 + 2x_{12}y_{12} \sin \phi_{12},$$

$$x^2 \cos^2 \phi - y^2 \sin^2 \phi = x_{12}^2 \cos^2 \phi_{12}.$$  

★ Four "experimental" parameters related to three "theoretical"
- a "constraint" equation is possible

$$\frac{x}{y} = \frac{1 - |q/p|}{\tan \phi} = \frac{1}{2} \frac{A_m}{\tan \phi}$$
Generic restrictions on NP from D\bar{D}-mixing

★ Comparing to experimental value of \( x \), obtain constraints on NP models
  - assume \( x \) is dominated by the New Physics model
  - assume no accidental strong cancellations b/w SM and NP

\[
H_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} z_i(\mu) Q_i'
\]

\[
Q_1^{cu} = \bar{u}_L^\alpha \gamma_\mu c_L^\alpha \bar{u}_L^\beta \gamma_\mu c_L^\beta,
Q_2^{cu} = \bar{u}_R^\alpha c_L^\alpha \bar{u}_R^\beta c_L^\beta,
Q_3^{cu} = \bar{u}_R^\alpha c_L^\beta \bar{u}_R^\beta c_L^\alpha
\]

★ ... which are

\[
|z_1| \lesssim 5.7 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
|z_2| \lesssim 1.6 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
|z_3| \lesssim 5.8 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
|z_4| \lesssim 5.6 \times 10^{-8} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
|z_5| \lesssim 1.6 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2.
\]

New Physics is either at a very high scales

- tree level: \( \Lambda_{NP} \geq (4 - 10) \times 10^3 \text{ TeV} \)
- loop level: \( \Lambda_{NP} \geq (1 - 3) \times 10^2 \text{ TeV} \)

or have highly suppressed couplings to charm!

★ Constraints on particular NP models available

Gedalia, Grossman, Nir, Perez
Phys.Rev.D80, 055024, 2009

E.Golowich, J. Hewett, S. Pakvasa and A.A.P.
**CP-violation I: indirect**

★ Assume that direct CP-violation is absent \((\text{Im}(\Gamma^*_f A_f/A_f) = 0, |A_f/A_f| = 1)\)
- experimental constraints on \(x, y, \varphi, |q/p|\) exist
- can obtain generic constraints on \(\text{Im}\) parts of Wilson coefficients

\[
\mathcal{H}_{NP}^{\Delta C=2} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} z_i(\mu) Q'_i
\]

★ In particular, from \(x_{12}^{NP} \sin \phi_{12}^{NP} \lesssim 0.0022\)

\[
\begin{align*}
\text{Im}(z_1) & \lesssim 1.1 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
\text{Im}(z_2) & \lesssim 2.9 \times 10^{-8} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
\text{Im}(z_3) & \lesssim 1.1 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
\text{Im}(z_4) & \lesssim 1.1 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
\text{Im}(z_5) & \lesssim 3.0 \times 10^{-8} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2.
\end{align*}
\]

New Physics is either at a very high scales
- tree level: \(\Lambda_{NP} \geq (4 - 10) \times 10^3 \text{ TeV}\)
- loop level: \(\Lambda_{NP} \geq (1 - 3) \times 10^2 \text{ TeV}\)

or have highly suppressed couplings to charm!

★ Constraints on particular NP models possible as well

**References**

- Bigi, Blanke, Buras, Recksiegel, JHEP 0907:097, 2009
★ Look at parameterization of CPV phases; separate absorptive and dispersive

\[ \lambda_f^2 = \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}} \left( \frac{A_f}{\bar{A}_f} \right)^2 \]

- consider \( f \) is CP eigenstate, can generalize later: \( \lambda_{CP}^2 = R_m^2 e^{2i\phi} \)

\[ \phi_{12f}^M = \frac{1}{2} \arg \left[ \frac{M_{12}}{M_{12}^*} \left( \frac{A_f}{\bar{A}_f} \right)^2 \right] \]
\[ \phi_{12f}^\Gamma = \frac{1}{2} \arg \left[ \frac{\Gamma_{12}}{\Gamma_{12}^*} \left( \frac{A_f}{\bar{A}_f} \right)^2 \right] \]

