The Quantum Critical Higgs

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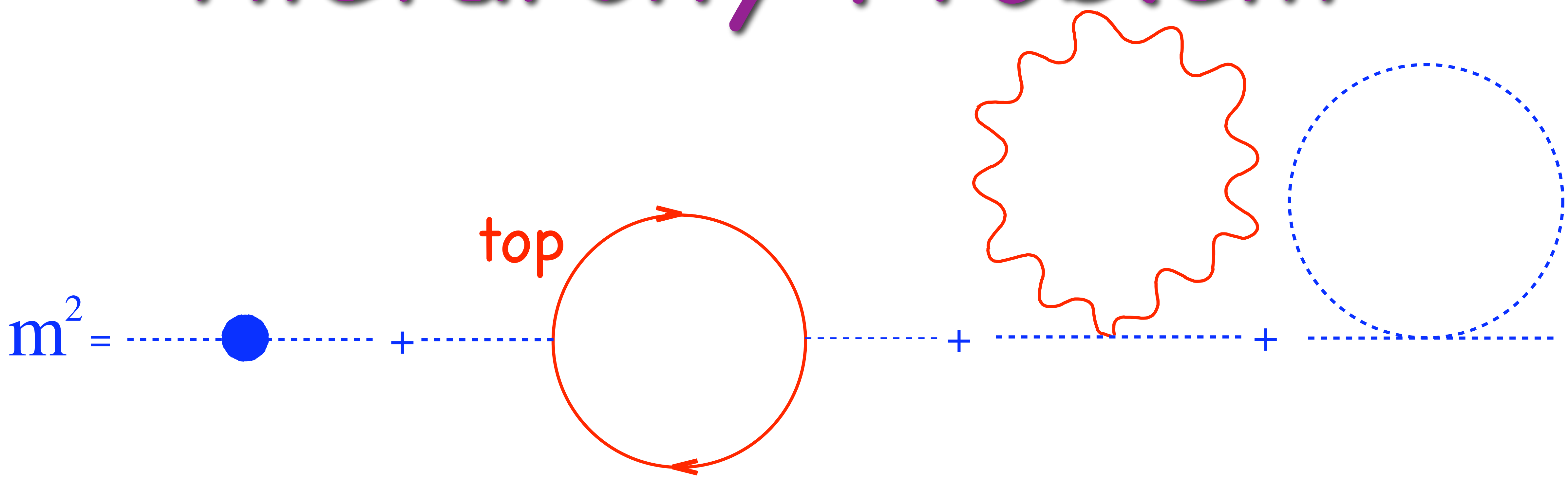
Outline

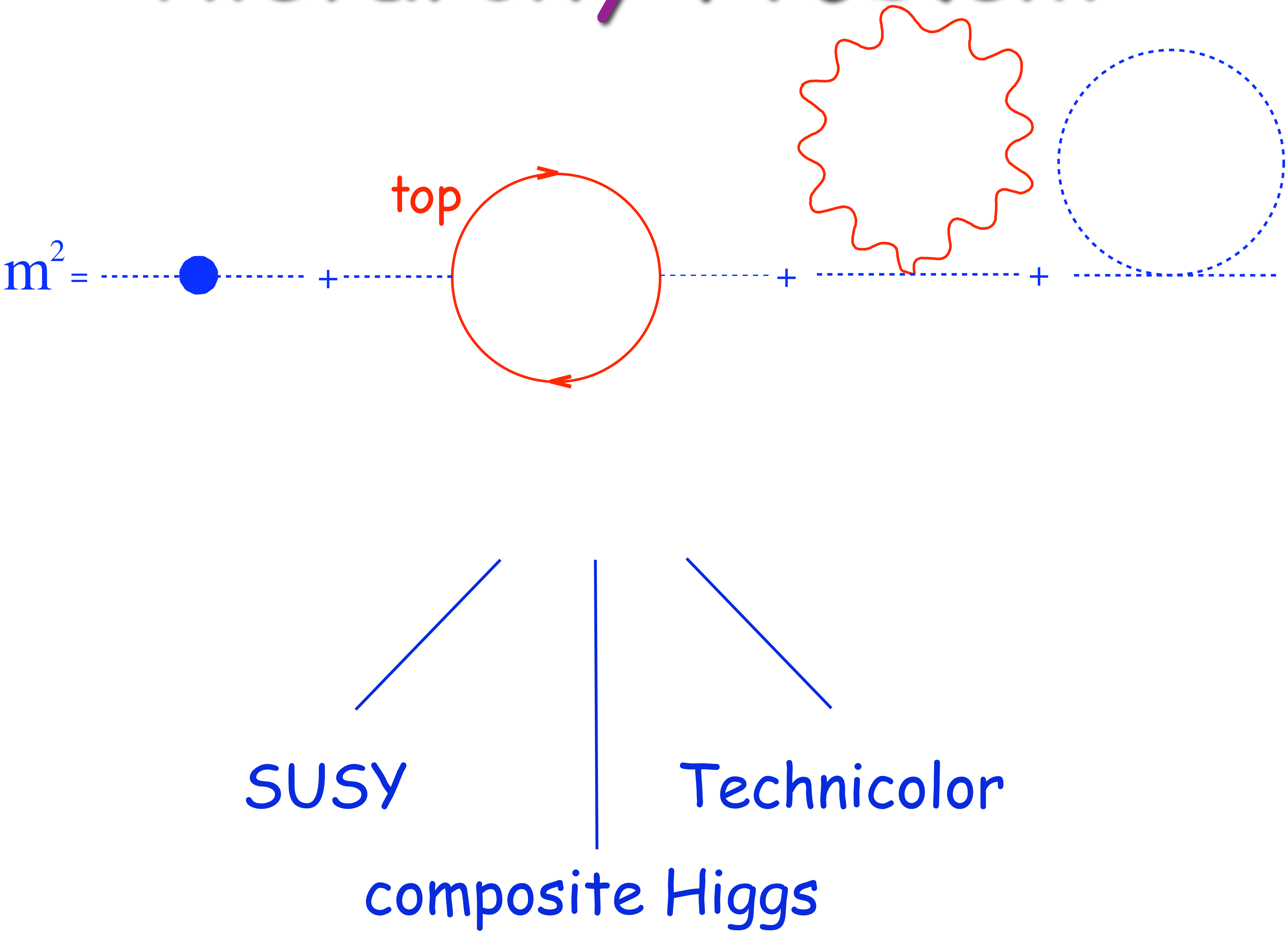
* Motivation
* Quantum Critical Points
* Effective Action for Quantum Critical Higgs
* Measuring of critical exponents at the LHC

Hierarchy Problem

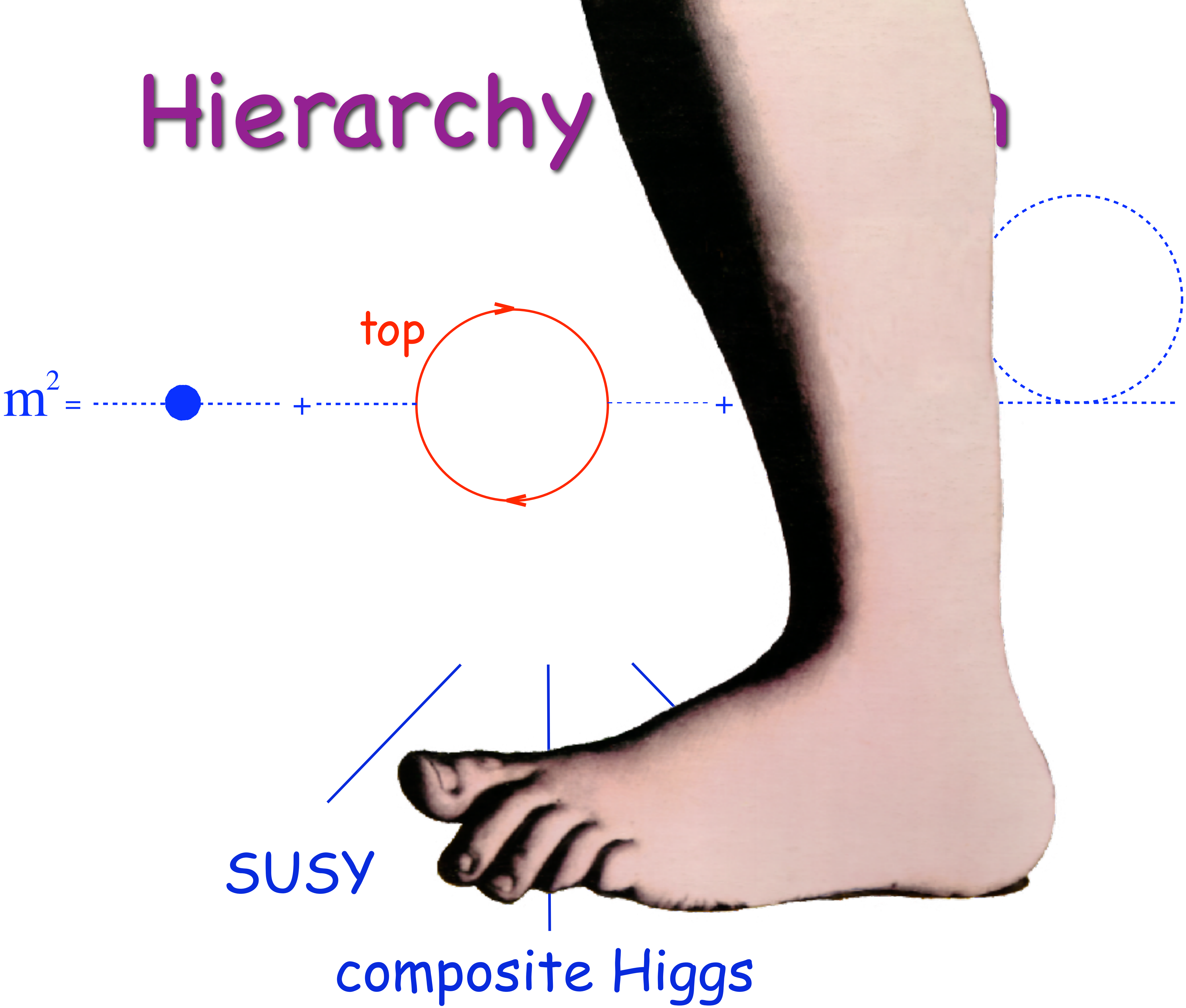


$$\left(\frac{125}{\sqrt{2}}\right)^2 = \begin{aligned} &16419971512763993607881093447038089115 \\ &-19402031160008016677277886179991476752 \\ &+24412810990665599549438182257396371142 \\ &+54077854817746311445297450721375149 5 \end{aligned}$$

