

Semileptonic B_c decays from full lattice QCD

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Intro & Motivation

- Obtain $|V_{cb}|$ from $b \rightarrow c$ transitions in semileptonic decays.
- Treatment of c and especially b quarks challenging in lattice simulations due to lattice artifacts which grow as $(am_q)^n$.
- We use two complementary approaches:
 - ▶ Highly improved relativistic action at small a , extrapolate $m_h \rightarrow m_b$.
 - ▶ Improved non-relativistic formalism (NRQCD) at m_b .
- First study:
 - ▶ $B_c \rightarrow \eta_c$
 - ▶ $B_c \rightarrow J/\psi$ [accessible at LHCb]
- More precise $b \rightarrow c$ currents used in $B \rightarrow D$, $B \rightarrow D^*$.

Outline

1. Intro & Motivation.
2. Calculation Framework.
 - ▶ Lattice QCD simulations.
 - ▶ HISQ action.
 - ▶ Improved NRQCD.
3. Semileptonic Decays.
 - ▶ Correlation functions.
 - ▶ $B_c \rightarrow \eta_c$ and results.
 - ▶ $B_c \rightarrow J/\psi$ and results.
4. Discussion & Future Work.

Lattice QCD simulations - I

Regulate QCD using a (Euclidean) spacetime lattice.
Integrate out fermionic degrees of freedom.

$$\begin{aligned}\mathcal{Z} &= \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\int (\mathcal{L}_{\text{YM}} + \bar{\psi} D\psi)} \\ &= \int \mathcal{D}U (\det D) e^{-\int \mathcal{L}_{\text{YM}}}\end{aligned}$$

Generate configurations using Monte Carlo techniques.

Lattice QCD simulations - II

Calculate quark propagators on gauge backgrounds.

$$D^{-1} = \longrightarrow$$

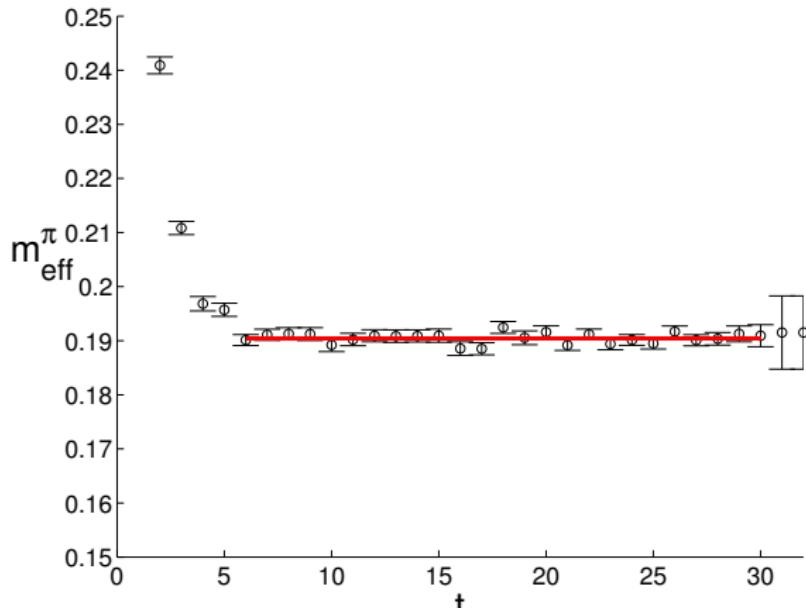
Use Wick's theorem to evaluate correlation functions.

$$\langle \pi \pi^\dagger \rangle = \pi \bullet \text{---} \circlearrowleft \text{---} \bullet \pi$$

Lattice QCD simulations - III

Energies and matrix elements can be determined by fitting exponentials.

$$\langle \pi(t) \pi^\dagger(0) \rangle \sim |\langle 0 | \pi | \pi \rangle|^2 e^{-m_\pi t}$$



DiRAC II computing

Computations carried out on the Darwin cluster at Cambridge.

Includes:

