Nonperturbative Input for $B \to K^{(*)}\ell\ell$ What do we know?

Thomas Mannel

Theoretische Physik I
Universität Siegen

6th LHCb Implications Workshop
CERN, October 11th-14th, 2016

Contents

Introduction

Basis of the Discussion: A couple of papers

- S. Descotes-Genon, L. Hofer, J. Matias and J. Virto: arXiv:1510.04239 [hep-ph]
- S. Jäger and J. M. Camalich: arXiv:1412.3183 [hep-ph]
- M. Ciuchini et al.: arXiv:1512.07157 [hep-ph]
- W. Altmannshofer and D. M. Straub: arXiv:1411.3161 [hep-ph]
- M. Beylich, G. Buchalla, and T. Feldmann: arXiv:1101.5118 [hep-ph]
- ....
- A. Khodjamirian, T. Mannel, A. A. Pivovarov and Y.-M. Wang: arXiv:1006.4945 [hep-ph]
- A. Khodjamirian, T. Mannel and Y. M. Wang: arXiv:1211.0234 [hep-ph]

In view of the tensions with data
this has triggered some discussions!

(I acknowledge some discussions with A. Khodjamirian, J. Matias, ...)

The main concern is the nonperturbative input

- Form factors (for both $B \to K$ and $B \to K^*$)
- Non-local contributions from “quark loops” (charm and light quarks)
- Use the relations of the “large energy limit” (Charles et al. 98)
- Observables become (more or less) indepenent of the form factors
- Power Corrections?
- Theoretical Uncertainties?

One Word on Form Factors

Two tools are available (to asses the power correction in the LE limit)

- Lattice QCD (@ large q^2)
- Light Cone QCD Sum Rules (@ small q^2)

Both approaches have limitations:

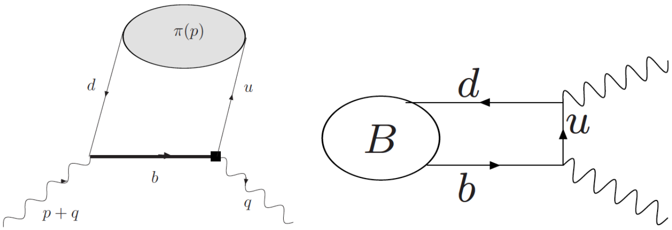
- K is much heavier than a pion:
  Good for LQCD
  bad for LCSR
- K^* is unstable: Difficult in both LQCD and LCSR

- K form factors are better known than ones for K^*
  = less theoretical uncertainty in the whole q^2 range

On the anatomy of LCSR

Two ways to proceed:

- Interpolate the B meson
  and use the light-cone distribution of the light meson
- Interpolate the light meson
  and use the light-cone distribution of the B meson



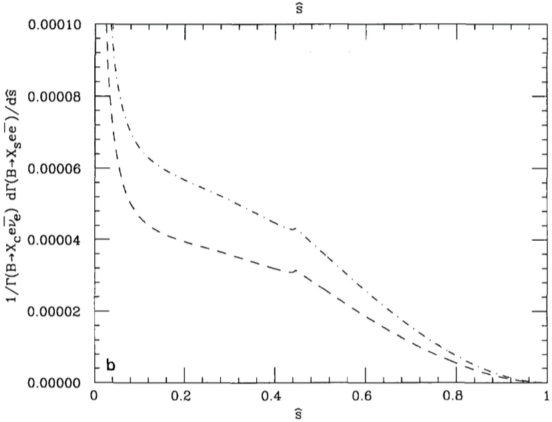
- LC distributions of the stable light mesons are quite well known
- LC distributions for the K^* are tricky: K^* is heavy and unstable

$\rightarrow$ Use LC distribution for the B meson and interpolate K^*

- All sum rule include subleading twists and in some cases also QCD corrections.