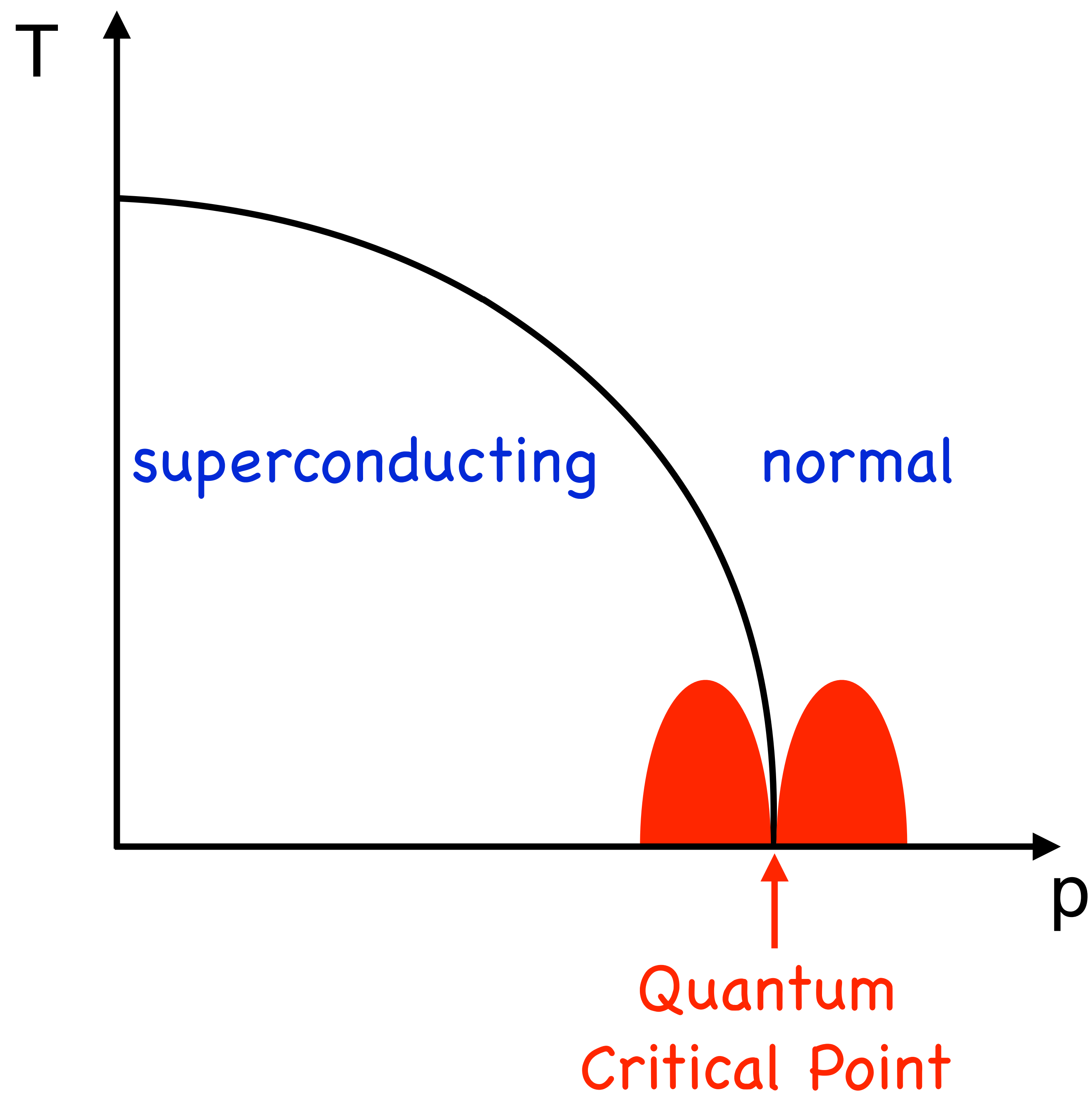
Hierarchy Problem



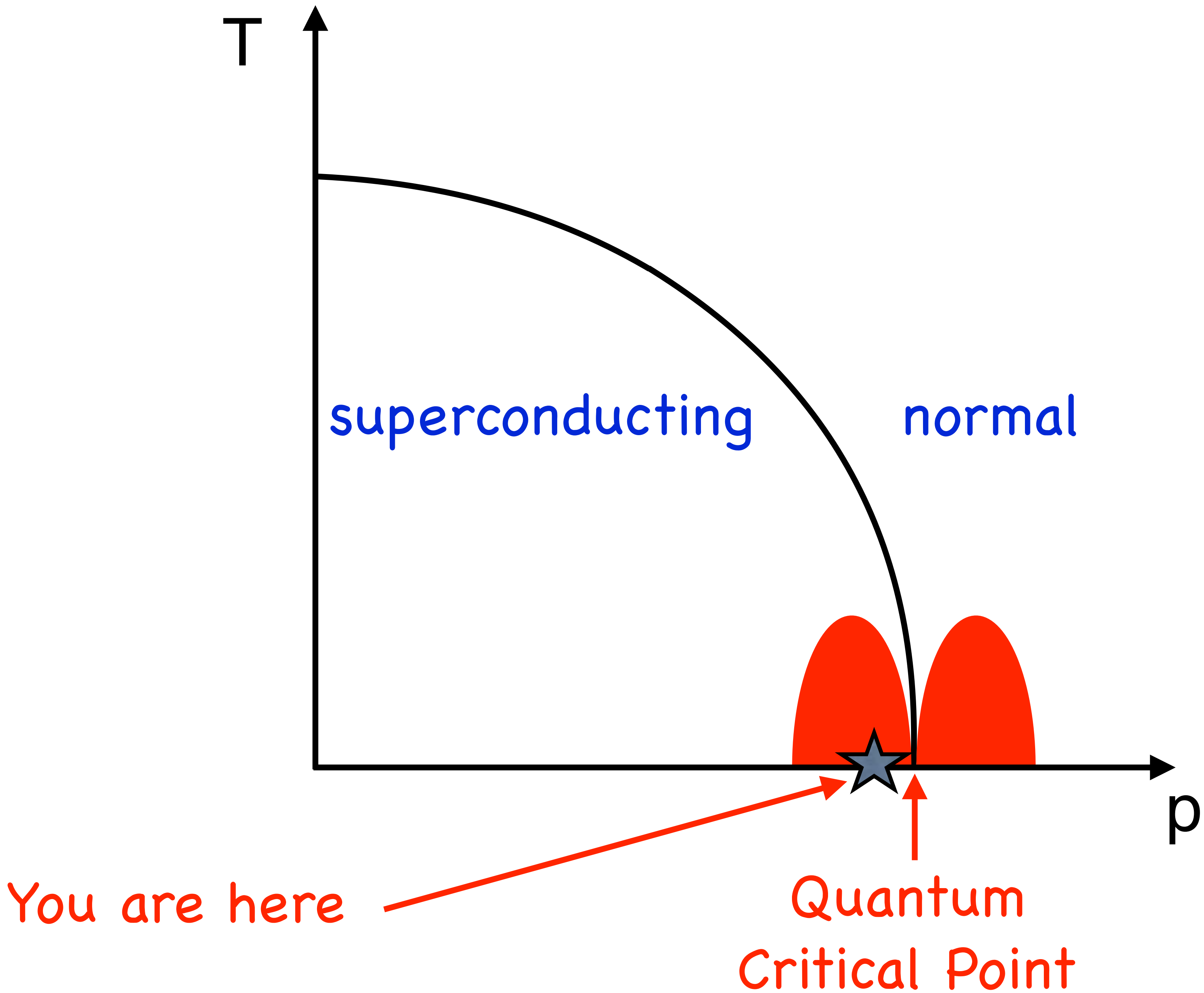
Hierarchy



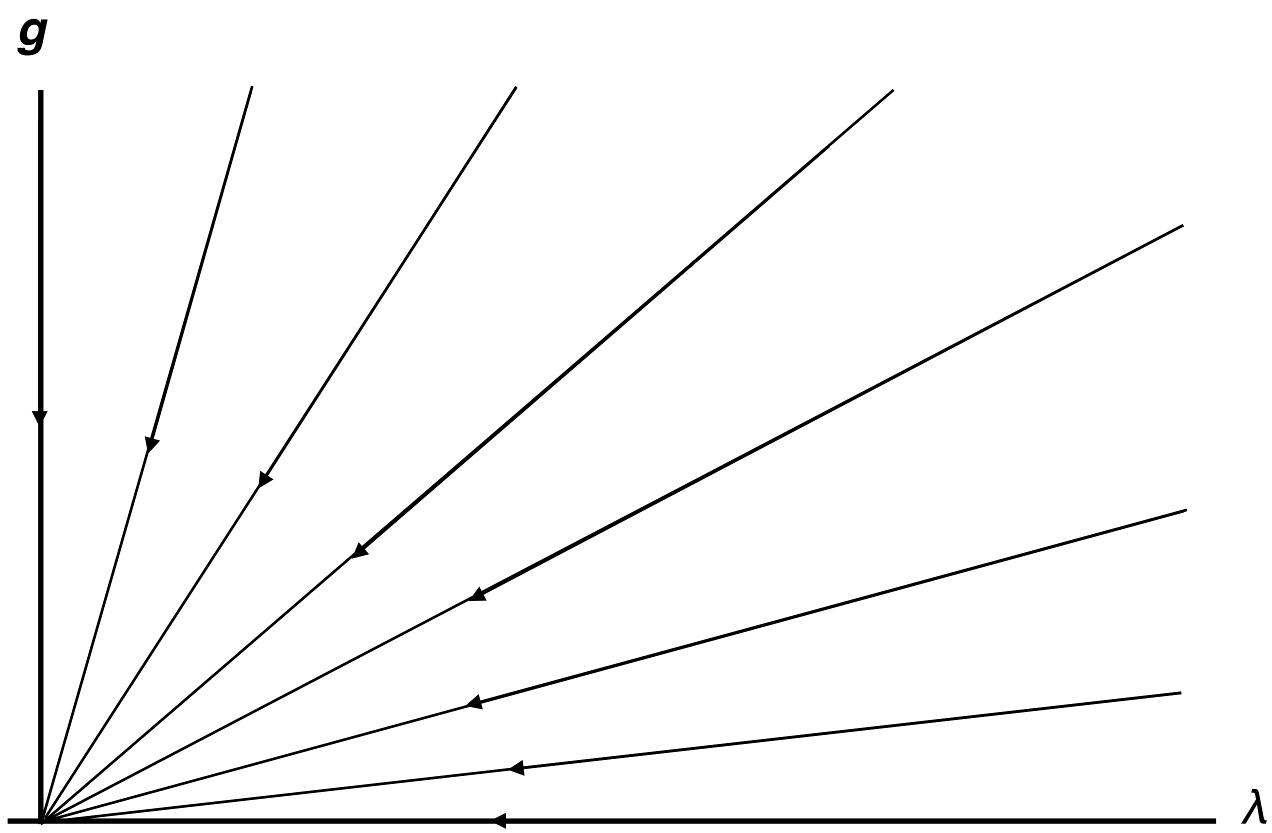
Quantum Phase Transition

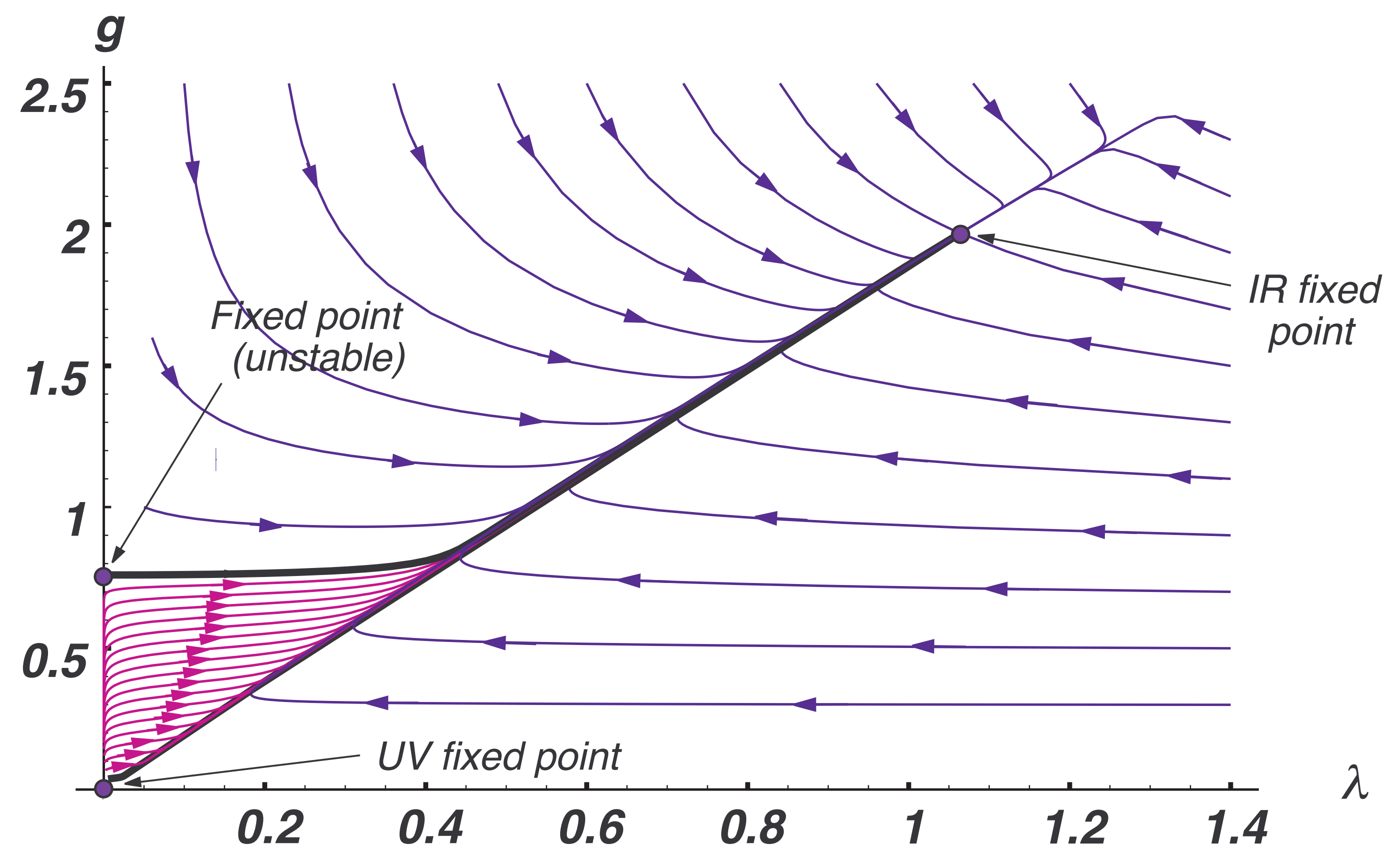


Quantum Phase Transition



Quantum Critical Point





arbitrarily long RG flow

AdS/CFT/Unparticle

CFT scaling dimension: $\Delta[\mathcal{O}]$

$$G(p) \equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle$$

Unparticle: $G(p) \propto \dfrac{1}{(p^2)^{2-\Delta}}$

AdS$_5$: $\Delta[\mathcal{O}] = 2 \pm \nu = 2 \pm \sqrt{4 + m^2 R^2}$

Why Broken CFT's are Interesting

pure CFT is equivalent to RS2

IR brane at TeV turns RS2 into RS1

IR brane is one type of scale breaking

other IR cutoffs will lead to new
LHC phenomenology

QC Higgs Model

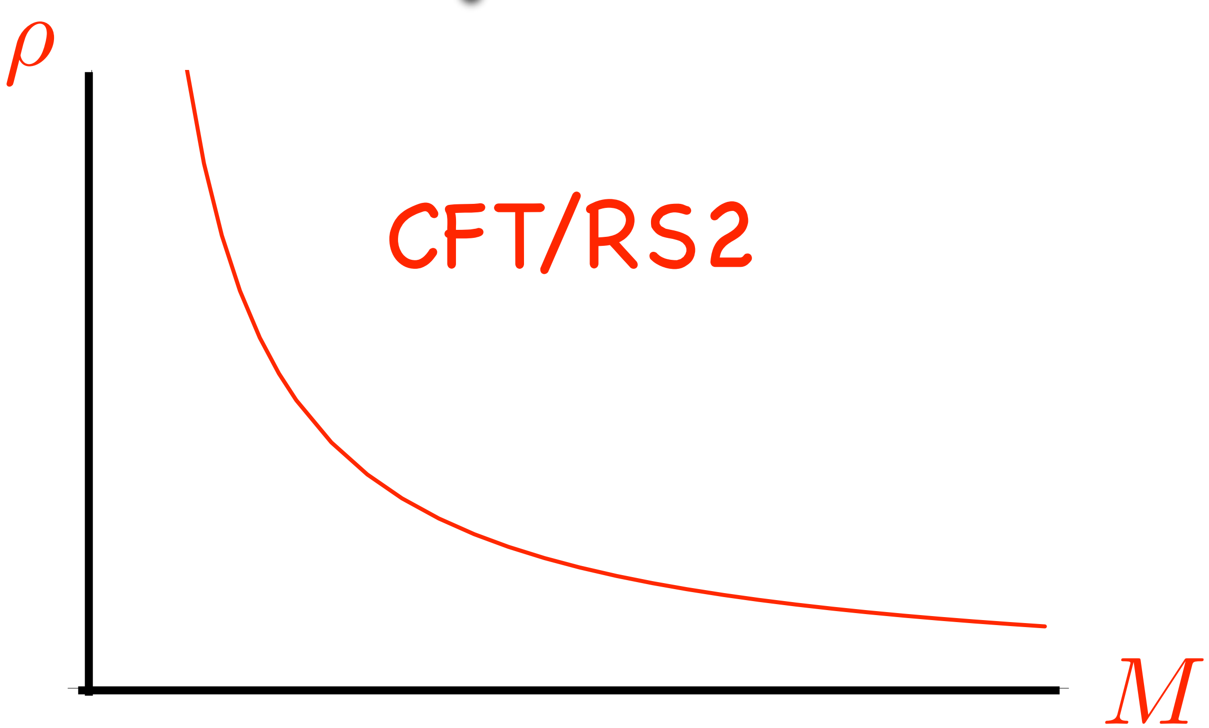
$$G = \frac{-i}{(\mu^2 - p^2)^{2-\Delta} - m^{4-2\Delta}}$$