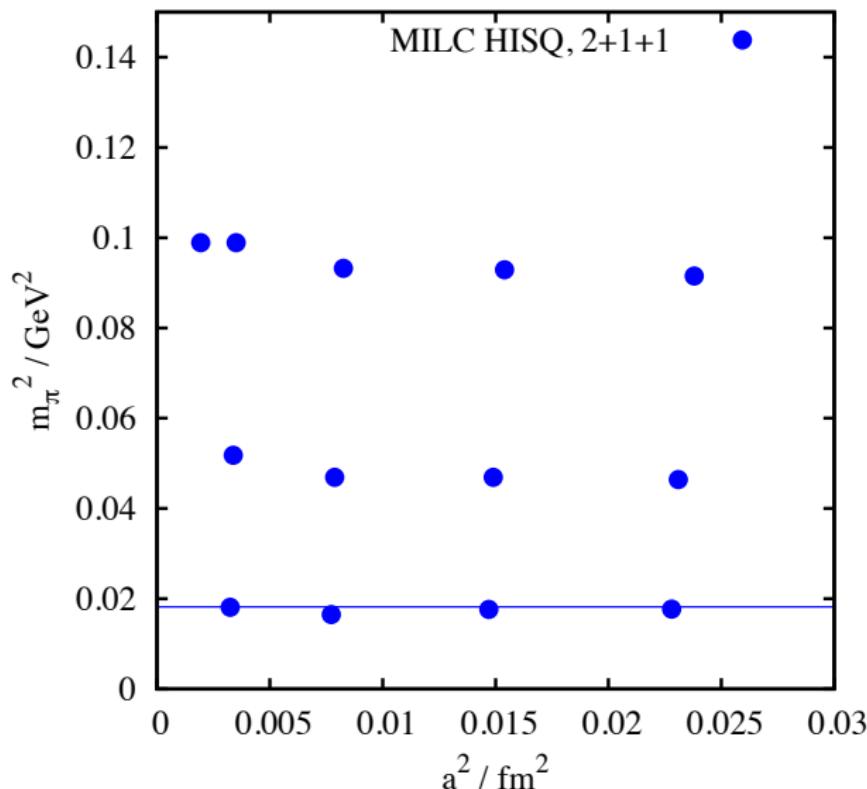
- 9600 Intel Sandy Bridge cores
- 2.6 GHz, 4 GB RAM/core
- 2 PB storage



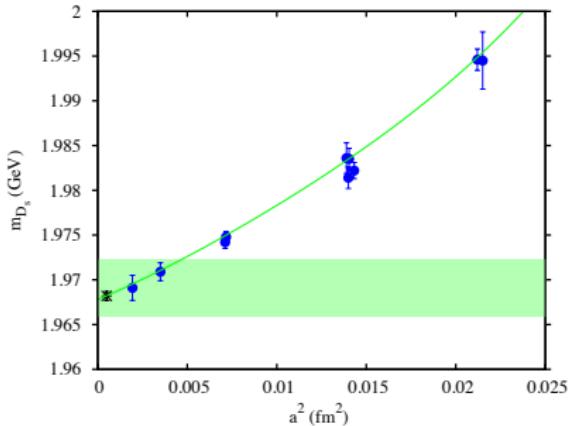
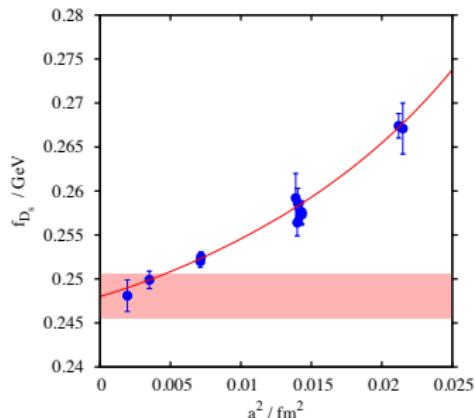
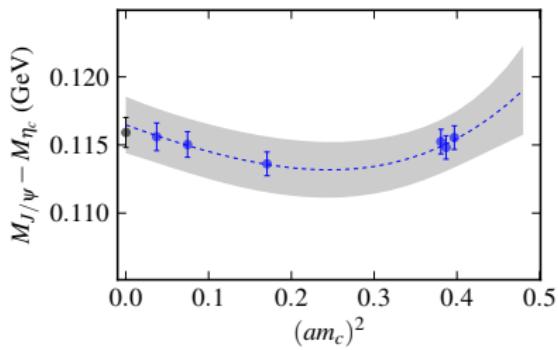
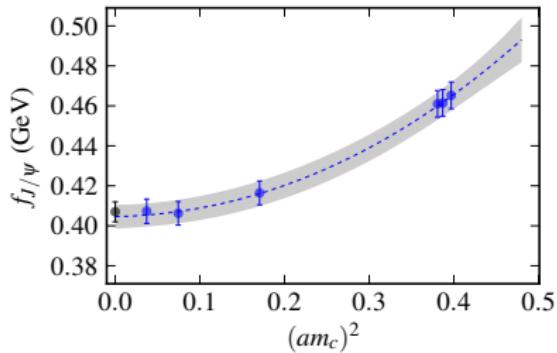
Part of STFC's HPC facility for theoretical particle physics and astronomy.

-
- HISQ fermion action.
 - Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]
 - Multiple lattice spacings down to ~ 0.045 fm.
 - Effects of u/d , s , and c quarks in the sea.
 - Multiple light-quark input parameters down to physical pion mass.
 - ▶ Chiral fits.
 - ▶ Reduce statistical errors.

MILC ensemble parameters



Charm physics with HISQ [1208.2855], [1008.4018]



NRQCD

Heavy quark propagators are calculated using a non-relativistic formalism.

