

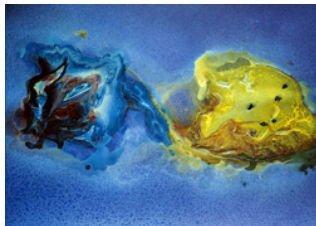
Heavy hybrids

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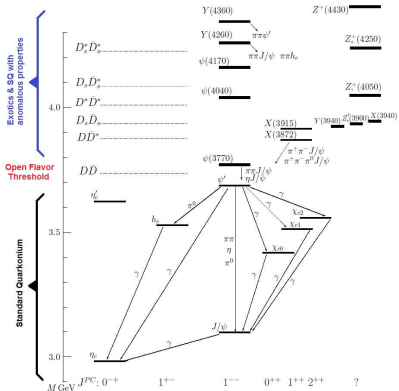
Implications of LHCb measurements and future prospects,
CERN, October 2016.



Physik-Department T30f

Exotic Quarkonium

- ▶ In the last decade many new unexpected states have been found close or above threshold.



Voloshin 2008

- ▶ The states that do not fit Quarkonium potential models are called Exotics and labeled Xs, Ys and Zs.
- ▶ These states are candidates for **non traditional** hadronic states, including **four constituent quark** or an **excited gluon** constituent.
- ▶ Large experimental effort to study normal and Exotic quarkonium: BaBar, Belle2, BESIII, LHCb and Panda (under construction).

Quarkonium Hybrids

What are quarkonium Hybrids?

- ▶ A quarkonium hybrid consists of Q, \bar{Q} in a color octet configuration and a gluonic excitation g .

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Key Characteristics

- ▶ Heavy quarks are non-relativistic, dynamical time-scale set by the heavy quarks mass.
- ▶ Gluons are fast, dynamical time-scale set by Λ_{QCD} .
- ▶ The hierarchy between dynamical time-scales can be exploited to describe the system.
- ▶ Quarkonium hybrids are a similar system to diatomic molecules.

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Our Aim

- **Systematize** the ideas behind the Born–Oppenheimer approximation for Quarkonium hybrids using **EFT techniques**.

Potential Non-Relativistic QCD

Pineda, Soto (1998); Brambilla, Pineda, Soto, Vairo (2000)

Motivation

- ▶ Quarkonium systems are non-relativistic bound states.
- ▶ **Multiscale system:** $m \gg mv \gg mv^2$, and Λ_{QCD} . m is the heavy-quark mass, $v \ll 1$ the heavy quark velocity.
- ▶ We can exploit the **scale hierarchies** by building an **Effective Field Theory** (EFT).

Matching procedure

- ▶ Integrating out the m scale leads to the well known NRQCD. Caswell, Lepage 1986; Bodwin, Braaten and Lepage 1995. Since $m \gg \Lambda_{QCD}$ the matching is perturbative.
- ▶ The degrees of freedom in pNRQCD are a **color singlet** (S) and **octet fields** (O^a) and the ultrasoft gluons.
- ▶ R is the CoM coordinate and r the relative coordinate of the quark pair.

Born-Oppenheimer EFT for QCD

Let us start from weakly-coupled pNRQCD:

pNRQCD Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{pNRQCD}} = & \int d^3r \text{Tr} \left[S^\dagger \left(i\partial_0 + \frac{\nabla_r^2}{M} - V_s(r) \right) S + O^\dagger \left(iD_0 + \frac{\nabla_r^2}{M} - V_o(r) \right) O \right] \\ & + gV_A(r) \text{Tr} \left[O^\dagger \mathbf{r} \cdot \mathbf{E} S + S^\dagger \mathbf{r} \cdot \mathbf{E} O \right] + \frac{g}{2} V_B(r) \text{Tr} \left[O^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger O \mathbf{r} \cdot \mathbf{E} \right] \\ & - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a}\end{aligned}$$

- Hierarchy of scales:
 - ▶ Weakly-coupled pNRQCD is valid for short distances: $mv \sim 1/r \gg \Lambda_{\text{QCD}}$.
 - ▶ Heavy quarks being slower than gluons implies $\Lambda_{\text{QCD}} \gg mv^2$.
- *Work plan*:
 - ▶ Integrate out the light d.o.f.

Born-Oppenheimer EFT for QCD

The Hamiltonian density corresponding to the light d.o.f at leading order in the multipole expansion is

$$\hat{h}_0(\mathbf{R}) = \frac{1}{2} (\mathbf{E}^a \mathbf{E}^a - \mathbf{B}^a \mathbf{B}^a)$$

Gluelump operators G^a

- G^a are a basis of color-octet eigenstates of $\hat{h}_0(\mathbf{R})$ with eigenvalues Λ_κ .

$$\hat{h}_0(\mathbf{R}) G_{i\kappa}^a(\mathbf{R})|0\rangle = \Lambda_\kappa G_{i\kappa}^a(\mathbf{R})|0\rangle$$

- Λ_κ is called the gluelump mass and it is a nonperturbative quantity.