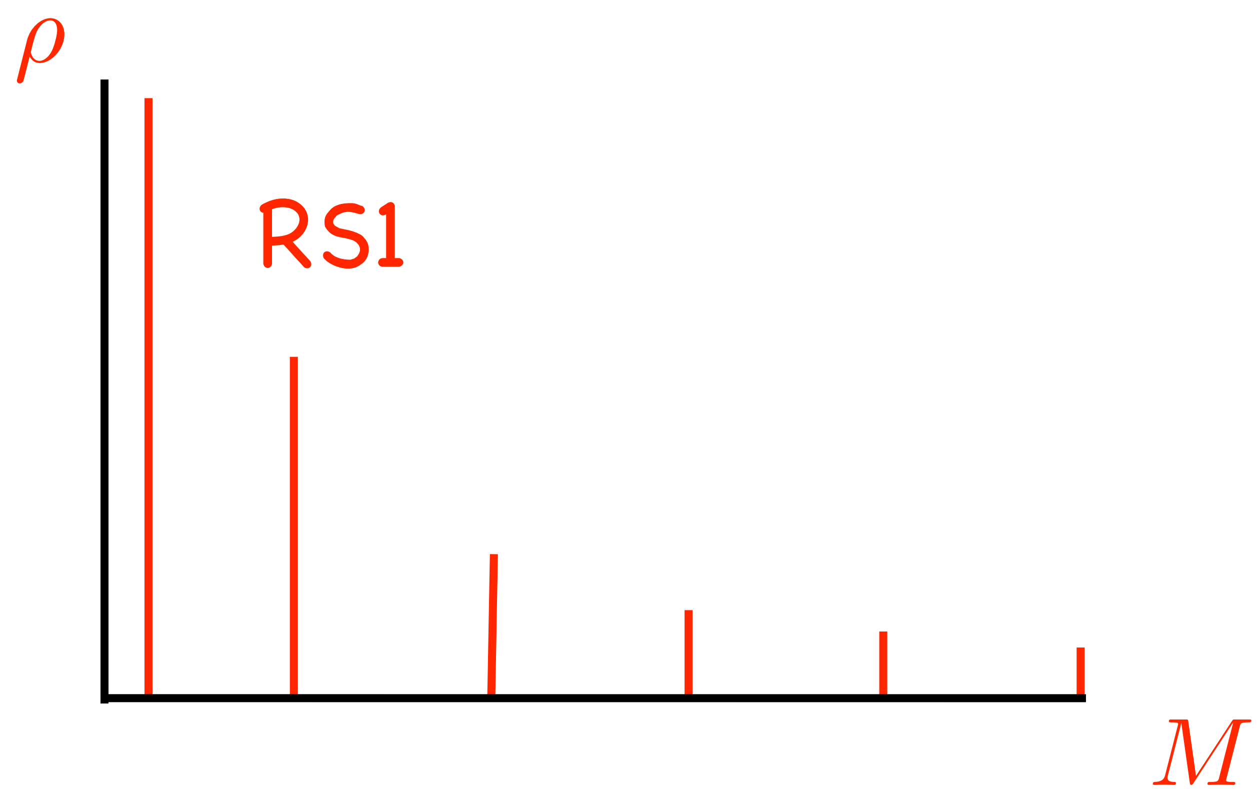
minimal parameterization requires
two mass scales: pole and cut threshold

approach the SM in two limits: $\Delta \to 1$ or $\mu \to \infty$

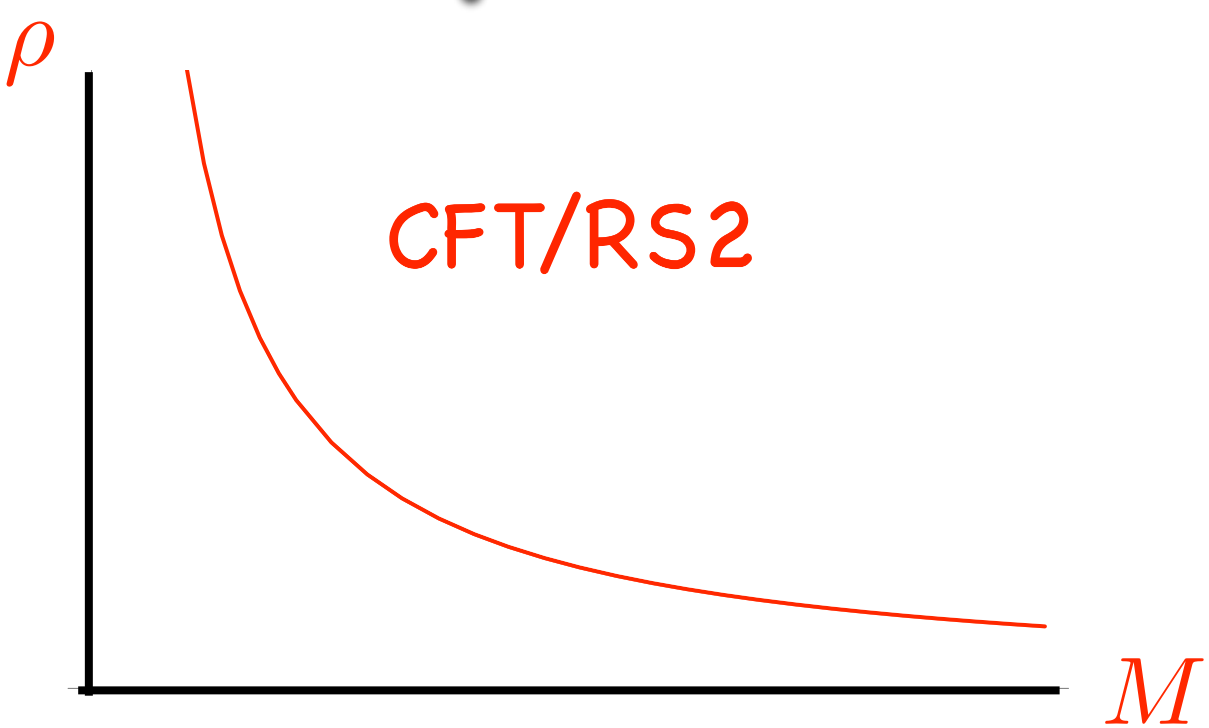
$$G = \frac{i}{p^2 - m_h^2}$$

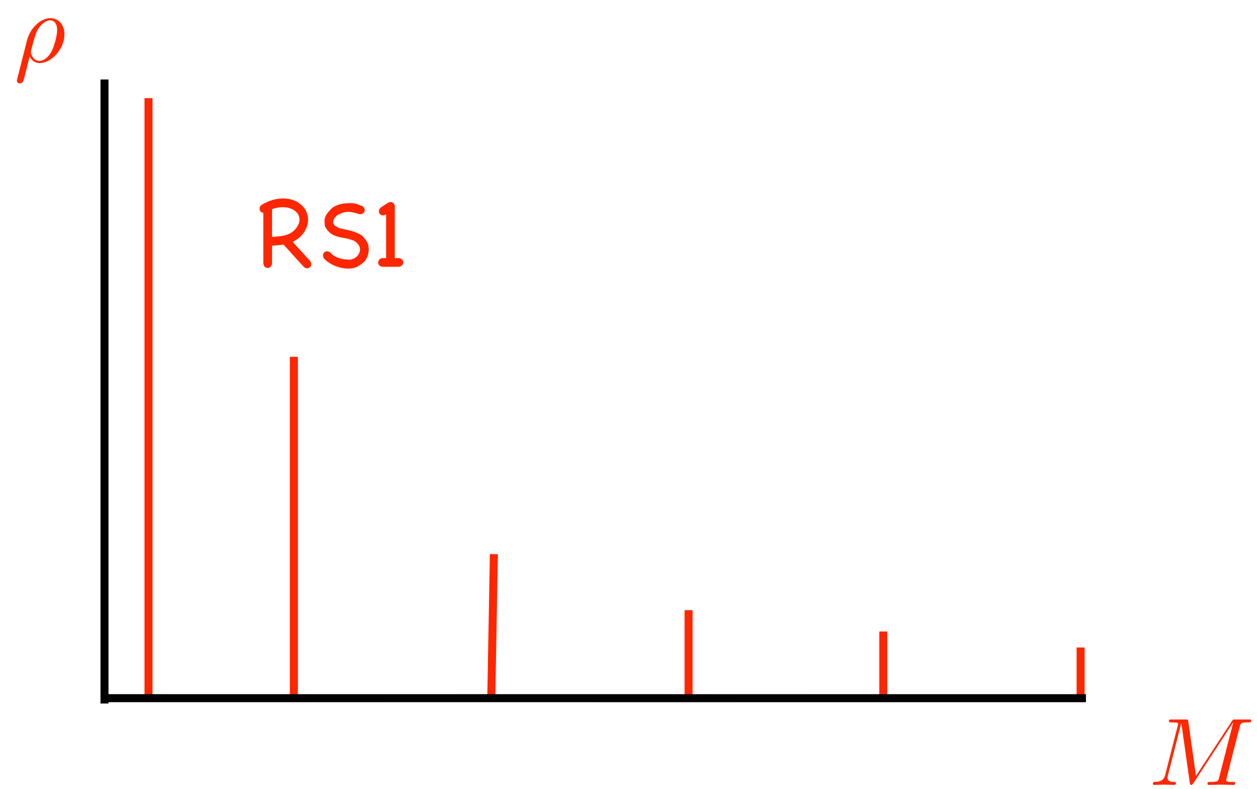
Spectral Densities



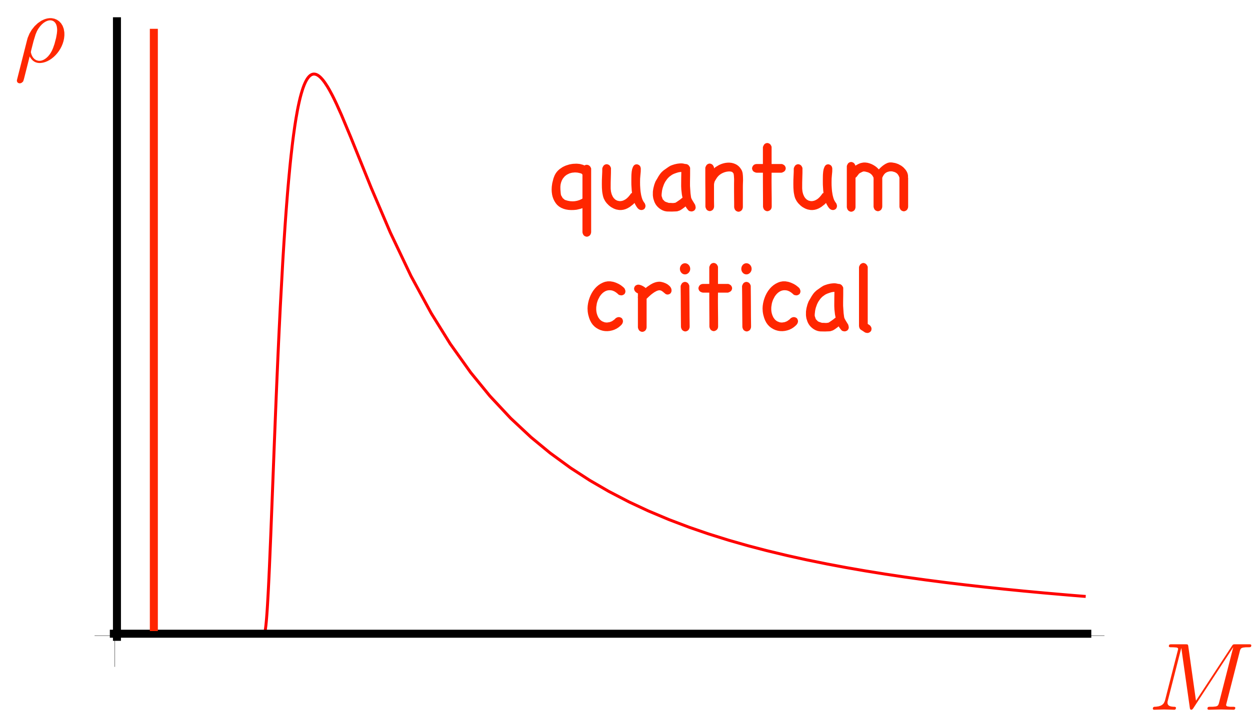


Spectral Densities





$$G = \frac{-i}{(\mu^2 - p^2)^{2-\Delta} - m^{4-2\Delta}}$$



Effective Action

$$S = -V[H^\dagger H] + \int \frac{d^4p}{(2\pi)^4}\, H^\dagger(p)\left[\mu^2 - p^2\right]^{2-\Delta} H$$