Improved Non-relativistic QCD action

- Accurate through $\mathcal{O}(\alpha_s v^4)$.
- Discretisation corrections through $\mathcal{O}(\alpha_s v^2 a^2 p^2)$.
- $v^2 \sim 0.1$ bottomonium, ~ 0.3 charmonium.
- $am > 1 \rightarrow b$ quarks on $a = 0.15 - 0.06$ fm (down to $m_b/2$ on $a = 0.15$ fm).

Propagators constructed via an evolution equation,
 $G(\mathbf{x}, t + a) = e^{-aH_{\text{eff}}} G(\mathbf{x}, t)$.

NRQCD

$$aH_{\text{NRQCD}} = aH_0 + a\delta H$$

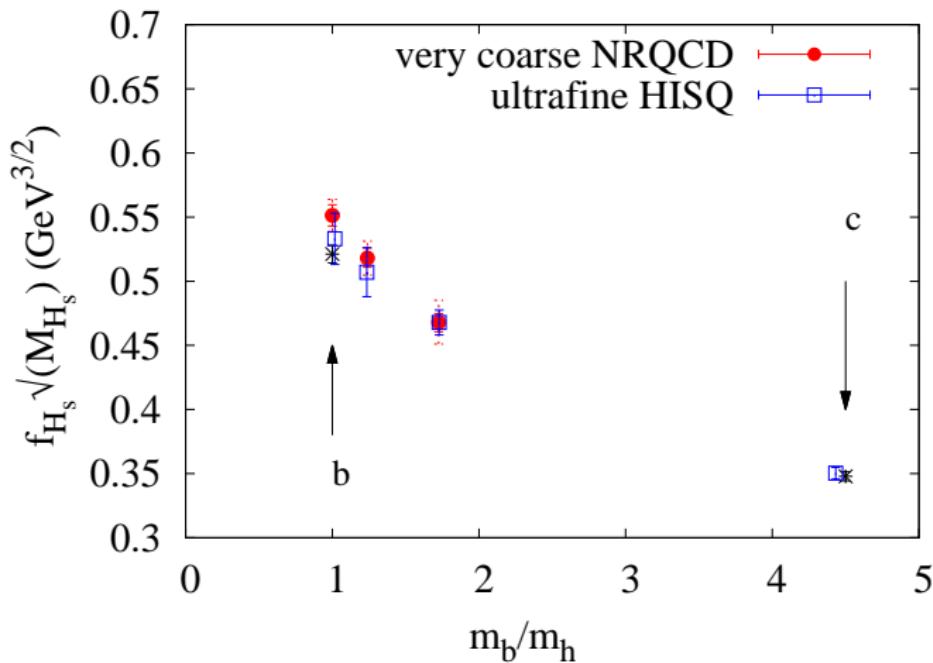
$$aH_0 = -\frac{\Delta^{(2)}}{2am_b}$$

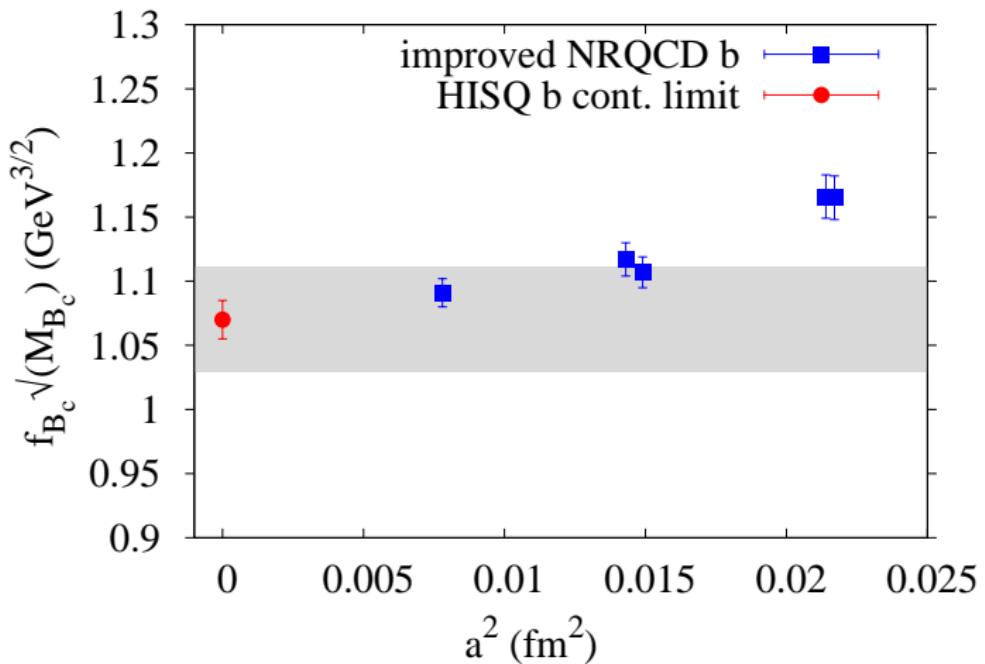
$$a\delta H = -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} (\nabla \cdot \mathbf{E} - \mathbf{E} \cdot \nabla)$$