The Charm Loop 1: Generalities

As a start: A historical plot:



(Grinstein, Savage, Wise 1989)

The kink (as well as most of the plot) is unphysical

- The kink comes form the (perturbatively calculated) charm loop at the point where the chamr quarks go on-shell
- This region is genuinly nonperturbative
- This requires nonperturbative input beyond form factors
- This requires nonperturbative input beyond the one form the $1/m_b$ expansion
- The expansion is really in

$$\frac{\Lambda_{\rm QCD}}{|\sqrt{q^2} - 2m_c|}$$

Contribution of a virtual photon (Virtuality q) =
Insertion of the electromagnetic current:

$$T_\mu(q) = \int d^4x \, e^{iqx} \, T[J_\mu^{\rm em}(x)(C_1 O_1(0) + C_2 O_2(0))]$$

Look in particular into the charm-contribution

$$T_\mu^{(c)}(q) = \int d^4x \, e^{iqx} \, T[\bar{c}(x)\gamma_\mu c(x)(C_1 O_1(0) + C_2 O_2(0))]$$

- This is a non-local operator
- Problems at the charm threshold $q^2 \sim 4m_c^2$

Expansion of the charm loop for $q^2 \leq 4m_c^2$:

- Heavy Quark Expansion:
  Define for positive q^2 the vector $v = q/\sqrt{q^2}$

$$c(x) = e^{-im_c(vx)} h_v^{(+)}(x) \qquad \text{Quark}$$
$$\bar{c}(x) = e^{-im_c(vx)} \bar{h}_v^{(-)}(x) \qquad \text{Antiquark}$$

- Insert this

$$T_\mu^{(c)}(q) = \int d^4x \exp[ix(q - 2m_c v)] \; T[\bar{h}_v^{(-)}(x)\gamma_\mu h_v^{(+)}(x)(C_1 O_1(0) + C_2 O_2(0))]$$

- The exponent becomes: $ix(q - 2m_c v) = ivx\left(\sqrt{q^2} - 2m_c\right)$
- The expansion parameter is

$$\frac{\Lambda_{\rm QCD}}{\sqrt{q^2} - 2m_c} \qquad (\text{for } 0 \leq q^2 \leq 4m_c^2)$$

Leading Term of the c-loop Expansion

Perform an OPE for $0 \leq q^2 \ll 4m_c^2$

$$T_\mu^{(c)}(q) = \sum_k \left(\frac{\Lambda_{\mathrm{QCD}}}{\sqrt{q^2} - 2m_c} \right)^k \sum_i C_{\mu\nu}^{k,i}(q) O_{k,i}^\nu$$

Leading term ($k = 0$):

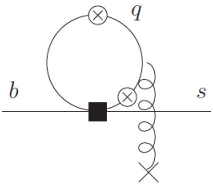
Perturbativly calculated charm loop: $y = 4m_c^2/q^2$

$$T_\mu^{(c),0}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \frac{9}{31\pi^2} g(q^2, m_c^2) \bar{s}_L \gamma^\nu b_L$$

$$g(q^2, m_c^2) = -\frac{8}{9} \ln\left(\frac{m_c}{m_b}\right) + \frac{8}{27} + \frac{4y}{9} - \frac{4}{9}(2+y)\sqrt{y-1} \arctan\left(\frac{1}{\sqrt{y-1}}\right)$$

Higher order terms

Soft Gluon insertions into the c loop:



The exponent is $(2m_c v - q) \cdot x$, so the Integral is dominated by the region

$$x^2 \sim \frac{1}{(2m_c v - q)^2} \sim \frac{1}{(2m_c - \sqrt{q^2})^2}$$

so in the region of interest this is dominated by the light cone $x^2 \sim 0$

Light cone kinematics:

$$v = \frac{1}{2}(n_+ + n_-) \qquad q = \frac{1}{2}[(n_- q)n_+ + (n_+ q)n_-]$$

Charm Propagator in an external Gluon field

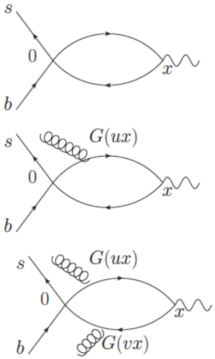
$$\langle 0 | Tc(x)\bar{c}(0) | 0 \rangle_G = \cdots \delta \left(\omega - \frac{in_+ D}{2} \right) G_{\alpha\beta} \qquad \text{(at leading twist)}$$

This leads to nonlocal operator of the form

$$T_\mu^{(c),1}(q) = \int d\omega \, I_{\mu\rho\alpha\beta}(q,\omega) \bar{s}_L \gamma^\rho \delta \left(\omega - \frac{in_+ D}{2} \right) \tilde{G}_{\alpha\beta} b_L$$

