Next steps with LHCb’s $b \to s\ell\ell$ data at low recoil

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LHCb measurements: Implications and future prospects

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Motivation

- $b \rightarrow s\ell\ell$ - FCNC processes, loop induced and suppressed in the SM, potentially sensitive to possible BSM effects.

- Better precision enables us to probe higher energy scales, but also requires better understanding of the long distance dynamics within the SM (e.g. form factors, charm effects).

- Here we talk about $B \rightarrow K^{*}\mu\mu$, for its rich angular information - larger potential to probe Dirac structure of the new physics effective operators, hadronic physics, providing the diagnosing power for the violation of CP etc.
Above $J/\psi$ and $\psi(2S)$ resonances, the complicated intermediate charm states (presumably dominated by the wide charm resonances $J^{PC} = 1^{--}$) show up as wiggles in $q^2$ distributions. These effects have been anticipated from the 1970s.

Theoretically controlled approach: Operator Product Expansion (OPE) in $1/Q^2$, $Q^2 \sim (q^2_{max}, m_b^2)$ of the non-local operator products

$$A_i^\mu \propto \frac{1}{q^2} \int d^4x e^{ikx} T\{O_i(0)j^\mu(x)\}$$

in terms of local operators [Grinstein, Pirjol (2004); Beylich, Buchalla, Feldmann (2011)]:

$$A^\mu = \sum_i C_i(q^2)Q_i^\mu.$$
Leading power corrections from the orders $\alpha_s/m_b$, $m_c^4/Q^4$ - only of order of few percents. Up to this precision charm contributions factorize, are universal and are absorbed into effective Wilson coefficients [Grinstein, Pirjol, (2004); Bobeth, Hiller, van Dyk, (2010)]:

$$C_{7}^{\text{eff}} = C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6 + \frac{\alpha_s}{4\pi} \left[ (C_1 - 6C_2) A(q^2) - C_8 F_8^{(7)}(q^2) \right],$$

$$C_{9}^{\text{eff}} = C_9 + \frac{1}{2} h(q^2, 0) \left[ \frac{8}{3} C_1 + 2C_2 + 11C_3 - \frac{4}{3} C_4 + 104C_5 - \frac{64}{3} C_6 \right] + \frac{8}{3} \frac{m_c^2}{q^2} \left[ \frac{4}{3} C_1 + C_2 + 6C_3 + 60C_5 \right] + \frac{\alpha_s}{4\pi} \left[ C_1 (B(q^2) + 4C(q^2)) - 3C_2 (2B(q^2) - C(q^2)) - C_8 F_8^{(9)}(q^2) \right] - \frac{1}{2} h(q^2, m_b^2) \left[ 7C_3 + \frac{4}{3} C_4 + 76C_5 + \frac{64}{3} C_6 \right] + \frac{4}{3} \left[ C_3 + \frac{16}{3} C_5 + \frac{16}{9} C_6 \right].$$

OPE expected to provide good description of charm effects within binned observables.

Quark-hadron duality violations enter at some level, but their magnitude is not known within the OPE itself.
Compare the predictions of the OPE with experimental data for angular observables and branching fractions at low recoil.

Use the Krüger-Sehgal (KS) data driven model of local $q^2$ distributions, to estimate the uncertainties (limitations of the OPE) for a chosen binning, independently of the underlying electro-weak model (SM or BSM).

\[ \mathcal{A}^{L,R}_\perp = +i C^{L,R} f_\perp, \quad \mathcal{A}^{L,R}_\parallel,0 = -i C^{L,R} f_{\parallel,0} \quad \text{where} \]

\[ C^{L,R} = C_{9}^{\text{eff}} \mp C_{10} + \kappa \frac{2\hat{m}_b}{\hat{S}} C_{7}^{\text{eff}} \tag{4} \]

