Heavy Flavour Spectroscopy and Exotic States From Lattice QCD

Graham Moir

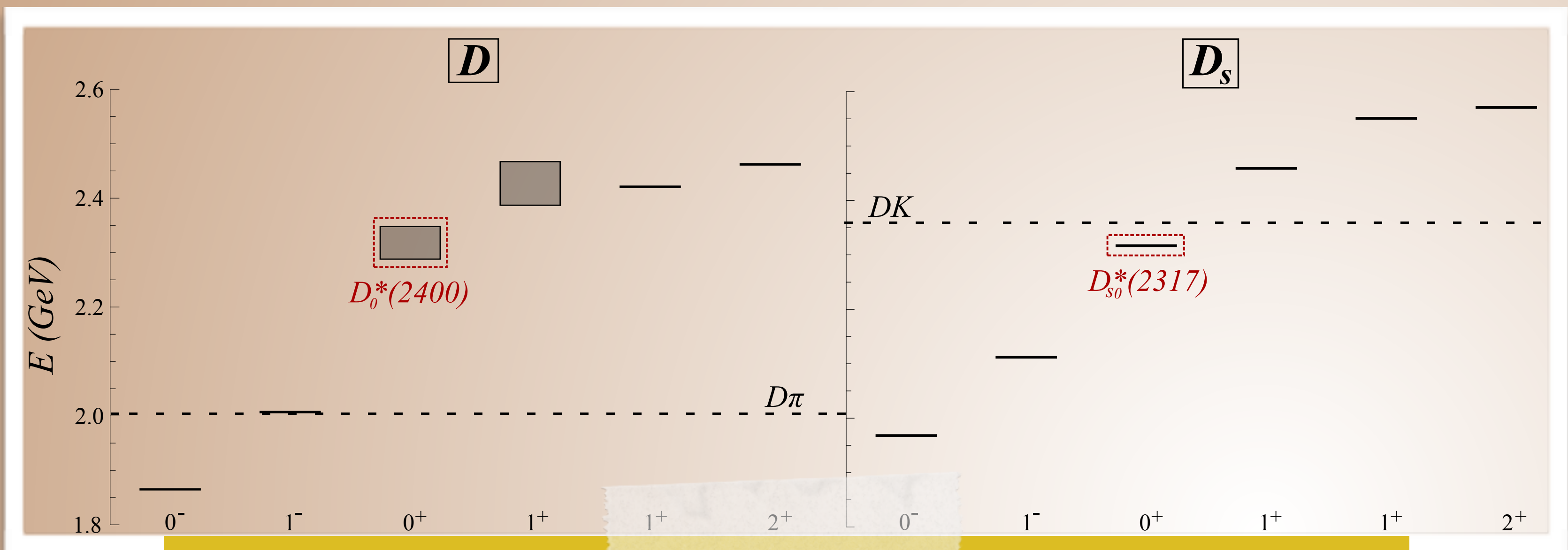
University of Cambridge

Implications of LHCb Measurements and Future Prospects

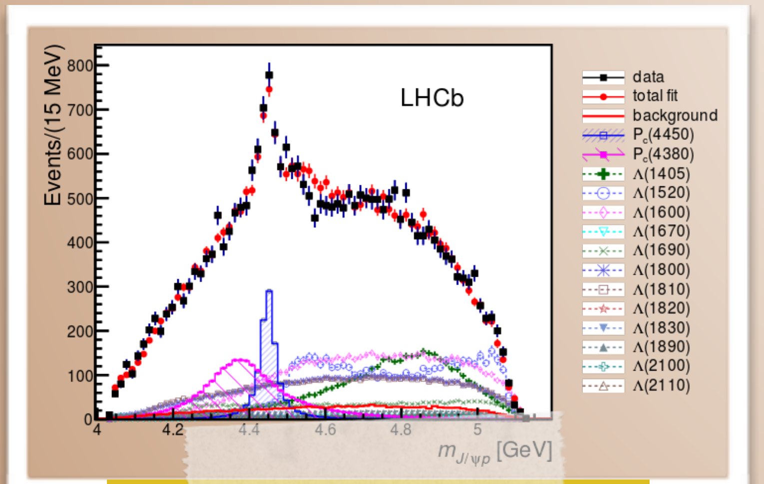
CERN, 13 October 2016

Motivation

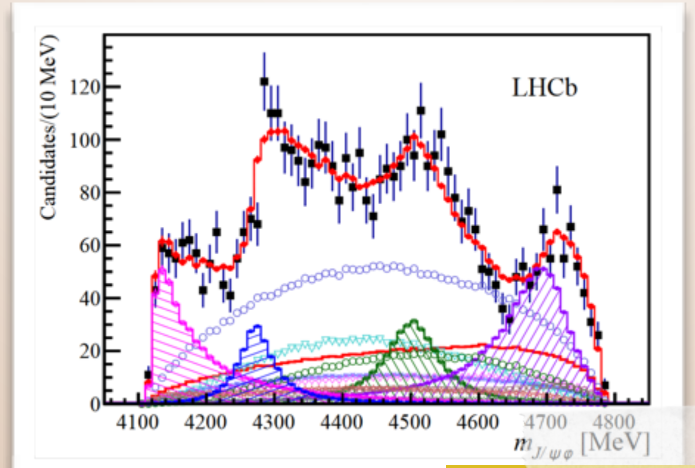
'A modern day November revolution'



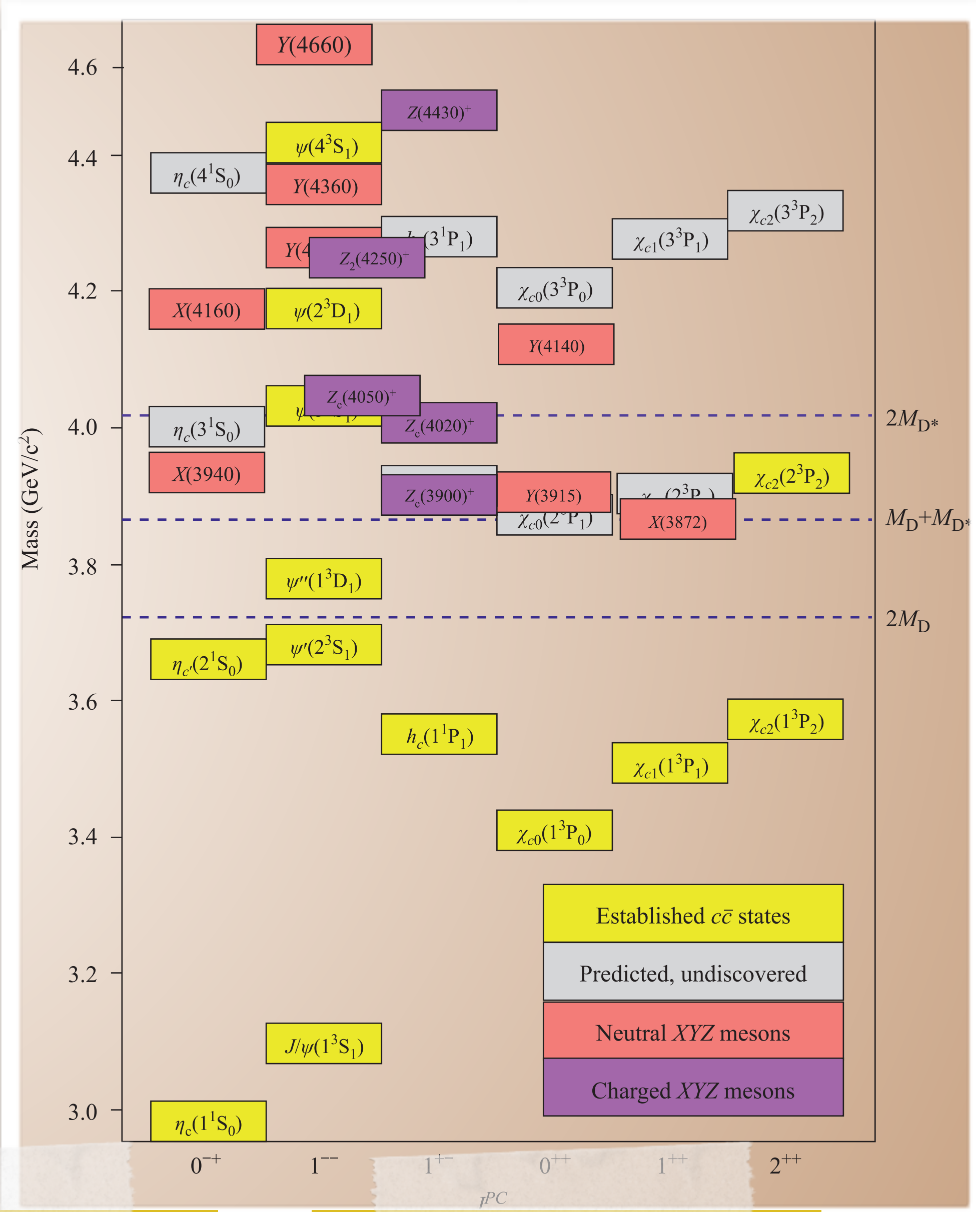
Quark-antiquark or something more?



Five-quarks



Four-quarks



Hybrids

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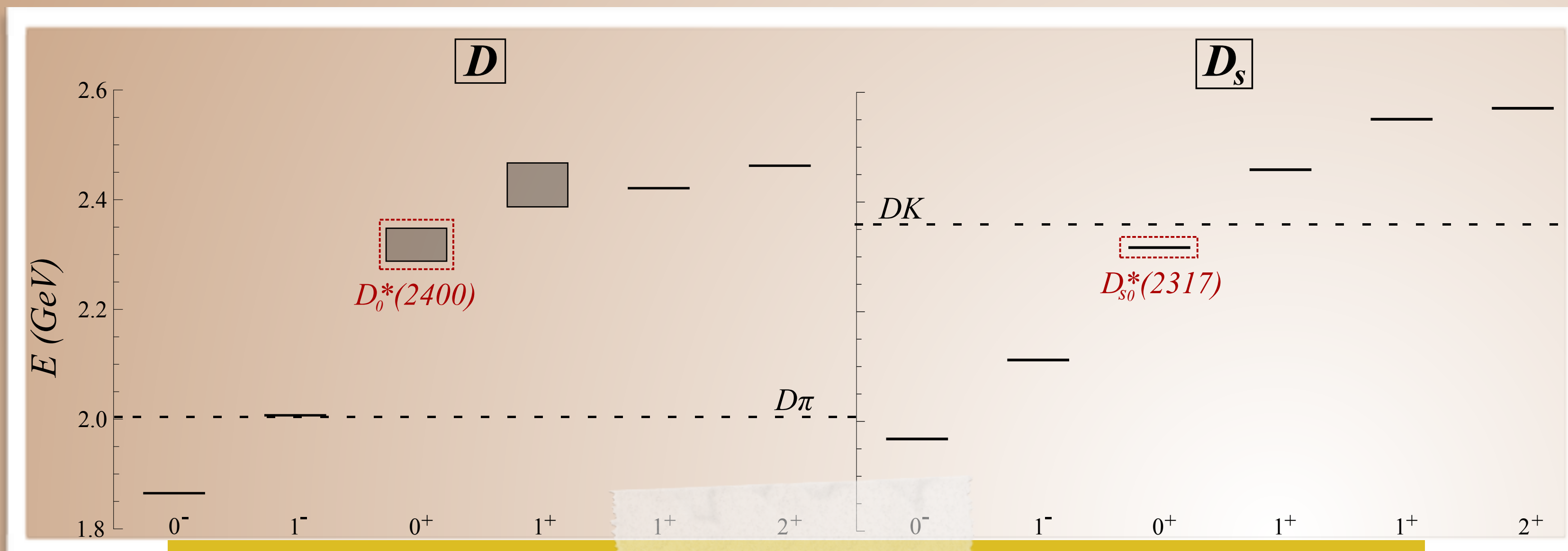

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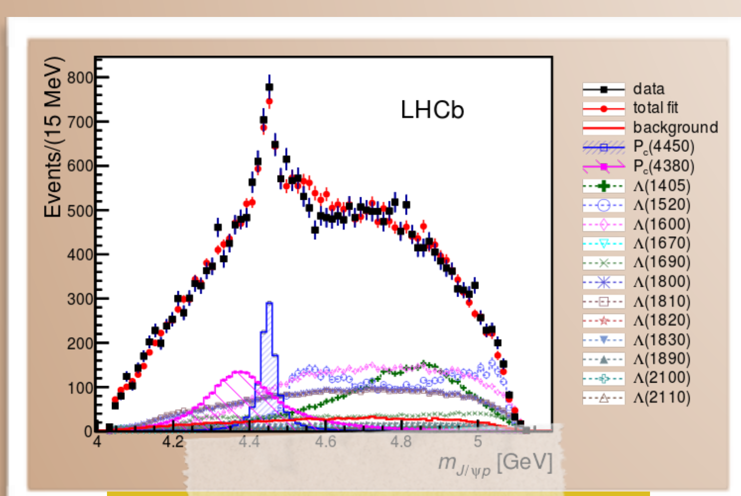

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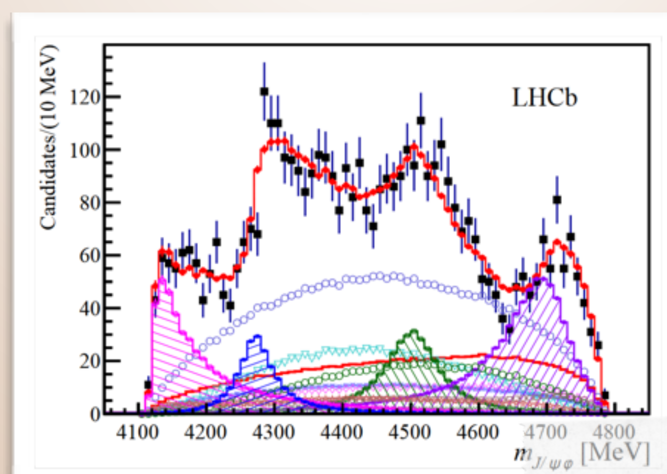

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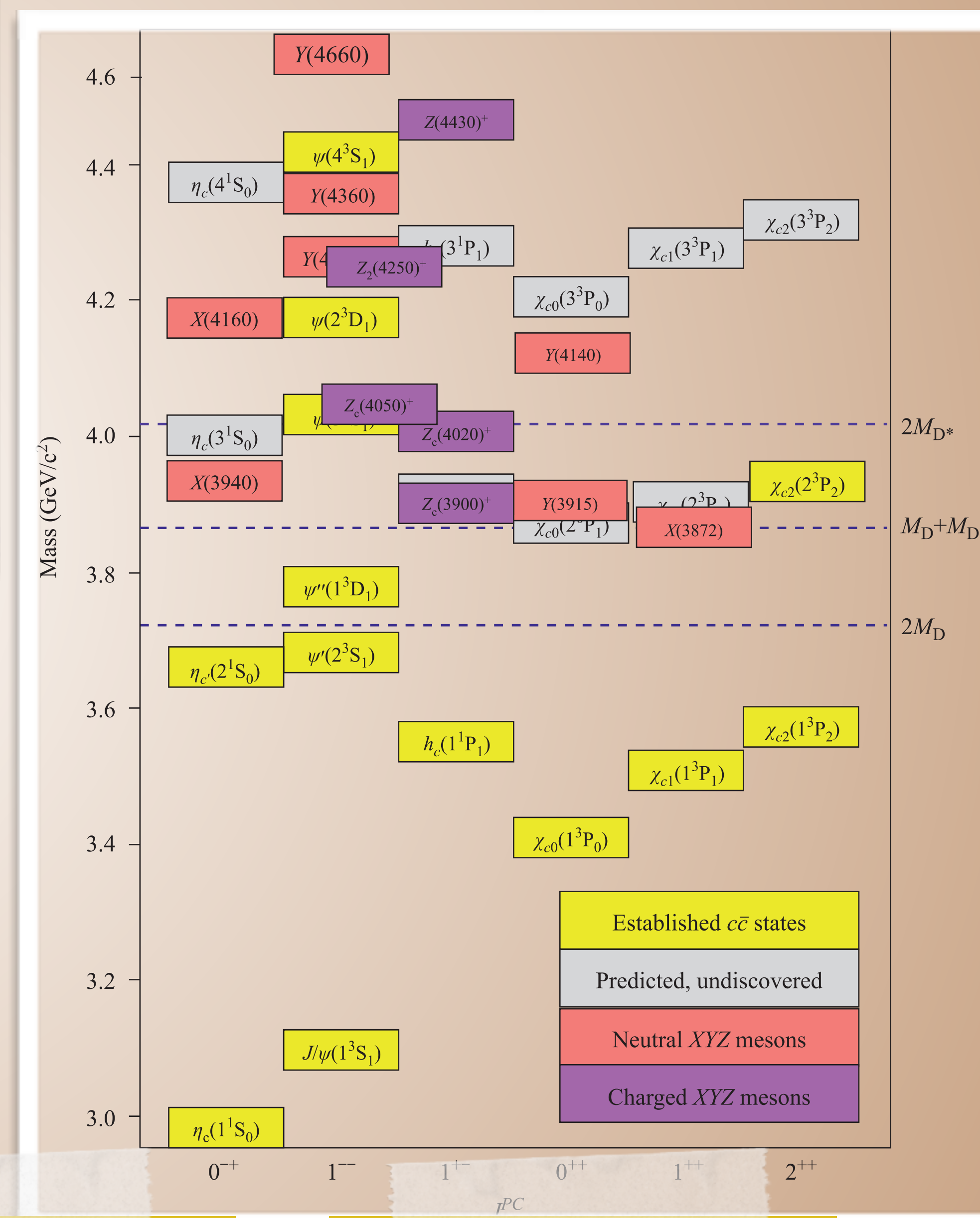


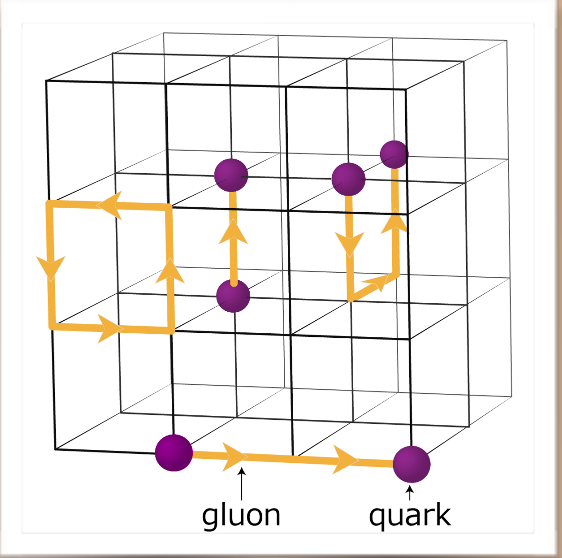
Hybrids

Similar story emerging in the bottom sector

Lattice QCD in a Nutshell

$$L = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

- **Gluons** → **SU(3) matrices ('links'):** $U_\mu(x) = e^{-iagA^b_\mu(x)T^b}$
- **Quarks live on 'sites' with colour, flavour and spinor indices**
- **Derivatives** → **finite differences:** $\nabla_\mu\psi(x) = \frac{1}{a}[U_\mu(x)\psi(x+a\hat{\mu}) - \psi(x)]$
- **Monte Carlo estimation of the path integral in a finite Euclidean space-time**
- **Measure desired observables . . .**



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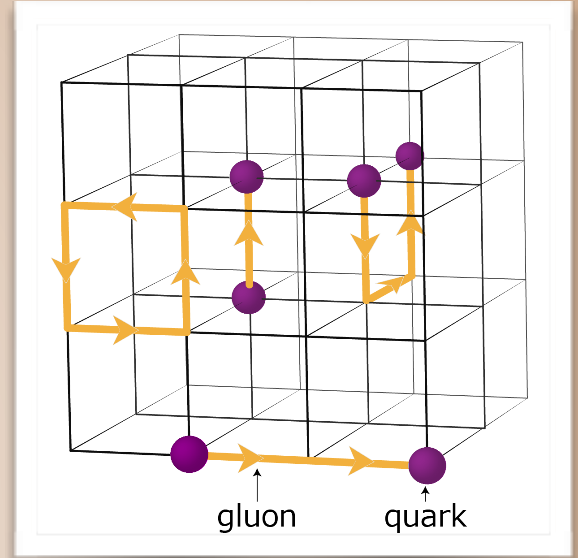
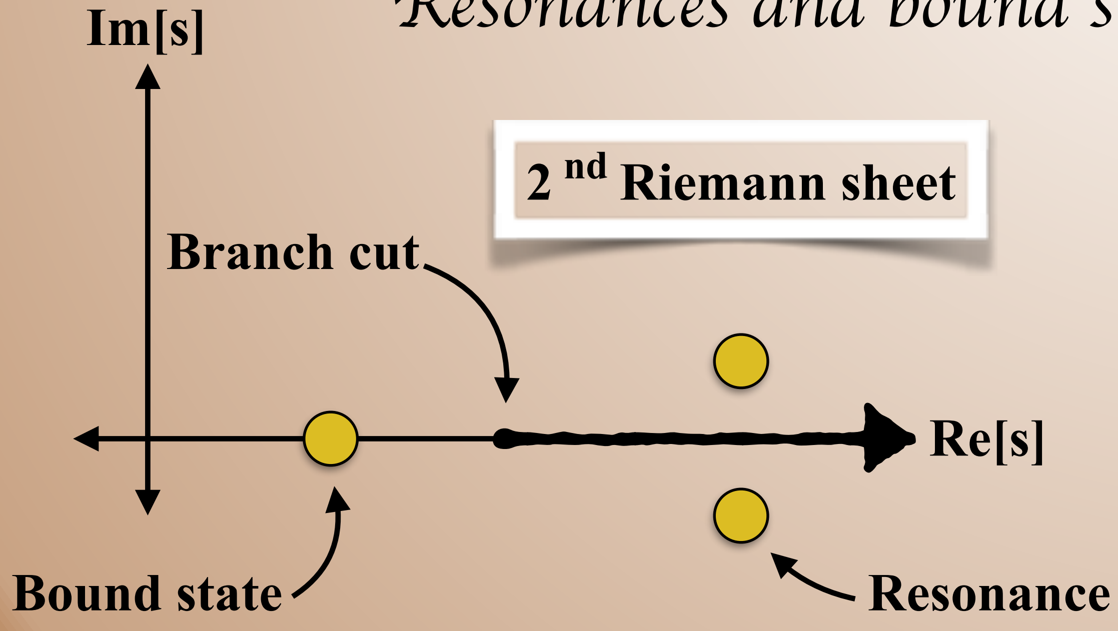


Spectroscopic observables

'Resonances and bound states appear as poles in the S-matrix'



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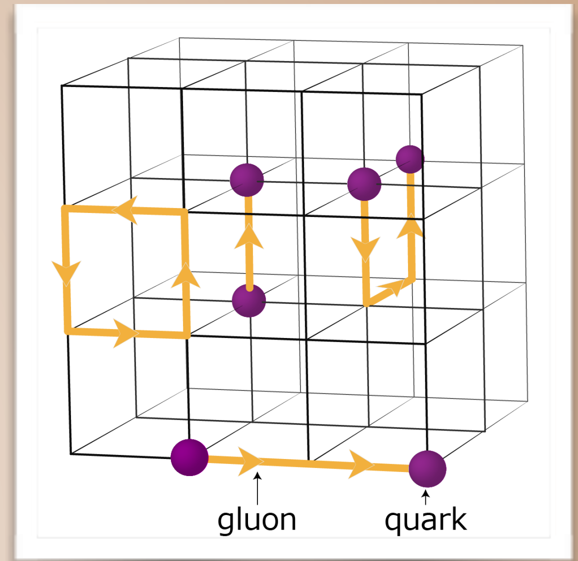

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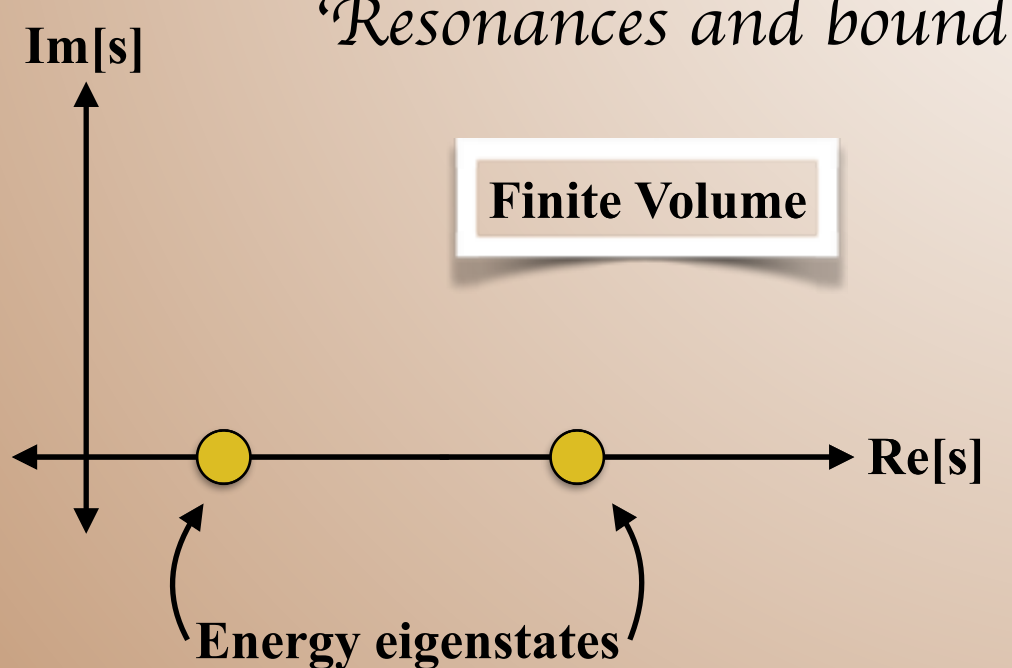

No continuum of states:

- **No cuts, sheets or resonances**

No asymptotic states:

- **No scattering**

'No-Go': Maiani-Testa theorem

Spectroscopic observables continued . . .