- CP-violating phase for the final state \( f \) is then

\[ \phi_{12} = \phi_{12f}^M - \phi_{12f}^\Gamma \]

★ Can we put a Standard Model theoretical bound on \( \phi_{12f}^M \) or \( \phi_{12f}^\Gamma \)?
Let us define convention-independent universal CPV phases. First note that
- for the absorptive part: \( \Gamma_{12} = \Gamma_{12}^0 + \delta \Gamma_{12} \)
  \[
  \Gamma_{12}^0 = -\lambda_s (\Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd}) \\
  \delta \Gamma_{12} = 2\lambda_b \lambda_s (\Gamma_{sd} - \Gamma_{ss}) + O(\lambda_b^2)
  \]
- ... and similarly for the dispersive part: \( M_{12} = M_{12}^0 + \delta M_{12} \)

**CP-violating mixing phase can then be written as**

\[
\phi_{12} = \arg \frac{M_{12}}{\Gamma_{12}} = \Im \left( \frac{\delta M_{12}}{M_{12}^0} \right) - \Im \left( \frac{\delta \Gamma_{12}}{\Gamma_{12}^0} \right) \equiv \phi_M^{\Gamma_{12}} - \phi^{\Gamma_{12}}
\]

**These phases can then be constrained; e.g. the absorptive phase**

\[
|\phi_{12}^{\Gamma_{12}}| = 0.009 \times \left| \frac{\Gamma_{sd}}{\Gamma} \right| \times \left| \frac{\Gamma_{sd} - \Gamma_{dd}}{\Gamma_{sd}} \right| < 0.01
\]

**Currently, \( \phi_{12} = 0.2 \pm 1.7 \) Need improvement!**
Charmed CKM triangle

★ Fundamental problem: observation of CP-violation in up-quark sector!

★ “Charmed” CKM triangle is very squashed in the Standard Model

\[
1 + \frac{V_{ub} V_{cb}}{V_{us} V_{cs}} + \frac{V_{ud} V_{cd}}{V_{us} V_{cs}} = 0
\]

\[
\left| \frac{V_{ud} V_{cd}}{V_{us} V_{cs}} \right| = 1 + \mathcal{O}(\lambda^4) \quad \left| \frac{V_{ub} V_{cb}}{V_{us} V_{cs}} \right| \sim \mathcal{O}(\lambda^4)
\]

★ ... with very small angles, e.g.

\[
\chi' = \arg \left( \frac{V_{ud} V_{cd}}{V_{us} V_{cs}} \right) \simeq A^2 \lambda^4 \eta \simeq 1.6 \cdot 10^{-3} \eta
\]
CP-violation II: direct

★ Form final state the asymmetry

\[ a_f = \frac{\Gamma(D \rightarrow f) - \Gamma(D \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(D \rightarrow \bar{f})} \]

★ Could consider the DIFFERENCE of decay rate asymmetries: \( D \rightarrow \pi\pi \) vs \( D \rightarrow KK \), as \( a_{mKK} = a_{m\pi\pi} \) and \( a_{iKK} = a_{i\pi\pi} \) (for CP-eigenstate final states), so, ideally, mixing asymmetries cancel!

\[ a_f^d = 2r_f \sin\phi_f \sin\delta_f \]

★ ... and the resulting DCPV asymmetry is \( \Delta a_{CP} = a_{KK}^d - a_{\pi\pi}^d \approx 2a_{KK}^d \) (double!)

\[ A_{KK} = \frac{G_F}{\sqrt{2}} \lambda [(T + E + P_{sd}) + a\lambda^4 e^{-i\gamma} P_{bd}] \]

\[ A_{\pi\pi} = \frac{G_F}{\sqrt{2}} \lambda [-(T + E) + P_{sd}) + a\lambda^4 e^{-i\gamma} P_{bd}] \]

★ ... so it is doubled in the limit of SU(3)\(_F\) symmetry

SU(3) is badly broken in D-decays
\[ e.g. \text{Br}(D \rightarrow KK) \sim 3 \text{ Br}(D \rightarrow \pi\pi) \]
Experimentally

![Diagram showing experimental results for CP violation in charm quark measurements.](Image)
Theoretical troubles