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$$S = -V[H^\dagger H] + \int \frac{d^4p}{(2\pi)^4}\, H^\dagger(p)\left[\mu^2 - p^2\right]^{2-\Delta} H$$

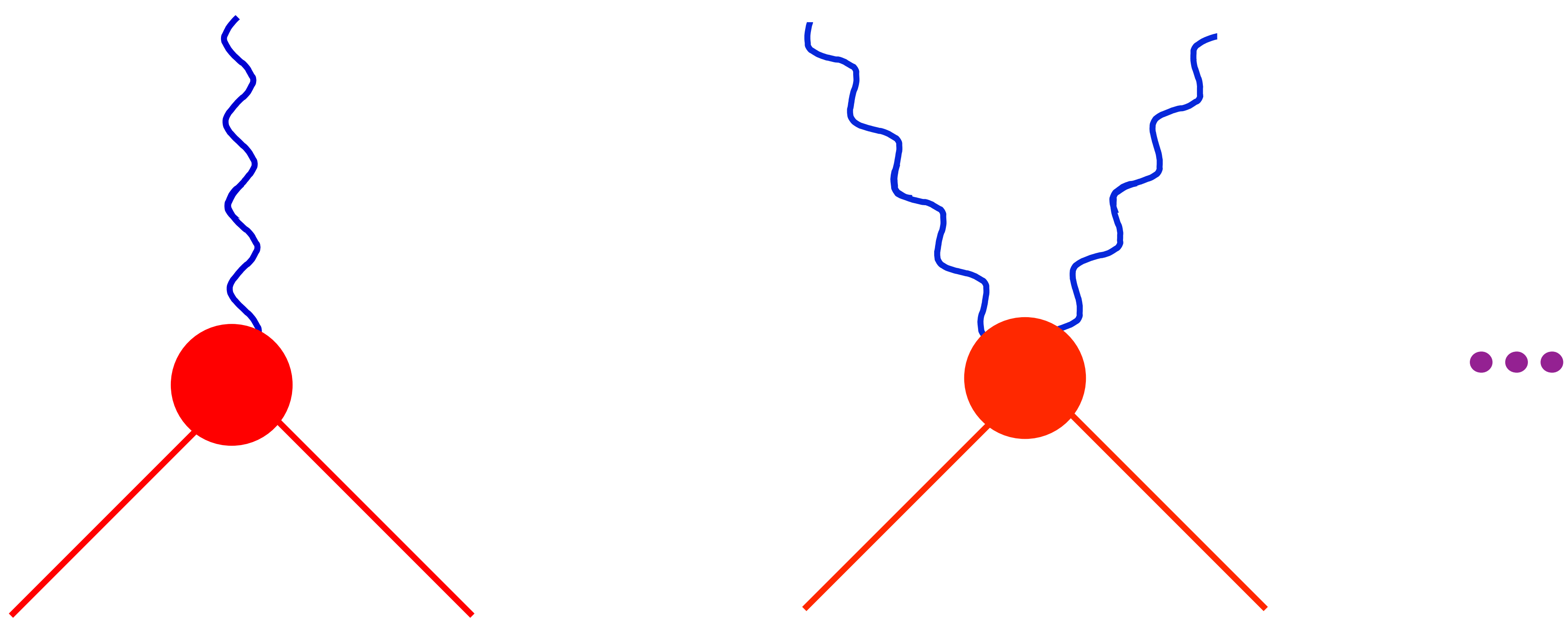
$$S = -V[H^\dagger H] + \int d^4x d^4y\, H^\dagger(x) F(x-y) H(y)$$

$$F(x-y) = (\partial^2 + \mu^2)^{2-\Delta}\delta^{(4)}(x-y)$$

Minimal Gauge Coupling

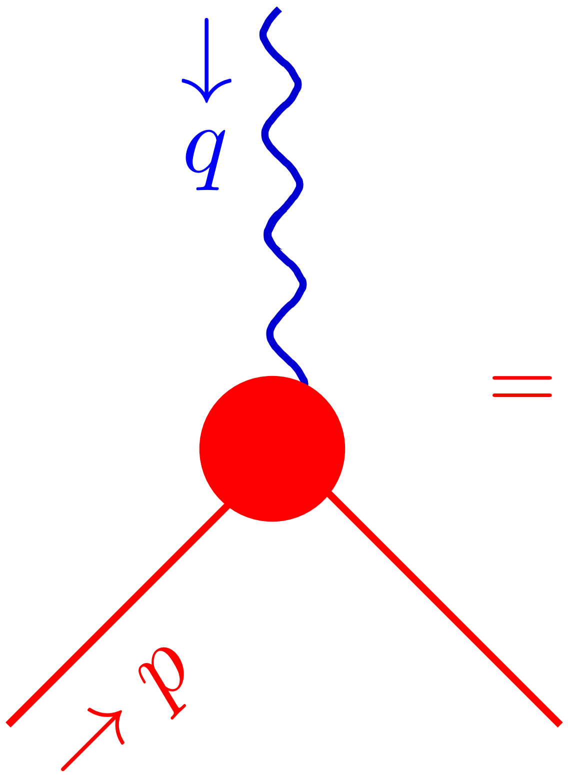
$$F(x-y) \to F(x-y)W(x,y)$$

$$W(x,y) = P\exp\left[-igT^a \int_x^y A_\mu^a dw^\mu\right]$$



cf Mandelstam Ann Phys **19** (1962) 1

Gauge Vertex

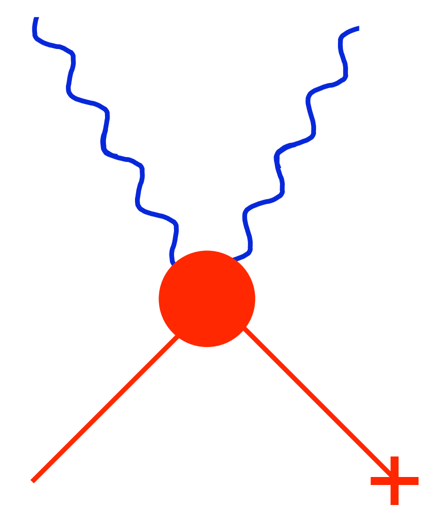


$$= \quad \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2}\left[\left(\mu^2 - (p+q)^2\right)^{2-\Delta} - \left(\mu^2 - p^2\right)^{2-\Delta}\right]$$

Higher Dimension Operator in AdS

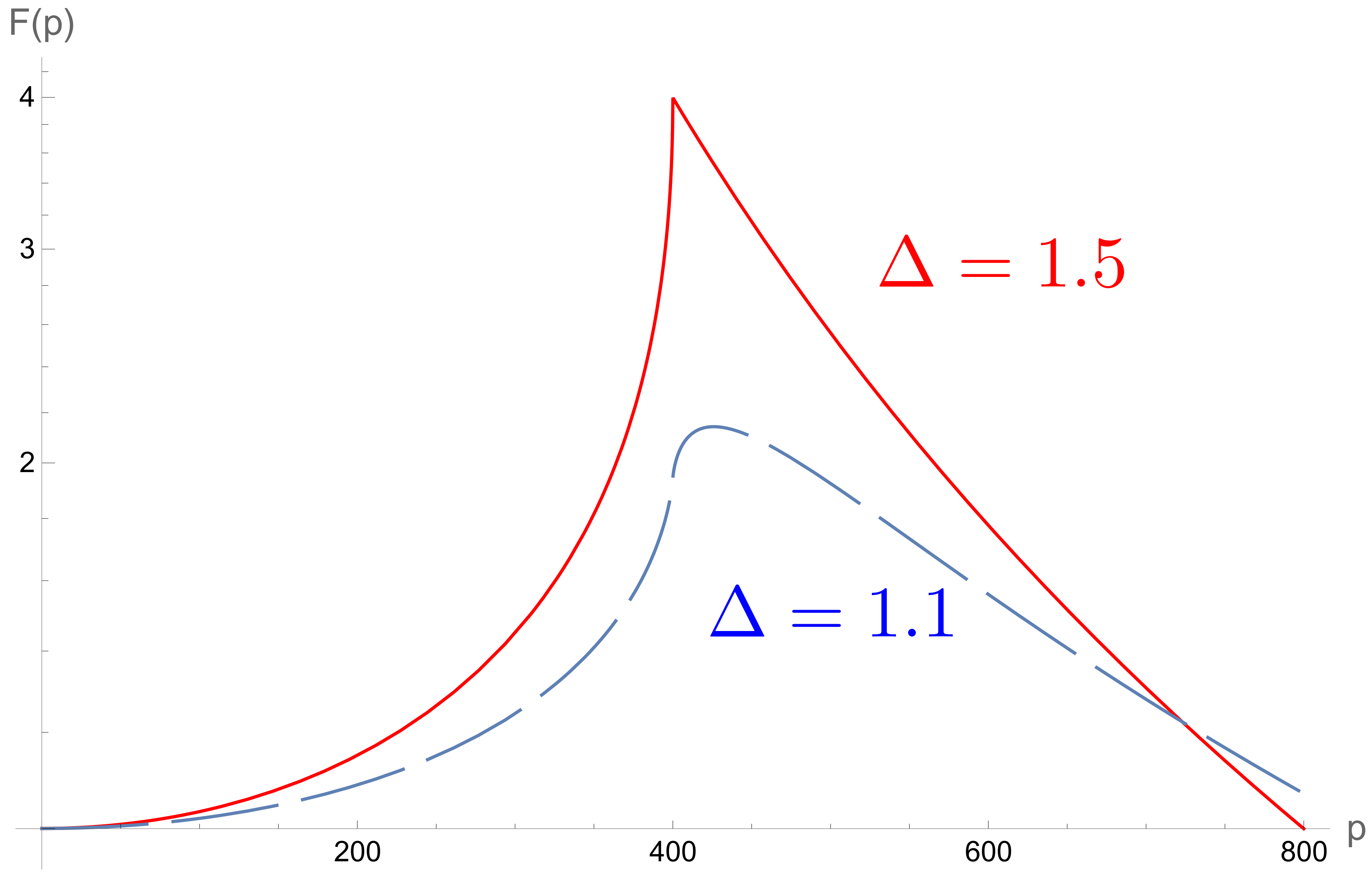
$$\int d^4x\, dz\, g_5^2\, H^\dagger F^a_{\alpha\beta} F^{b\alpha\beta} H$$

$$\mathcal{M} = \{T^a, T^b\} \left(g^{\alpha\beta} p_1 \cdot p_2 - p_1^\beta p_2^\alpha \right) F^{ab}_{VVh}$$