So what can we do?

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'Lüscher formalism'

Finite-volume energy eigenstates ⟺ **infinite-volume scattering amplitudes**

- **Calculations becoming reliable**
- **Sill in 'R&D' stage!**

Spectroscopic observables continued . . .

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'Single hadron spectroscopy'

Excitation spectrum of finite-volume energy eigenstates

- **Extensive calculations for mesons and baryons**
- **Access to different structures and states**

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'Limiting cases'

'Static' heavy-quarks, (p)NRQCD

Recent calculations of 4 and 5 quark states

‘Limiting cases’

‘Static’ heavy-quarks, (p)NRQCD

The static limit and (p)NRQCD

'Study of 4 and 5 quark states in full QCD is difficult'

m QCD

mv NRQCD

mv^2 pNRQCD

The static limit and (p)NRQCD

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1. **Use a non-relativistic action for heavy quarks**
2. **Set finite bottom mass via non-rel. dispersion relation**
3. **Proceed as ‘normal’**

The static limit and (p)NRQCD

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mv^2 pNRQCD

1. Quarkonium $\longrightarrow$ NRQM problem (leading order)
2. Interaction potential V(r) between static quarks
3. V(r) from Wilson loops (non-perturbatively)

Hadro-quarkonium in the static limit

$$\langle W(r,t)\rangle = \left\langle 0 \left| \mathcal{Q}_r \ \mathcal{T}^{t/a} \ \mathcal{Q}_r^\dagger \right| 0 \right\rangle$$

Wilson loop

Fundamental static colour charge

Static potential

$$V_0(r) = -\lim_{t\to\infty} \frac{d}{dt} \langle W(r,t)\rangle$$

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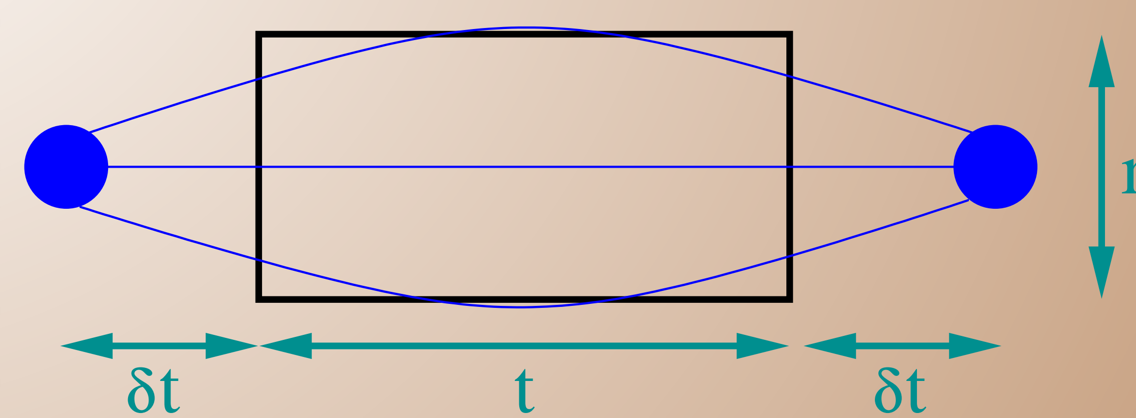
Does the static potential become more attractive in the presence of light hadrons?

$$\begin{aligned}
\Delta V_H(r) &= V_H - V_0 \\
&= -\lim_{t\to\infty} \frac{d}{dt} \ln \left[\frac{\langle H \,|\mathcal{Q}_r \; \mathcal{T}^{t/a} \; \mathcal{Q}_r^\dagger |\, H\rangle}{\left\langle 0 \middle| \mathcal{Q}_r \; \mathcal{T}^{t/a} \; \mathcal{Q}_r^\dagger \middle| 0 \right\rangle} \right] \\
&= -\lim_{t\to\infty} \frac{d}{dt} \ln \left[\frac{\left\langle W(r,t) \; C_H^{2pt}(t+2\delta t) \right\rangle}{\langle W(r,t)\rangle \;\; \left\langle C_H^{2pt}(t+2\delta t) \right\rangle} \right]
\end{aligned}$$



Hadro-quarkonium in the static limit

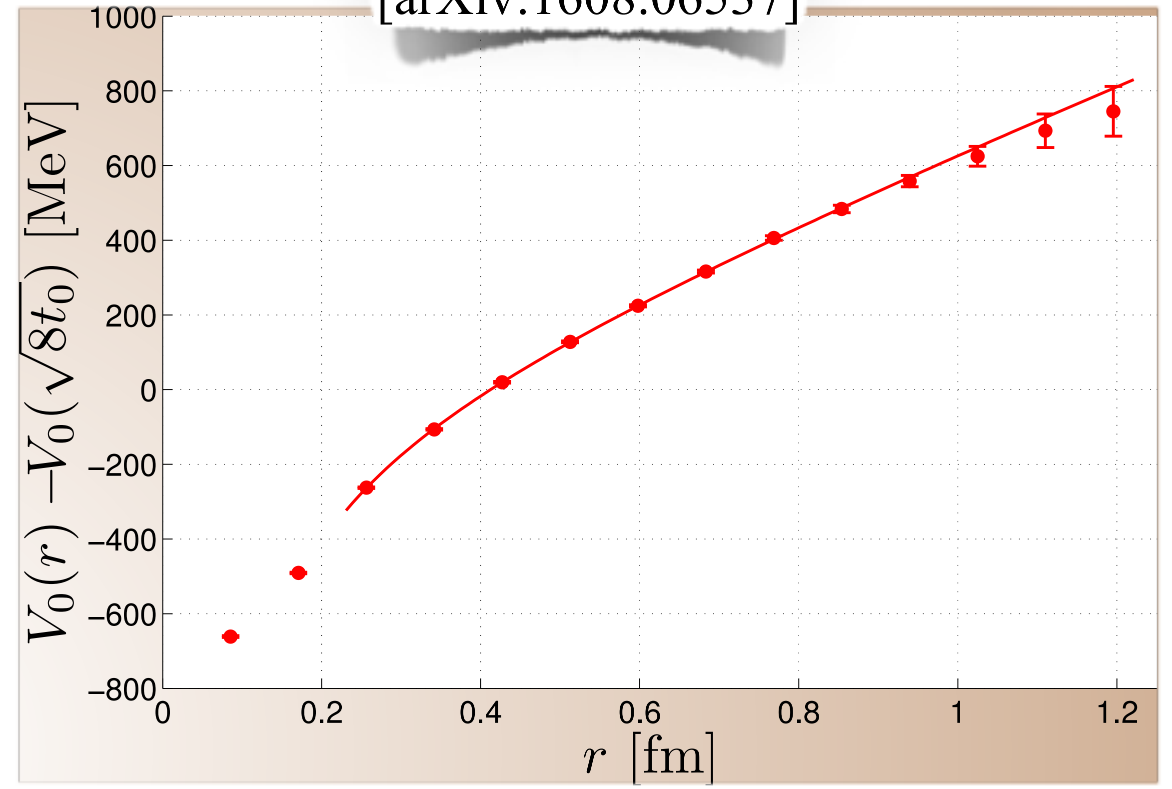
High statistics:

- 1552 configurations
- 12 time-sources

Ensemble:

- $M_\pi \approx 220$ MeV
- $M_K \approx 480$ MeV

[arXiv:1608.06537]



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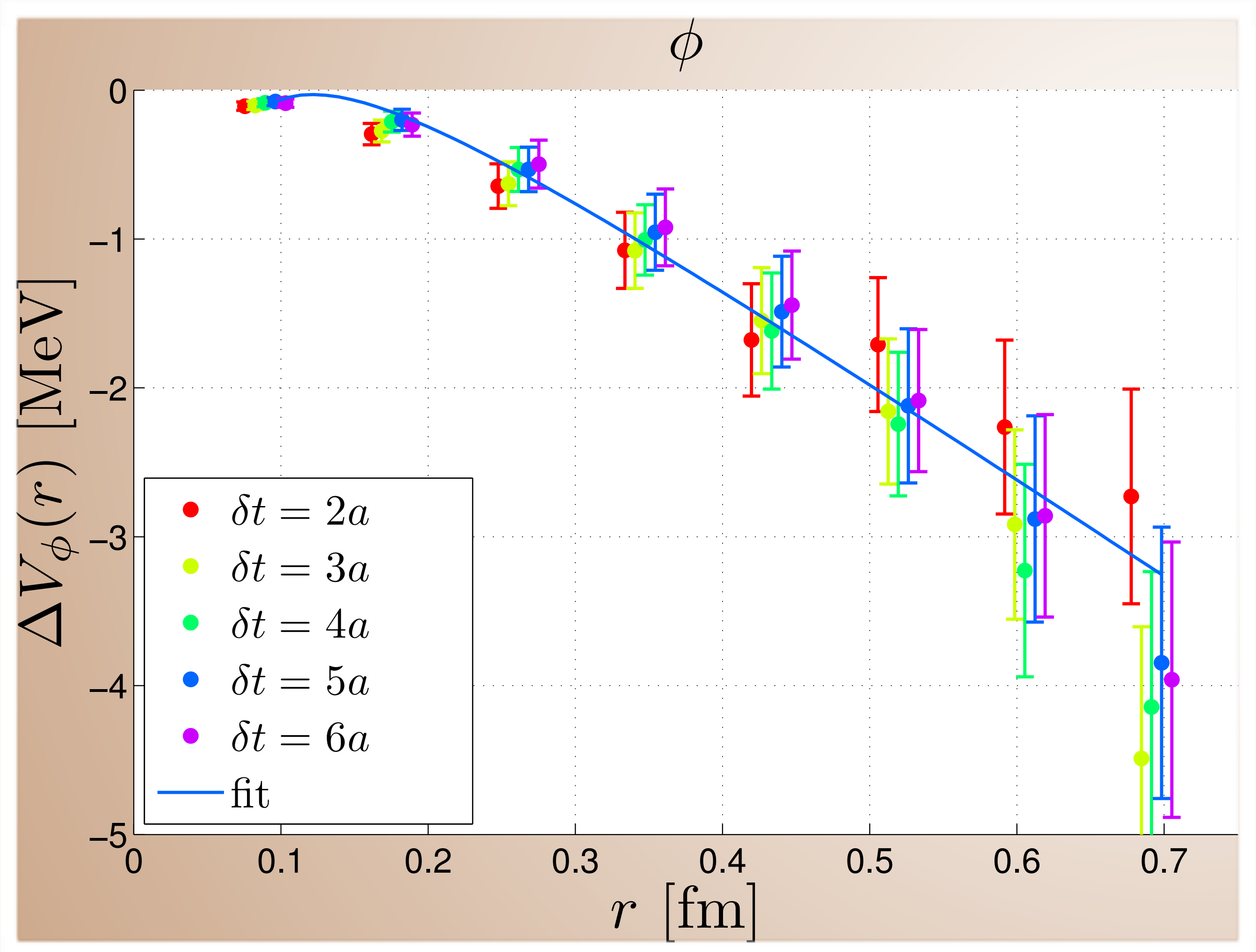

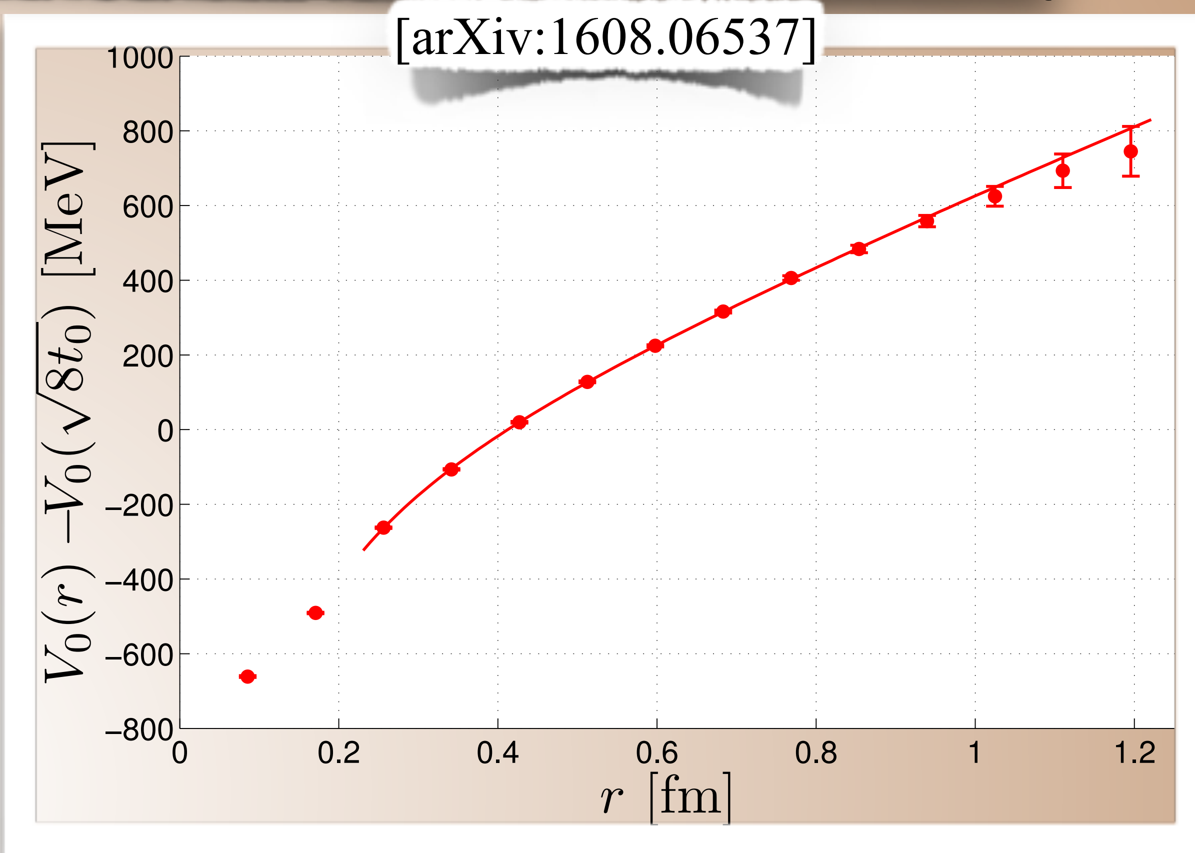

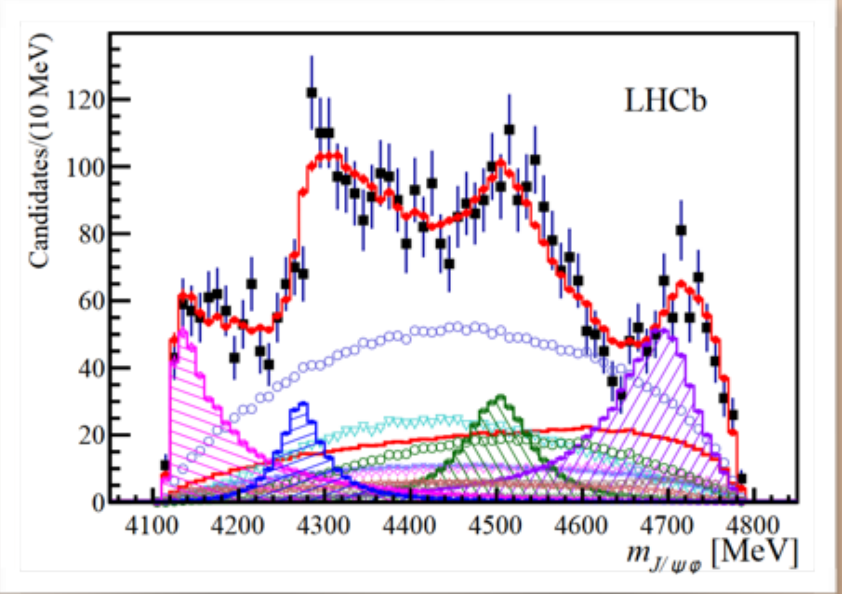

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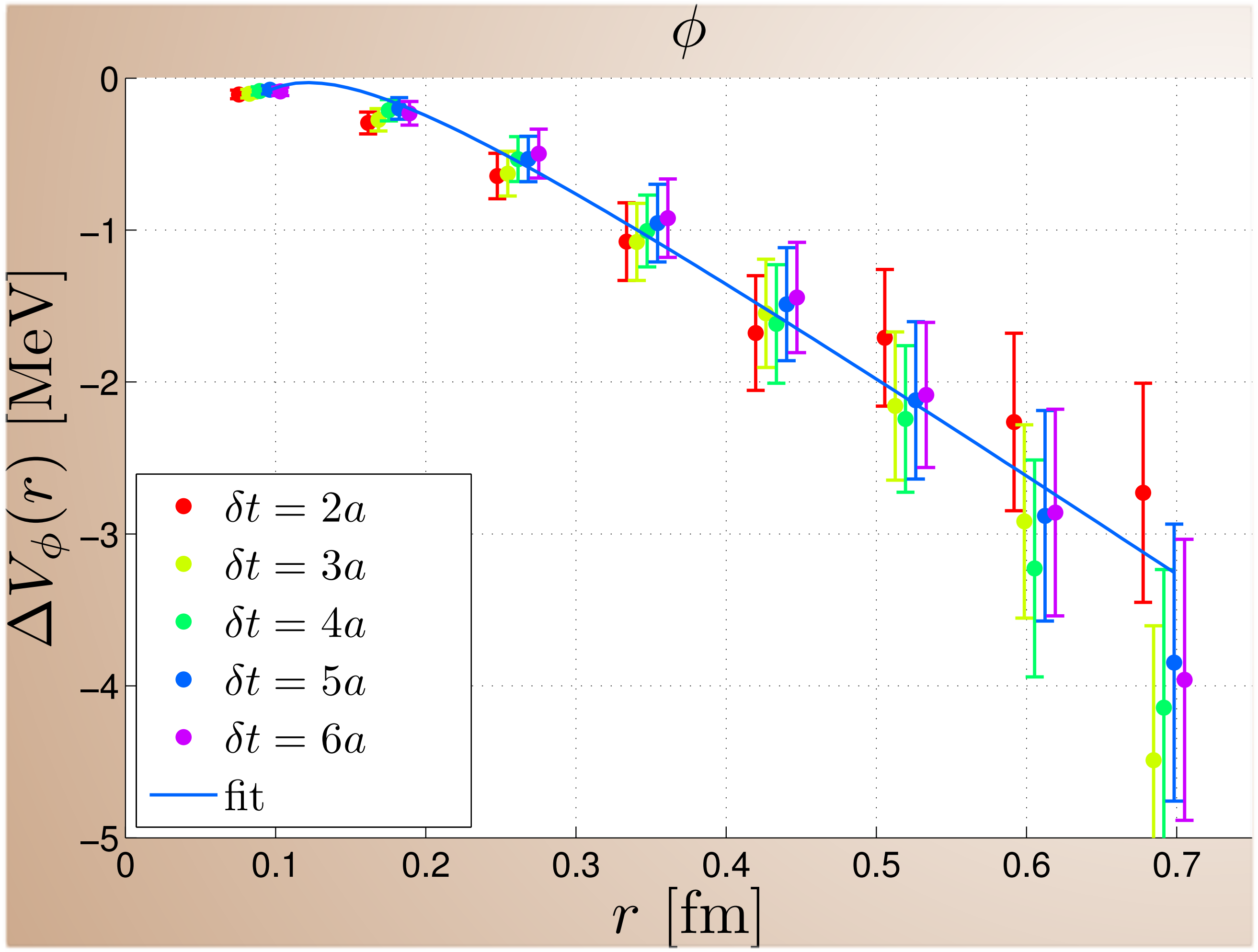