★ These asymmetries are notoriously difficult to compute

★ In the Standard Model
  - need to estimate size of penguin/penguin contractions vs. tree
  - unknown penguin contribution (similar to $\Delta I = 1/2$)
    - SU(3) analysis: some ME are enhanced?
      Golden & Grinstein PLB 222 (1989) 501; Pirtshalava & Uttayarat 1112.5451
    - could expect large $1/m_c$ corrections
      Isidori et al PLB 711 (2012) 46; Brod et al 1111.5000
    - no assumptions, flavor-flow diagrams
      Broad et al 1203.6659; Bhattacharya et al PRD 85 (2012) 054014;
      Cheng & Chiang 1205.0580

★ Theoretical progress in the coming months?
  - QCD sum rule calculations of $\Delta a_{CP}$
    Khodjamirian, AAP
  - SU(3) breaking analyses of $D \rightarrow PV, VV$
  - constant (but slow) lattice QCD progress in $D \rightarrow \pi\pi, \pi\pi\pi$
    Hansen, Sharpe

★ Other points: $CP$-fractions in Dalitz plot analyses
  - e.g., why is $D \rightarrow \pi^+\pi^-\pi^0 \sim 97\%$ CP-even?
    Bhattacharya, et al Gronau, Rosner
Is it a penguin or a tree?

Without QCD

With QCD
Charming “triangle analyses”?

★ “Triangle analyses” require a lot of data, but only rely on isospin relations

- several final states possible, for $D \rightarrow \pi^i \pi^k$

\[
\frac{1}{\sqrt{2}} A^{+-} = A^{+0} - A^{00},
\]
\[
\frac{1}{\sqrt{2}} A^{-+} = A^{-0} - A^{00},
\]

- others include $D \rightarrow \pi\pi, \rho\pi, \rho\rho$

<table>
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<th>Pionic modes</th>
<th>$\Gamma$</th>
<th>$\pi^+\pi^-$</th>
<th>$\pi^0\pi^0$</th>
<th>$\rho^+\rho^-$</th>
<th>$\rho^0\rho^0$</th>
<th>$\rho^-\rho^+$</th>
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<td>$\Gamma_1$</td>
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<tr>
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<tr>
<td>$\Gamma_3$</td>
<td>$\pi^+\pi^-\pi^0$</td>
<td>$(1.47 \pm 0.09)$%</td>
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<tr>
<td>$\Gamma_4$</td>
<td>$\rho^+\rho^-$</td>
<td>$(1.00 \pm 0.06)$%</td>
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<td>$\Gamma_5$</td>
<td>$\rho^0\rho^0$</td>
<td>$(3.82 \pm 0.29) \times 10^{-3}$</td>
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<tr>
<td>$\Gamma_6$</td>
<td>$\rho^-\rho^+$</td>
<td>$(5.09 \pm 0.34) \times 10^{-3}$</td>
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New Physics: operator analysis

★ Factorizing decay amplitudes, e.g.

\[ \mathcal{H}^{\text{eff-NP}}_{|\Delta c|=1} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2,5,6} \sum_q \left( C_i^q Q_i^q + C_i^{q'} Q_i^{q'} \right) + \frac{G_F}{\sqrt{2}} \sum_{i=7,8} \left( C_i Q_i + C_i^{Q'} Q_i^{Q'} \right) + \text{H.c.} \]

\[ Q_1^q = (\bar{u}q)_{V-A} (\bar{q}c)_{V-A} \]
\[ Q_2^q = (\bar{u}_\alpha q_\beta)_{V-A} (\bar{q}_\beta c_\alpha)_{V-A} \]
\[ Q_5^q = (\bar{u}c)_{V-A} (\bar{q}q)_{V+A} \]
\[ Q_6^q = (\bar{u}_\alpha c_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A} \]
\[ Q_7 = -\frac{e}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} c \]
\[ Q_8 = -\frac{g_s}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) T^a G^\mu\nu_a c \]

★ one can fit to \( \epsilon'/\epsilon \) and mass difference in D-anti-D-mixing
- LL are ruled out
- LR are borderline
- RR and dipoles are possible

Constraints from particular models also available


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<th>Disfavored</th>
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<td>( Q_{7,8} ), ( Q_{7,8}' )</td>
<td>( Q_{1,2}^{(c-u,8d,b,0)} ), ( Q_{1,2}^{(s-d,8d,b,0)} ), ( Q_{5,6}^{(s-d,8d,b)} ), ( Q_{5,6}^{(s-d,8d,b,0)} ), ( Q_{5,6}^{(8d,b)} ), ( Q_{5,6}^{(8d,b,0)} )</td>
<td></td>
</tr>
<tr>
<td>( \forall f )</td>
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</table>
"Having nothing, nothing can he lose."