$$- c_3 \frac{1}{8(am_b)^2} \sigma \cdot (\nabla \times \mathbf{E} - \mathbf{E} \times \nabla)$$

$$- c_4 \frac{1}{2am_b} \sigma \cdot \mathbf{B} + c_5 \frac{\Delta^{(4)}}{24am_b}$$

$$- c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}$$





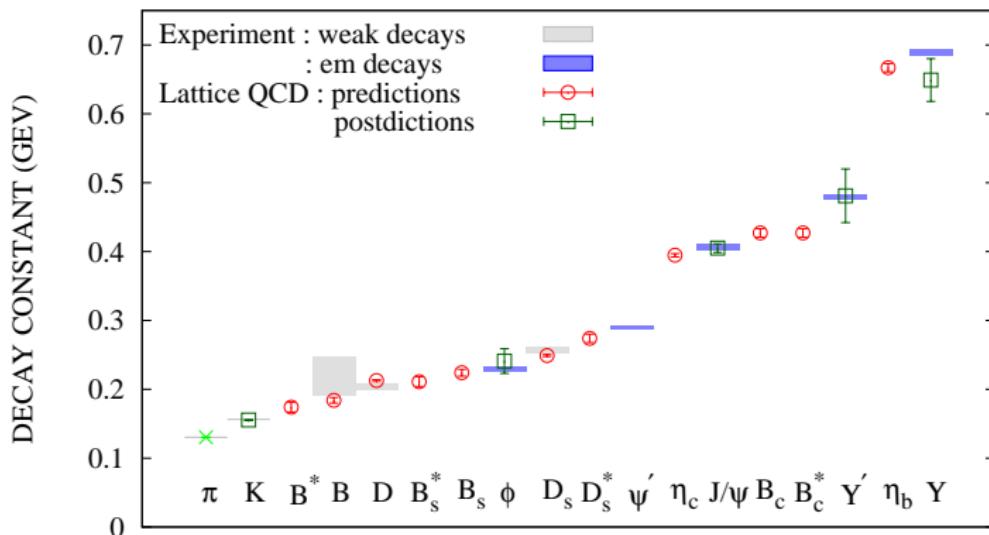
General strategy

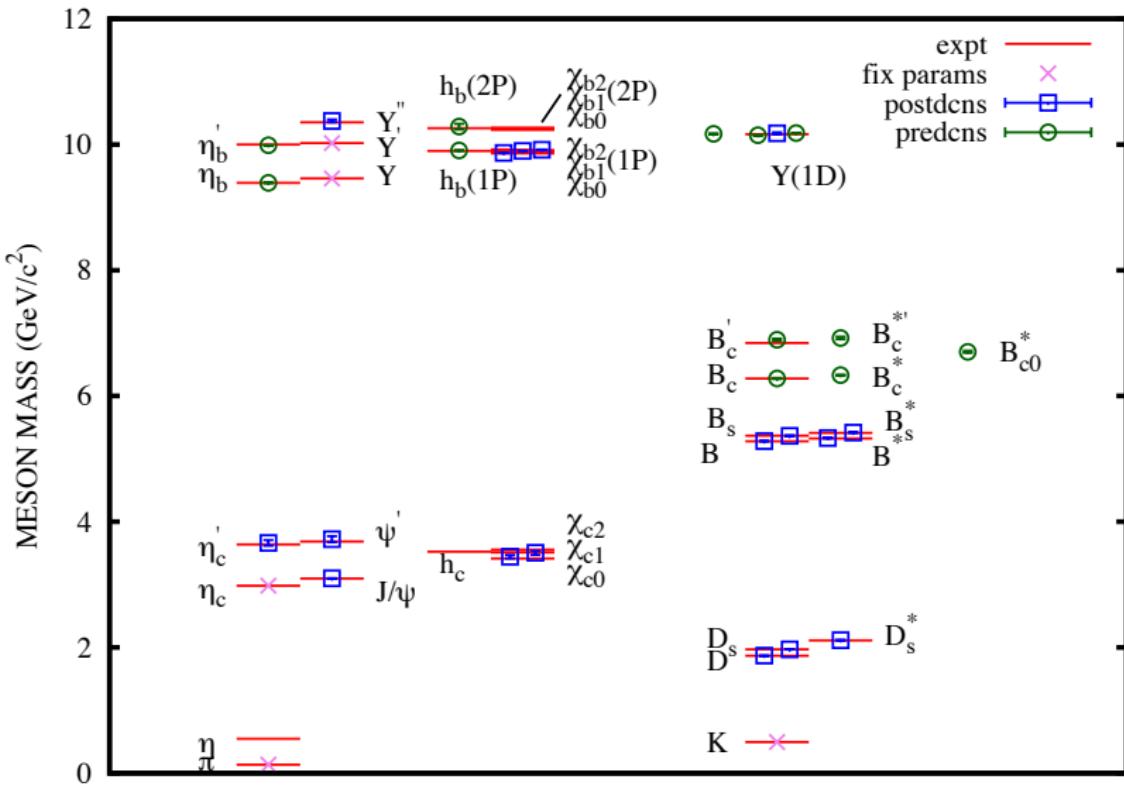
Strategy pursued by HPQCD collaboration:

- Staggered quarks → small a , physical pions, multiple lattice spacings..
- Highly improved action
 - discretisation effects under control at m_c
 - reduced taste-splittings
 - physical point ensembles with dynamical u/d , s , and c quarks.
- Compute heavy quarks using (improved) NRQCD.

These two approaches are complementary. Ideally there is a range of overlap in applicability to check the approaches are mutually consistent.

Decay constants – summary plot.





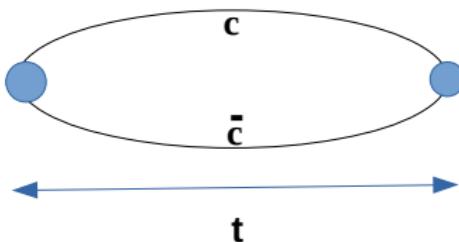
B_c semileptonic decays

Semileptonic decays

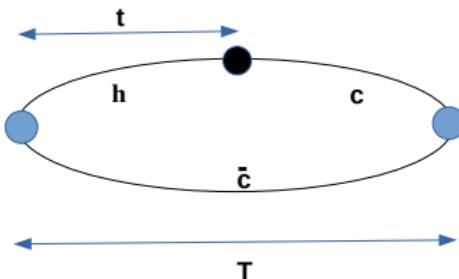
- Study of $B_c \rightarrow \eta_c$, $B_c \rightarrow J/\psi$ decay matrix elements.
- We work in the frame where the B_c is at rest.
- The form factors which parametrise the matrix elements are functions of q^2 , where q is the four-momentum transferred to the leptons.
 - ▶ $q_{\max}^2 = (M - m)^2$, zero recoil of decay hadron.
 - ▶ $q^2 = 0$, maximum recoil of decay hadron.
- Matrix elements are determined by simultaneous fitting of three-point and two-point functions.

Semileptonic decays – meson correlators

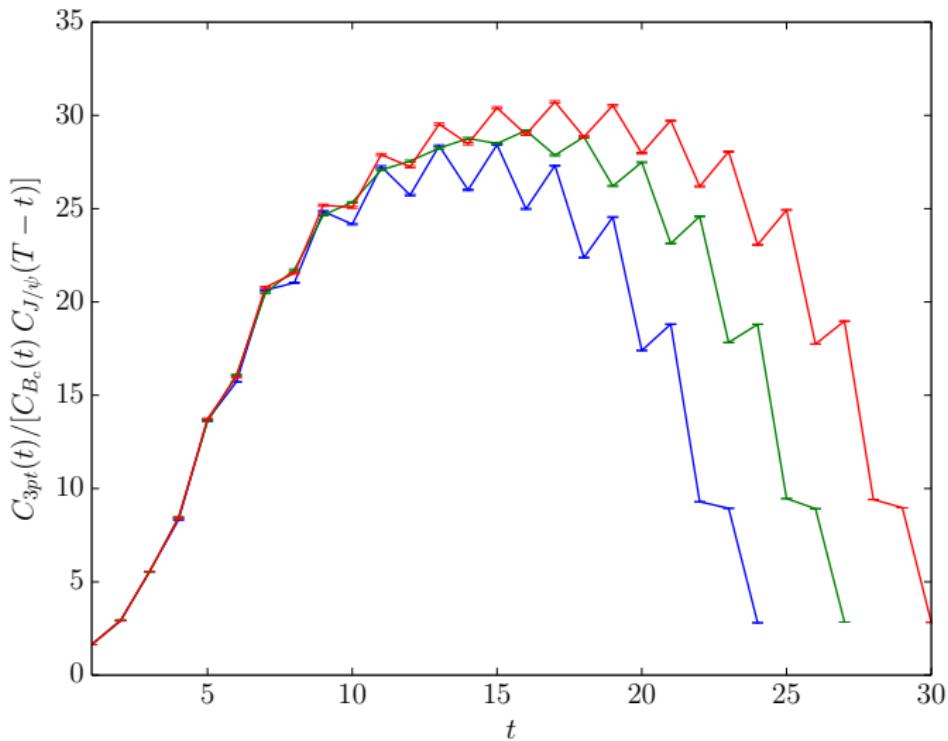
Two-point functions:



Three-point functions:



Semileptonic decays



$$B_c \rightarrow \eta_c$$

$$\begin{aligned} Z\langle\eta_c(p)|V^\mu|B_c(P)\rangle = & f_+(q^2) \left[P^\mu + p^\mu - \frac{M^2 - m^2}{q^2} q^\mu \right] + \\ & f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu, \end{aligned}$$

From PCVC,

$$\langle\eta_c(p)|S|B_c(P)\rangle = \frac{M^2 - m^2}{m_{b0} - m_{c0}} f_0(q^2)$$

Find Z by calculating both matrix elements at q_{\max}^2 .