New “shape function” (Details in arXiv:1006.4945 [hep/ph])

- T- product of $\bar{c}c$-operators expanded near $x^2 \sim 0$:



the simple loop

loop-function $\otimes$ $\bar{s}\Gamma b$

one-gluon emission:

nonlocal operator $\sim G_{\mu\nu}(ux)$, $0 < u < 1$

$$\widetilde{\mathcal{O}}(q) \sim \bar{s}\left(\frac{1}{4m_c^2 - q^2 - q\cdot(iD)}\right) G_{\mu\nu}\, b$$

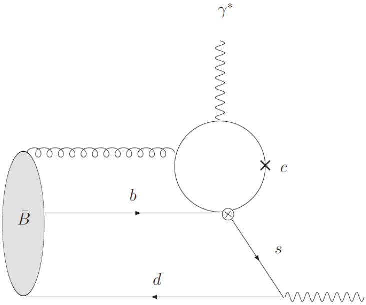
two-gluon emission

$$\sim \frac{\Lambda_{QCD}^2}{4m_c^2 - q^2} \times \{\text{one-gluon term}\}$$

(Plots taken from Alex Khodjamirian)

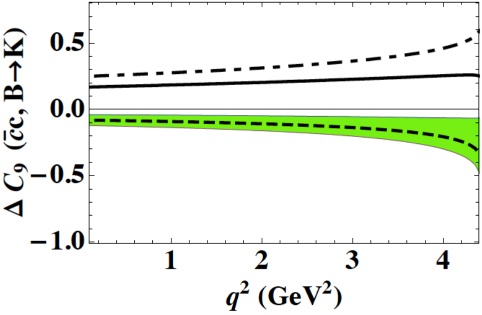
The Charm Loop 2: Estimates

How to compute the matrix element? LCSR!

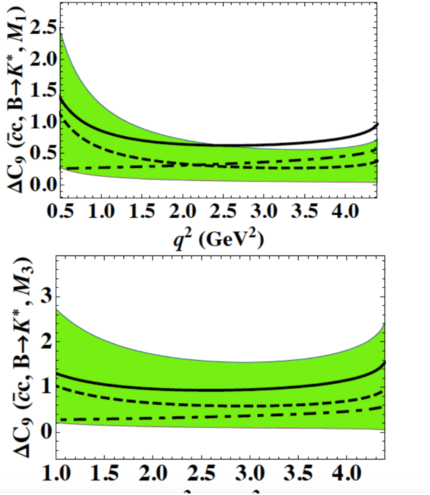


Use the standard way of estimating the uncertainties in QCD SR

$B \to K\ell\ell$



$B \to K^*\ell\ell$



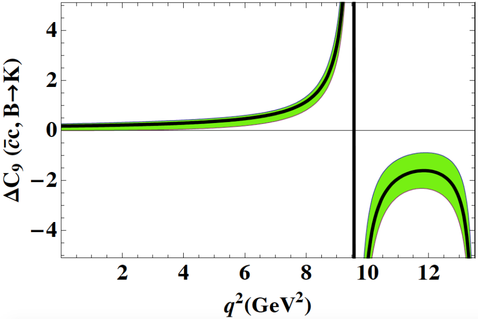
Sneaking up to large q^2

- Use the LCSR at small and negative q^2
- Use a dispersion relation at positive q^2

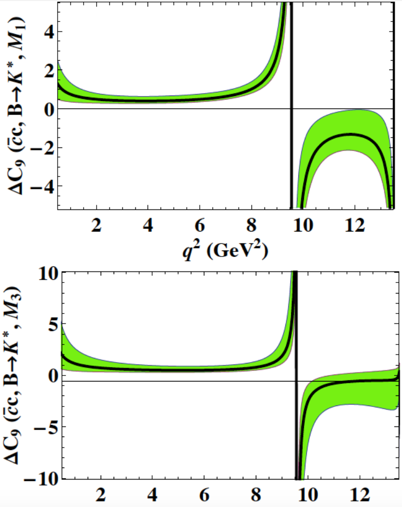
$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2\Big[\sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - im_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)}\Big],$$