Foster, Michael 1999; Bali, Pineda 2004

- κ labels the $O(3) \times C$ representation (K^{PC} quantum numbers).

The eigenstates of the octet sector Hamiltonian in the static limit are

$$|\kappa\lambda\rangle = P_{\kappa\lambda}^i O^a(\mathbf{r}, \mathbf{R}) G_{i\kappa}^a(\mathbf{R})|0\rangle,$$

The $P_{\kappa\lambda}^i$ are projectors that select different polarizations of $\Psi_{i\kappa}$. We can expand the Lagrangian this basis by projecting into the subspace spanned by

$$\int d^3r d^3R \sum_{\kappa\lambda} |\kappa\lambda\rangle \Psi_{\kappa\lambda}(t, \mathbf{r}, \mathbf{R})$$

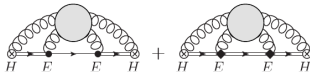
Born-Oppenheimer EFT for QCD

- After projecting and integrating out Λ_{QCD} :

$$\mathcal{L}_{BO}^o = \int d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi_{\kappa\lambda}^{\dagger}(t, \mathbf{r}, \mathbf{R}) \left\{ \left[i\partial_t - V_o(r) - \Lambda_{\kappa} - b_{\kappa\lambda} r^2 + \dots \right] \delta_{\lambda\lambda'} - P_{\kappa\lambda}^j \frac{\nabla_r^2}{M} P_{i\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t, \mathbf{r}, \mathbf{R})$$

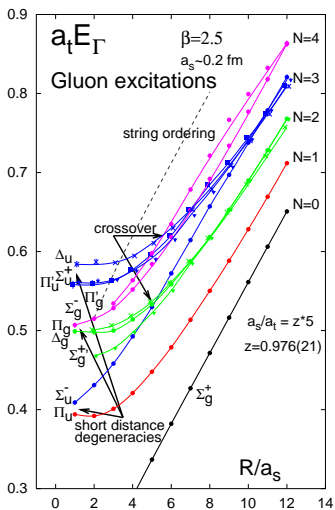
NLO term: $b_{\kappa\lambda}$

- ▶ At finite r the eigenstates must be organized in representations of $D_{\infty h}$.
- ▶ Proportional to r^2 due to the multipole expansion.



- ▶ $b_{\kappa\lambda}$ is a non-perturbative quantity.
- ▶ We obtain it from a fit to the lattice data.
- ▶ Breaks the $O(3) \times C \rightarrow D_{\infty h}$, $b_{\kappa\lambda} = b_{\kappa-\lambda}$.
- ▶ Responsible for the attractive part of the potential.

Lattice data on hybrid static energies



- ▶ The gluonic static energies are the eigenvalues of the NRQCD Hamiltonian in the static limit.
- ▶ The most recent data by [Juge, Kuti, Morningstar, 2002](#) and [Bali and Pineda 2003](#).
- ▶ Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- ▶ The rest of the static energies correspond to excited gluonic states that generate hybrids.
- ▶ The two lowest hybrid static energies are Π_U and Σ_U^- , they are nearly degenerate at short distances.
- ▶ Quenched and unquenched calculations for Σ_g^+ and Π_U were compared in [Bali et al 2000](#) and good agreement was found below string breaking distance.

Quenched lattice NRQCD: [Juge, Kuti, Morningstar 2002](#)

Lowest energy multiplet $\Sigma_u^- - \Pi_u$

- ▶ The lowest mass glueball has quantum numbers 1^{+-} and $\Lambda_{1^{+-}}^{RS} = 0.87 \pm 0.15$ GeV. **Bali, Pineda 2004**
- ▶ It generates the two lowest lying hybrid static energies Π_u and Σ_u^- which are degenerate at short distances.
- ▶ The nonadiabatic coupling mixes these two static energies.

Coupled radial equations for $\Sigma_u^- - \Pi_u$

$$\left[-\frac{\partial_r^2}{m} + \frac{1}{mr^2} \begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma}^{\text{light}} & 0 \\ 0 & E_{\Pi}^{\text{light}} \end{pmatrix} \right] \begin{pmatrix} \Psi_{\epsilon, \Sigma}^N \\ \Psi_{\epsilon, \Pi}^N \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \Psi_{\epsilon, \Sigma}^N \\ \Psi_{\epsilon, \Pi}^N \end{pmatrix}$$
$$\left[-\frac{\partial_r^2}{m} + \frac{l(l+1)}{mr^2} + E_{\Pi}^{\text{light}} \right] \psi_{-\epsilon, \Pi}^N = \mathcal{E}_N \psi_{-\epsilon, \Pi}^N.$$

- ▶ The coupled Schrödinger equations can be solved numerically.