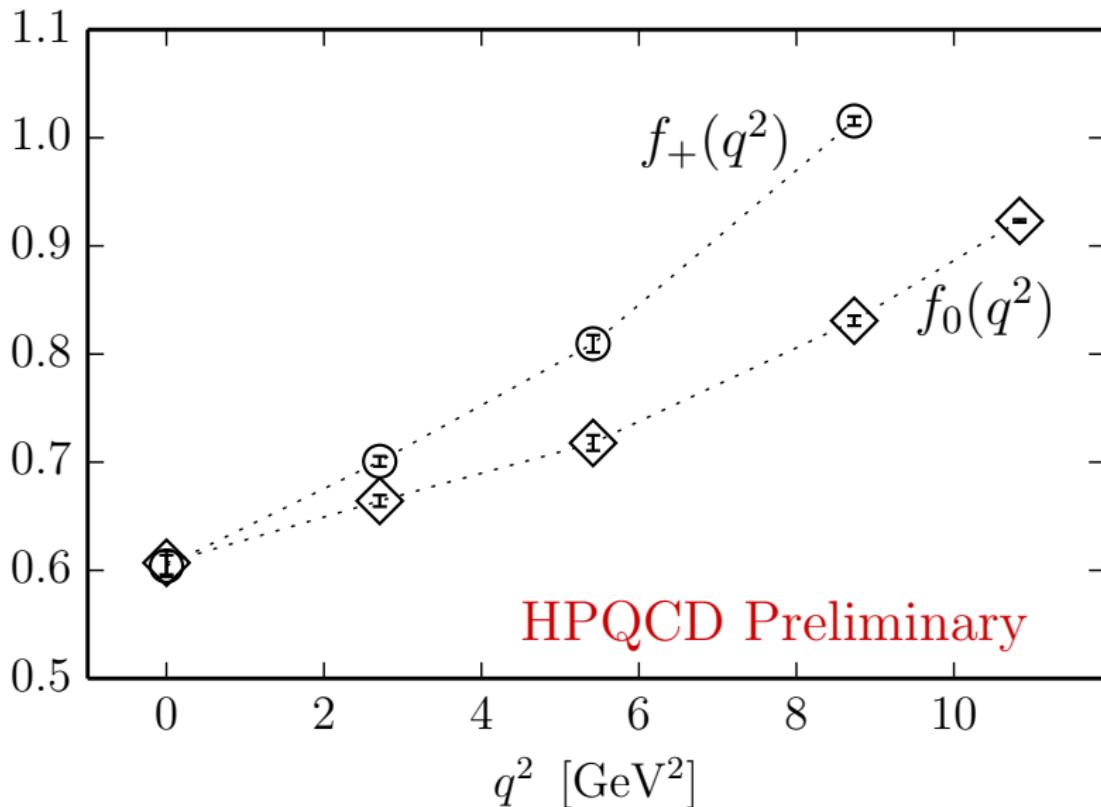
f_0 and f_+ are determined in the NRQCD formalism from matrix elements of the vector current $\langle V_\mu^{\text{nrqcd}} \rangle$, where

$$V_0^{\text{nrqcd}} = (1 + \alpha_s z_0^{(0)}) \left[V_0^{(0)} + (1 + \alpha_s z_0^{(1)}) V_0^{(1)} + \alpha_s z_0^{(2)} V_0^{(2)} \right]$$