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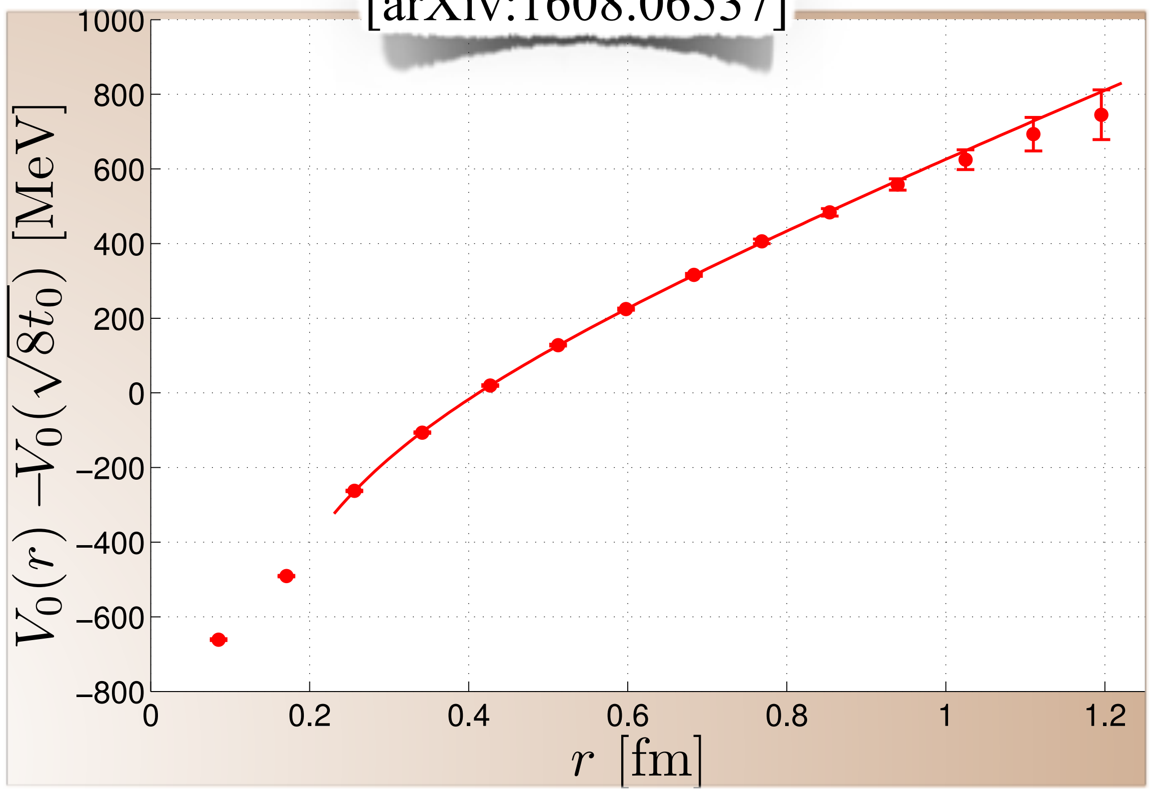
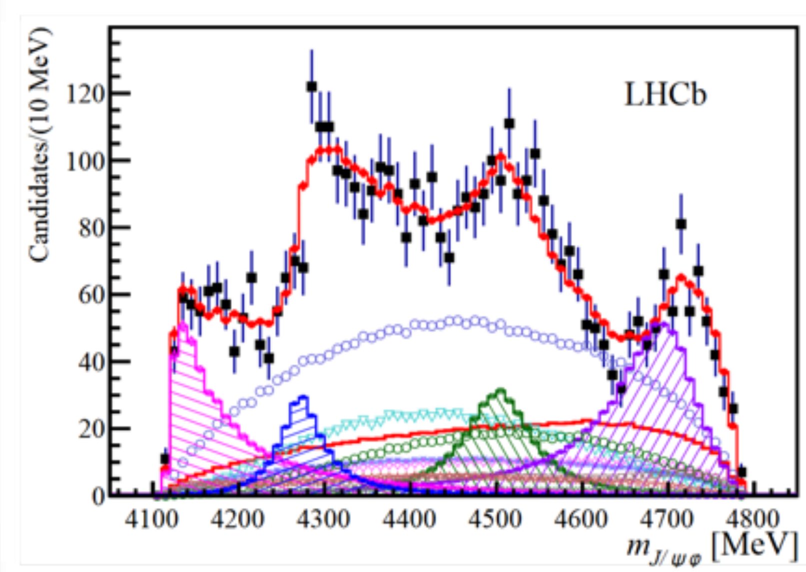
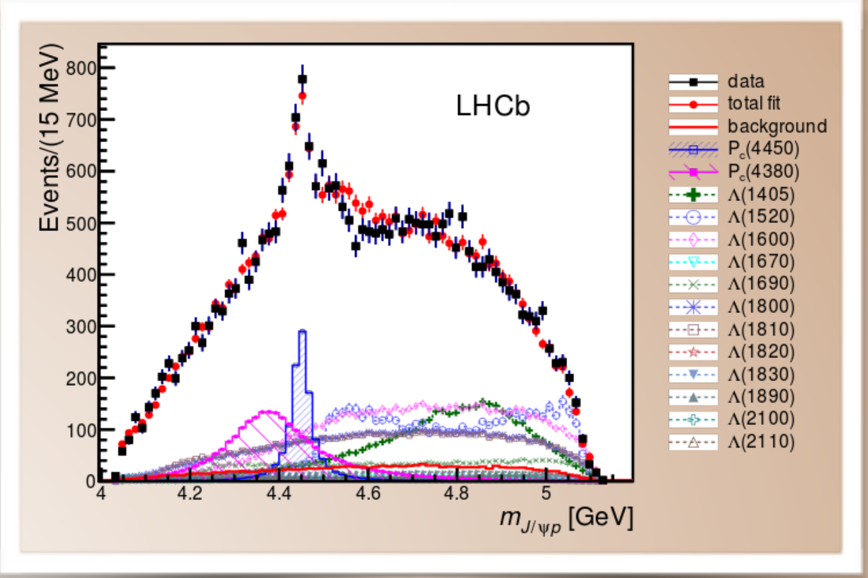




Similar effects for the $\pi,\ K,\ \rho,\ K^*$ mesons

Hadro-quarkonium in the static limit



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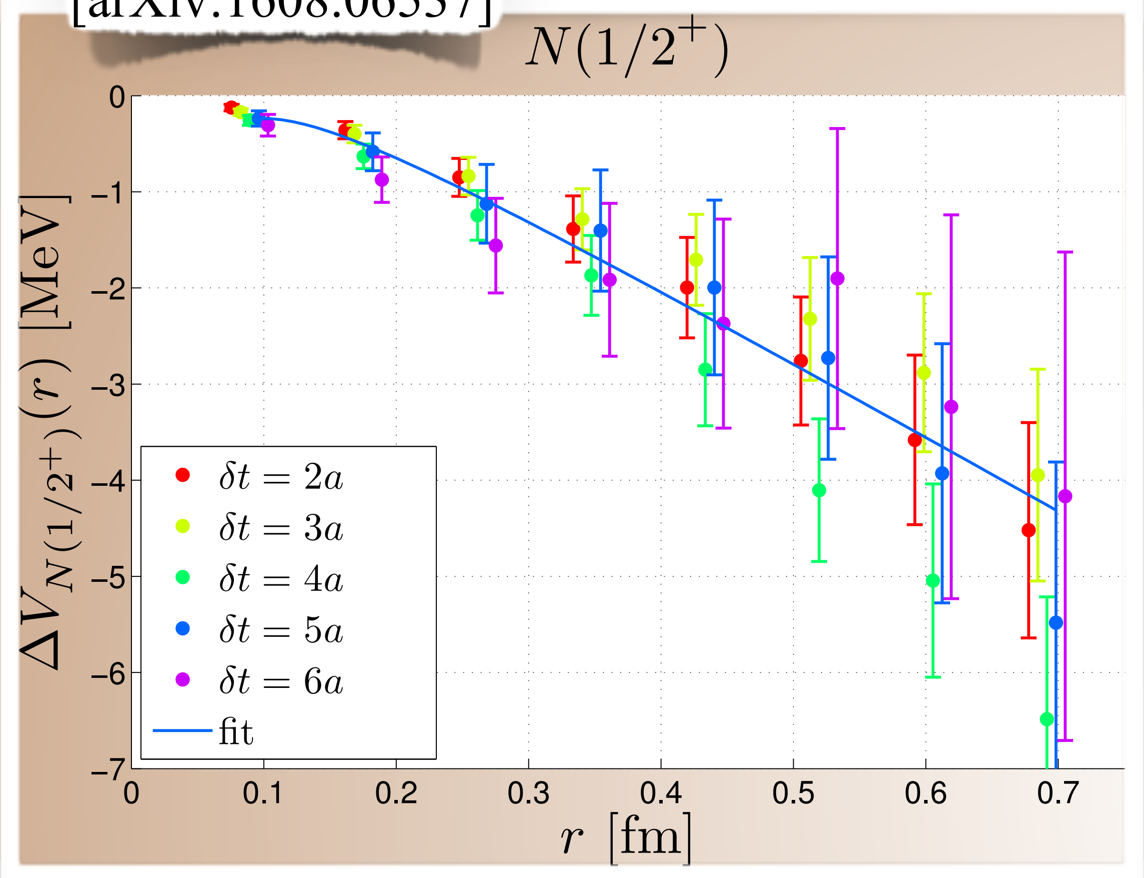

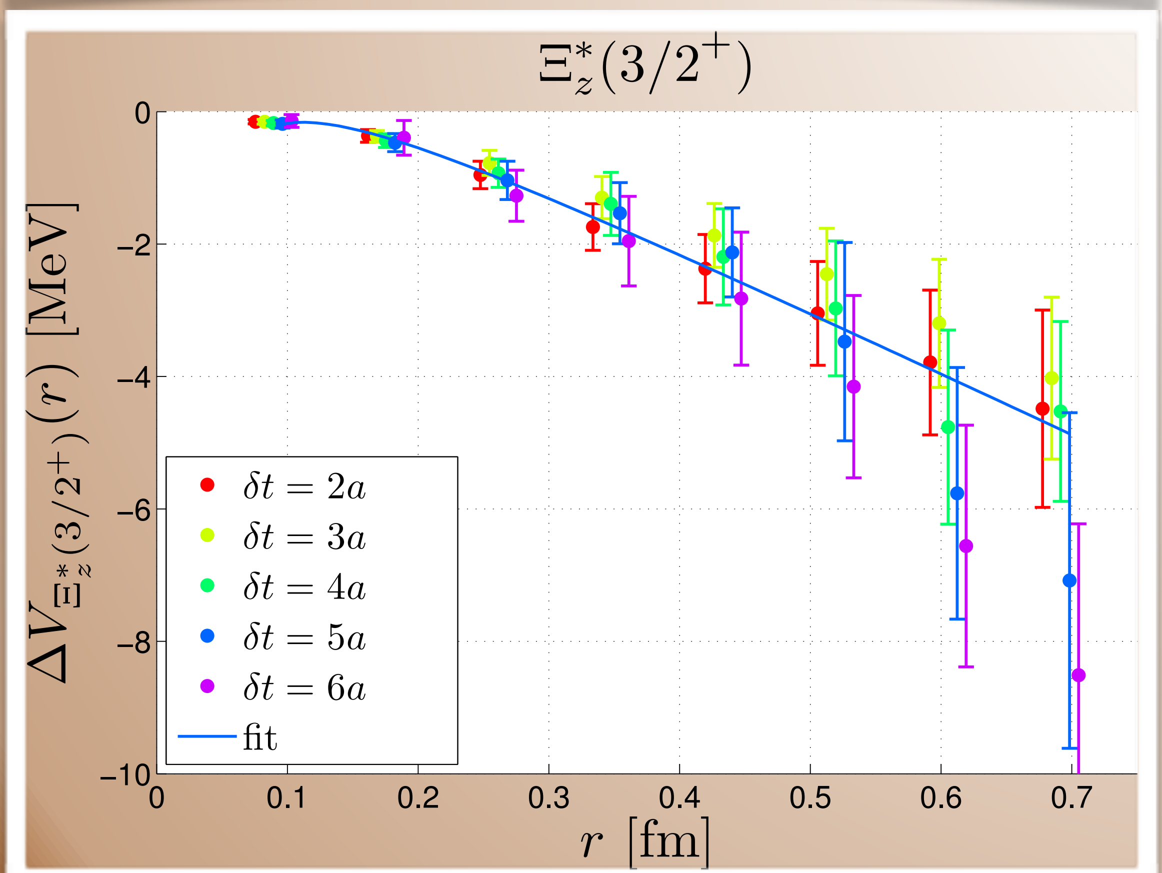

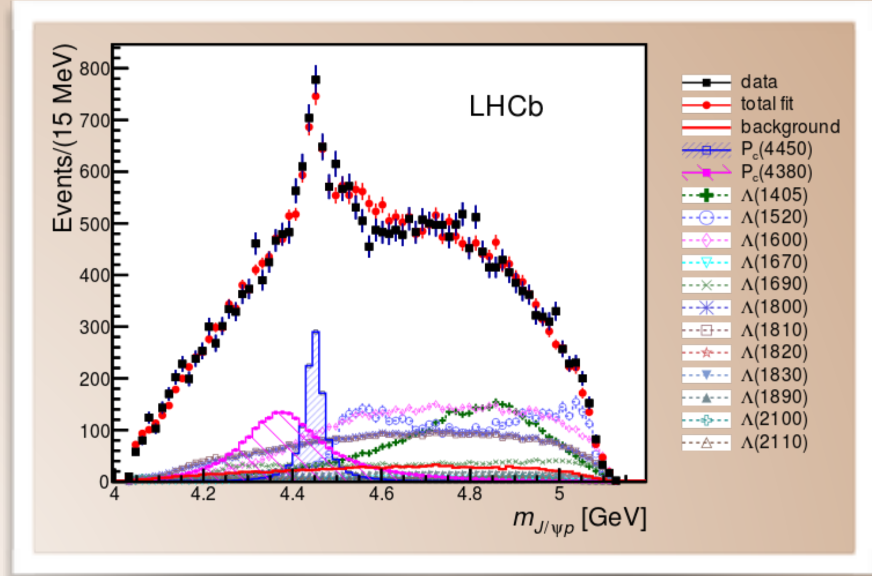

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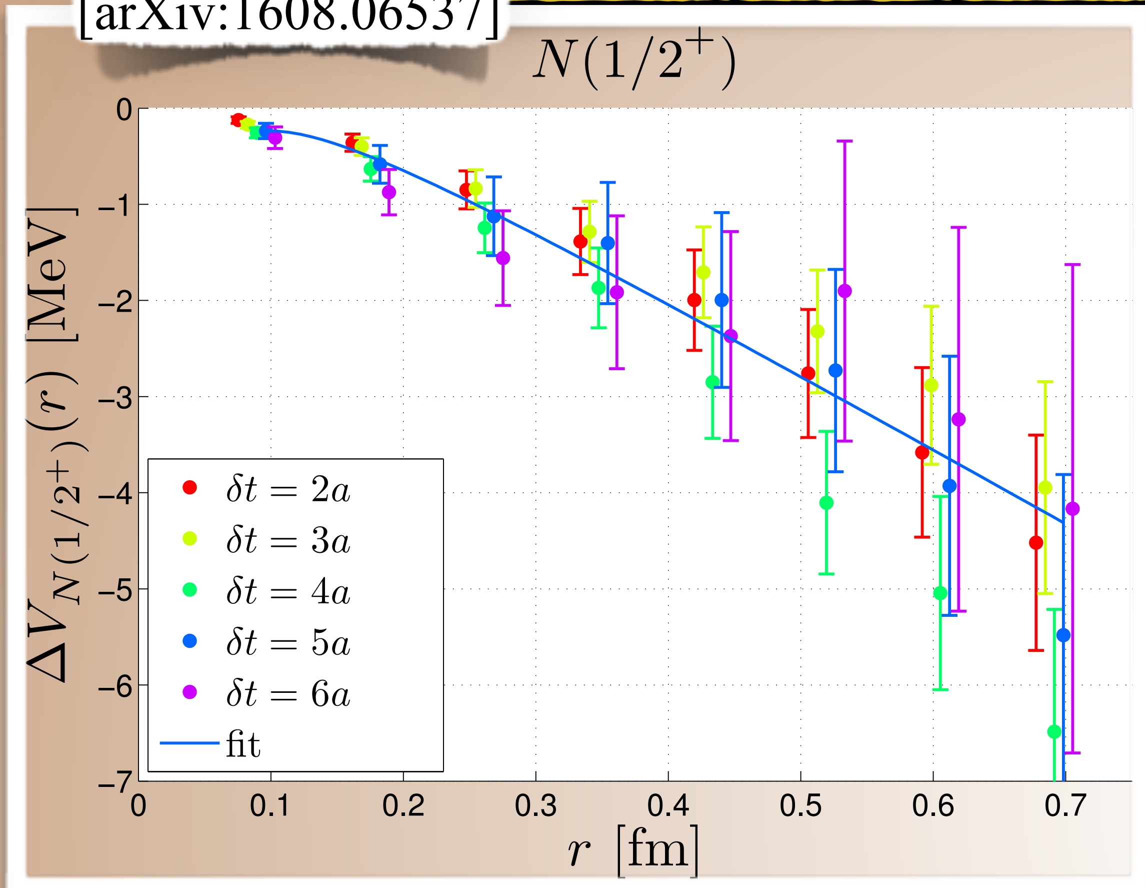

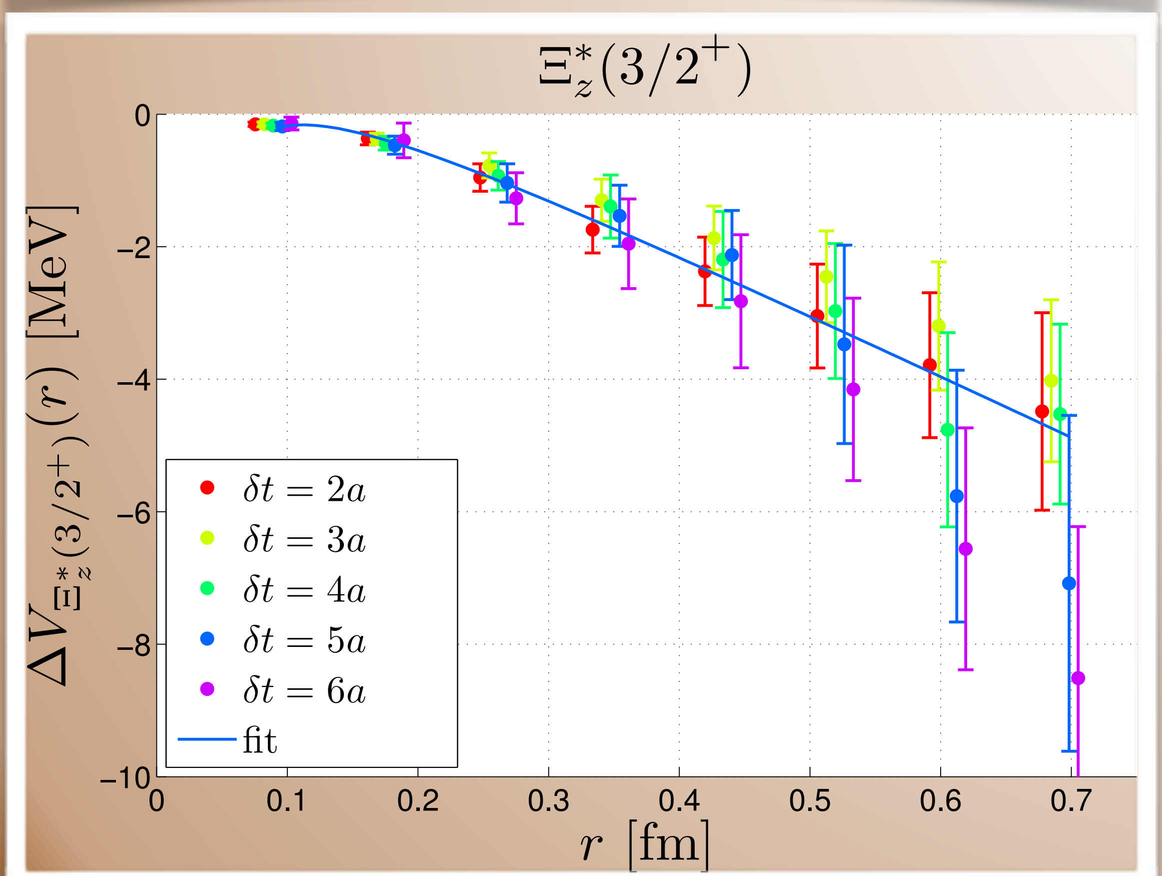

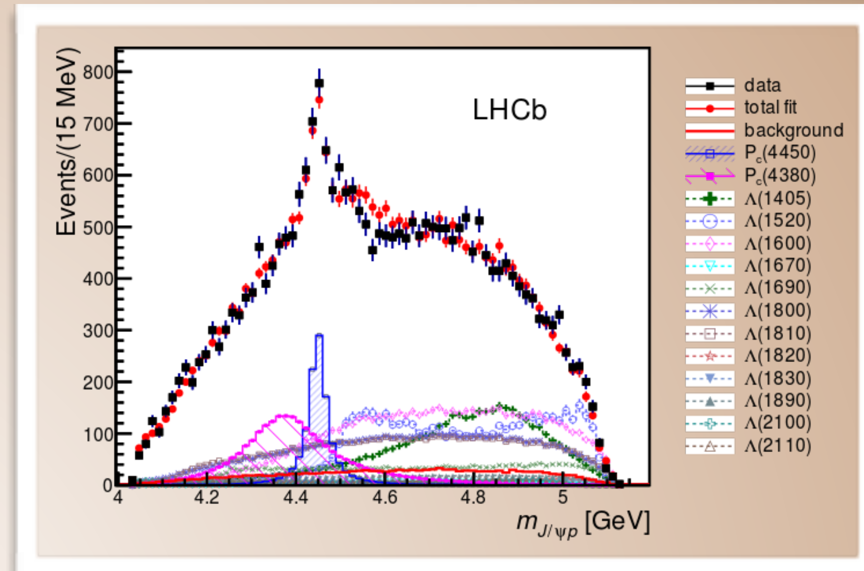

Similar effects for all octet and decouplet baryons

Hadro-quarkonium in the static limit

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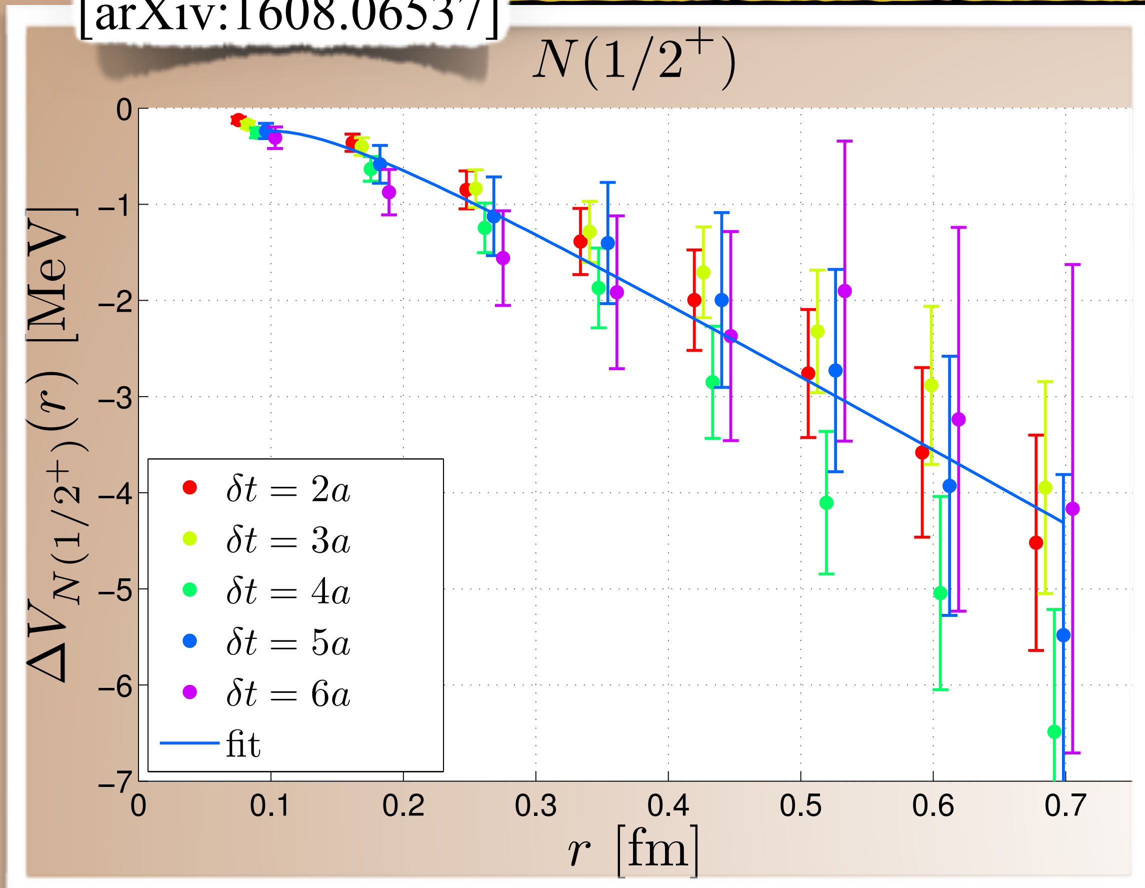
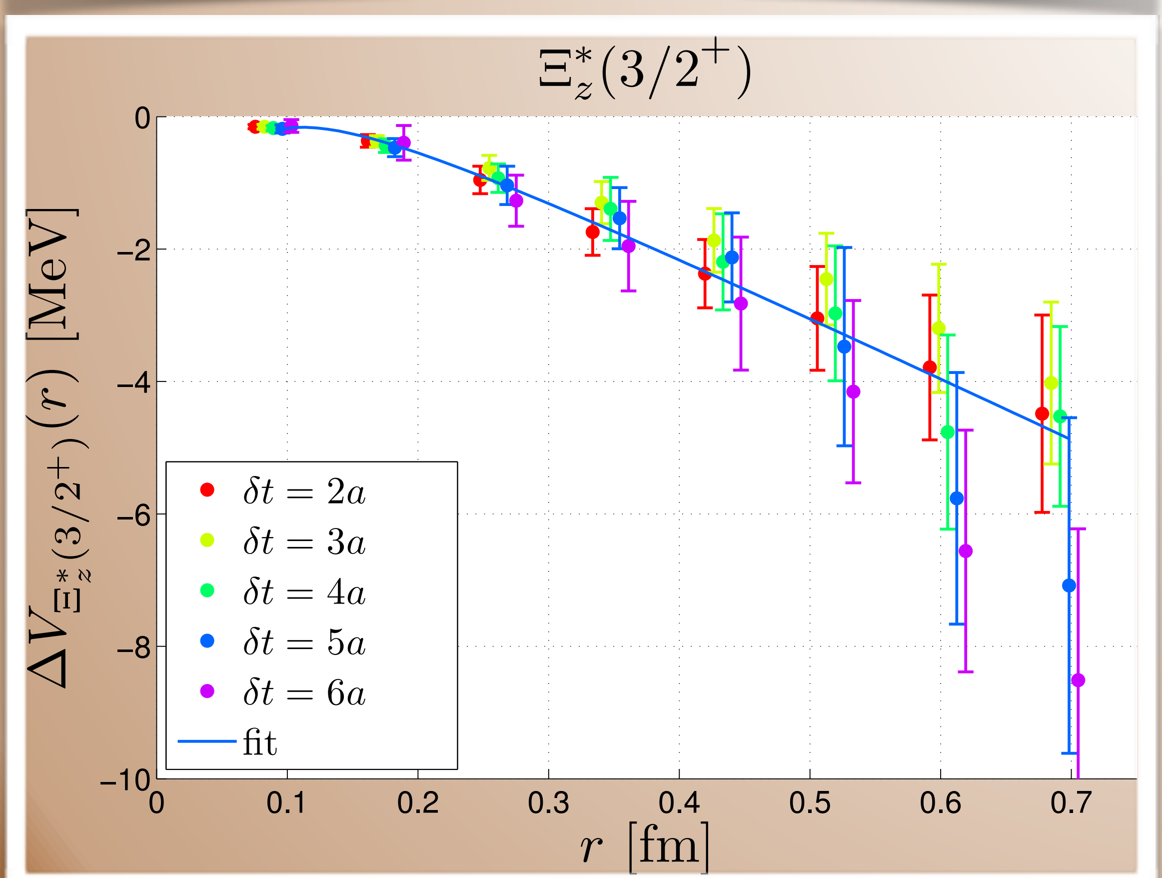
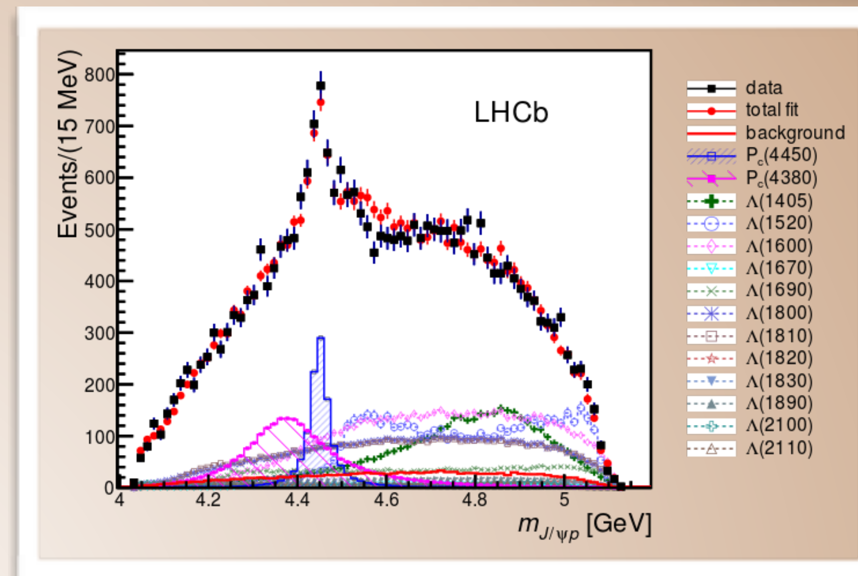