Hybrid state masses from $V^{(0,25)}$

M. Berwein, N. Brambilla, J.T., A. Vairo. arXiv:1510.04299 Phys.Rev. D92 (2015) 11, 114019

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H'_1	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H'_2	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

- The multipole expansion requires $\langle 1/r \rangle \gtrsim E_{kin}$.

Conclusion:

- As expected our approach works better in bottomonium than charmonium

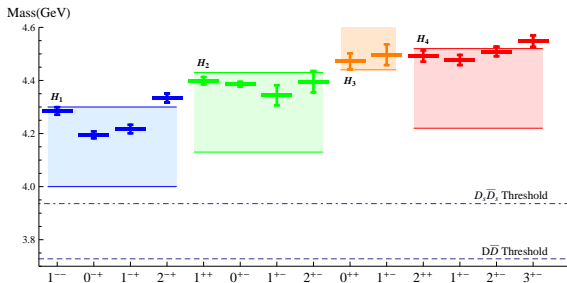
Spin symmetry multiplets

H_1	$\{1^{--}, (0, 1, 2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0, 1, 2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1, 2, 3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1, 2, 3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2, 3, 4)^{-+}\}$	Σ_u^-, Π_u
H_7	$\{3^{++}, (2, 3, 4)^{+-}\}$	Π_u

Comparison with direct lattice computations

Charmonium sector

- ▶ Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. [Liu et al 2012](#)
- ▶ The small mass difference of the H_2 , H_3 , H_4 multiplets suggests the constituent gluon picture ($r \rightarrow 0$ limit of BO picture).



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

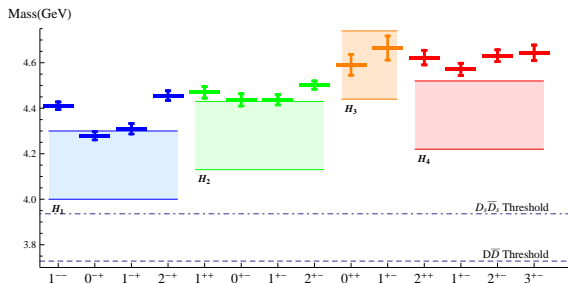
Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- ▶ Our masses are 0.1 – 0.14 GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^- .
- ▶ Good agreement with the mass gaps between multiplets, in particular the Λ -doubling effect ($\delta m_{H_2-H_1}$).

Comparison with direct lattice computations

Charmonium sector

- Recent new data from the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 240 MeV. [Cheung et al 2016](#)



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

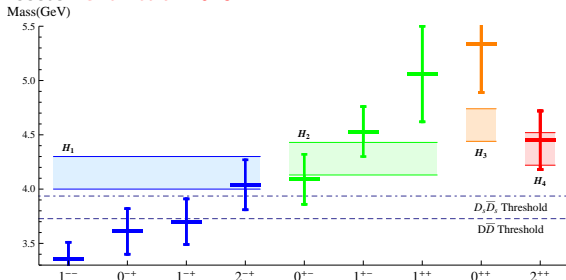
Split (GeV)	Cheung	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.08	0.13
$\delta m_{H_4-H_1}$	0.23	0.22
$\delta m_{H_4-H_2}$	0.15	0.09
$\delta m_{H_3-H_1}$	0.18	0.44
$\delta m_{H_3-H_2}$	0.10	0.31

- Our masses are 0.2 – 0.25 GeV lower except for the H_3 multiplet which now matches!
- Comparison of mass gaps between multiplets slightly worse.

Comparison with QCD sum rules

- ▶ A recent analysis of QCD sum rules for hybrid operators has been performed by [Chen et al 2013, 2014](#) for $b\bar{b}$ and $c\bar{c}$ hybrids, and $b\bar{c}$ hybrids respectively.
- ▶ Correlation functions and spectral functions were computed up to dimension six condensates which stabilized the mass predictions compared to previous calculations which only included up to dimension 4 condensates.