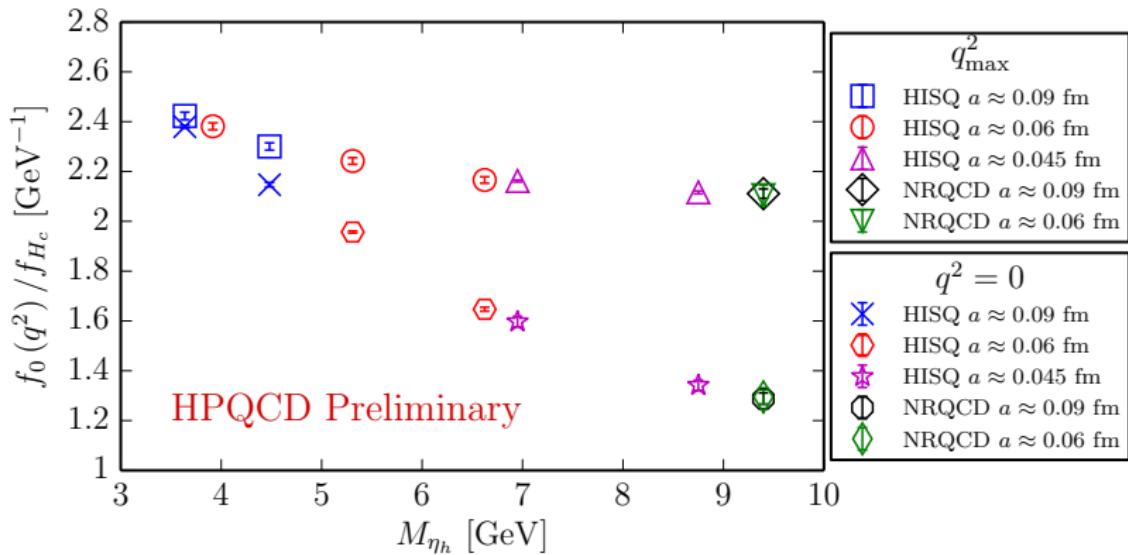
$$\begin{aligned} V_k^{\text{nrqcd}} = & (1 + \alpha_s z_k^{(0)}) \left[V_k^{(0)} + (1 + \alpha_s z_k^{(1)}) V_k^{(1)} + \alpha_s z_k^{(2)} V_k^{(2)} + \right. \\ & \left. \alpha_s z_k^{(3)} V_k^{(3)} + \alpha_s z_k^{(4)} V_k^{(4)} \right]. \end{aligned}$$

One goal of the present work is to constrain the coefficients entering V_μ^{nrqcd} using fully relativistic HISQ data.

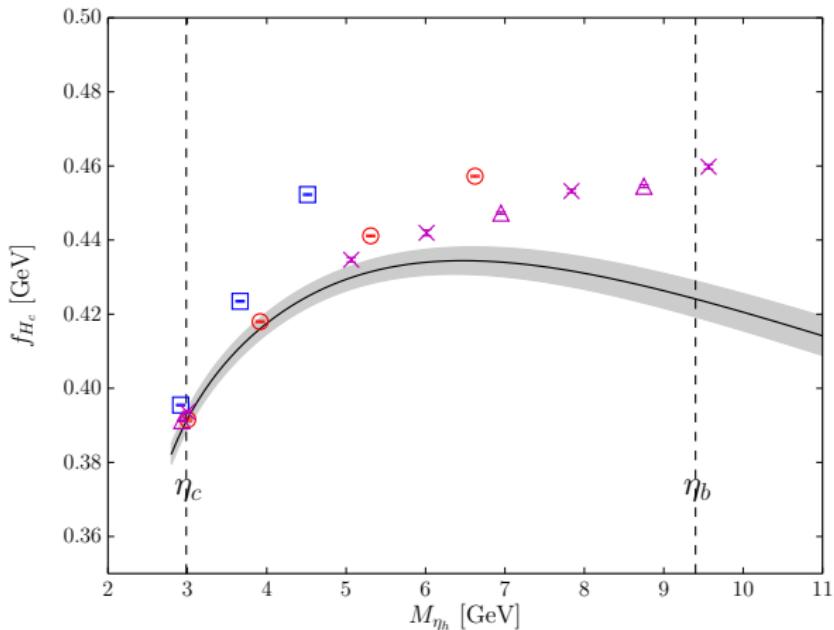
NRQCD form factors.



f_0 from HISQ.



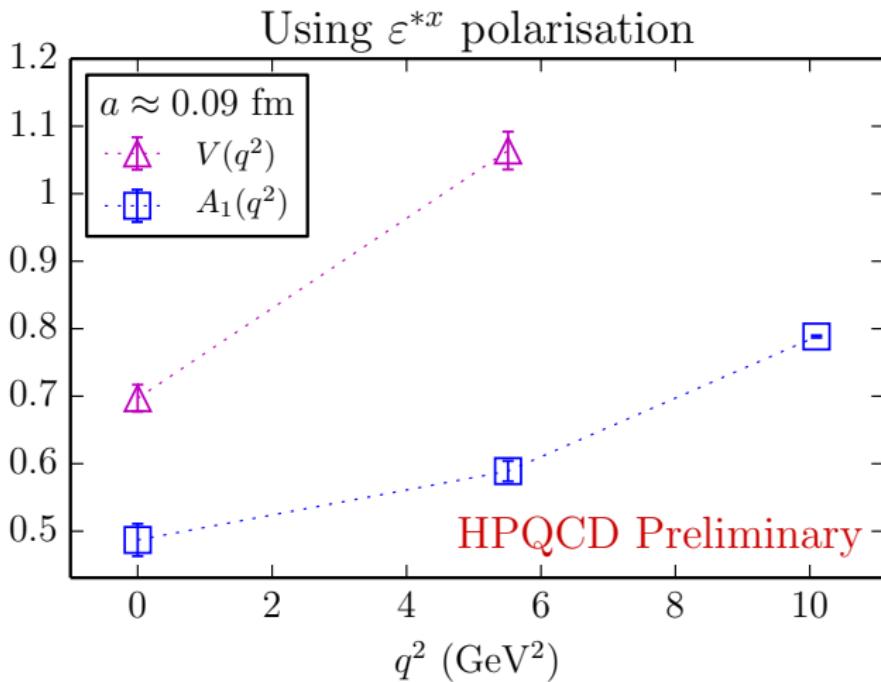
f_{H_c} from HISQ.