Similar effects for all octet and decuplet baryons

Modified potentials $\longrightarrow$ Schrödinger equation:

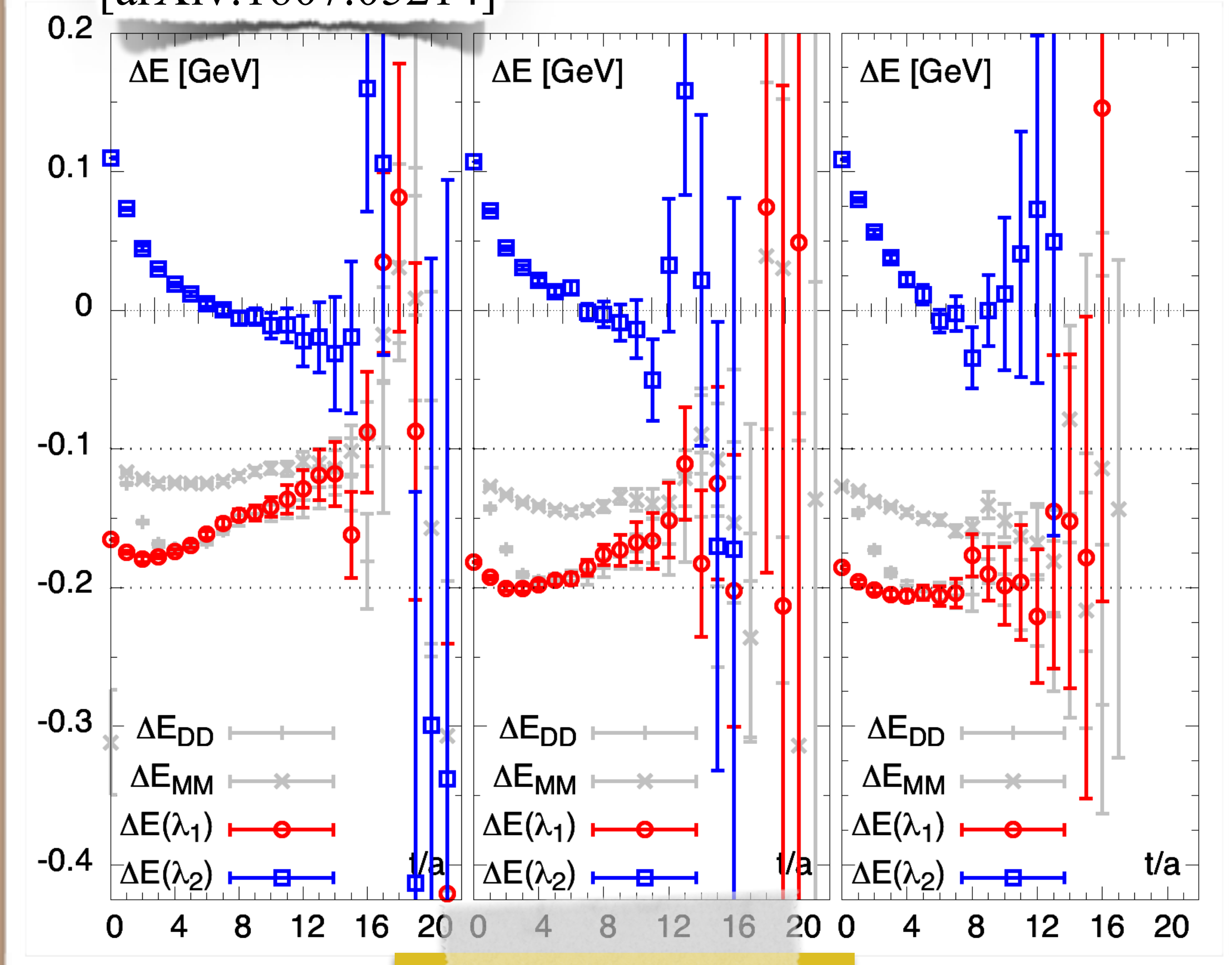
Charmonium 1S, 1P and 2S states reduce in mass by < 10 MeV

Tetra-quarks from NRQCD

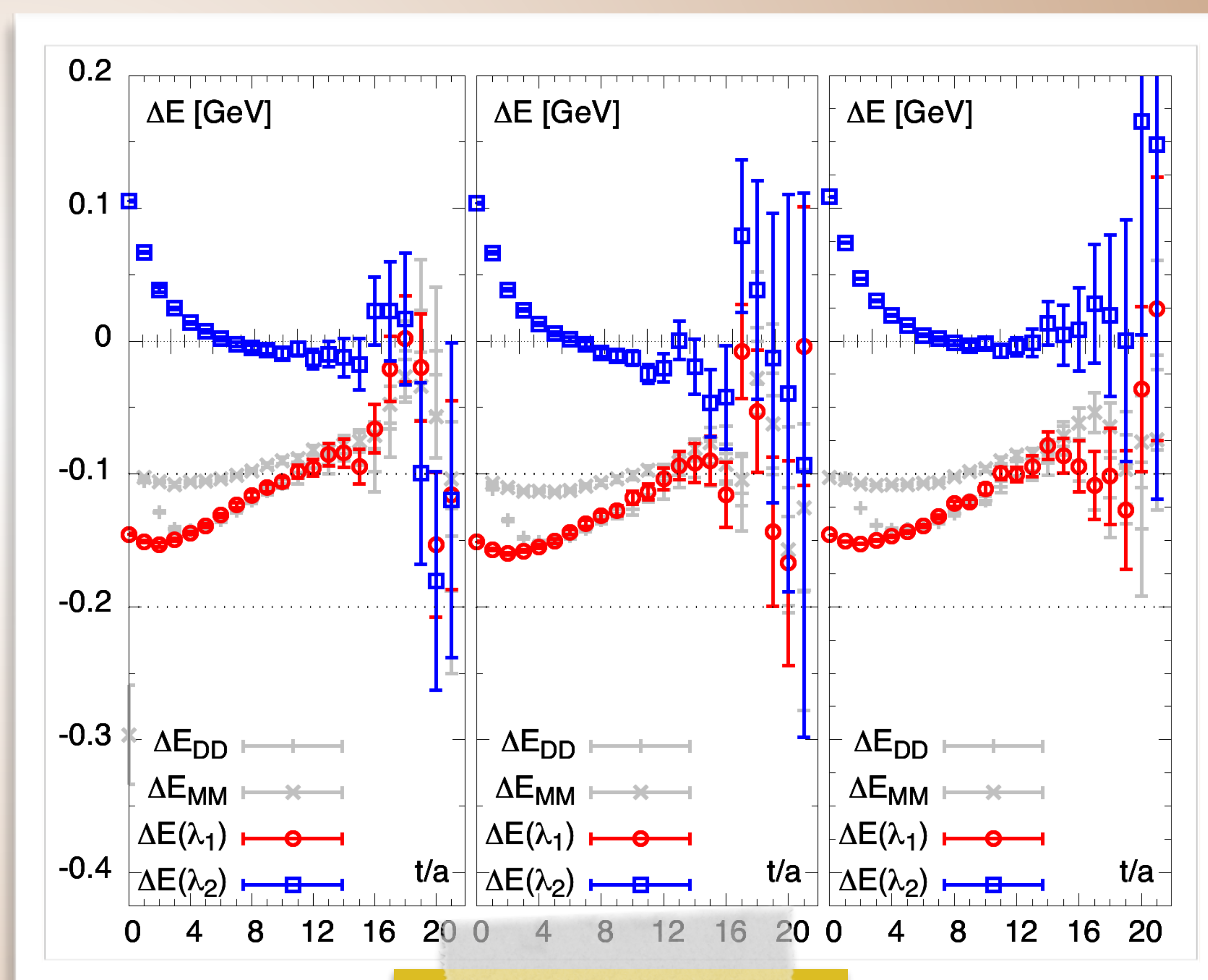
$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t)\ \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle = \sum_n A e^{-E_n t}$$

- A single 'meson-meson' type operator
- A single 'diquark-antidiquark' operator

[arXiv:1607.05214]



$ll\bar{b}\bar{b}$



$ls\bar{b}\bar{b}$

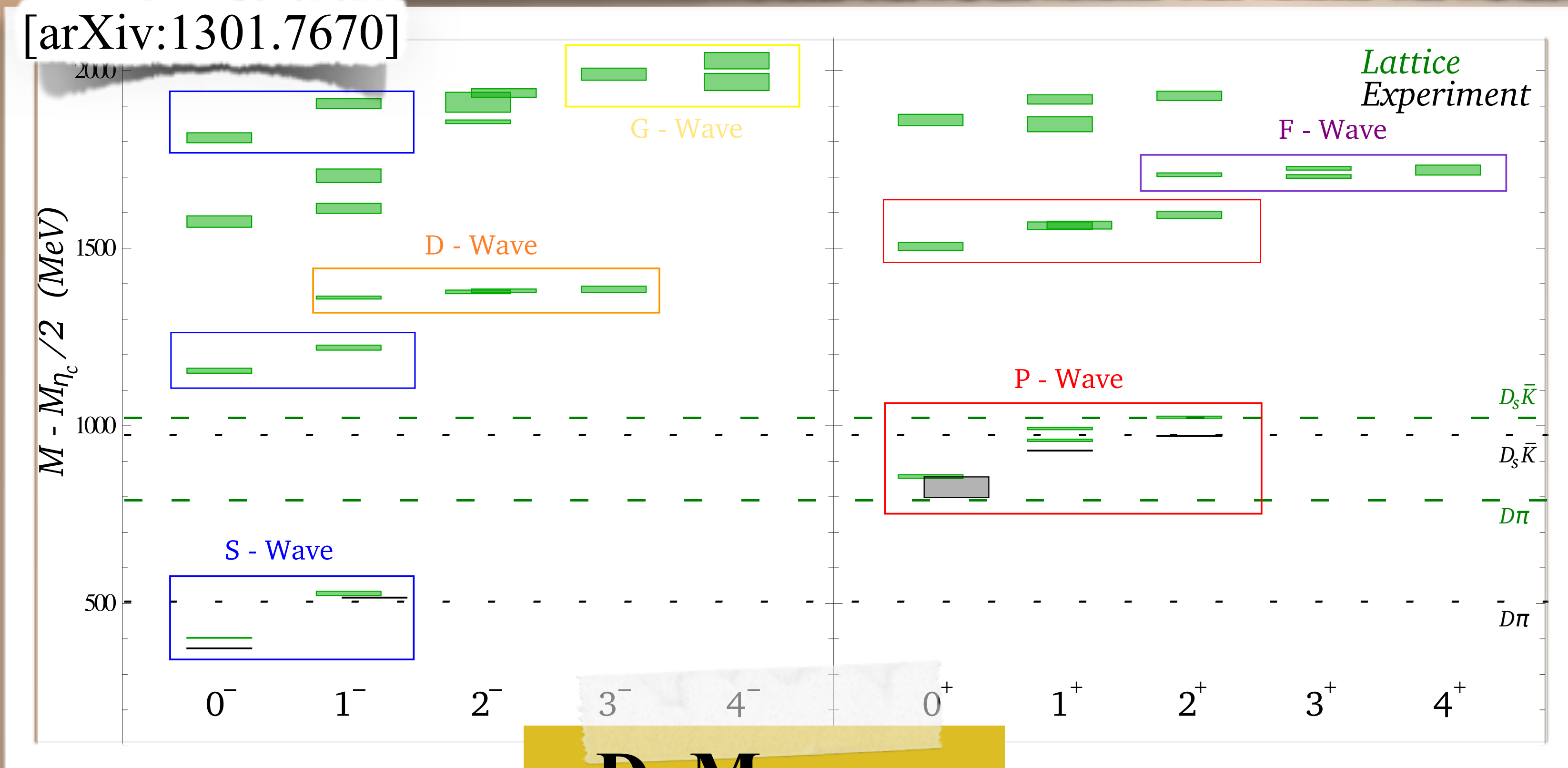
Predict tetra-quarks ~ 10.5 GeV that only decay weakly

'Single hadron spectroscopy'

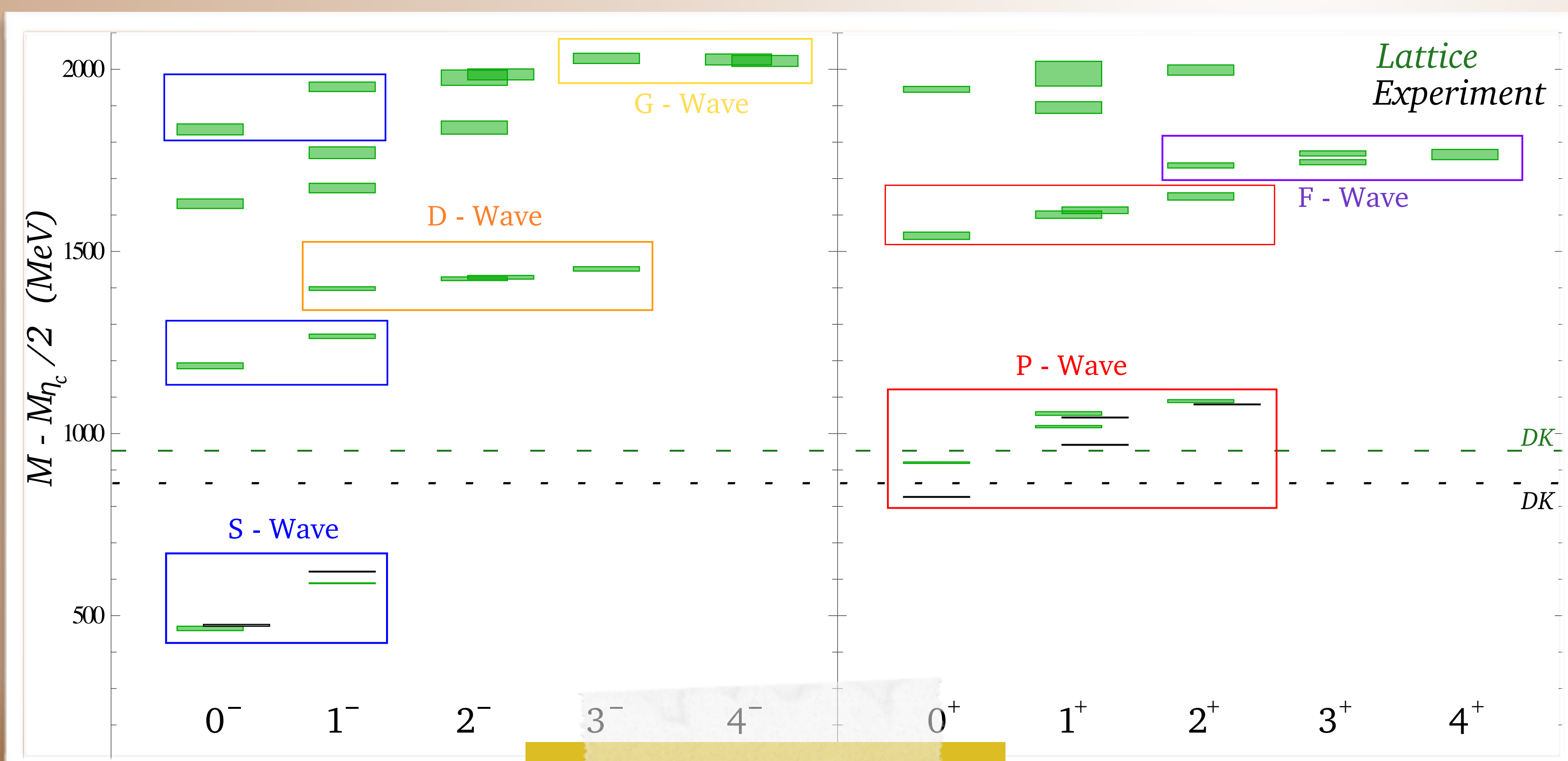
Excitation spectrum of finite-volume energy eigenstates

The Charm Sector - Mesons

[arXiv:1301.7670]



D Mesons



Ds Mesons

Operators:

- 'quark-antiquark'
- 'quark-glue-antiquark'

'Excess states' → hybrid mesons

The Charm Sector - Mesons

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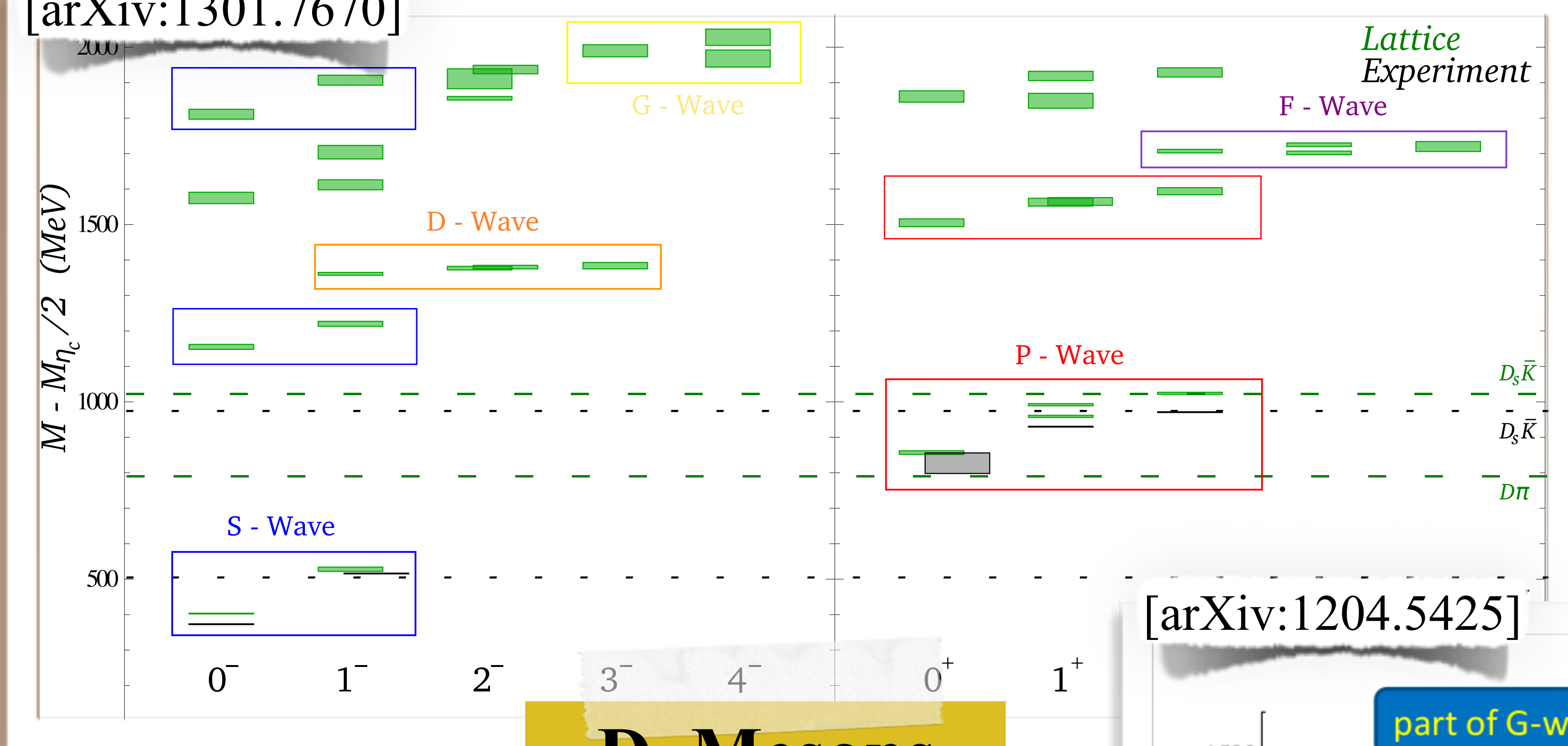