William Shakespeare, "Henry VI"
Future: CP-violation in charmed baryons

Other observables can be constructed for baryons, e.g.

\[ A(\Lambda_c \rightarrow N\pi) = \bar{u}_N (p, s) [A_s + A_P \gamma_5] u_{\Lambda_c} (p_{\Lambda_c}, s_{\Lambda_c}) \]

These amplitudes can be related to “asymmetry parameter”

\[ \alpha_{\Lambda_c} = \frac{2 \text{Re}(A_s^* A_P)}{|A_s|^2 + |A_P|^2} \]

... which can be extracted from

\[ \frac{dW}{d \cos \theta} = \frac{1}{2} \left( 1 + P \alpha_{\Lambda_c} \cos \theta \right) \]

Same is true for \( \bar{\Lambda}_c \)-decay

If CP is conserved \( \alpha^{CP}_{\Lambda_c} \Rightarrow -\alpha_{\Lambda_c} \), thus CP-violating observable is

\[ A_f = \frac{\alpha_{\Lambda_c} + \overline{\alpha}_{\Lambda_c}}{\alpha_{\Lambda_c} - \overline{\alpha}_{\Lambda_c}} \]

FOCUS[2006]: \( A_{\Lambda_c \pi} = -0.07 \pm 0.19 \pm 0.24 \)
Things to take home

- Computation of charm amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
  - D-mixing is a second order effect in SU(3) breaking ($x,y \sim 1\%$ in the SM)

- For indirect CP-violation studies
  - constraints on Wilson coefficients of generic operators are possible, point to the scales much higher than those directly probed by LHC
  - consider new parameterizations that go beyond the “superweak” limit

- For direct CP-violation studies
  - unfortunately, large DCPV signal is no more; need more results in individual channels, especially including baryons
  - hit the “brown muck”: future observation of DCPV does not give easy interpretation in terms of fundamental parameters
  - need better calculations: lattice?

- Lattice calculations can, in the future, provide a result for $a_{CP}$!
- Need to give more thought on how large SM CPV can be…
"I'm looking for a lot of men who have an infinite capacity to not know what can't be done."

Henry Ford

"Strong reasons make strong actions."

William Shakespeare, King John (1598), Act III, scene 4, line 182
Rare radiative decays of charm

★ Can radiative charm decays help with $\Delta a_{CP}$?

★ In many NP models, there is a link between chromomagnetic and electric-dipole operators

\[
Q_8 = \frac{m_c}{4\pi^2} \bar{u}_L \sigma_{\mu\nu} T^a \epsilon_8 G^\mu\nu c_R
\]

\[
Q_7 = \frac{m_c}{4\pi^2} \bar{u}_L \sigma_{\mu\nu} Q_\epsilon e F^\mu\nu c_R
\]

Same is true for operators of opposite chirality as well

★ There are many operators that can generate $\Delta a_{CP}$

- one possibility is that NP affects $Q_8$ the most; the asymmetry then

\[
|\Delta a_{CP}^{NP}| \approx -1.8 |\text{Im}[C_8^{NP}(m_c)]|
\]

- e.g. in SUSY, gluino-mediated amplitude satisfies $C_7^{SUSY}(m_{SUSY}) = (4/15)C_8^{SUSY}(m_{SUSY})$

- then at the charm scale,

\[
|\text{Im}[C_7^{NP}(m_c)]| = (0.2 - 0.8) \times 10^{-2}
\]

\[
|C_7^{SM-eff}(m_c)| = (0.5 \pm 0.1) \times 10^{-2}
\]

What about LD effects?
CP-violation in radiative decays of charm

★ Probing $a_{CP}$ in radiative D-decays can probe $\text{Im} \ C_7 \to \text{Im} \ C_8 \to \Delta a_{CP}$
- problem is, radiative decays are dominated by LD effects

\[
\Gamma(D \to V\gamma) = \frac{m_D^3}{32\pi} \left(1 - \frac{m_V^2}{m_D^2}\right)^3 \left[|A_{PV}|^2 + |A_{PC}|^2\right].
\]