Then the observables \( F_L, S_3, S_4 \) turn out independent of \( C^{L,R} \) and only depend on form factors (within OPE, IW and no RH quark currents assumption) [Hambrock, Hiller, (2012); Hambrock, Hiller, Schacht, Zwicky, (2013)]. If universal, also the wiggles cancel in \( F_L, S_{3,4} \) for infinitesimal bin size, their appearance signals the non-universal effects.
Testing the SM (OPE)

One can then test the SM with $d\mathcal{B}/dq^2$ and the observables $S_5, A_{FB}$:

$$S_5 = \frac{3\sqrt{2}}{2} \frac{\rho_2(q^2)f_0f_\perp}{\rho_1(q^2)(f_0^2 + f_\perp^2 + f_\parallel^2)}, \quad A_{FB} = \frac{3\rho_2(q^2)f_\parallel f_\perp}{\rho_1(q^2)(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

where

$$\rho_1(q^2) \equiv \frac{1}{2}(|C^R|^2 + |C^L|^2) = \left| C^\text{eff}_9 + \kappa \frac{2m_b m_B}{q^2} C^\text{eff}_7 \right|^2 + |C_{10}|^2,$$

$$\rho_2(q^2) \equiv \frac{1}{4}(|C^R|^2 - |C^L|^2) = \text{Re} \left[ \left( C^\text{eff}_9 + \kappa \frac{2m_b m_B}{q^2} C^\text{eff}_7 \right) C^*_{10} \right].$$
$F_L, S_3, S_4$: OPE + ($C'_{9,10} = 0$) - comparison with Experiment LHCb, 1512.04442
Only $S_4, S_5$ show some tension with the OPE in the highest 1GeV$^2$ bin.
Krueger-Sehgal parametrisation

- Krüger-Sehgal (KS) model [Kruger, Sehgal, (1996)] - data from $e^+e^- \to \text{hadrons}$ is used to obtain charm vacuum polarization, which is then plugged in the $B \to K^*$ and corrected with fudge factors $\eta_c$.

- Extraction of the charm vacuum polarization $h_c(q^2)$ from the $e^+e^- \to h_i$ data [BES Collaboration, (2007)]:

$$R(s) = \frac{\sigma(e^+e^- \to h_i)(s)}{\sigma(e^+e^- \to \mu^+\mu^-)(s)}.$$  \hspace{1cm} (7)

Using the optical theorem and the dispersion relation:

$$\text{Im}[A(e^+e^- \to h \to e^+e^-)] = s\sigma(e^+e^- \to h)(s),$$

$$\text{Im}[h_c(s)] = \frac{\pi}{3} R_c(s),$$

$$\text{Re}[h_c(s)] = \text{Re}[h_c(s_0)] + \frac{s - s_0}{\pi} P \int_{t_0}^{\infty} \frac{ds'}{(s' - s)(s' - s_0)} \text{Im}[h_c(s')].$$  \hspace{1cm} (8)
Krüger-Sehgal parametrisation ctd.

\[ C_9^{\text{eff}}(q^2) = C_9 + 3a_2 \eta_c h_c(q^2) + \ldots \]  


\[ a_2 = \frac{1}{3} \left( \frac{4}{3} C_1 + C_2 + 6C_3 + 60C_5 \right) = 0.2 \text{ at NNLO at the } b\text{-mass scale.} \]

- Introduce ”fudge function” \( \eta_c \equiv \eta_c(K^*_j, q^2), j = \perp, \parallel, 0 \), that corrects for effects beyond NFA (\( |\eta_c = 1| \) and universal). We take \( \eta_c(j) \in \mathbb{R} \), more generally \( \eta_c(j) \in \mathbb{C} \).
Krüeger-Sehgal parametrisation ctd.