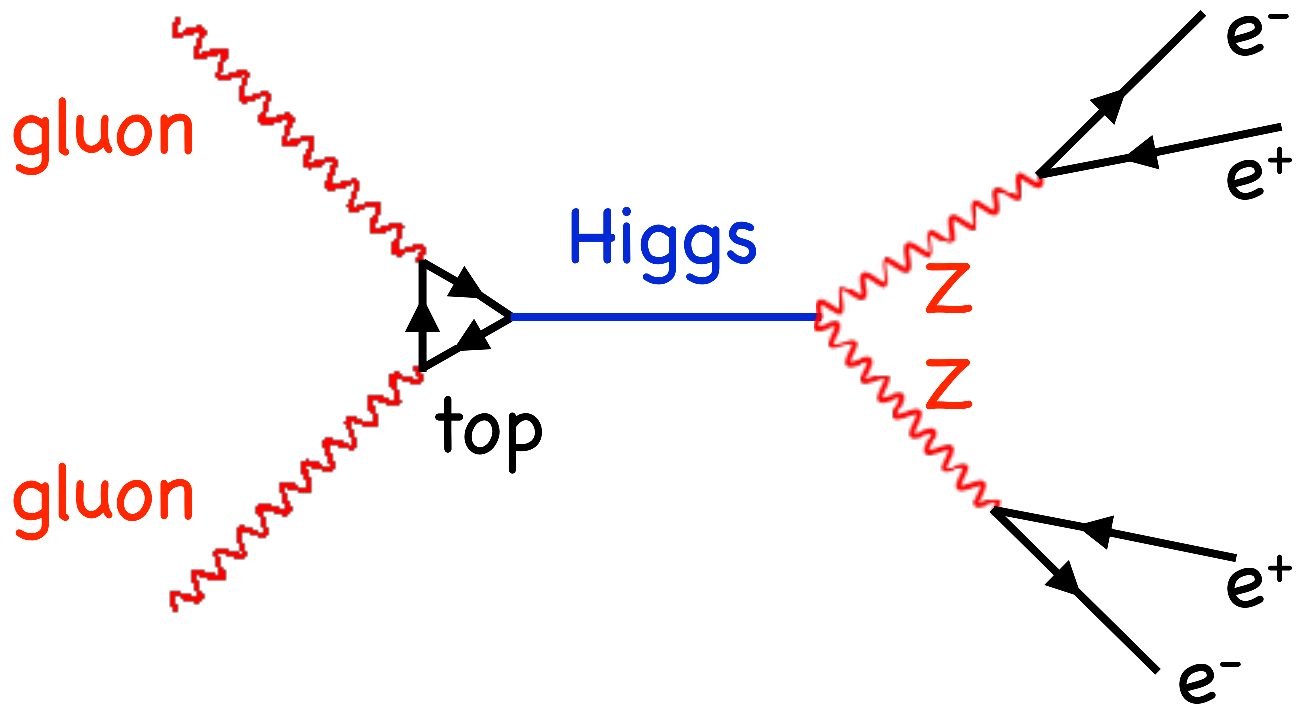


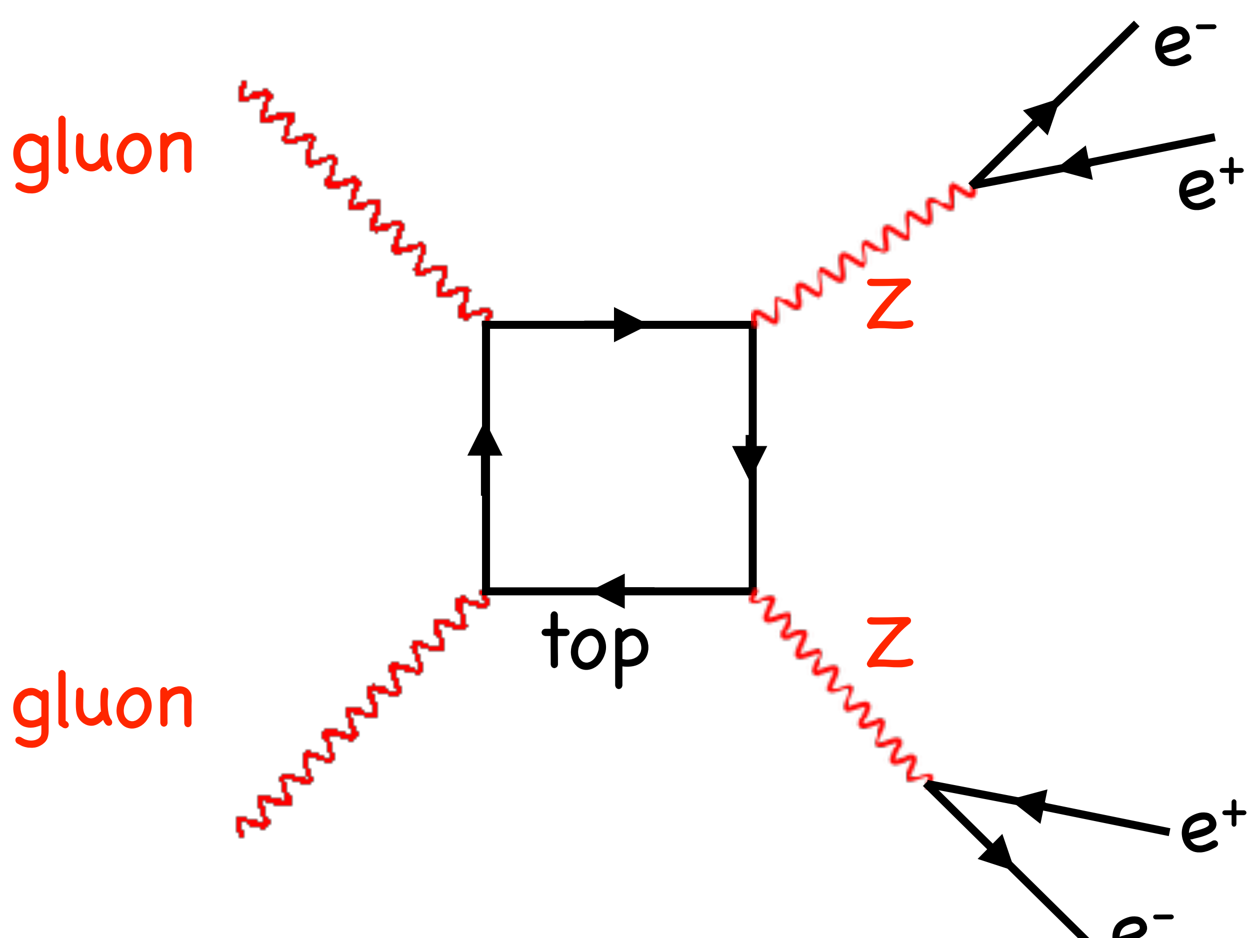
$$F^{ab}_{VVh} \propto \tilde{v}^\Delta g_5^2 \int_R^\infty dz\, z^3 \frac{K_{2-\Delta}(\sqrt{\mu^2 - (p_1+p_2)^2}\, z) K_{2-\Delta}(\mu\, z)}{K_{2-\Delta}(\sqrt{\mu^2 - (p_1+p_2)^2}\, R) K_{2-\Delta}(\mu\, R)} \,,$$

AdS Form Factor F_{VVh}

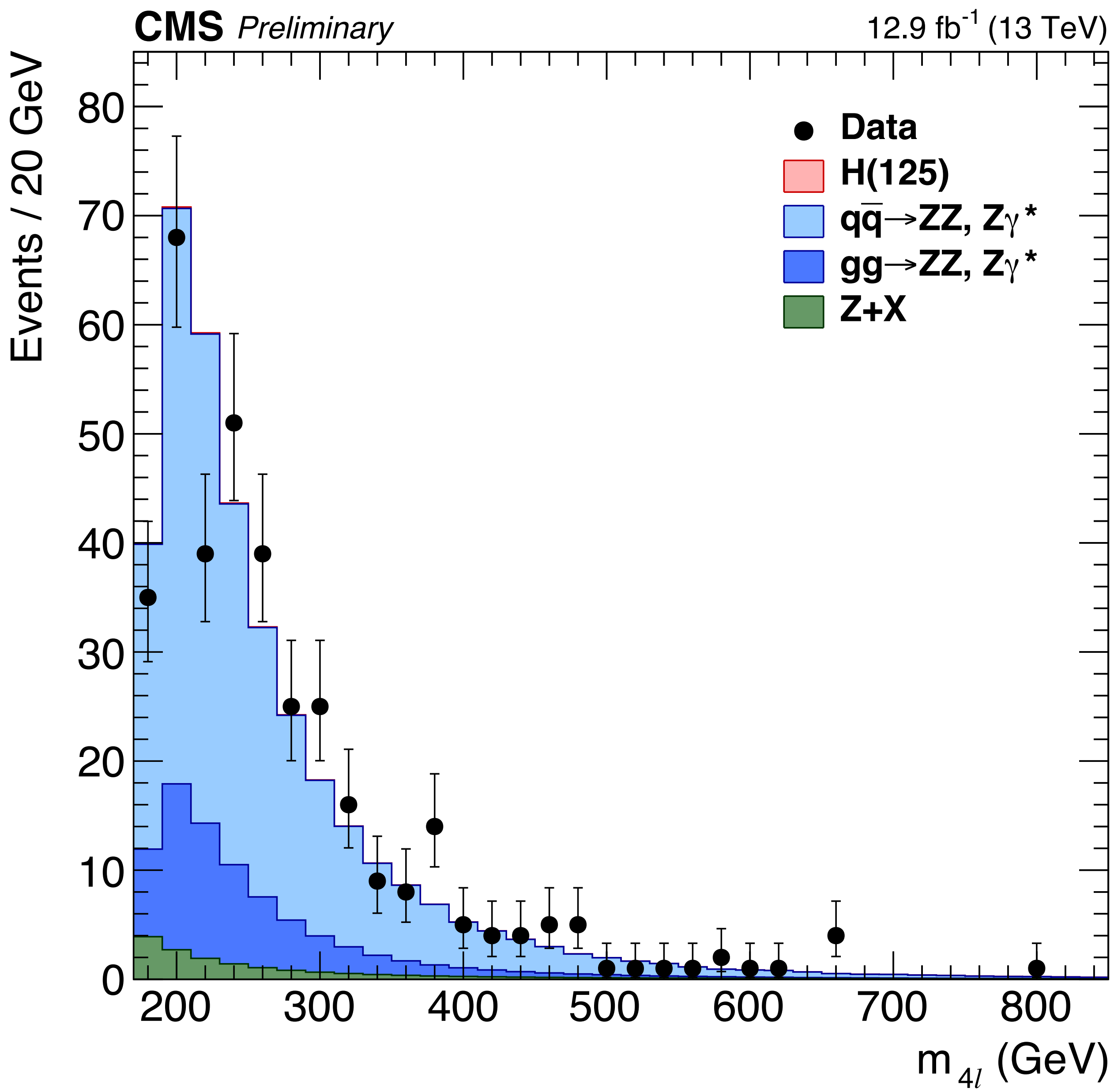


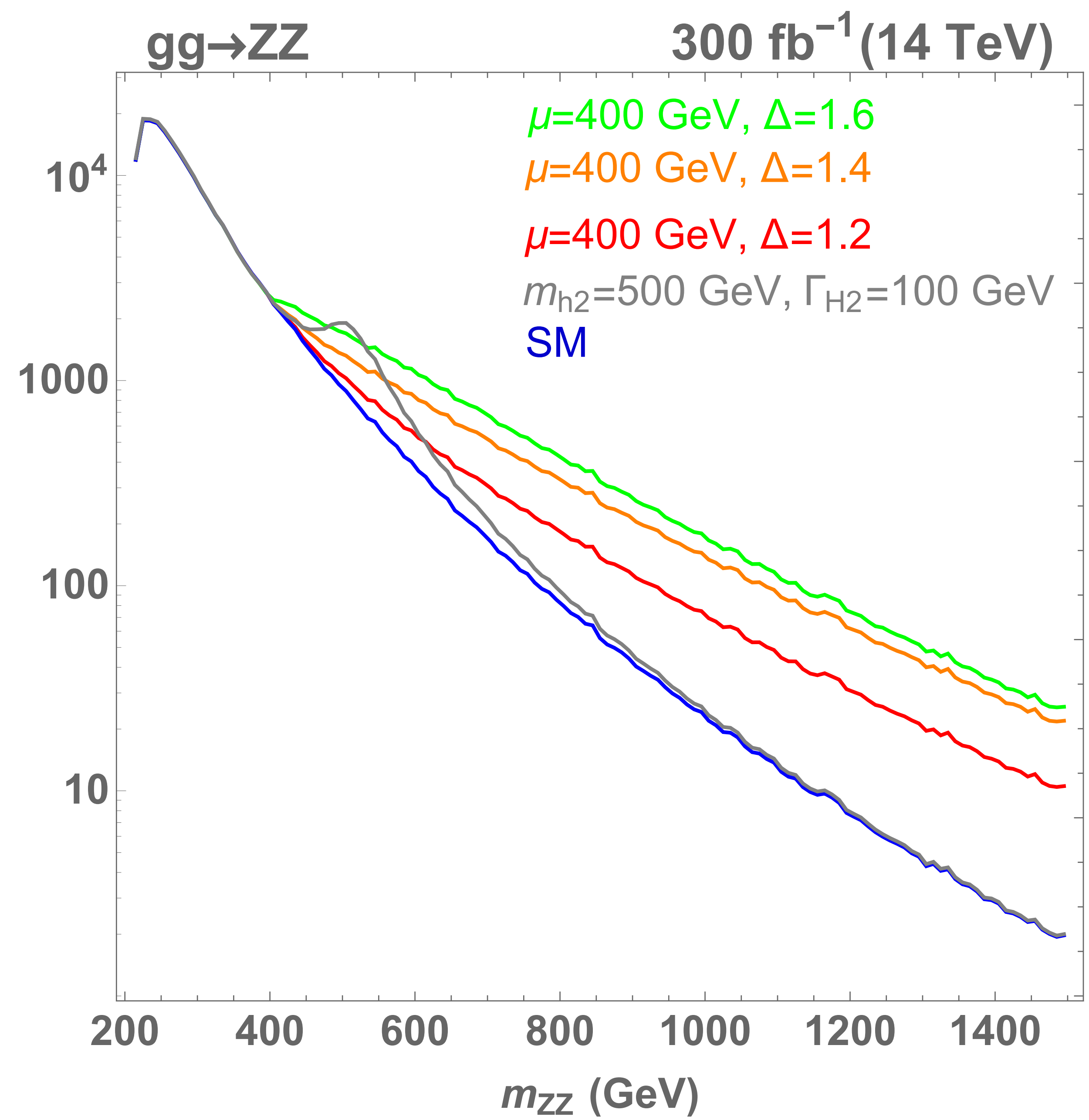
LHC Interference





LHC Experiment





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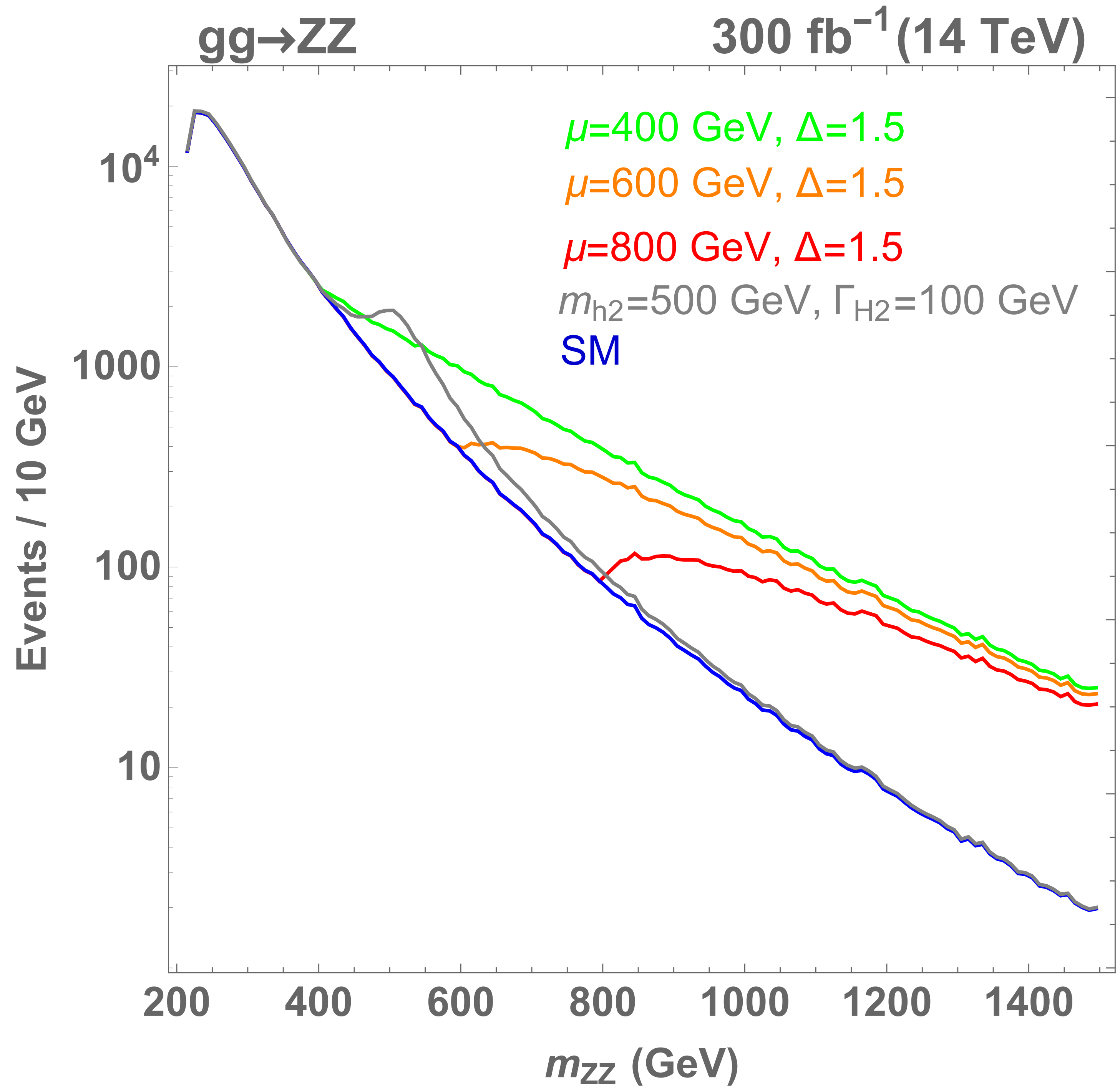
hep-ph/1511.08218