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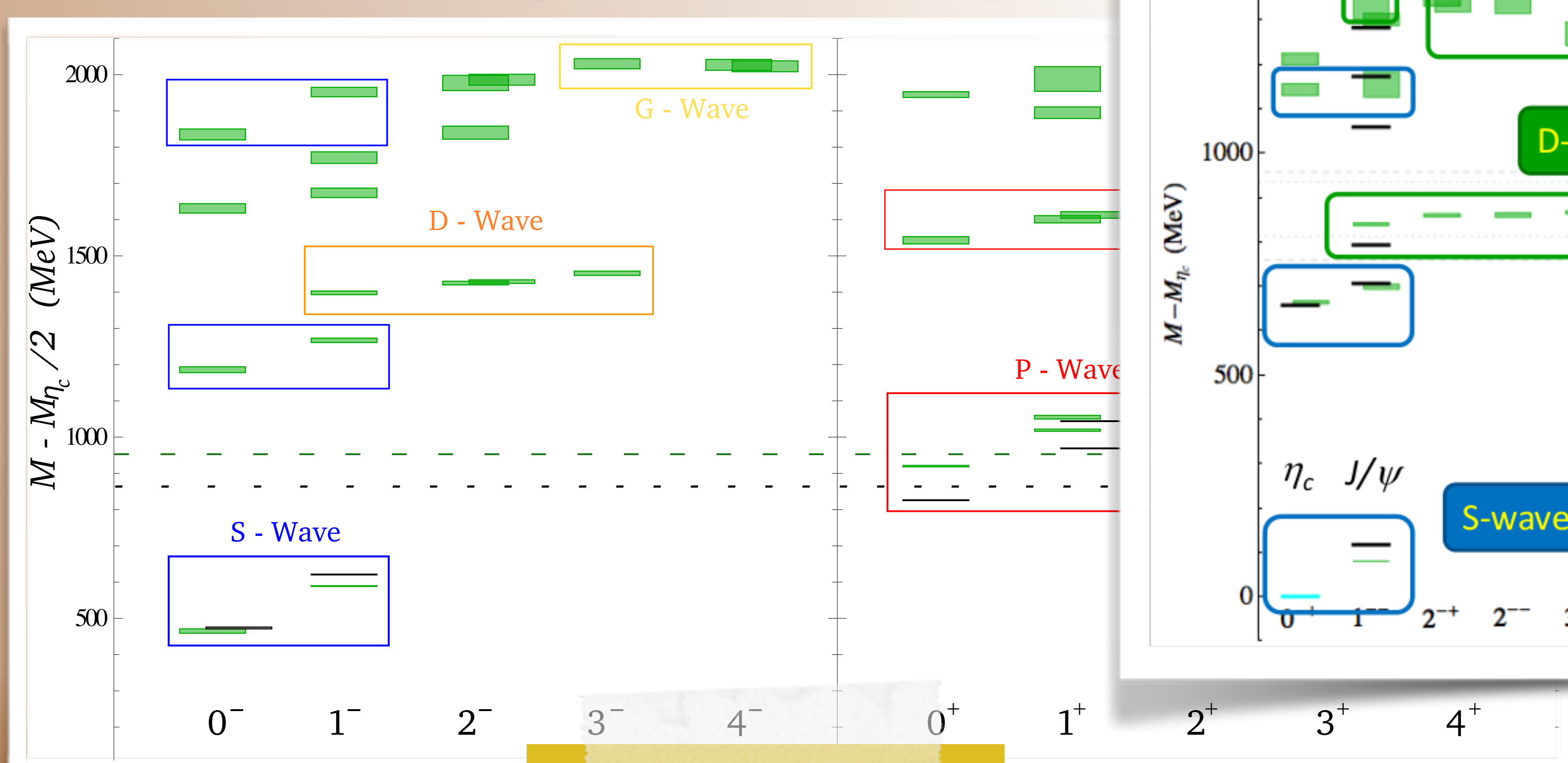

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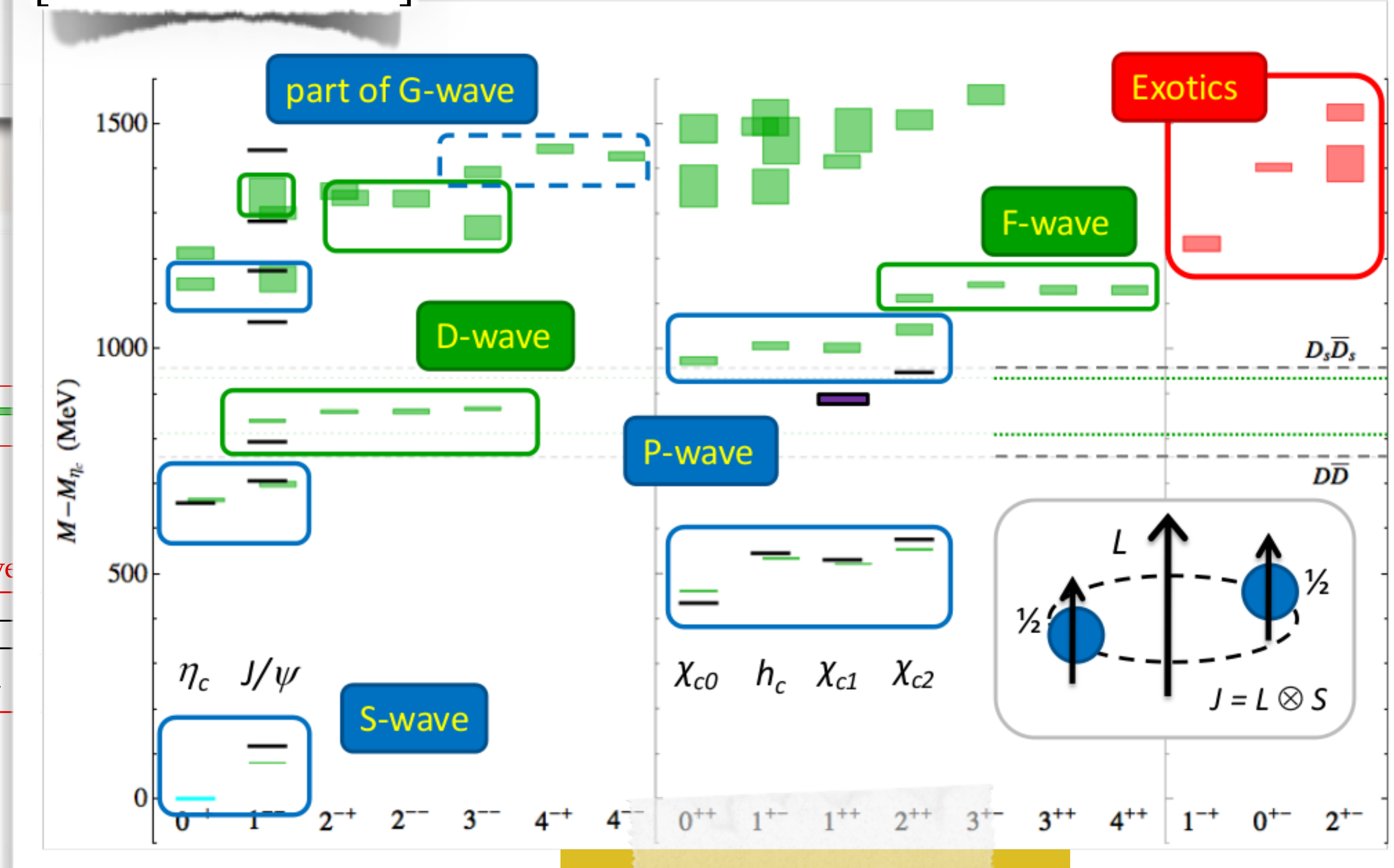
States with ‘exotic’ J^{PC}

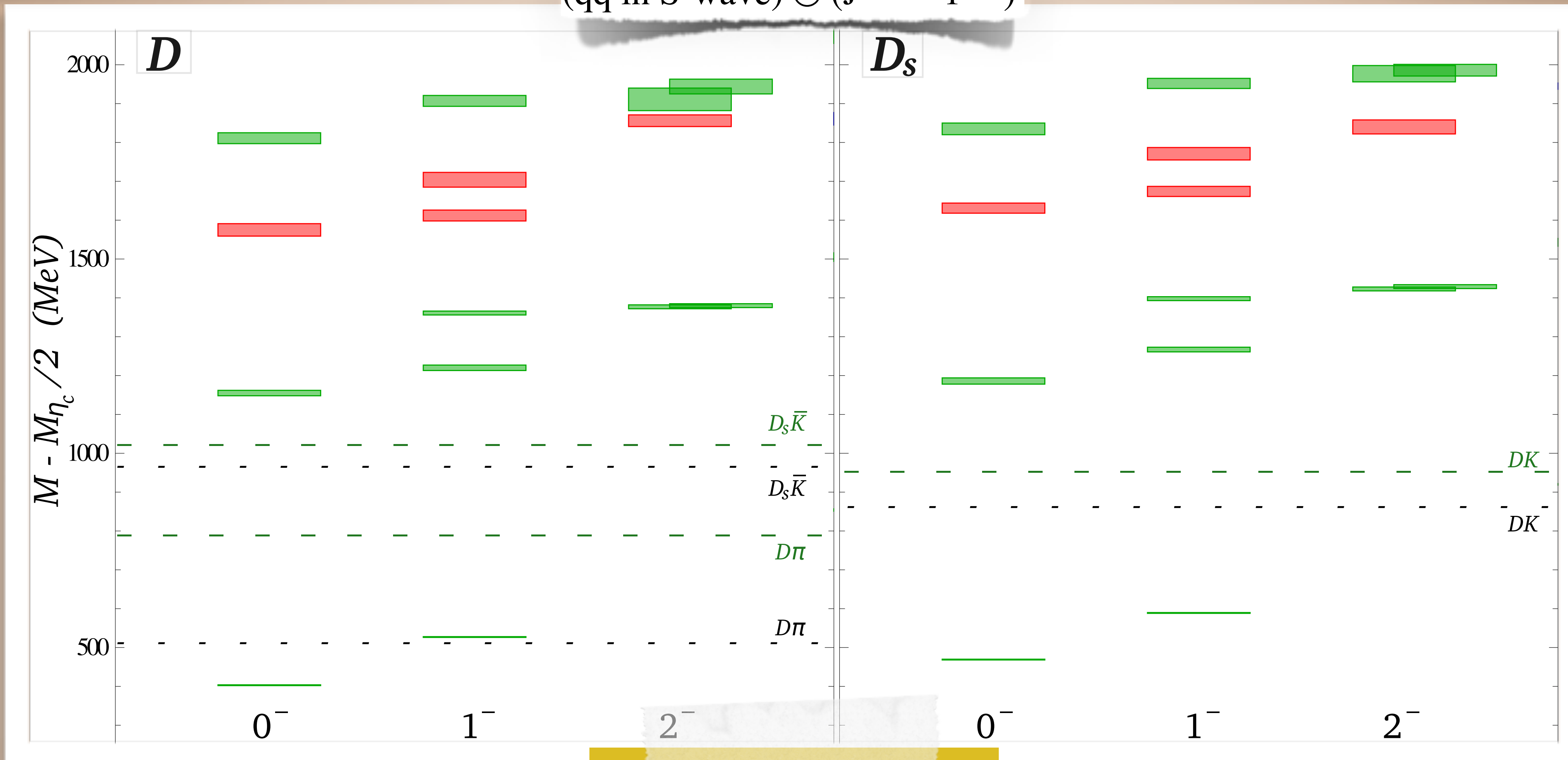
[arXiv:1204.5425]



Charmonium

The Charm Sector - Mesons

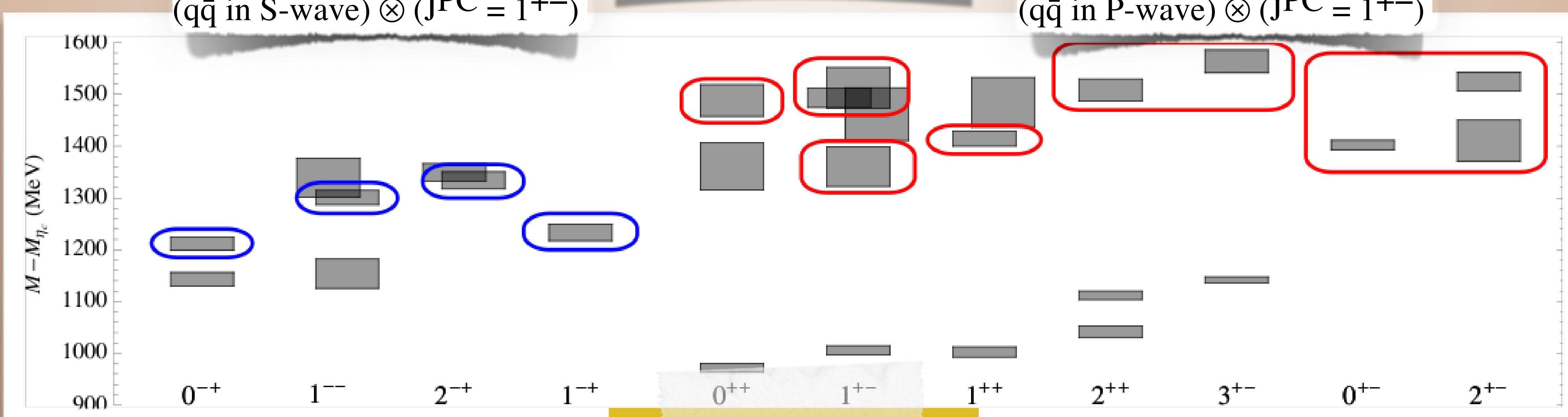
(q$\bar{q}$ in S-wave) ⊗ ($J^{PC} = 1^{+-}$)



Open-Charm

(q$\bar{q}$ in S-wave) ⊗ ($J^{PC} = 1^{+-}$)

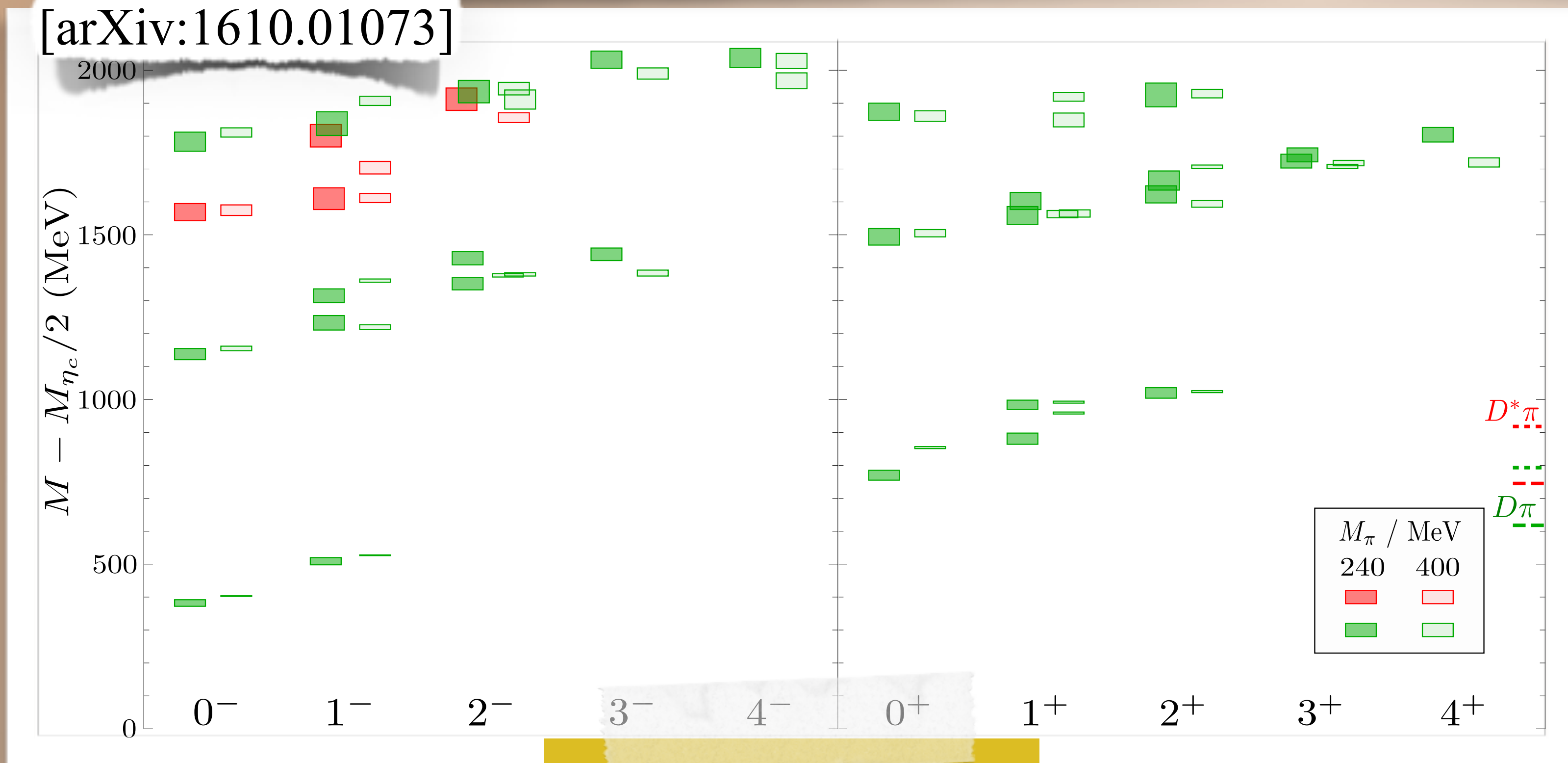
(q$\bar{q}$ in P-wave) ⊗ ($J^{PC} = 1^{+-}$)



Charmonium

The Charm Sector - Mesons

[arXiv:1610.01073]



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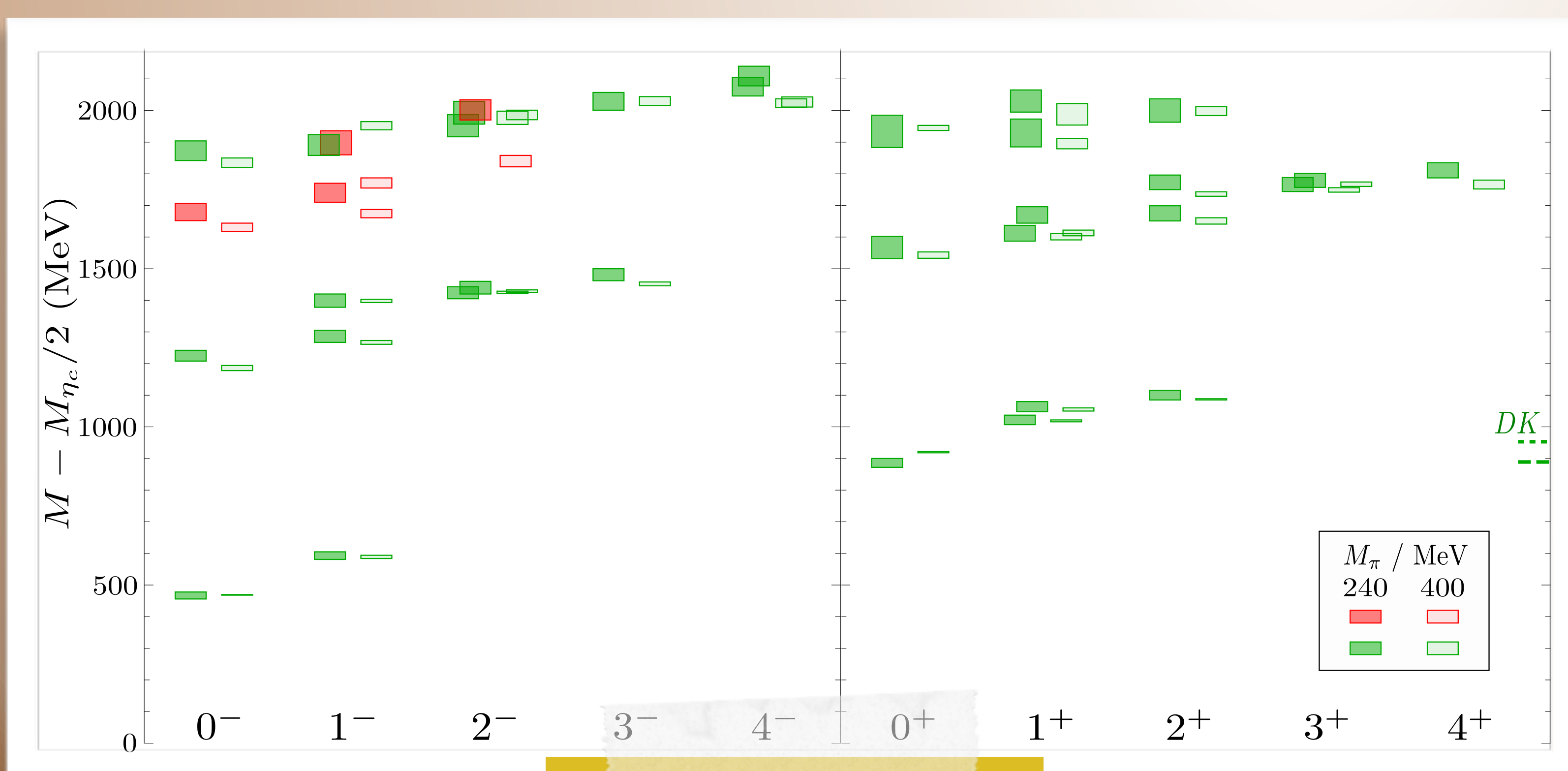


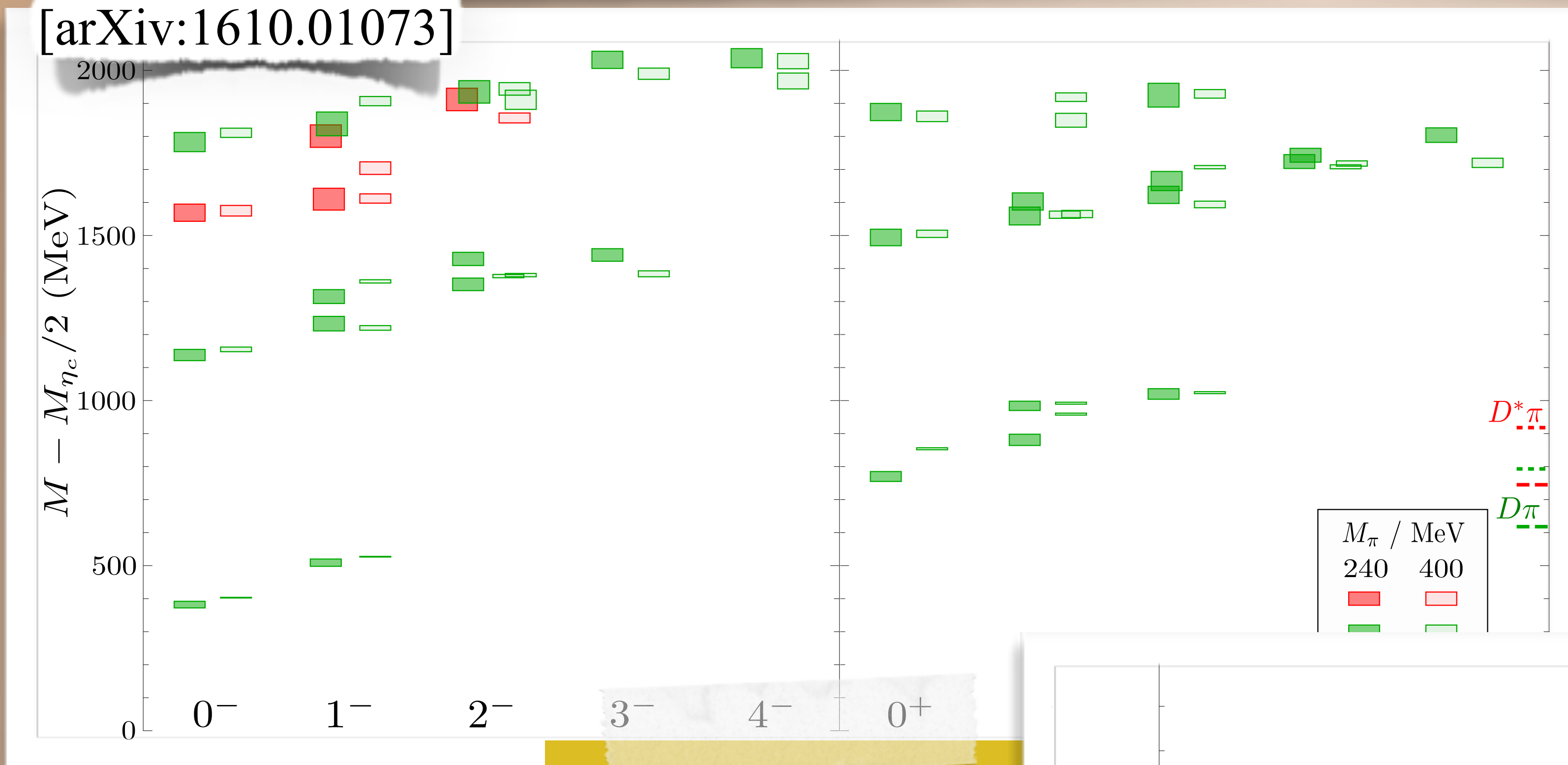
D_s Mesons

Light quark mass dependence:

- M_π: **400 MeV → 240 MeV**
- **Small quantitative changes**
- **No qualitative changes!**

The Charm Sector - Mesons

[arXiv:1610.01073]



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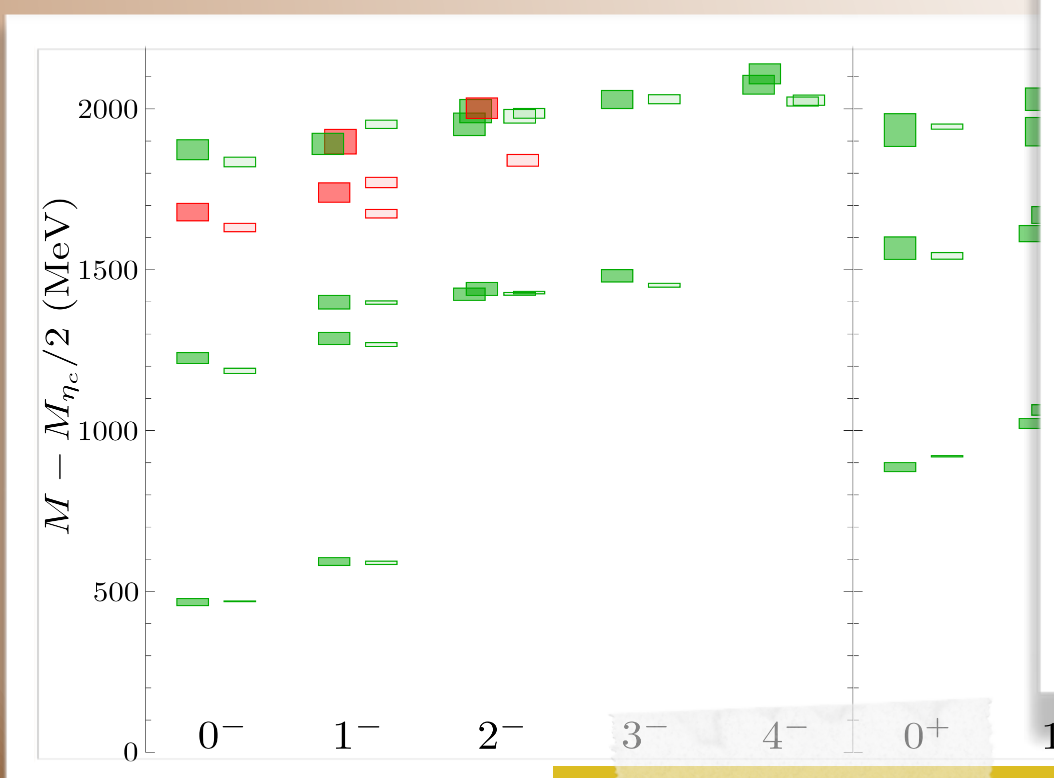

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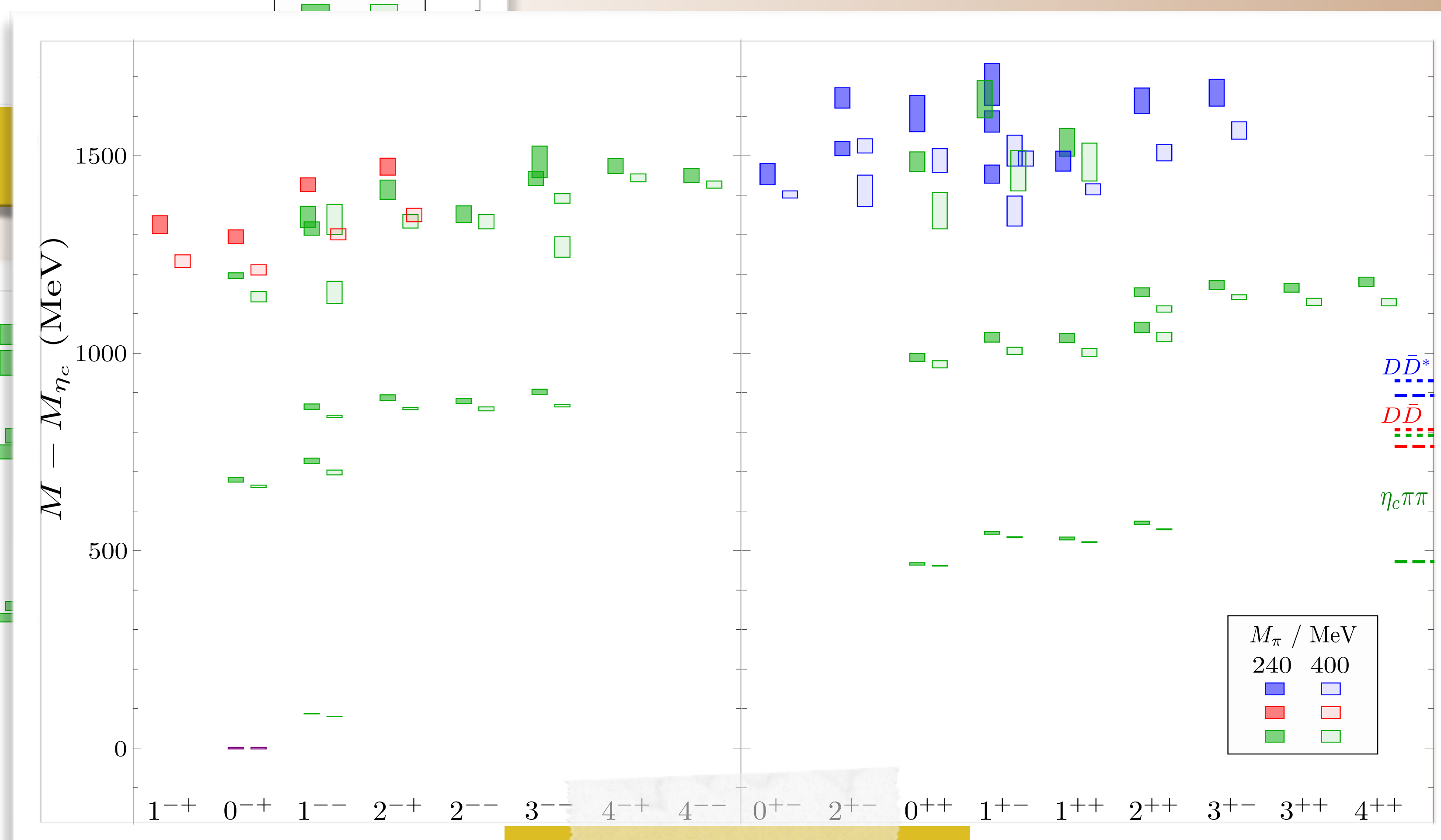

Charmonium

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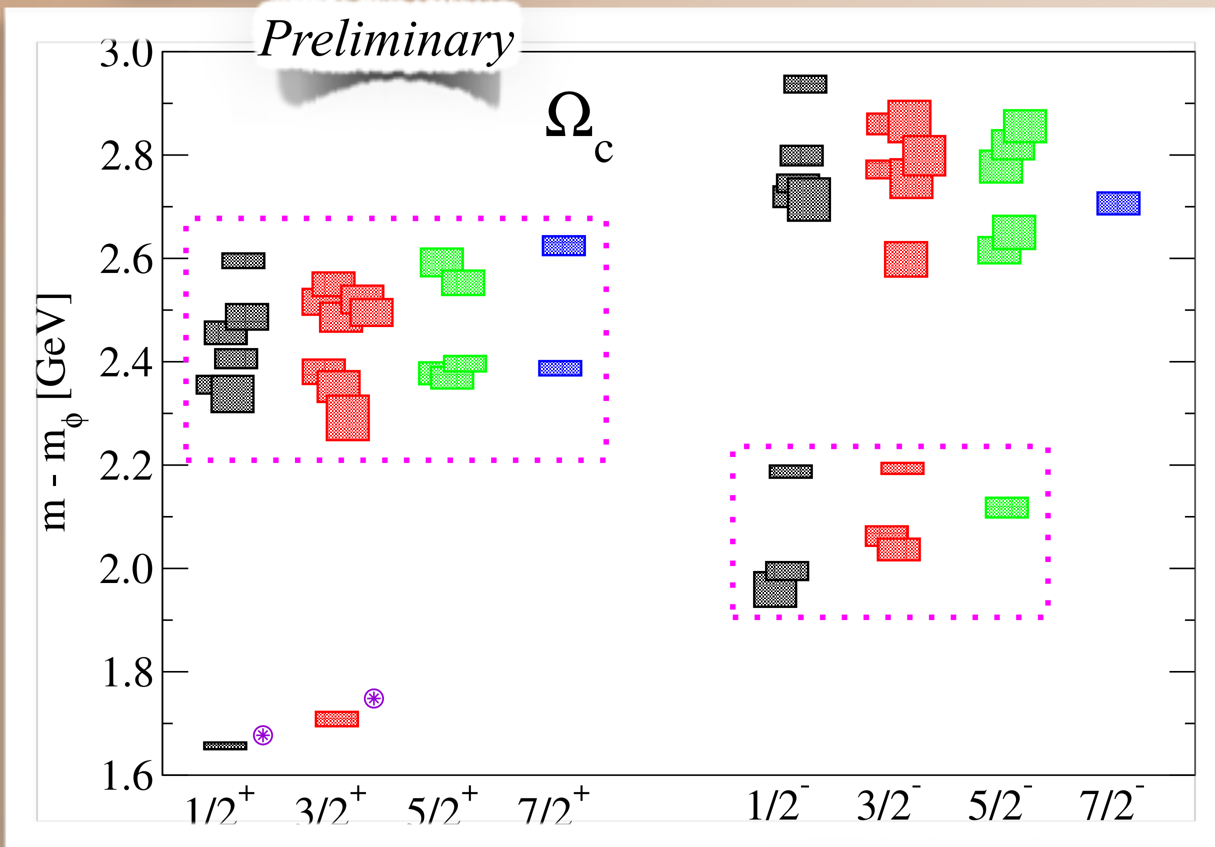
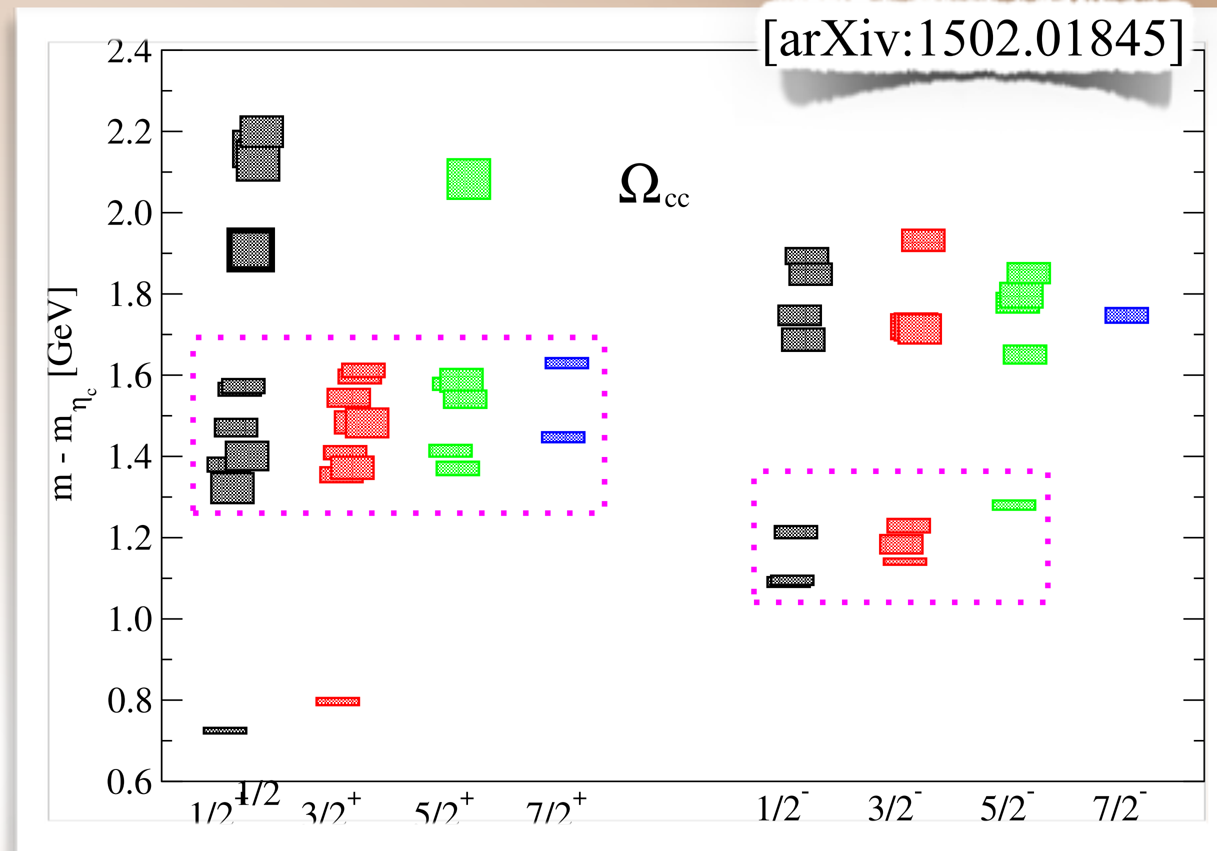
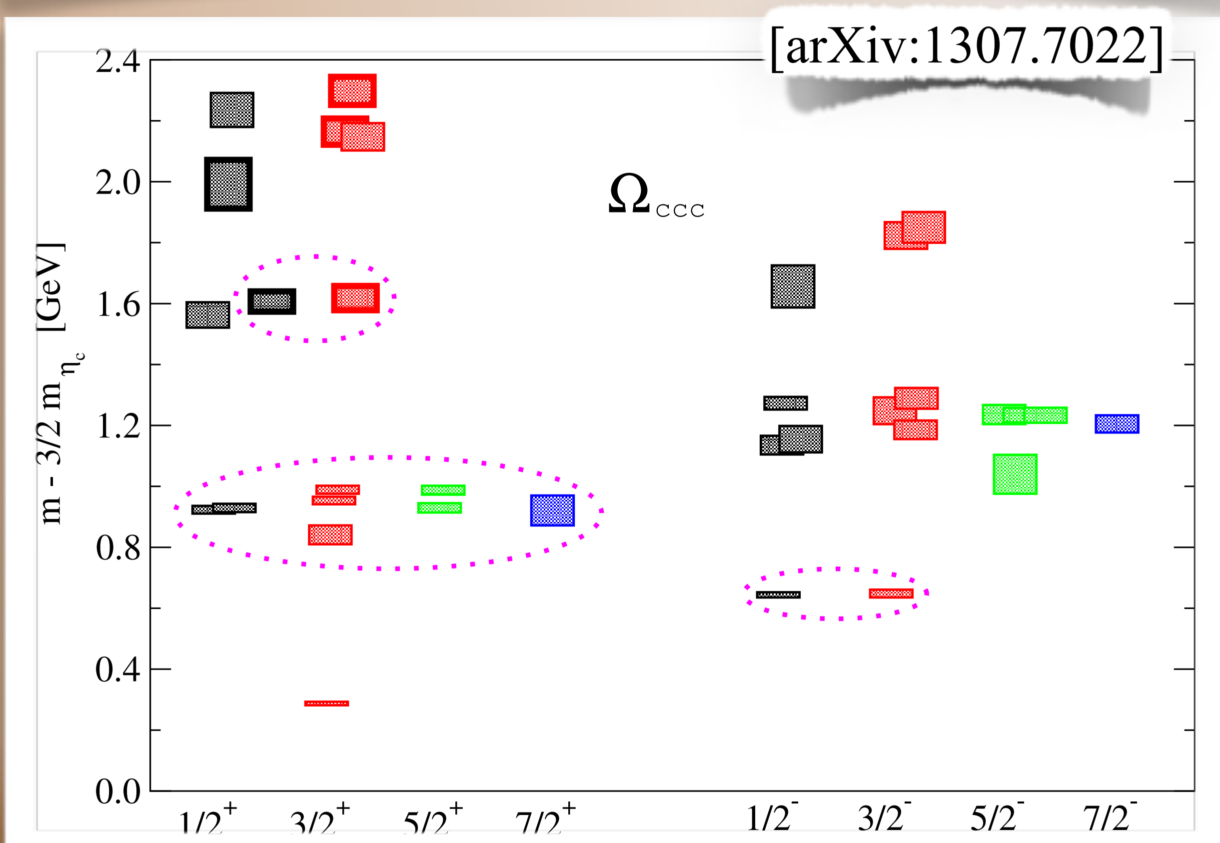
The Charm Sector - Baryons



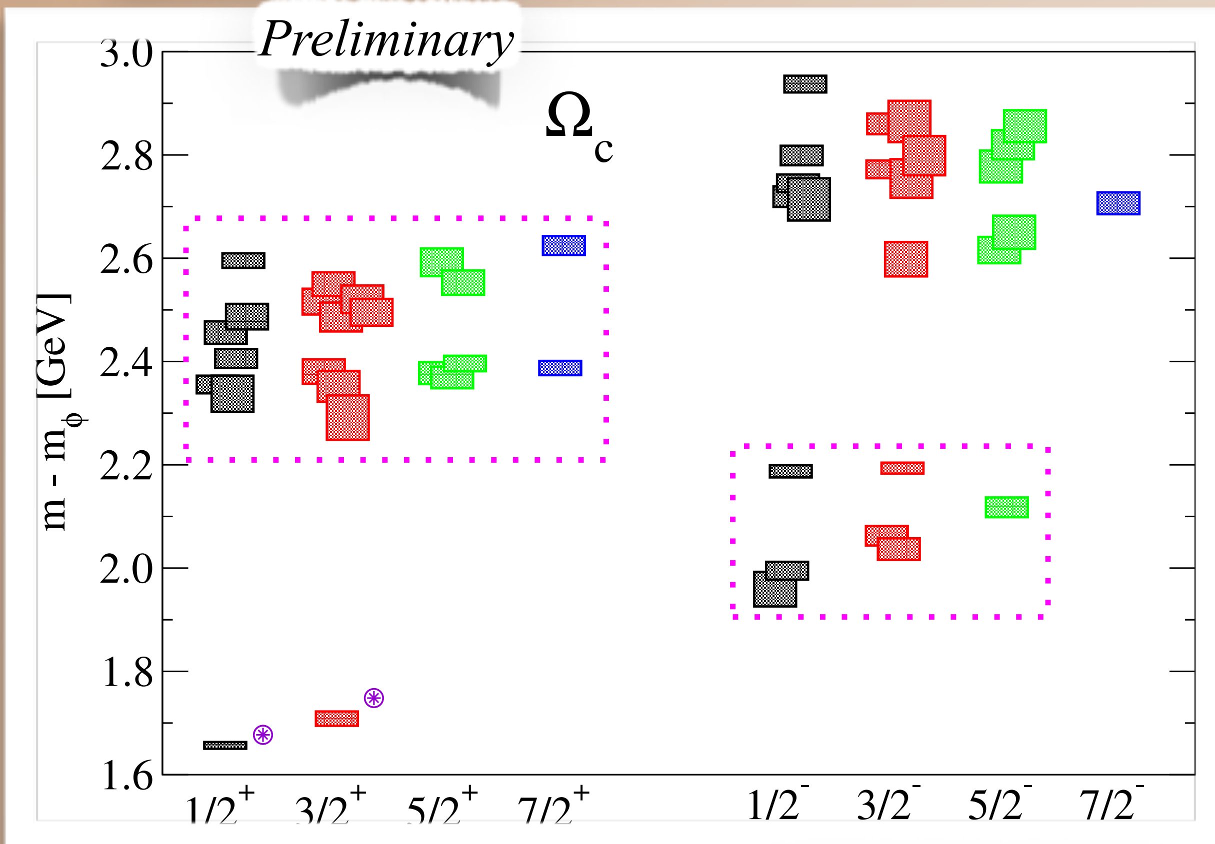


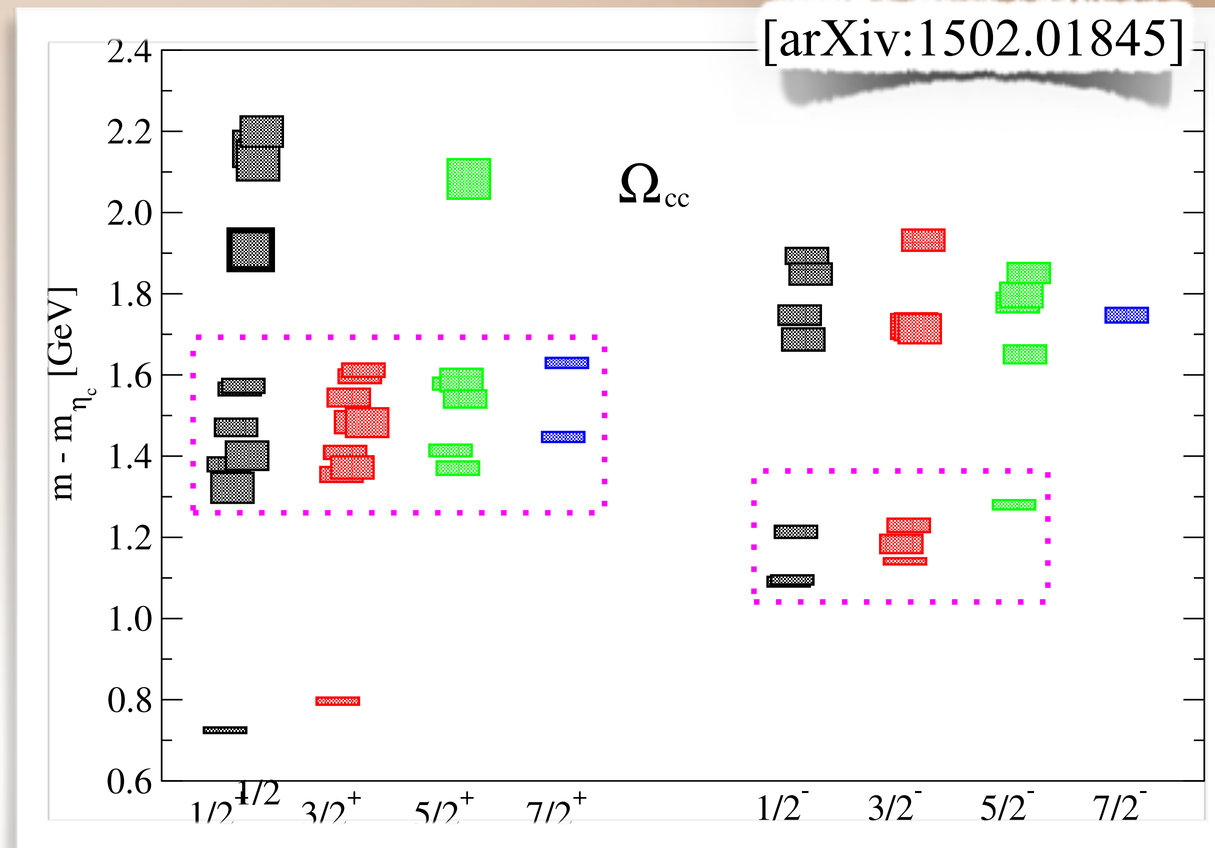


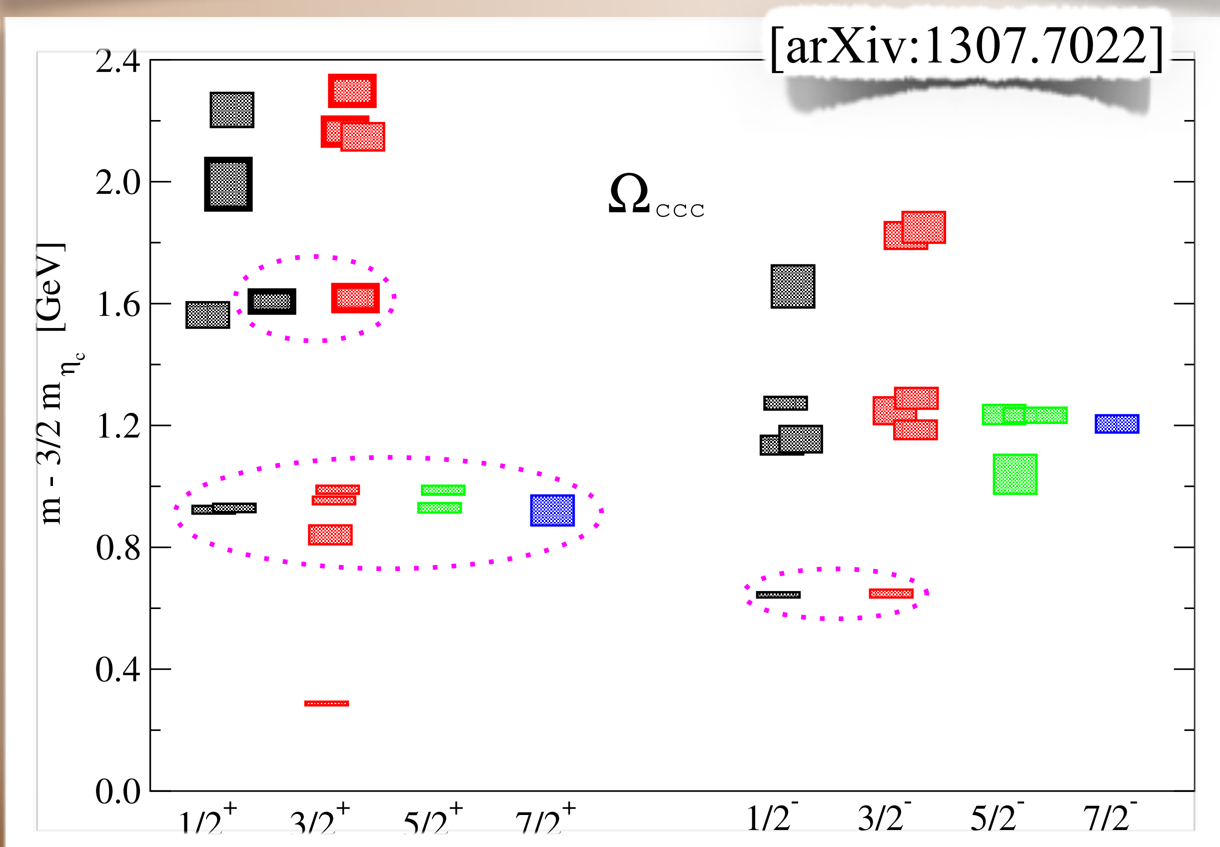
Operators:

- **'quark-quark-quark'**
- **'quark-quark-quark-glue'**

The Charm Sector - Baryons







<u>Operators:</u>

- **'quark-quark-quark'**
- **'quark-quark-quark-glue'**

Low-lying spectrum consistent with the non-relativistic $SU(6) \otimes O(3)$ quark models

Scale of hybrids similar to the meson sector!