- $B \to K\mu^+\mu^-$ wiggles observed in the high $q^2$ measured by LHCb [LHCb, 1307.7595]. [Lyon, Zwicky, (2014)] : require large fudge factor $\sim -2.5$; one can also probe $\psi_i$ dependent correction factors.
- We expect the correction factor(s) for $B \to K^*\psi_i$ to differ wrt the $B \to K\psi_i$, e.g.

$$|\eta_{J/\psi(\psi(2S))^*} K^*| \simeq 1, \text{ while } |\eta_{J/\psi(\psi(2S))^*} K| \simeq 1.5. \quad (11)$$

- We take in our analysis $\eta'$'s constant but polarization dependent. With more experimental information, we could be more general.
- Kinematic relations at the endpoint ($q^2 = q^2_{\text{max}}$) [Hiller, Zwicky, (2013)],

$$A_\parallel = -\sqrt{2} A_0, \quad A_\perp = 0, \quad (12)$$

and our ansatz $C_{9,i}^{\text{eff}}(q^2) = C_9 + 3a_2 \eta_i h_c(q^2) + \ldots (i = 0, \parallel, \perp)$, imply:

$$\eta_0 = \eta_\parallel, \eta_\perp \to \text{not constrained} \quad (13)$$

since $f_\perp(q^2_{\text{max}}) = 0$. 

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Krüeger-Sehgal parametrisation ctd.

- The observables $S_{7,8,9}$ provide null tests for the hadronic universality (vanish in the OPE), (see also Bobeth, Hiller, van Dyk (2012)), for example:

$$S_8 \sim f_0 f_\perp (\tilde{C}_9^{\text{eff}} + \kappa \frac{2m_bm_B}{q^2} C_7^{\text{eff}}) a_2 \text{Im}[h_c(q^2)(\eta_0 - \eta_\perp)].$$

(14)

- We perform the simultaneous fit for $\eta_0, \eta_\perp, C_9, C_{10}$
OPE vs Resonance model for 3 different binnings

$\chi^2$/dof = (1.3, 0.8, 1.3) for OPE and (1.0, 0.6, 1.2) for KS
Differences between shaded areas (OPE) and dashed lines (KS) can serve as a binning dependent systematic uncertainty for OPE.

To estimate the uncertainties of the OPE predictions for a given binning, we suggest the ratios:

\[
\begin{align*}
\epsilon_1 &= \frac{\int_{\text{bin}} \rho_1^{KS} dq^2}{\int_{\text{bin}} \rho_1^{OPE} dq^2}, & \epsilon_2 &= \frac{\int_{\text{bin}} \rho_2^{KS} dq^2}{\int_{\text{bin}} \rho_2^{OPE} dq^2}, & \epsilon_{12} &= \frac{\int_{\text{bin}} \rho_2^{KS} dq^2}{\int_{\text{bin}} \rho_1^{OPE} dq^2} \cdot \frac{\int_{\text{bin}} \rho_1^{KS} dq^2}{\int_{\text{bin}} \rho_2^{OPE} dq^2}. \\
\end{align*}
\]

(15)

<table>
<thead>
<tr>
<th>Bin [GeV^2]</th>
<th>15 − 19</th>
<th>15 − 17</th>
<th>17 − 19</th>
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<th>17 − 18</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon_1)</td>
<td>(0.85,1.16)</td>
<td>(0.81,1.30)</td>
<td>(0.87,1.03)</td>
<td>(0.76,1.20)</td>
<td>(0.84,1.38)</td>
<td>(0.84,1.03)</td>
<td>(0.86,1.05)</td>
</tr>
<tr>
<td>(\epsilon_2)</td>
<td>(0.82,1.00)</td>
<td>(0.74,1.13)</td>
<td>(0.85,0.91)</td>
<td>(0.71,1.17)</td>
<td>(0.78,1.08)</td>
<td>(0.76,0.95)</td>
<td>(0.84,0.97)</td>
</tr>
<tr>
<td>(\epsilon_{12})</td>
<td>(0.86,1.05)</td>
<td>(0.87,1.05)</td>
<td>(0.84,1.05)</td>
<td>(0.95,1.06)</td>
<td>(0.78,1.05)</td>
<td>(0.75,1.05)</td>
<td>(0.93,1.05)</td>
</tr>
</tbody>
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**Table:** Ratios \(\epsilon_k\) for different \(q^2\)-bins and 1σ ranges of parameters \(\eta_0, \eta_\perp\) and \(C_{9,10}\). The coefficients \(C'_{9,10} \to 0\).