LHC Experiment



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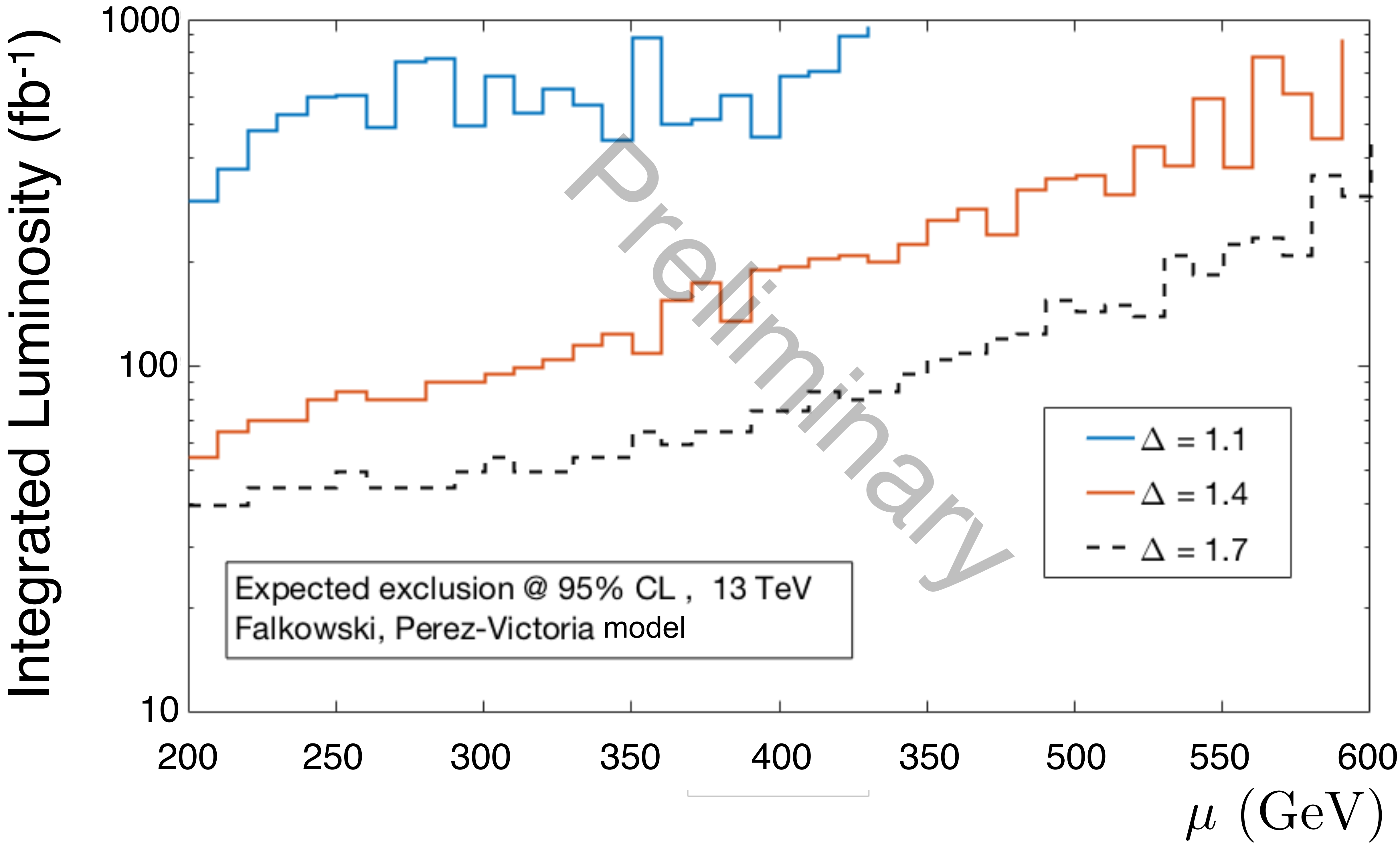
hep-ph/1511.08218

Future Sensitivity



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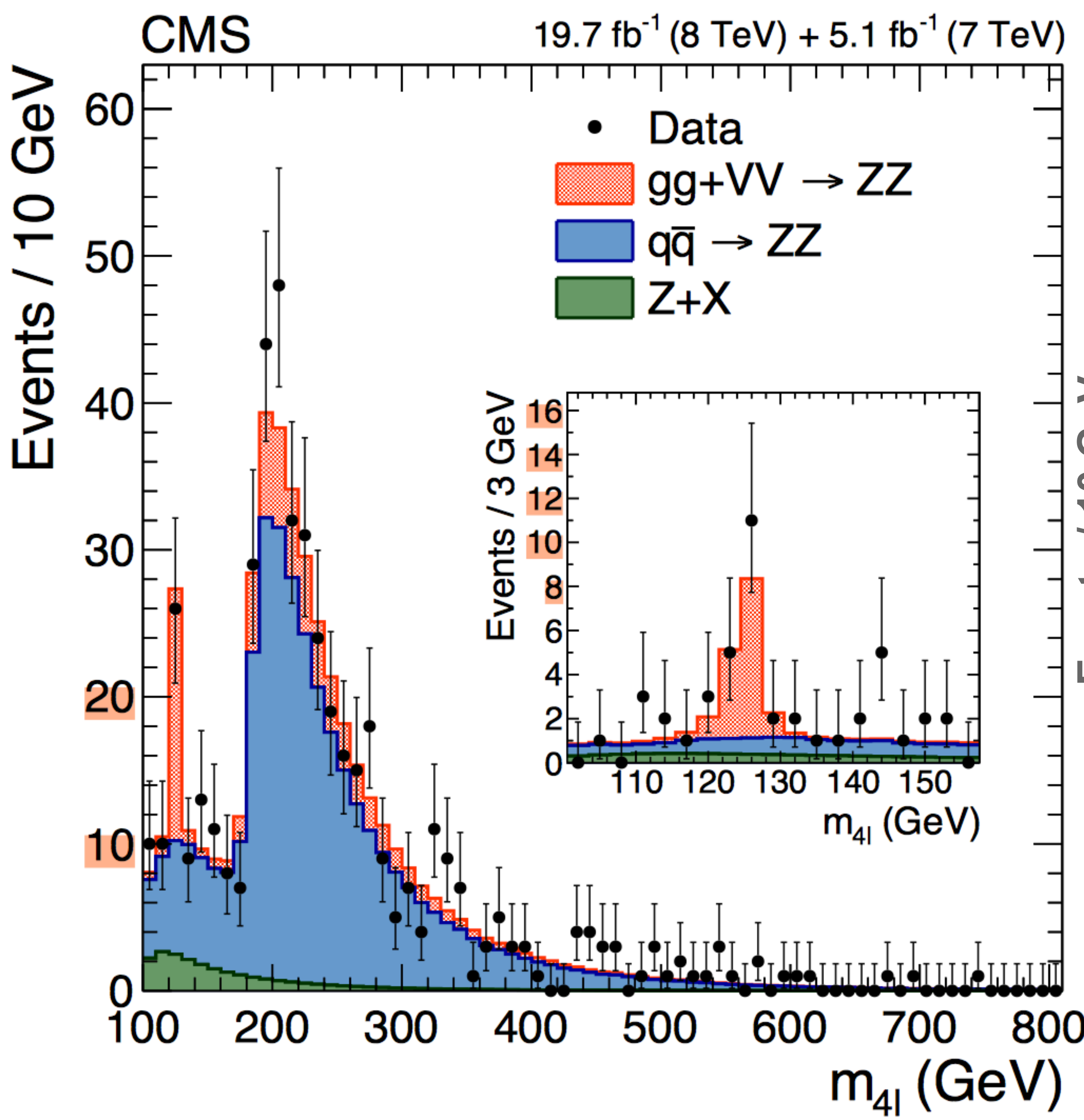
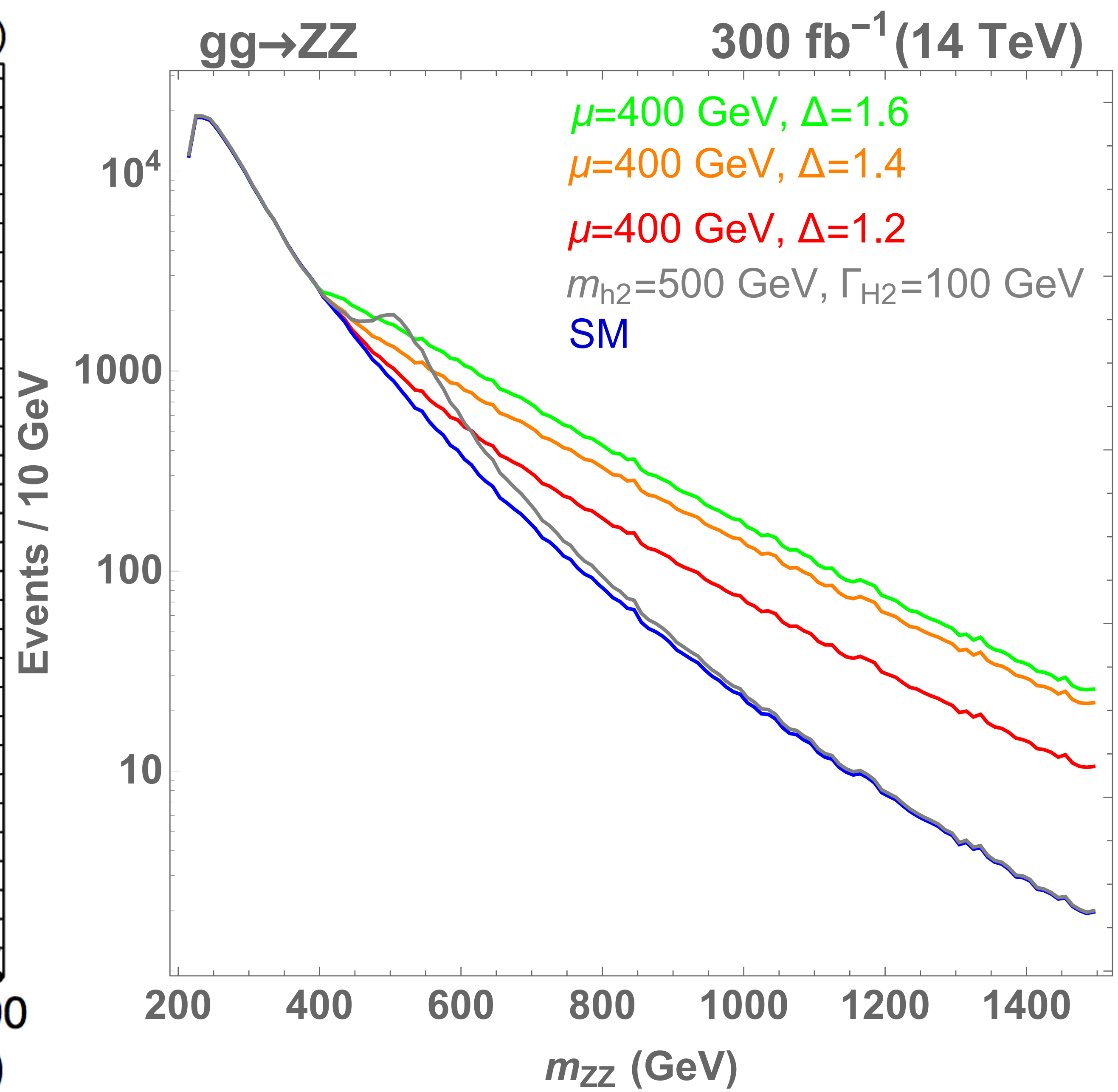
Conclusions