'Lüscher formalism'

Finite-volume energy eigenstates ⟷ **infinite-volume scattering amplitudes**

Scattering on the Lattice

'Lüscher formalism'

$$\det[t_{ij}^{-1}(E) + M_{ij}(E,L)] = 0$$

Scattering on the Lattice

Infinite-volume t-matrix

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Channels

Known finite-volume function

$$C_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t)\ \mathcal{O}_j^\dagger(0) \middle| 0 \right\rangle = \sum_n A e^{-E_n t}$$

Lattice QCD spectrum → **infinite-volume t-matrix**

Scattering on the Lattice

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Lattice QCD spectrum $\longrightarrow$ infinite-volume t-matrix

The bad news:

- N channels $\Longrightarrow$ N(N+1)/2 unknowns per energy!
- Under-constrained for N > 1

A work-around:

- Parametrise the t-matrix with a 'few' free parameters
- Use >> 'few' parameters to constrain the t-matrix as a function of energy

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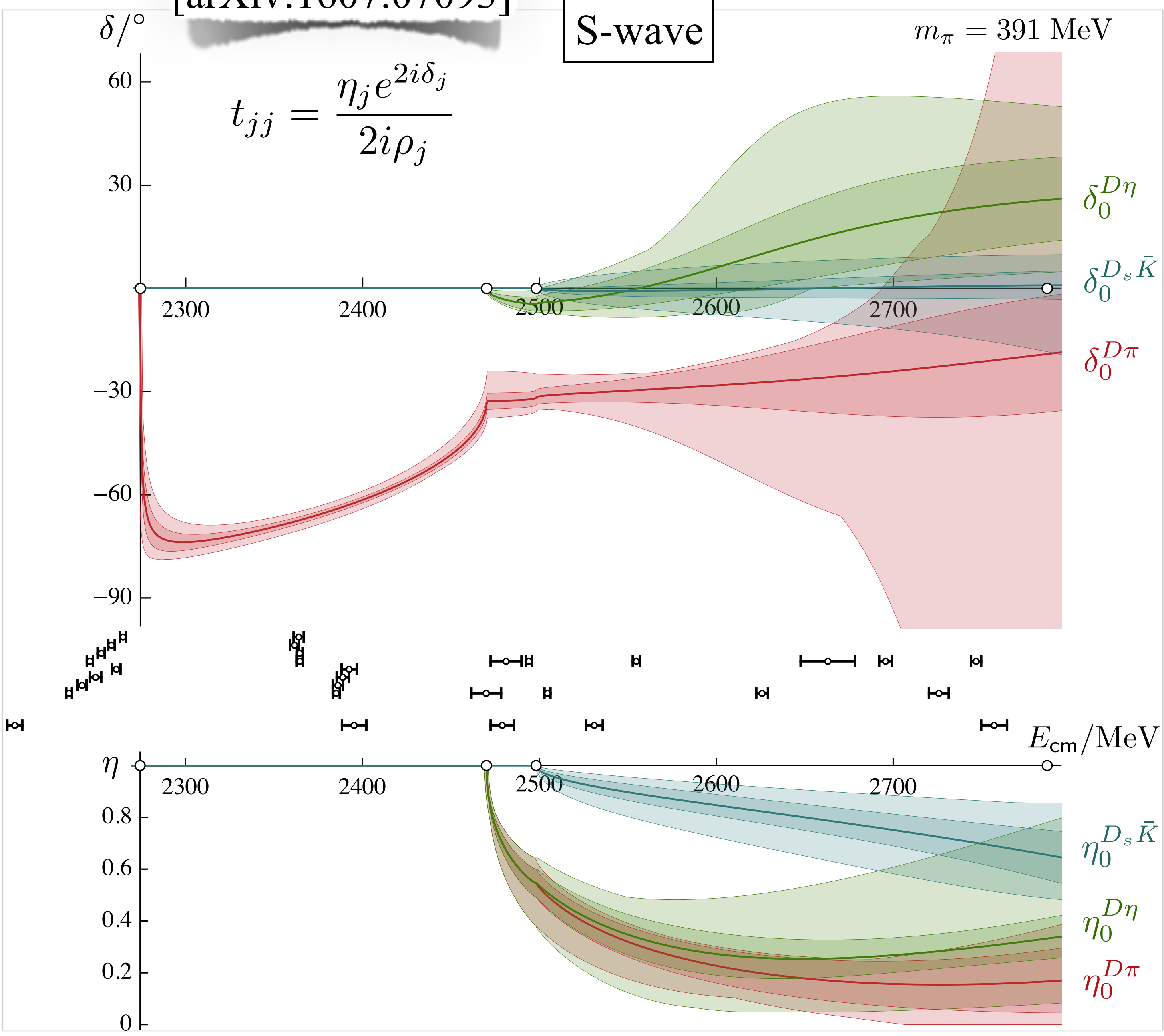
- **Preserve Unitarity**
- **Examine pole content of t-matrix**

$D\pi, D\eta, D_s\bar{K}$ Scattering

[arXiv:1607.07093]



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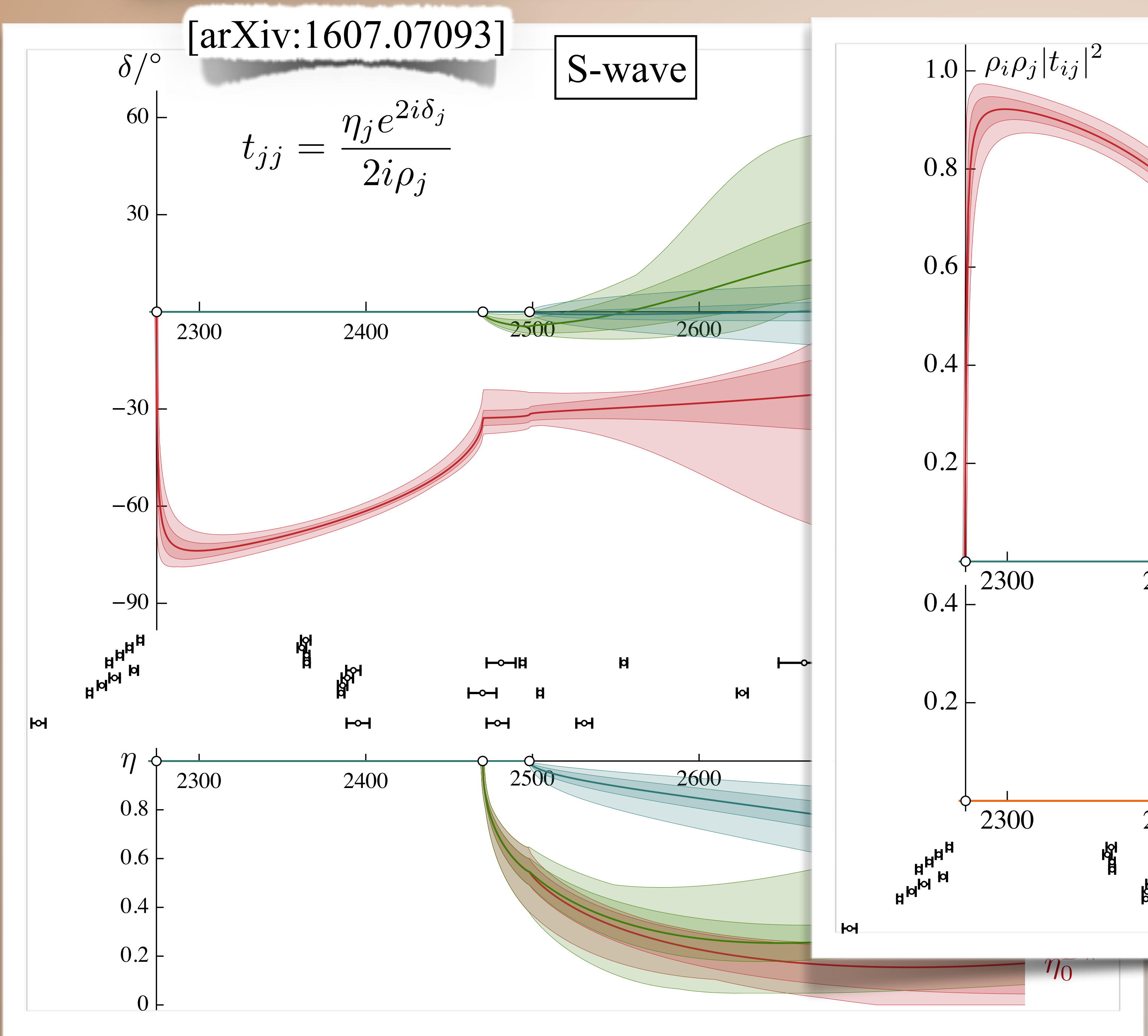
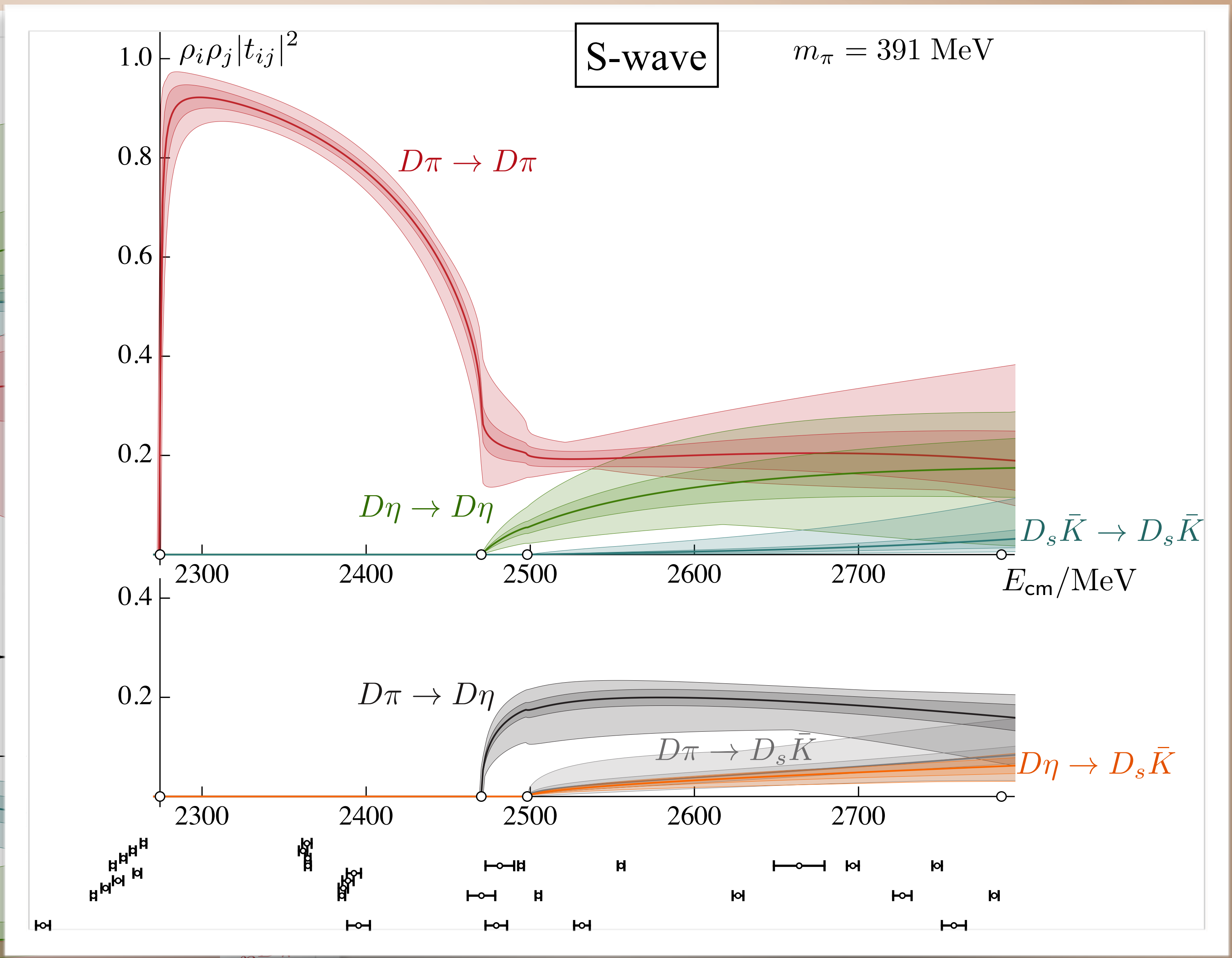




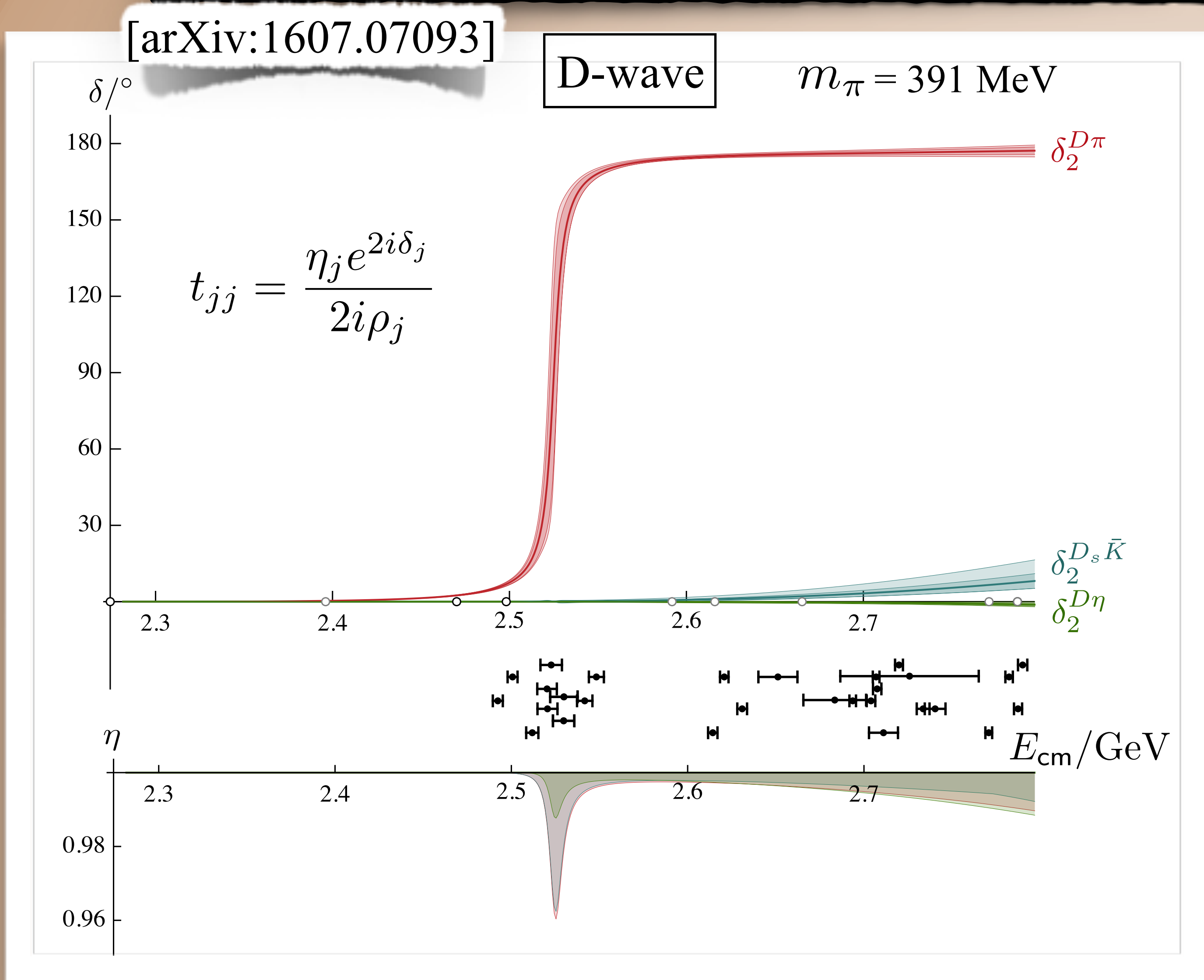
S-wave:

- **Near threshold bound-state pole**

P-wave:

- **Deeply bound pole**

$D\pi, D\eta, D_s\bar{K}$ Scattering



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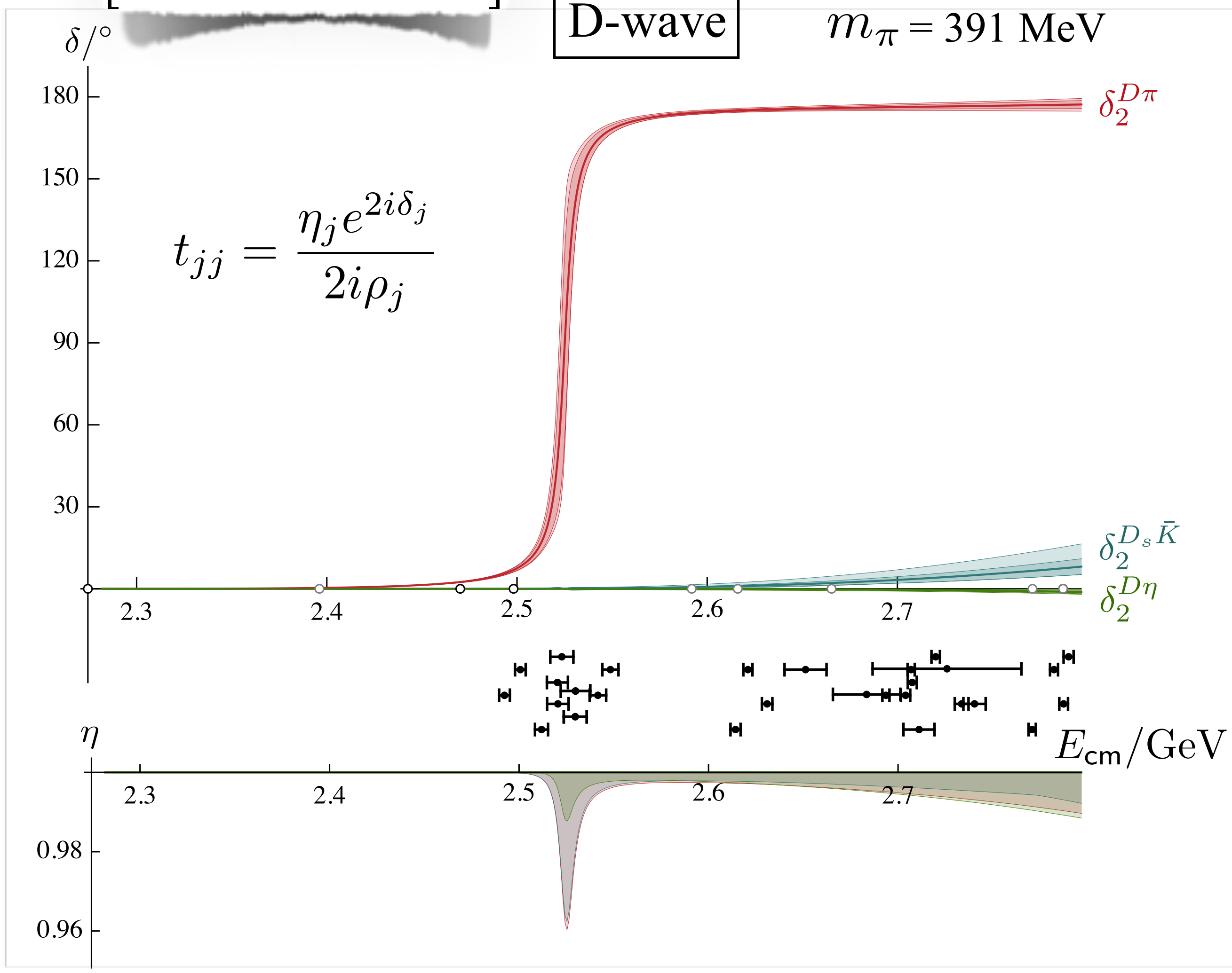