- fit the parameters to the LCSR
- We express the result trough a change in C_9

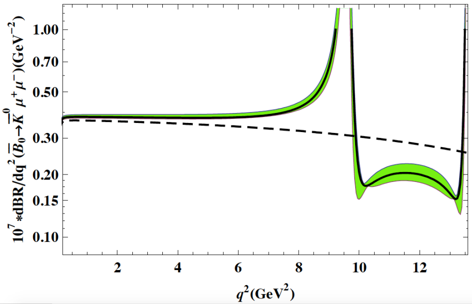
$B \to K\ell\ell$



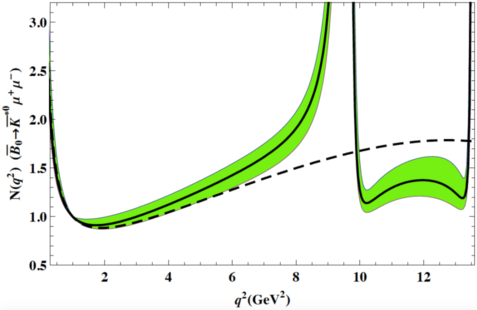
$B \to K^*\ell\ell$



$B \to K\ell\ell$

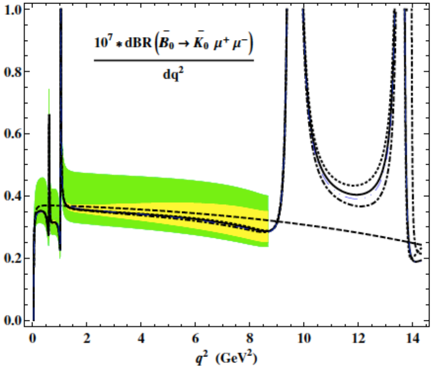


$B \to K^*\ell\ell$



Including also light-quark loops

$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins



solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

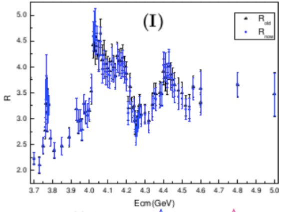
long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.

(Plot from A. Khodjamirian)

Even larger q^2

- charmonium states above $\psi(2S)$: $\psi(3770), \psi(4040), \psi(4160), \psi(4415)$



measurement of $R(e^+e^- \to hadrons)$,
from: BESS Collab. 0705.4500 [hep-ex]

$\Uparrow$ $\sqrt{q^2} = m_B - m_{K*} = 4.389\text{GeV}$ $\quad$ $\Uparrow$ $\sqrt{q^2} = m_B - m_K = 4.784\text{GeV}$

- nonlocal effects: local OPE valid at $|q^2| \sim m_b^2 \gg 4m_c^2$,

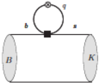
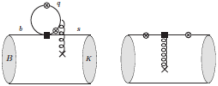
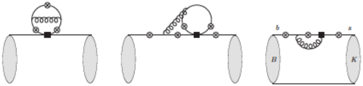
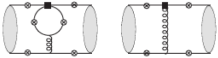
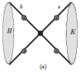
B.Grinstein, D. Pirjol (2004); Beilich,Buchalla, Feldmann (2011)

- a duality ansatz *Beilich,Buchalla, Feldmann (2011)*

nonfactorizable effects (FSI phases in $B \to \psi K$) not included...

(taken from Alex Khodjamirian)

What is calculated/estimated?

Contribution	$B \to K\ell\ell$	$B \to K^*\ell\ell$
	✓	✓
	✓	✓
	✓	—
	✓	—
	✓	—

What we know and what we don't know ...

Hadronic Uncertainties:

- $B \to K$ Form factors known at the level of $\sim$ (5-10)%
  - Can be improved by LQCD
  - Eventually no need for HQE/LE expansion any more
- $B \to K^*$ Form factors are less well known!
  - K^* unstable: Hard to treat in LQCD / LCSR
  - Eventually one needs to deal with $B \to K\pi\ell\ell$
  - Treatment with stable K^* can only be approximate!

- Nonlocal quark loop contributions
  - Harder than form factors: No LQCD calculation
  - Uncertainties difficult to estimate (beyond the LCSR standards)
  - Below charm threshold: new “shape functions”
  - Not much is known about these ....
  - Above charm threshold: Global Duality!

This needs to be scrutinized further before we can make a definite statement on phsyics beyond the Standard Model!