The Electroweak Phase Transition is close to a Quantum Critical Point

The LHC can test whether the Higgs has a non-trivial critical exponent

LHC Experiment





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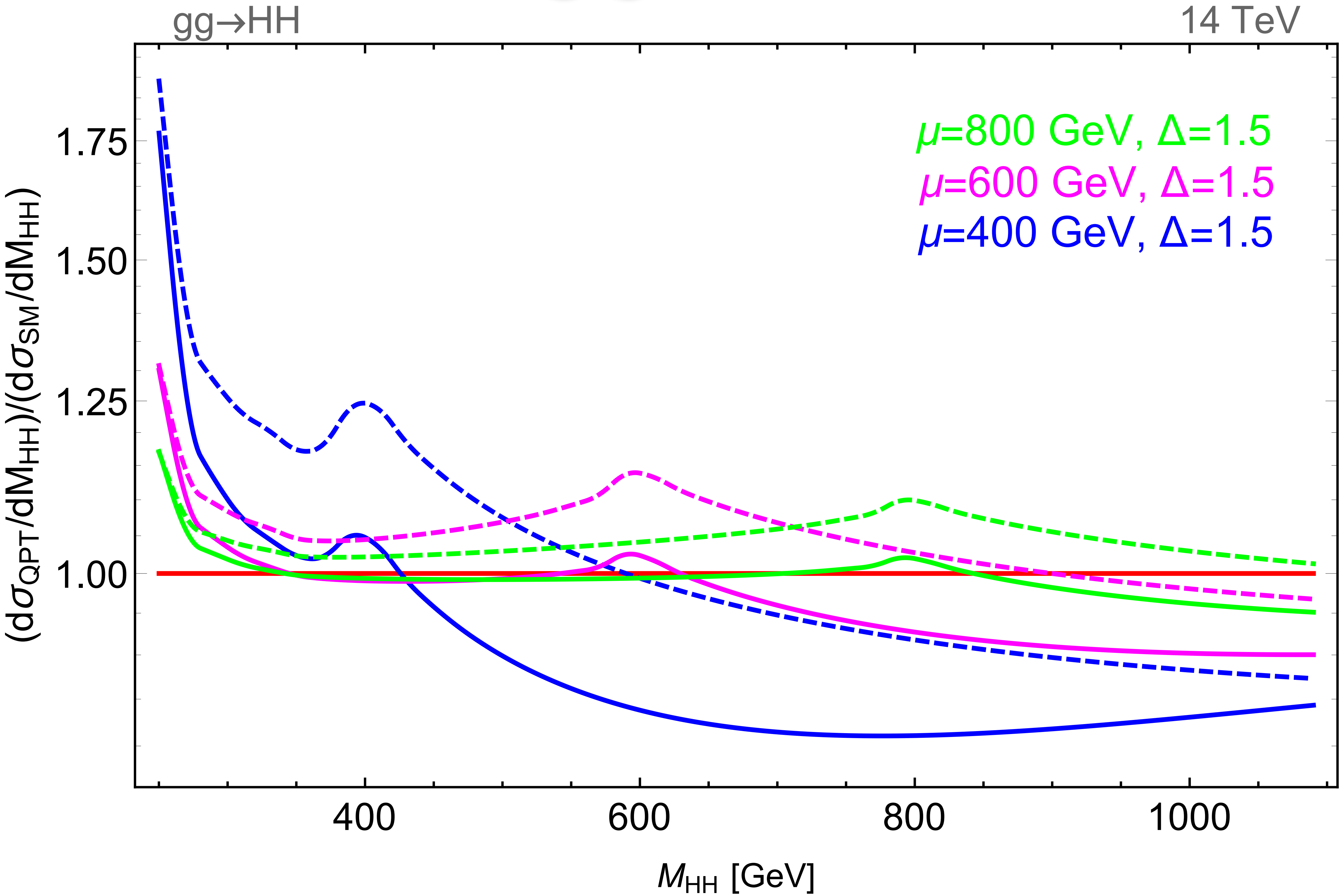
Double Higgs Production



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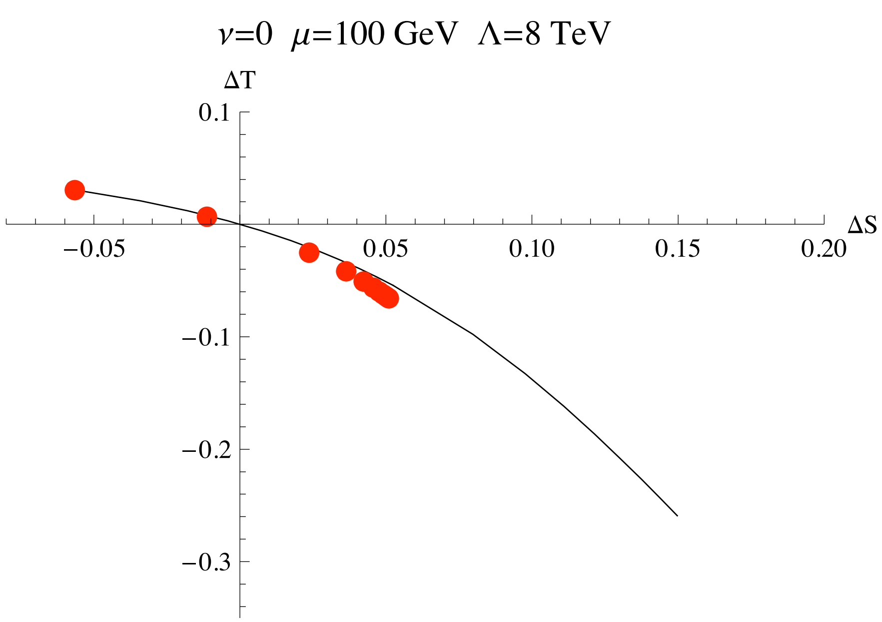
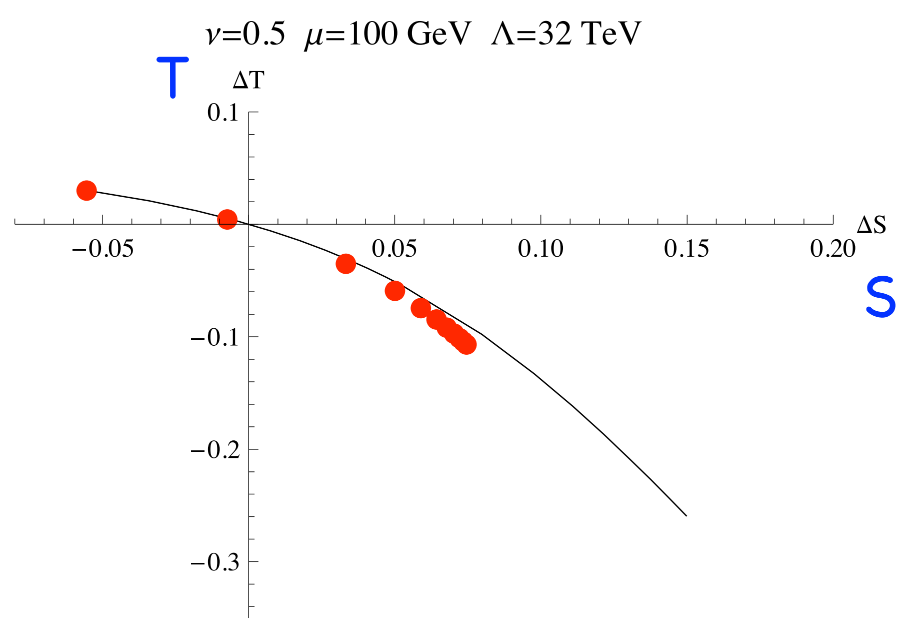
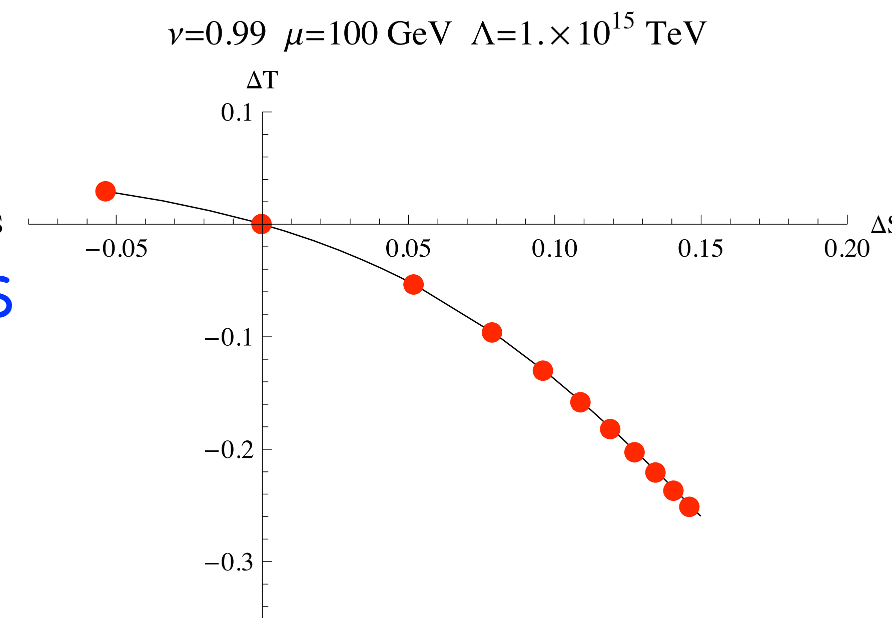
Precision Measurements



$\Delta = 2$



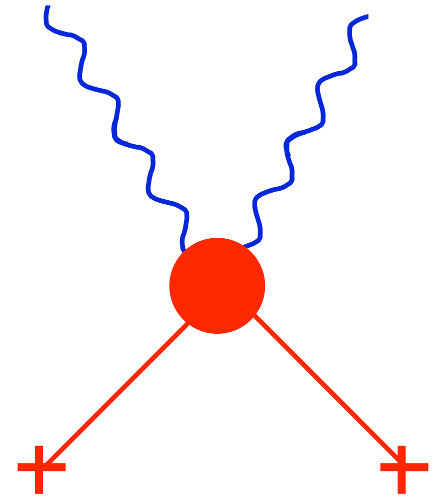
$\Delta = 1.5$



$\Delta = 1.01$

Falkowski & Perez-Victoria, hep-ph/0901.3777

QC Higgs and M_W

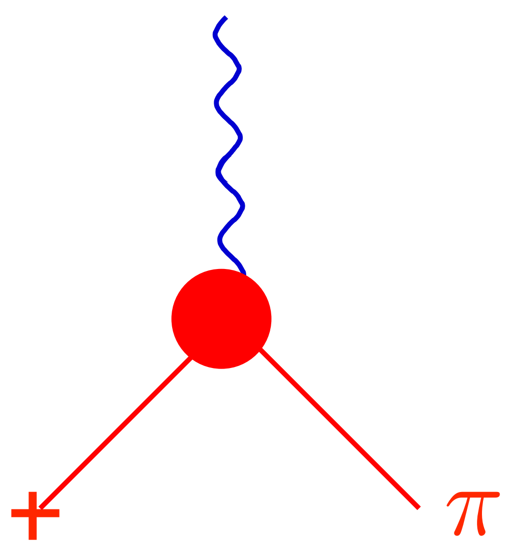


$$-g^2 A^a_\alpha A^b_\beta \langle \mathcal{H}^\dagger \rangle T^a T^b \langle \mathcal{H} \rangle \Bigg\{ \; g^{\alpha\beta}(\Delta - 2)\mu^{2-2\Delta}$$

$$-\frac{q^\alpha q^\beta}{q^2}\left[(\Delta - 2)\mu^{2-2\Delta} - \frac{\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta}}{q^2}\right]\Bigg\}$$

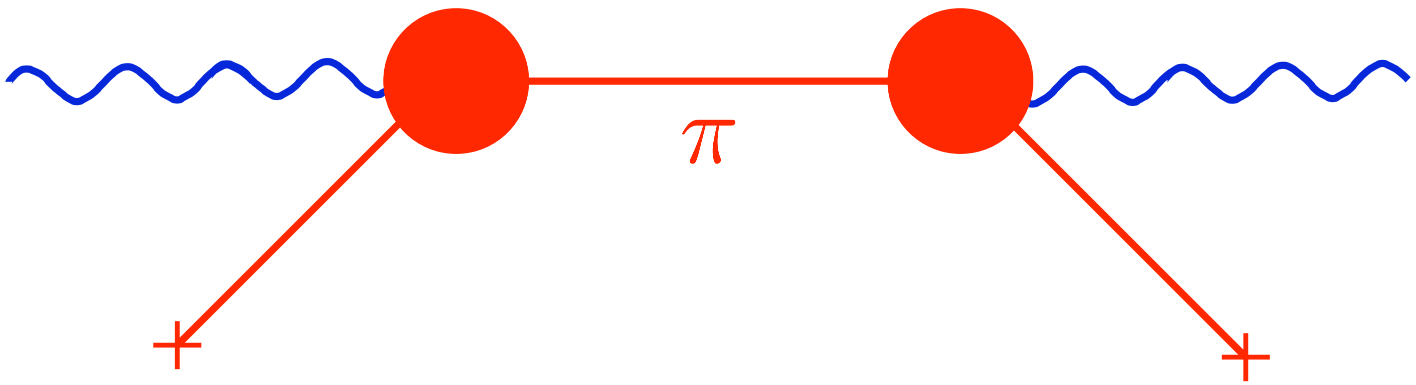
$$M_W^2 \quad = \quad \frac{g^2(2-\Delta)\mu^{2-2\Delta}v^{2\Delta}}{4}$$

GB mixing



$$g\left(\langle\mathcal{H}^\dagger\rangle A^a_\alpha T^a \Pi - \Pi^\dagger A^a_\alpha T^a \langle\mathcal{H}\rangle\right)\left[\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta}\right] q^\alpha / q^2$$