Isidori, Kamenik (12)

★ CP-violating asymmetry in radiative transitions would be

\[
\left|a_{(\rho,\omega)\gamma}\right|_{\text{max}} = 0.04(1) \left|\frac{\text{Im}[C_7(m_c)]}{0.4 \times 10^{-2}}\right| \times \left[\frac{10^{-5}}{\mathcal{B}(D \to (\rho,\omega)\gamma)}\right]^{1/2} \lesssim 10\%.
\]

Isidori, Kamenik (12)
Cappiello, Cata, D’Ambrosio (12)

★ Better go off-resonance (consider $K^+K^-\gamma$) or even $h^+h^-\mu^+\mu^-$ final states
- the LD effects would be smaller, but the rate goes down as well
Experimental analyses of mixing

★ In principle, can extract mixing $(x, y)$ and CP-violating parameters $(A_m, \varphi)$

★ In particular, time-dependent $D^0(t) \rightarrow K^+\pi^-$ analysis

$$
\Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |A_{K^+\pi^-}|^2 \left[ R + \sqrt{R} R_m \left( y' \cos \phi - x' \sin \phi \right) \Gamma t + \frac{R_m^2}{4} \left( x'^2 + y'^2 \right) (\Gamma t)^2 \right]
$$

$$
R_m^2 = \frac{q^2}{p}, \quad x' = x \cos \delta + y \sin \delta, \quad y' = y \cos \delta - x \sin \delta
$$

LHCb: $x'^2 = (-0.9 \pm 1.3) \times 10^{-4}$, $y' = (7.2 \pm 2.4) \times 10^{-3}$

★ The expansion can be continued to see how well it converges for large $t$

$$
\Gamma[D^0(t) \rightarrow K^+\pi^-] |A_{K\pi}|^{-2} e^{\Gamma t} = R - \sqrt{R} R_m \left( x \sin(\delta + \phi) - y \cos(\delta + \phi) \right) (\Gamma t)
$$

$$
+ \frac{1}{4} \left( (R_m - R) x^2 + (R + R_m) y^2 \right) (\Gamma t)^2
$$

$$
+ \frac{1}{6} \sqrt{R} R_m \left( x^3 \sin(\delta + \phi) + y^3 \cos(\delta + \phi) \right) (\Gamma t)^3
$$

$$
- \frac{1}{48} R_m \left( x^4 - y^4 \right) (\Gamma t)^4
$$
\[ \Delta c = 2 \text{ example: mixing} \]

★ Main goal of the exercise: understand physics at the most fundamental scale

★ It is important to understand relevant energy scales for the problem at hand

physics of beauty

\[ m_t \]
\[ m_W \]
\[ m_b \]
\[ m_c \]

dominant

small

c,u
t

physics of charm

\[ b, s, d \]
\[ s, d \]
\[ b \]

b,s,d

c,u
t
dominant

small
**Mixing: short vs long distance**

★ How can one tell that a process is dominated by long-distance or short-distance?

★ It is important to remember that the expansion parameter is $1/E_{\text{released}}$

$$y_D = \frac{1}{2M_D \Gamma_D} \text{Im} \langle D^0 | i \int d^4 x \, T \{ \mathcal{H}_{w_{\Delta C} = 1}(x) \mathcal{H}_{w_{\Delta C} = 1}(0) \} | D^0 \rangle$$

OPE-leading contribution:

★ In the heavy-quark limit $m_c \to \infty$ we have $m_c \gg \sum m_{\text{intermediate quarks}}$, so $E_{\text{released}} \sim m_c$
  - the situation is similar to B-physics, where it is “short-distance” dominated
  - one can consistently compute pQCD and $1/m$ corrections

★ But wait, $m_c$ is NOT infinitely large! What happens for finite $m_c$???
  - how is large momentum routed in the diagrams?
  - are there important hadronization (threshold) effects?
Threshold (and related) effects in OPE

★ How can one tell that a process is dominated by long-distance or short-distance?

★ Let’s look how the momentum is routed in a leading-order diagram
- injected momentum is $p_c \sim m_c$, so
- thus, $p_1 \sim p_2 \sim m_c/2 \sim O(\Lambda_{QCD})$?