This suggests that OPE performs better at endpoint bins than at the lower \(q^2\) bins and the bin of maximal size (15 − 19)GeV^2.
Conclusions

- So far, good performance of the SM+OPE, although large BSM effects are allowed.
- Small binnings and more precise data is going to probe the limits of the OPE.
- $B \rightarrow K^* \ell \ell$ provides large(r) number of angular observables to hopefully disentangle the SD and LD effects.
- CP-averages $S_{7,8,9}$ important for testing the hadronic universality of charm effects.
- Consistency between fits to Wilson coefficients between high $q^2$, low $q^2$ and inclusive decays are important to decide the fate of the $B \rightarrow K^* \mu \mu$ anomaly.
- Looking forward for the future precise data.
Let us review the notation and the conventions.

- The effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V^*_{ts} V_{tb} \sum_i C_i(\mu) \mathcal{O}_i + h.c$$

- We use CMM basis $\mathcal{O}_1 - \mathcal{O}_8$ [Chetyrkin, Misiak, Münz (1996)], e.g.

$$\mathcal{O}_1^c = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma_\mu T^a b_L), \quad \mathcal{O}_2^c = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L).$$

- EM and QCD dipole operators:

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2} (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{g_s}{(4\pi)^2} m_b (\bar{s}\sigma^{\mu\nu} P_R T^a b) G_{\mu\nu}^a$$

- The semileptonic operators:

$$\mathcal{O}_9 = \frac{\alpha_{em}}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \quad \mathcal{O}_{10} = \frac{\alpha_{em}}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell).$$

$\mathcal{O}'_{9,10}$ with $P_L \rightarrow P_R$ in quark currents
Angular Observables in $B \rightarrow K^* \ell \ell$

- Complete information about the decay in full four-fold angular distributions:

$$\frac{d^4 \Gamma}{dq^2 d \cos \theta_\ell d \cos \theta_K d\phi} = \frac{3}{8\pi} J(q^2, \cos \theta_\ell, \cos \theta_K, \phi),$$

$$\frac{d^4 \bar{\Gamma}}{dq^2 d \cos \theta_\ell d \cos \theta_K d\phi} = \frac{3}{8\pi} \bar{J}(q^2, \cos \theta_\ell, \cos \theta_K, \phi),$$

with

$$J(q^2, \theta_\ell, \theta_K, \phi) = J_1^s \sin^2 \theta_K + J_1^c \cos^2 \theta_K + (J_2^s \sin^2 \theta_K + J_2^c \cos^2 \theta_K) \cos 2\theta_\ell$$

$$+ J_3 \sin^2 \theta_\ell \sin^2 \theta_K \cos 2\phi + J_4 \sin 2\theta_\ell \sin 2\theta_K \cos \phi$$

$$+ J_5 \sin \theta_\ell \sin 2\theta_K \cos \phi + J_6 \cos \theta_\ell \sin^2 \theta_K$$

$$+ J_7 \sin \theta_\ell \sin 2\theta_K \sin \phi + J_8 \sin 2\theta_\ell \sin 2\theta_K \sin \phi$$

$$+ J_9 \sin^2 \theta_\ell \sin^2 \theta_K \sin 2\phi.$$

- The LHCb Collaboration measured the CP-averaged ratios\(^2\)

$$S_i \equiv \frac{J_i + \bar{J_i}}{d\Gamma/dq^2 + d\bar{\Gamma}/dq^2}.$$  

\(^2\)Note the different convention $F_L = F_L^{\text{LHCb}}$, $S_{3,5,7,9} = \frac{3}{4} S_{3,5,7,9}^{\text{LHCb}}$, $S_{4,8} = -\frac{3}{4} S_{4,8}^{\text{LHCb}}$, $S_5 = \frac{3}{4} S_5^{\text{LHCb}}$, $A_{FB} = -A_{FB}^{\text{LHCb}}$.