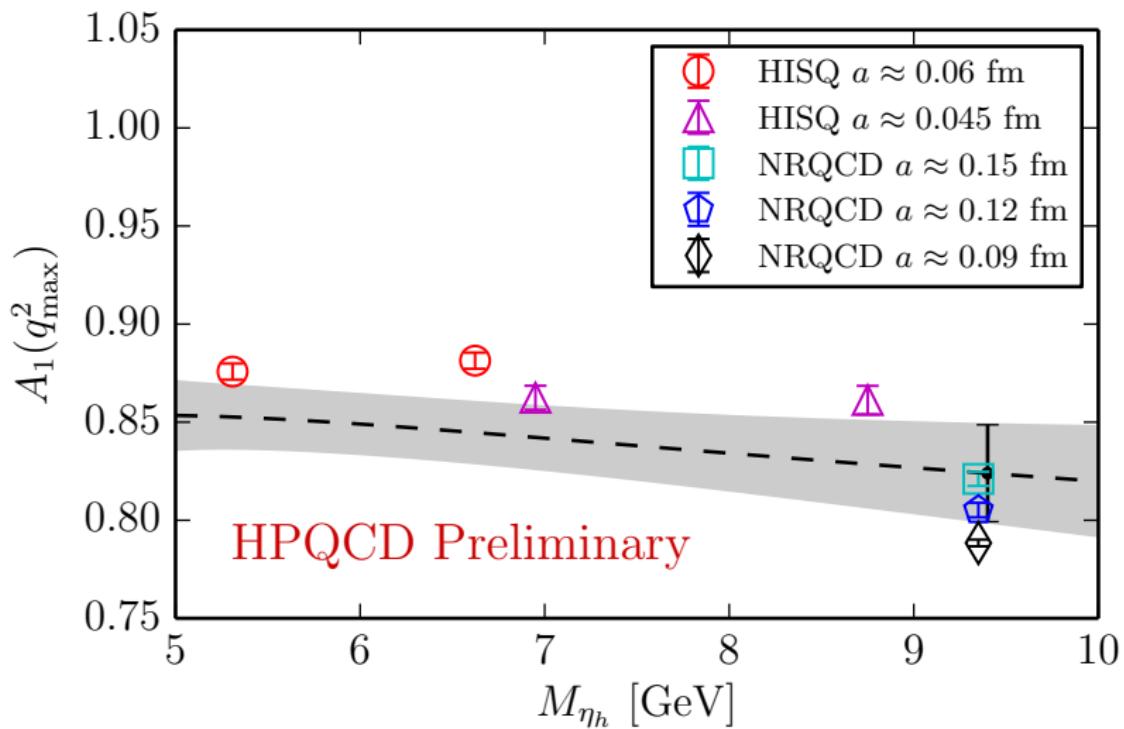
$$B_c \rightarrow J/\psi$$

$$\begin{aligned} \langle J/\psi(p, \varepsilon) | V^\mu - A^\mu | B_c(P) \rangle = \\ \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m} \varepsilon_\nu^* p_\rho P_\sigma V(q^2) - (M+m) \varepsilon^{*\mu} A_1(q^2) + \\ \frac{\varepsilon^* \cdot q}{M+m} (p+P)^\mu A_2(q^2) + 2m \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_3(q^2) - 2m \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_0(q^2) \end{aligned}$$

$B_c \rightarrow J/\psi$.



A_1 from HISQ.



Summary

- A promising approach to study of $b \rightarrow c$ transitions:
 - ▶ Lattice NRQCD with HISQ quarks, plus
 - ▶ Fully relativistic formulation, extrapolate m_h to m_b .
- Proof-of-principle demonstrated for f_0 .
 - ▶ Controlled calculation over full q^2 range.
 - ▶ Good agreement seen with NRQCD results.
- Outputs:
 - ▶ B_c to $J/\Psi \rightarrow$ new possible determination of $|V_{cb}|$.
 - ▶ Improved understanding of NRQCD currents feeds into additional calculations (B to D , B to D^* , ...).

Summary

- Expt'l results in μ and τ channel $\rightarrow R(B_c \rightarrow J/\psi)$ can be tested against SM prediction.
- Combine lattice results + V_{cb} with measurements of σBr to pin down B_c production.
- Binning in q^2 gives more tests for theory. Often lattice results are not as precise near $q^2 = 0$ but we think we are controlling this region.
- Angular information: A_0 appears $\propto m_l^2 \cos \theta_l$