Charmonium sector [Chen et al 2013](#)



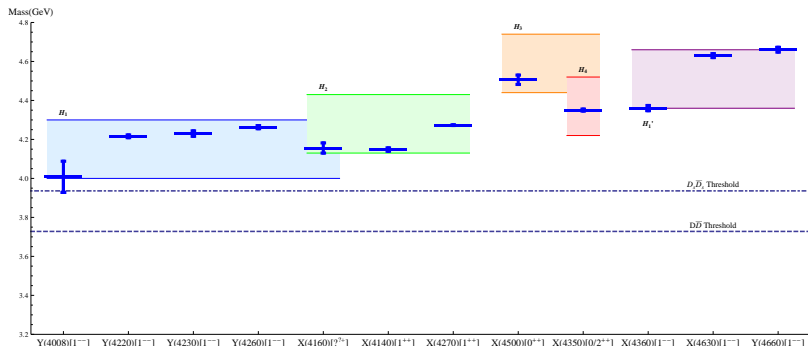
Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- ▶ The spin average of the H_1 multiplet is 0.4 GeV lower than our mass.
- ▶ H_2 , H_3 and H_4 multiplets are incomplete.
- ▶ Large uncertainties compared to direct lattice calculations.

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively. B decays are a source of 1^{++} states.

- ▶ Neutral exotic charmonium states (Belle, CDF, BESIII, Babar, LHCb):



- ▶ Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, some of the candidates decay modes violate Heavy Quark Spin Symmetry.

Conclusions

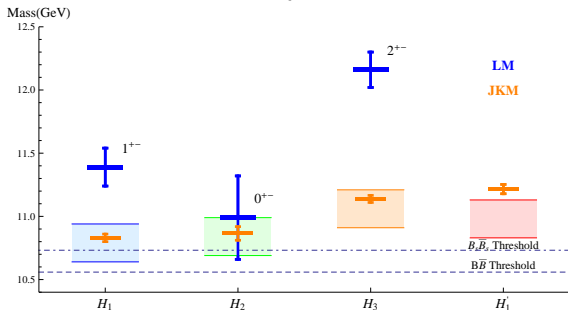
- The static energies computed in Lattice NRQCD can be used to obtain the quarkonium hybrid spectrum in a EFT framework.
- The nonadiabatic coupling mixing terms are important due to the short range degeneracy of the static energies.
- There are several experimental candidates for charmonium hybrids and one candidate for bottomonium hybrids.
- Substantial (but not complete) qualitative agreement between our work and direct lattice and sum rules calculations.
- More theoretical work required to obtain spin splits, mixing with standard quarkonium states and decay patterns.

Thank you for your attention

Comparison with direct lattice computations

Bottomonium sector

- ▶ Calculations done by **Juge, Kuti, Morningstar 1999** and **Liao, Manke 2002** using quenched lattice QCD.
- ▶ **Juge, Kuti, Morningstar 1999** included no spin or relativistic effects.
- ▶ **Liao, Manke 2002** calculations are fully relativistic.

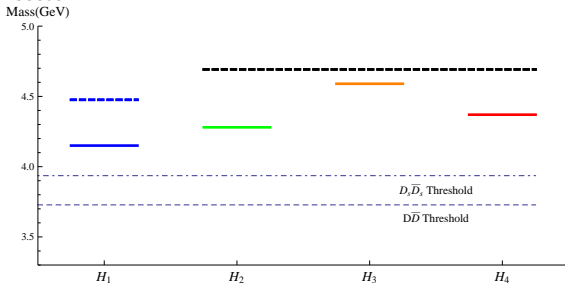


Error bands take into account the uncertainty on the glueball mass ± 0.15 GeV

Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- ▶ Our masses are 0.15 – 0.25 GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- ▶ Good agreement with the mass gaps between multiplets, in particular the Λ -doubling effect ($\delta m_{H_2-H_1}$).

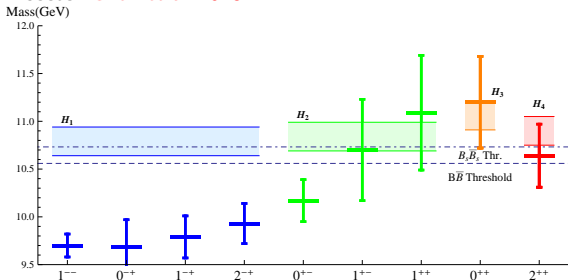
Charmonium sector



- ▶ For the H_1 multiplet they are about 300 MeV higher, and for the H_2 , H_3 and H_4 , (degenerate in the constituent gluon picture) they are 100 – 400 MeV higher depending on the multiplet.
- ▶ Different multiplet structure (constituent gluon vs Born-Oppenheimer pictures).
- ▶ Overall mass shift due to the different origin of the potential.

Comparison with QCD sum rules

Bottomonium sector *Chen et al 2013*



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- ▶ The spin average of the H_1 multiplet is 0.98 GeV lower than our mass.
- ▶ H_2 , H_3 and H_4 multiplets are incomplete.
- ▶ Large uncertainties compared to direct lattice calculations.