★ For a particular example of the lifetime difference, have hadronic intermediate states
- let’s use an example of KKK intermediate state
- in this example, $E_{\text{released}} \sim m_D - 3m_K \sim O(\Lambda_{QCD})$

★ Similar threshold effects exist in B-mixing calculations
- but $m_b \gg \sum m_{\text{intermediate quarks}}$, so $E_{\text{released}} \sim m_b$ (almost) always
- quark-hadron duality takes care of the rest!

Maybe a better approach would be to work with hadronic DOF directly?
**Generic restrictions on NP from D̄D-mixing**

★ Comparing to experimental value of $x$, obtain constraints on NP models...
- assume $x$ is dominated by the New Physics model
- assume no accidental strong cancellations b/w SM and NP

\[
\frac{H_{NP}}{\Lambda_{NP}} = \frac{1}{\Lambda_{NP}^2} \sum_{i=1}^{8} z_i(\mu)Q_i'
\]

... which are

\[
\begin{align*}
|z_1| &\lesssim 5.7 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
|z_2| &\lesssim 1.6 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
|z_3| &\lesssim 5.8 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
|z_4| &\lesssim 5.6 \times 10^{-8} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
|z_5| &\lesssim 1.6 \times 10^{-7} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2.
\end{align*}
\]

★ Constraints on particular NP models also available!

---

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$R_D$ (x10^{-3})</th>
<th>$y^*$ (x10^{-3})</th>
<th>$x'^2$ (x10^{-3})</th>
<th>Excl. No-Mix Significance</th>
<th>$R_B$ (x10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle (2006)</td>
<td>3.64 ± 0.17</td>
<td>0.6 ± 4.0</td>
<td>0.18 ± 0.22</td>
<td>2.0</td>
<td>3.77 ± 0.09</td>
</tr>
<tr>
<td>BaBar (2007)</td>
<td>3.03 ± 0.19</td>
<td>9.7 ± 5.4</td>
<td>-0.22 ± 0.37</td>
<td>3.9</td>
<td>3.53 ± 0.09</td>
</tr>
<tr>
<td>LHCb</td>
<td>3.52 ± 0.15</td>
<td>7.2 ± 2.4</td>
<td>-0.09 ± 0.13</td>
<td>9.1</td>
<td>4.25 ± 0.04</td>
</tr>
<tr>
<td>CDF (9.6/fb)</td>
<td>3.51 ± 0.35</td>
<td>4.27 ± 4.30</td>
<td>0.08 ± 0.18</td>
<td>6.1</td>
<td>4.30 ± 0.06</td>
</tr>
</tbody>
</table>

M. Mattson, 2013

New Physics is either at a very high scales
- tree level: $\Lambda_{NP} \geq (4 - 10) \times 10^3$ TeV
- loop level: $\Lambda_{NP} \geq (1 - 3) \times 10^2$ TeV

or has highly suppressed couplings to charm!

Gedalia, Grossman, Nir, Perez
Phys.Rev.D80, 055024, 2009

E.Golowich, J. Hewett, S. Pakvasa and A.A.P.
Transitions forbidden w/out CP-violation

★ Recall that CP of the states in $D^0\overline{D^0} \rightarrow (F_1)(F_2)$ are anti-correlated at $\psi(3770)$:
★ a simple signal of CP violation: $\psi(3770) \rightarrow D^0\overline{D^0} \rightarrow (CP_\pm)(CP_\pm)$

$CP[F_1] = CP[F_2]$

$\Gamma_{F_1F_2} = \frac{\Gamma_{F_1} \Gamma_{F_2}}{R_m^2} \left[ (2 + x^2 + y^2) |\lambda_{F_1} - \lambda_{F_2}|^2 + (x^2 + y^2) |1 - \lambda_{F_1} \lambda_{F_2}|^2 \right]$}

★ CP-violation in the rate → of the second order in CP-violating parameters.
★ Cleanest measurement of CP-violation!

τ-charm factory

I. Bigi, A. Sanda; H. Yamamoto; Z.Z. Xing; D. Atwood, AAP

AAP, Nucl. Phys. PS 142 (2005) 333
hep-ph/0409130