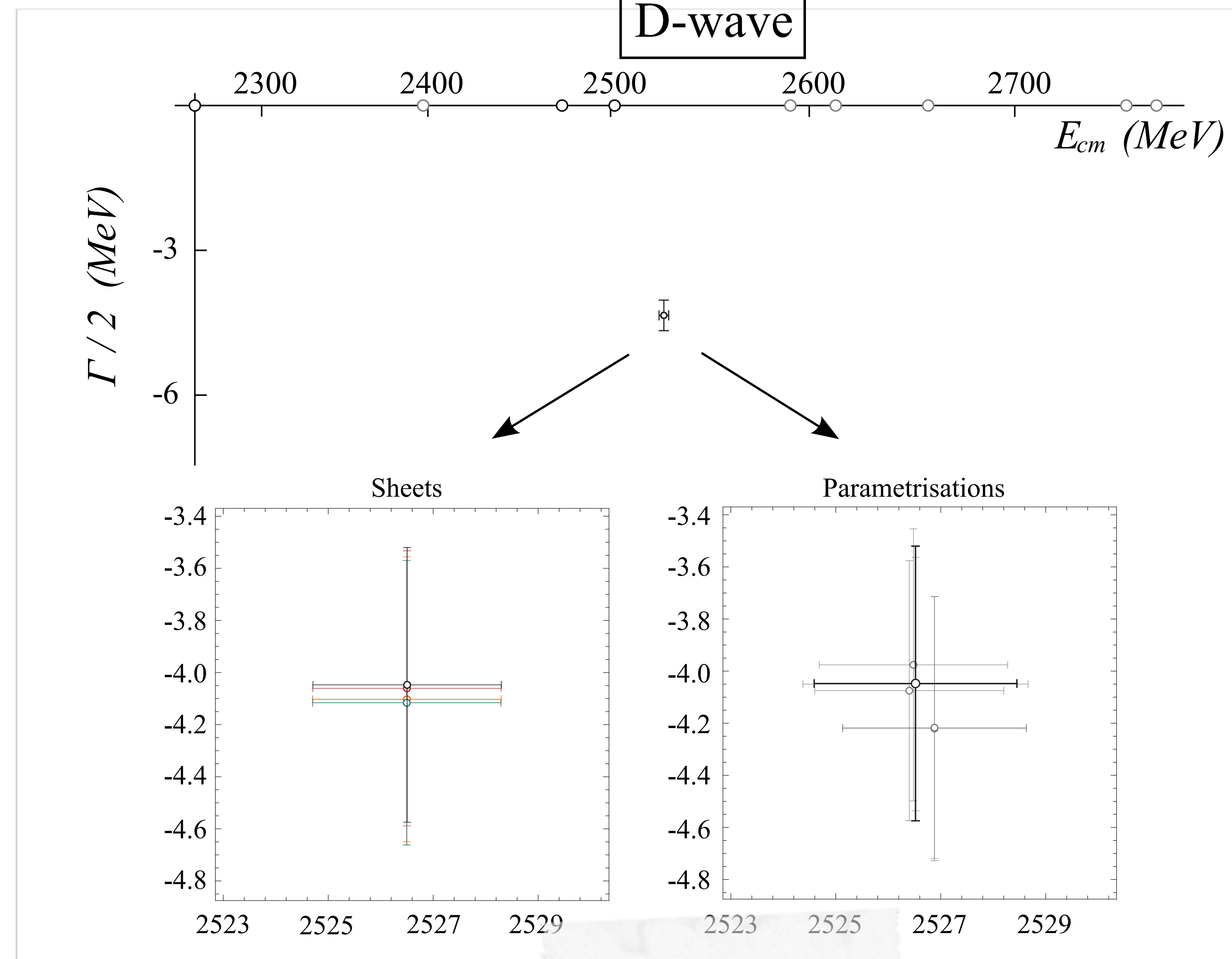

Poles on all sheets with $\mathrm{Im}[k_{D\pi}] < 0$

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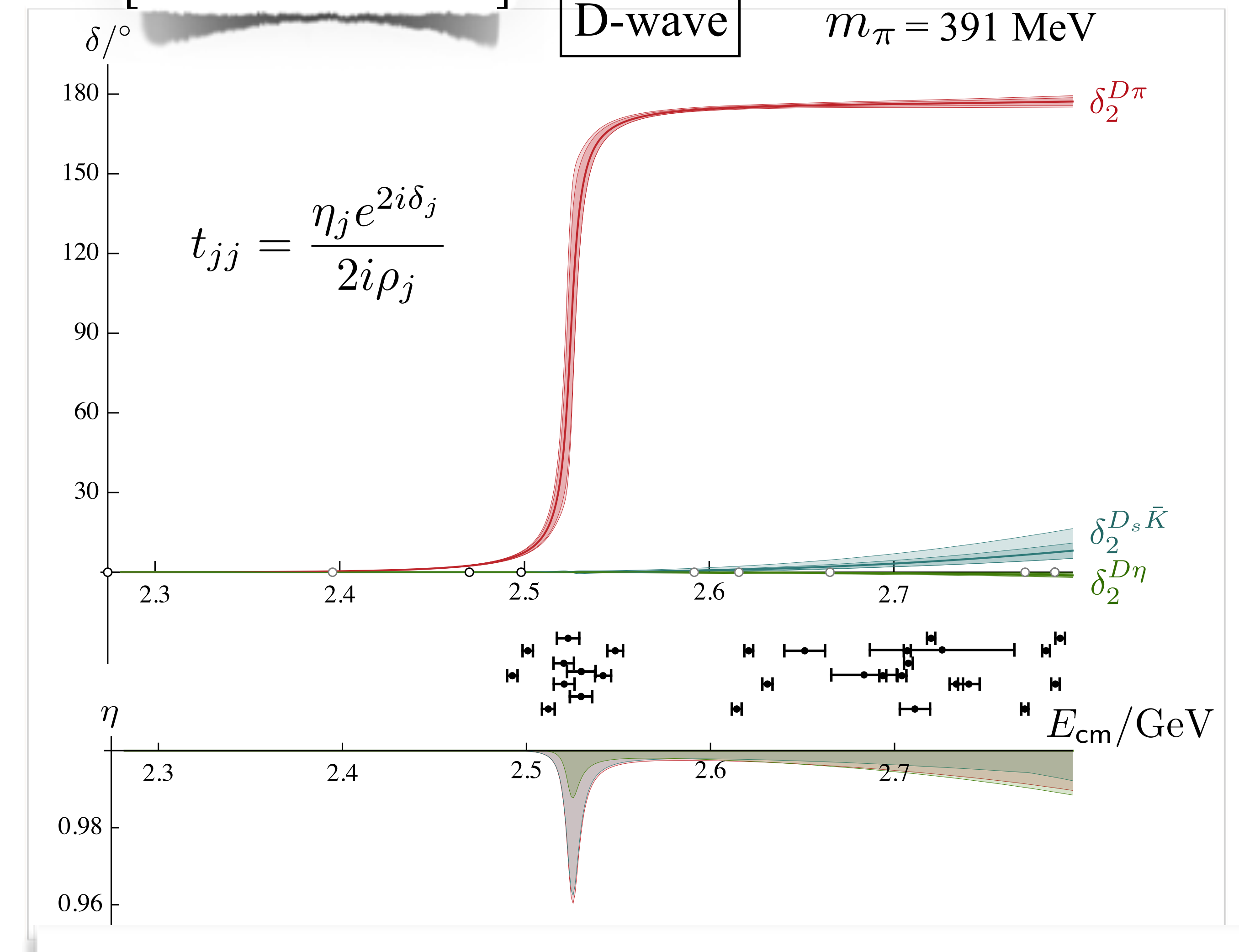
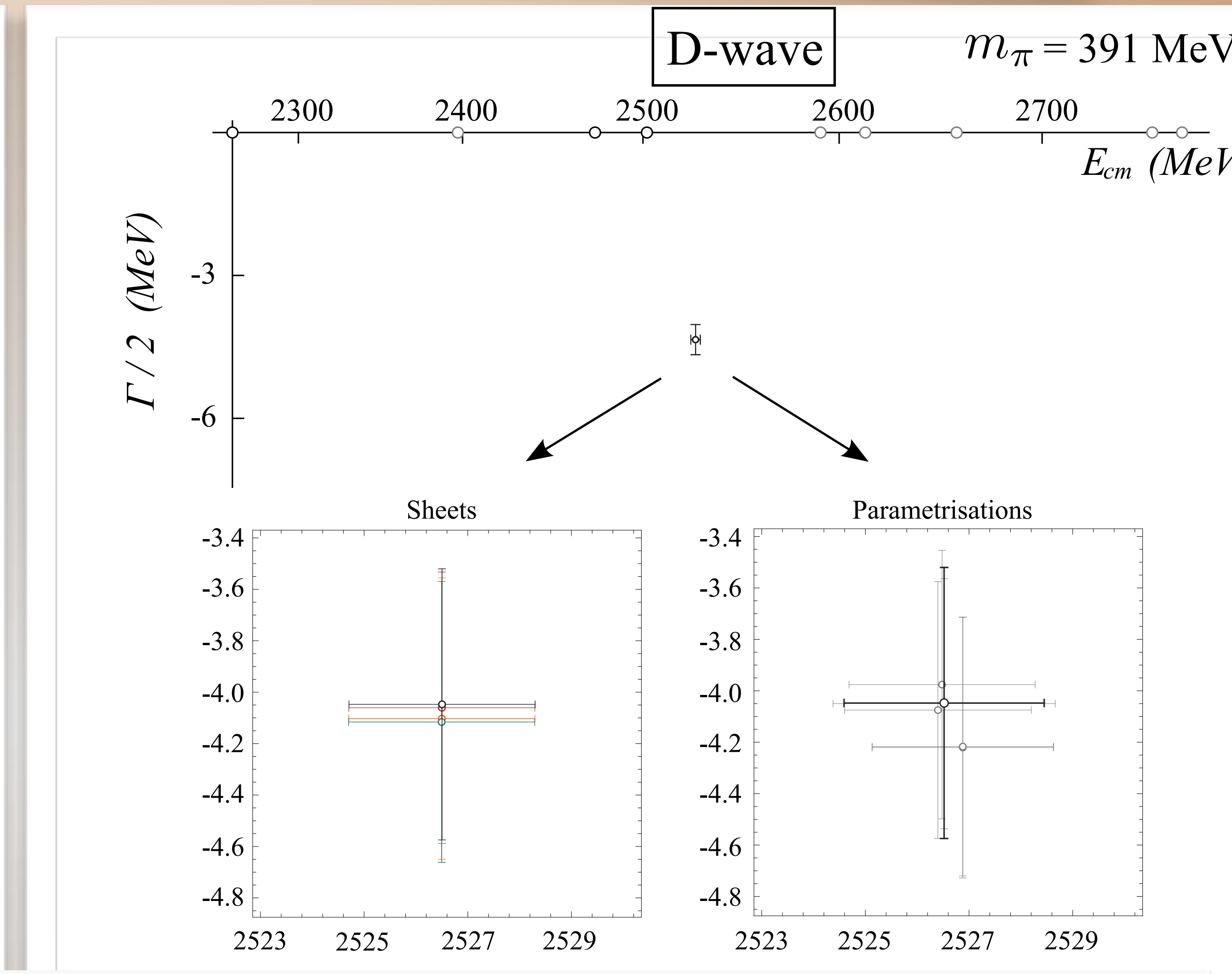
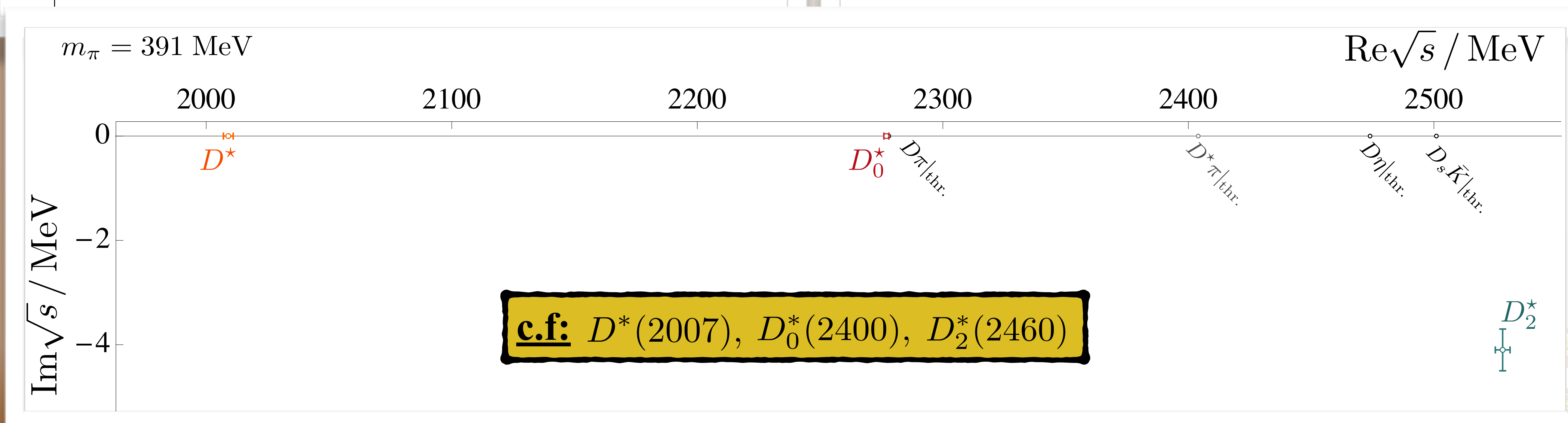
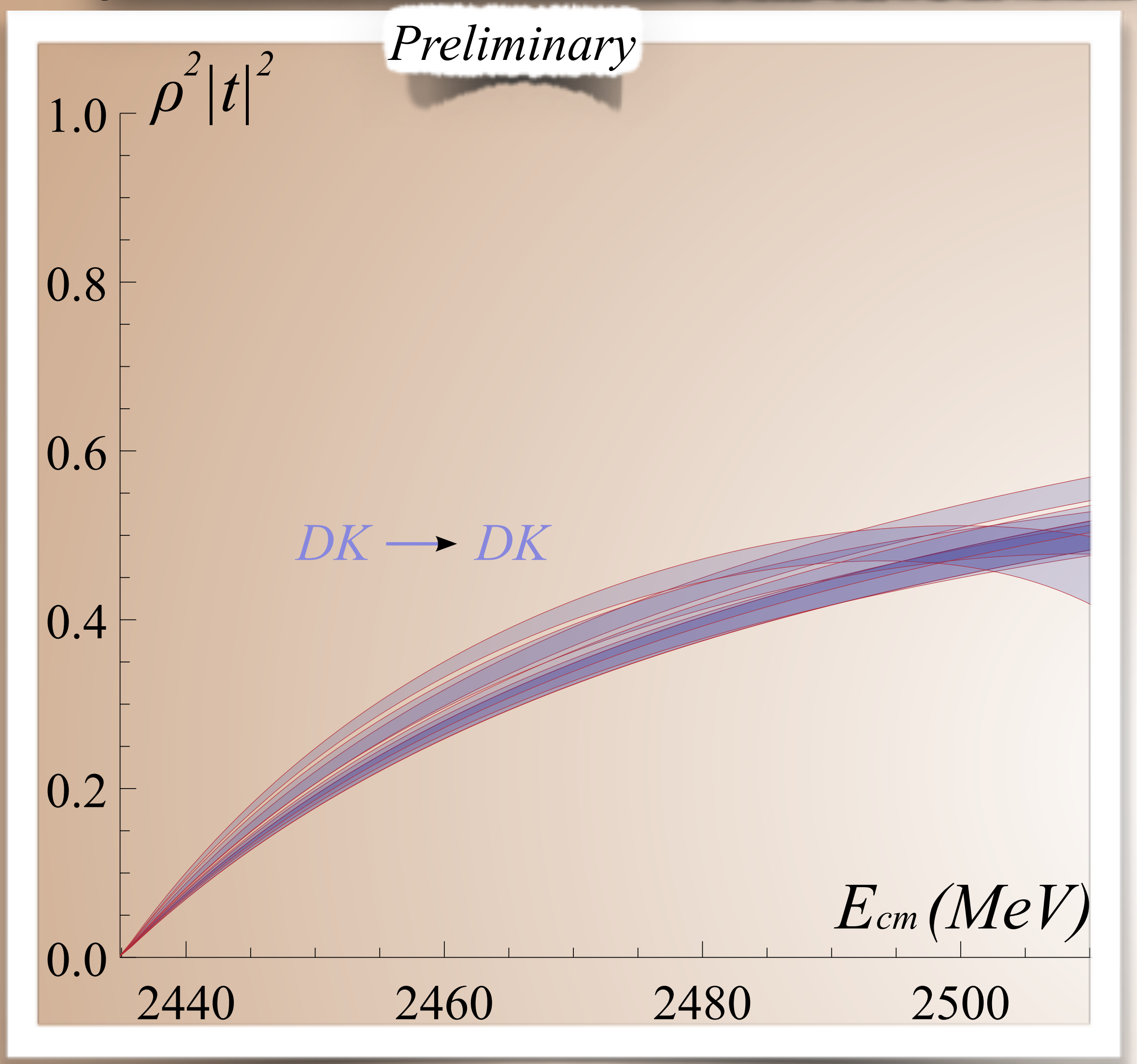






c.f: $D^*(2007),\ D_0^*(2400),\ D_2^*(2460)$

DK Scattering



S-wave:

- Bound-state pole $\approx 2380\text{MeV}$; ≈ 55 MeV below DK threshold (at $M_\pi = 391$ MeV)
- Expt: $D^*_{s0}(2317) = 2317.7 \pm 0.6$ MeV ; ≈ 45 MeV below DK threshold

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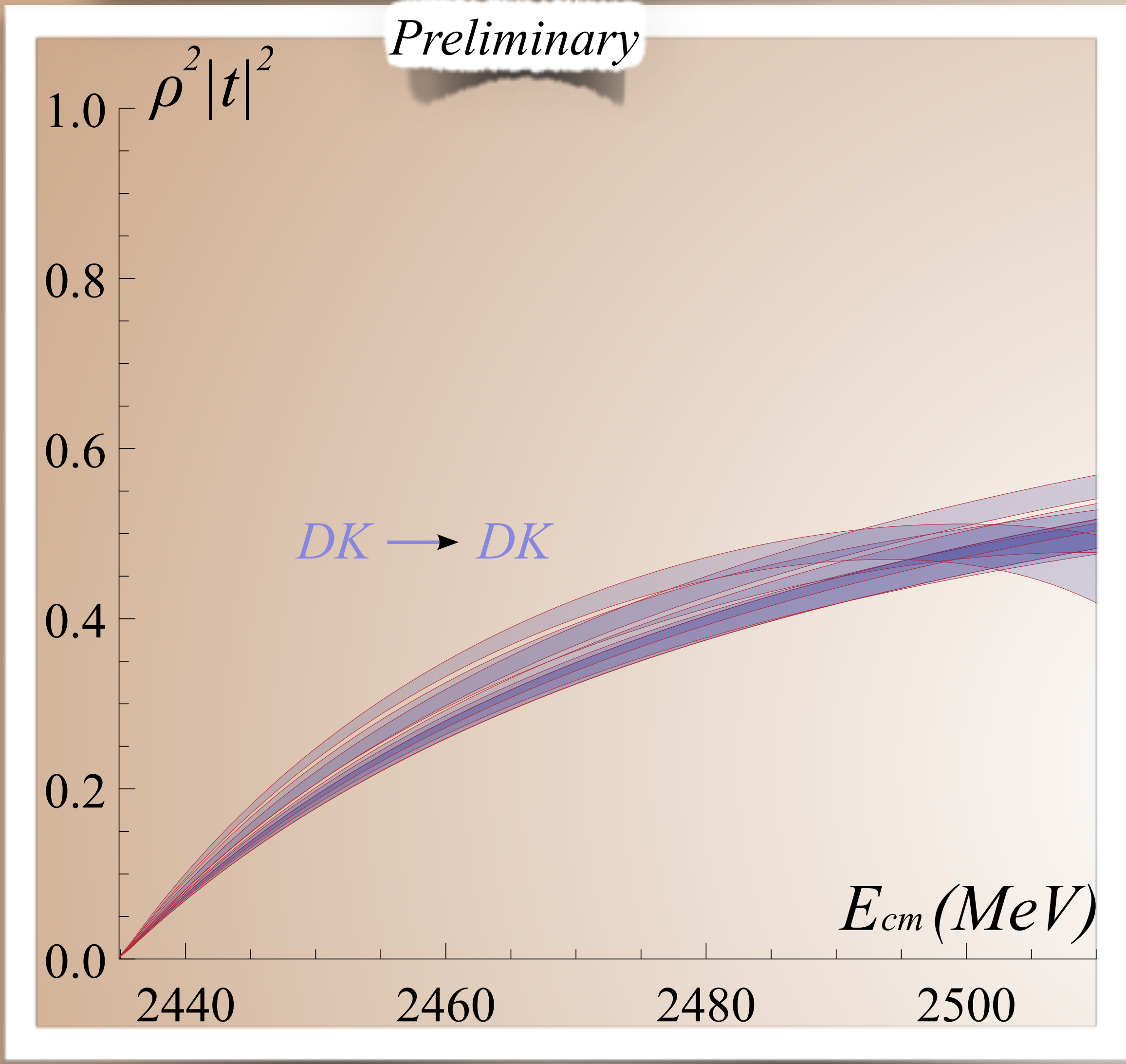

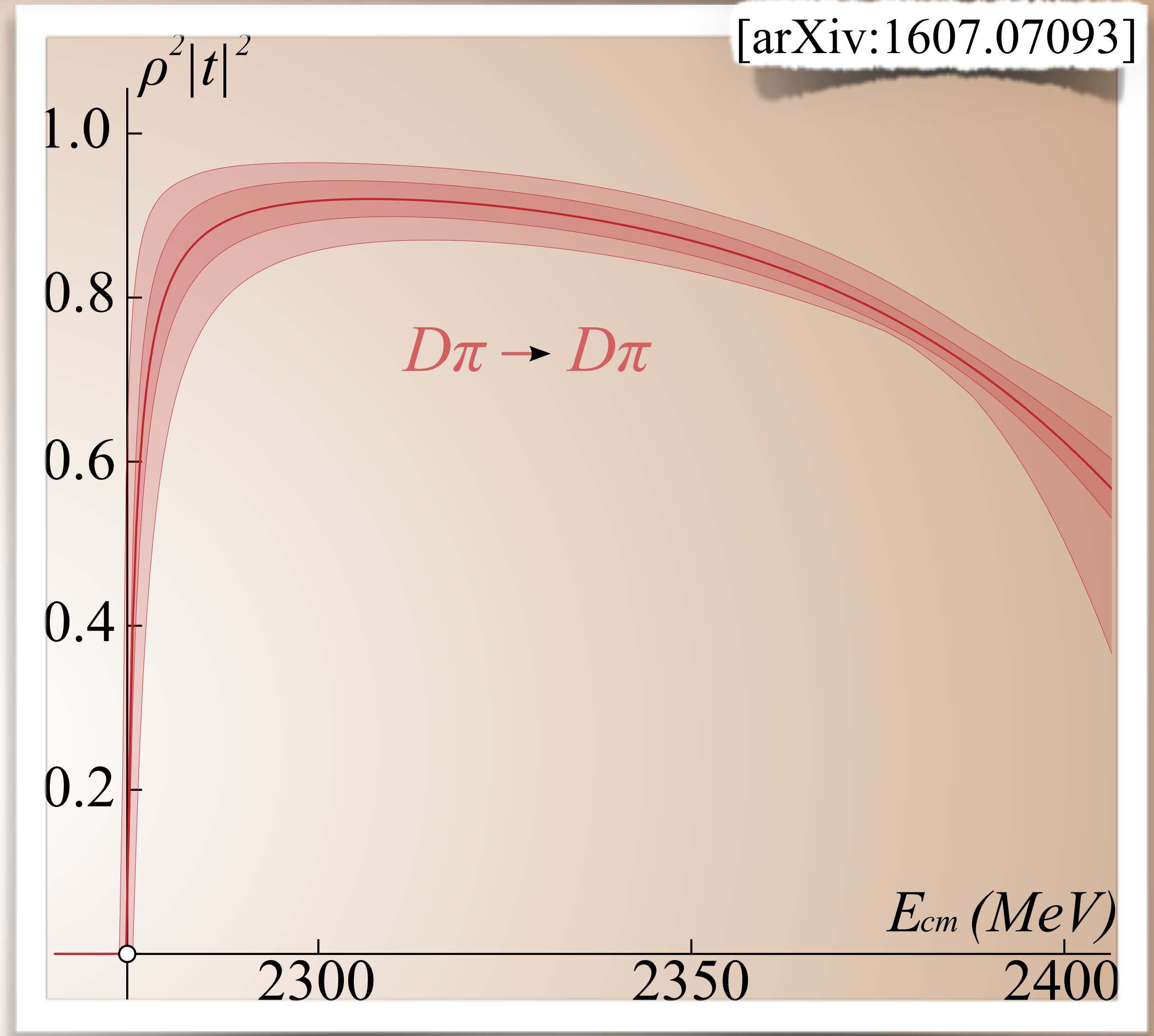

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- Expt: $D_{s0}^*(2317) = 2317.7 \pm 0.6$ MeV ; ≈ 45 MeV below DK threshold
- c.f: S-wave pole in the $D\pi$ channel $\approx 1\text{MeV}$ below threshold

Summary and Outlook

Lattice QCD calculations now probing exotic states and structures with heavy quarks

'Static' limit, NRQCD → **Probing four and five-quark states**

'Single hadron spectroscopy' → **Hybrid mesons and baryons and states with exotic quantum numbers**

'Lüscher formalism' → **First coupled-channel scattering calculation including heavy quarks**

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Thank you for your attention!