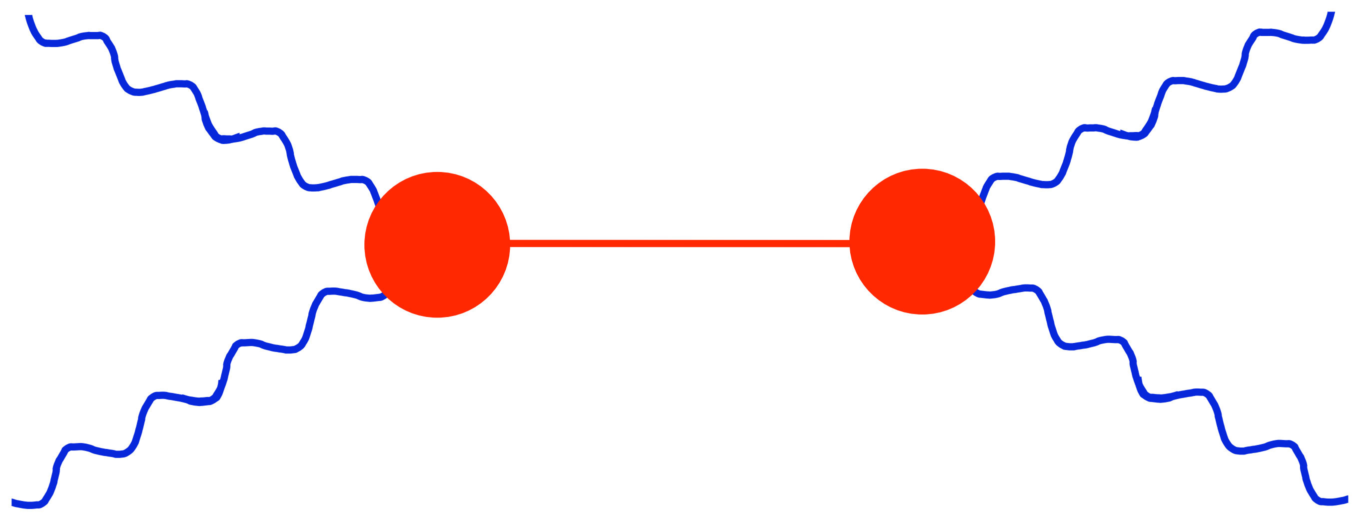
Gauge invariance is maintained



$$\Pi^{ab\alpha\beta}(q) = -g^2 \langle \mathcal{H}^\dagger \rangle T^a T^b \langle \mathcal{H} \rangle \frac{q^\alpha q^\beta}{q^4} \times \left[\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta} \right]^2 G_{GB}(q)$$

$$G_{GB}(q) = -\frac{i}{(\mu^2 - q^2 - i\epsilon)^{2-\Delta} - \mu^{4-2\Delta}}$$

WW Scattering

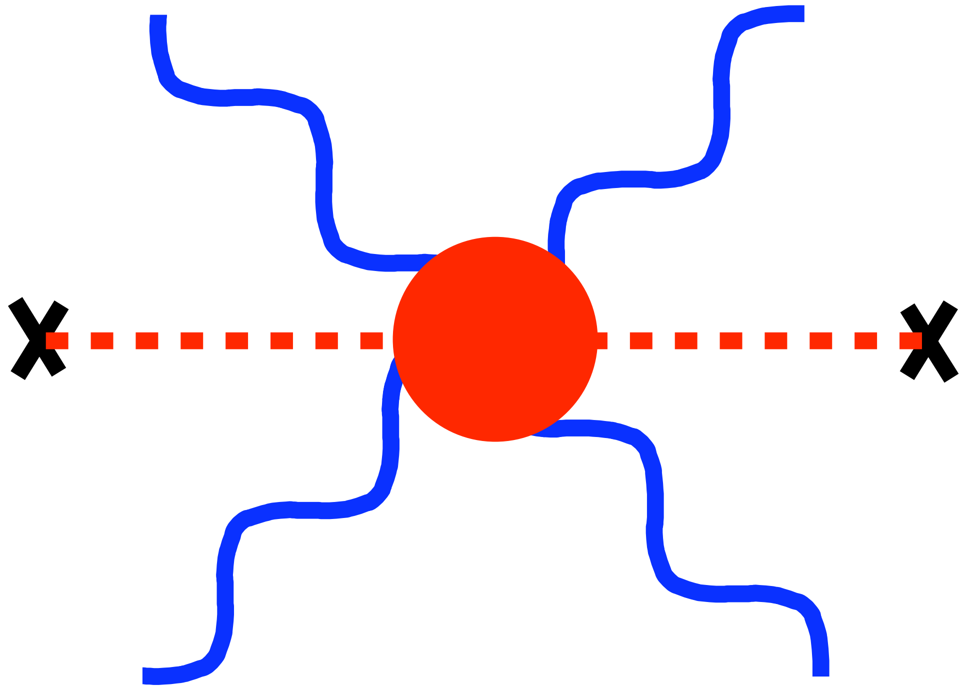


at large s

$$\mathcal{M}_h = -i\frac{g^4}{4M_W^2(2-\Delta)\mu^{2-2\Delta}}(-s)^{2-\Delta}$$

QC Higgs exchange is insufficient
to unitarize WW scattering

WW Scattering



$$\mathcal{M}_{hh} = -i\frac{g^2}{4M_W^2}\left[s+\frac{(-s)^{2-\Delta}}{(2-\Delta)\mu^{2-2\Delta}}\right]$$

QC Higgs 6 point vertex does unitarize WW scattering

Stancato JT, hep-ph/0807.3961

AdS/CFT

$$\langle\, e^{\int d^4x\, \underset{\uparrow\atop\text{source}}{\phi_0}(x)\mathcal{O}(x)} \rangle_{\rm CFT} \approx e^{-S_{\rm 5Dgrav}[\phi(x,z)|_{z=0}=\phi_0(x)]}$$

$$ds^2 = \frac{R^2}{z^2}\left(dx^2 - dz^2\right)$$

$\mathcal{O} \subset$ CFT $\leftrightarrow \phi$ AdS$_5$ field, $\phi_0(x)$ is boundary value

AdS/CFT

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right)$$

$$z > \epsilon$$

$$S_{bulk} = \frac{1}{2}\int d^4x\,dz\sqrt{g}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + m^2\phi^2)$$

$$\phi(p,z) = az^2 J_\nu(pz) + bz^2 J_{-\nu}(pz)$$

$$\Delta[\mathcal{O}] = 2 \pm \nu = 2 \pm \sqrt{4 + m^2R^2}$$

AdS/CFT/Unparticles

$$\phi(p,\epsilon) = \epsilon^{-\nu} R^{-3/2}\, \phi_0(p)$$

$$S = \frac{1}{2}\int d^4x\, dz\, \partial_z \left(\frac{R^3}{z^3} \phi \partial_z \phi \right)$$

surface term

AdS/CFT/Unparticles

$$\phi(p,\epsilon) = \epsilon^{-\nu} R^{-3/2}\, \phi_0(p)$$

$$S = \frac{1}{2}\int d^4x\, dz\, \partial_z \left(\frac{R^3}{z^3}\phi\partial_z\phi\right)$$

surface term

$$S = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(-p)\phi_0(p)K(p)$$

$$K(p) = (2-\nu)\epsilon^{-2\nu} + b\,p^{2\nu} + c\,p^2\epsilon^{2-2\nu} + \ldots$$

$$K(p) = G(p) \qquad\qquad \Delta = 2+\nu$$

unparticle propagator

$$G(p) \equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle$$

unparticle propagator

$$
\begin{aligned}
G(p) &\equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle \\
&= \frac{A_d}{2\pi}\int_0^\infty (M^2)^{\Delta\,-\,2}\frac{i}{p^2 - M^2 + i\epsilon}dM^2
\end{aligned}
$$

spectral dens

unparticle propagator

$$
\begin{aligned}
G(p) &\equiv \int d^4x\, e^{ipx} \langle 0 | T \mathcal{O}(x) \mathcal{O}^\dagger(0) | 0 \rangle \\
&= \frac{A_d}{2\pi} \int_0^\infty (M^2)^{\Delta - 2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2 \\
&= i \frac{A_d}{2} \frac{\left(-p^2 - i\epsilon\right)^{\Delta - 2}}{\sin d\pi}
\end{aligned}
$$

spectral dens

$$
A_d = \frac{16\pi^{5/2}}{(2\pi)^{2\Delta}} \frac{\Gamma(\Delta + 1/2)}{\Gamma(\Delta - 1)\Gamma(2\Delta)}
$$

Legendre Transform

$$\Delta = 2 - \nu$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(-p)K\phi_0(p) + \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(p)A(p)$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}A(-p)\,K^{-1}A(p)$$

A is the source

$K(p)^{-1} = G(p)$ ϕ_0 is the field

Klebanov, Witten hep-th/9905104

Legendre Transform

$$\Delta = 2 - \nu$$

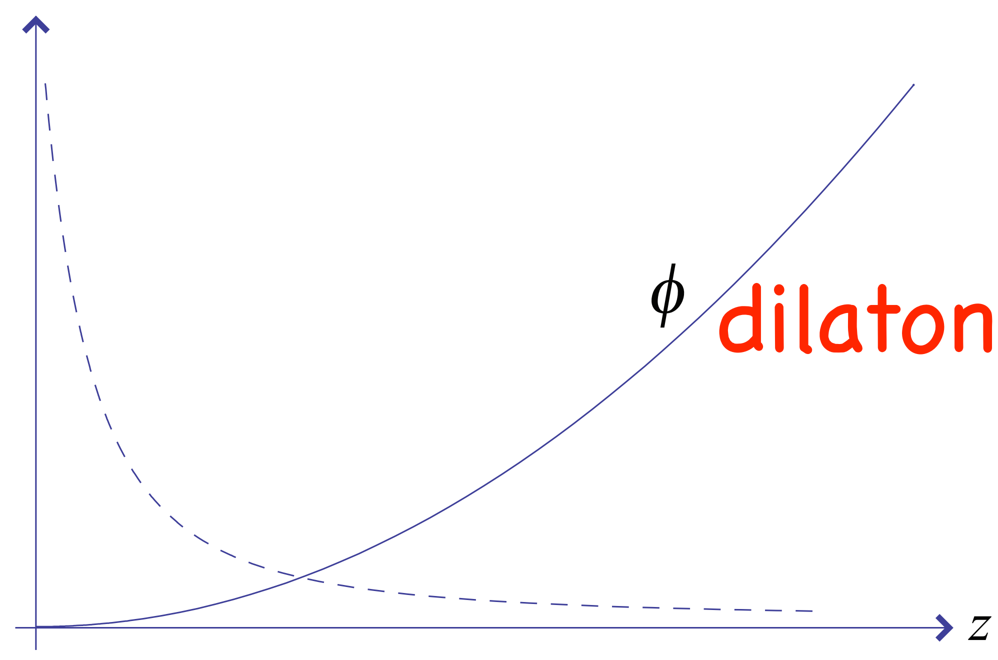
$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(-p)K\phi_0(p) + \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(p)A(p)$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}A(-p)\,K^{-1}A(p)$$

$$\langle \mathcal{O}(p')\mathcal{O}(p)\rangle \quad \propto \quad \frac{\delta^2 S'}{\delta A(p')\,\delta A(p)} \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4}\,(p^2)^{\Delta-2}$$

Klebanov, Witten hep-th/9905104

Soft-Wall



Karch, Katz, Son, Stephanov hep-ph/0602229
Gherghetta, Batell hep-th/0801.4383

Ward-Takahashi Identity

$$ig\Gamma^{a\alpha}(p,q) \; = \; \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2}\left[\left(\mu^2 - (p+q)^2\right)^{2-\Delta} - \left(\mu^2 - p^2\right)^{2-\Delta}\right]$$

$$iq_\mu\Gamma^{a\mu} = G^{-1}(p+q)T^a - T^a G^{-1}(p)$$

AdS/CFT/Unparticles IR Cutoff

$$S_{int} = \frac{1}{2}\int d^4x\, dz\, \sqrt{g}\phi\mathcal{H}^\dagger\mathcal{H}$$

$$\phi = \mu z^2$$

$$z^5\partial_z\left(\frac{1}{z^3}\partial_z\mathcal{H}\right) - z^2(p^2-\mu^2)\mathcal{H} - m^2R^2\mathcal{H} = 0$$

$$\langle\mathcal{O}(p')\mathcal{O}(p)\rangle \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4}(p^2-\mu^2)^{\Delta-2}$$

Quantum Critical Higgs Model

$$\mathcal{L} = -\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

$$-\frac{Y}{\Lambda_F^{\Delta-1}} \bar{\psi}_L \mathcal{H} \psi_R + h.c$$

$$\langle \mathcal{H} \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v^\Delta \end{array} \right)$$

QC Higgs Model

$$\mathcal{L} = -\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

$$-\frac{Y}{\Lambda_F^{\Delta-1}} \bar{\psi}_L \mathcal{H} \psi_R + h.c$$

$$G(p) = \frac{i\, Z_h}{p^2 - m_h^2} + i \int_{\mu^2}^{\infty} \frac{\rho_h(M^2) dM^2}{p^2 - M^2}$$

minimal parameterization requires
two mass scales: pole and cut threshold

QC Higgs Model

$$G(p) = \frac{i\,Z_h}{p^2 - m_h^2} + i\int_{\mu^2}^{\infty} \frac{\rho_h(M^2)dM^2}{p^2 - M^2}$$

$$\mathcal{H} \to \frac{1}{\sqrt{2-\Delta}}\mu^{\Delta-1}H$$

$$Z_h = \left(\frac{\mu^2}{\mu^2 - m_h^2}\right)^{1-\Delta} = 1 - (\Delta - 1)\frac{m_h^2}{\mu^2} + \mathcal{O}\left(\frac{m_h^4}{\mu^4}\right)$$

approach the SM in two limits: $\Delta \to 1$ or $\mu \to \infty$