20 Ways to Solve the Hierarchy Problem

Nathaniel Craig
UC Santa Barbara

Aspen 2017: From the LHC to Dark Matter and Beyond

20 Ways to Solve the Hierarchy Problem?

Nathaniel Craig
UC Santa Barbara

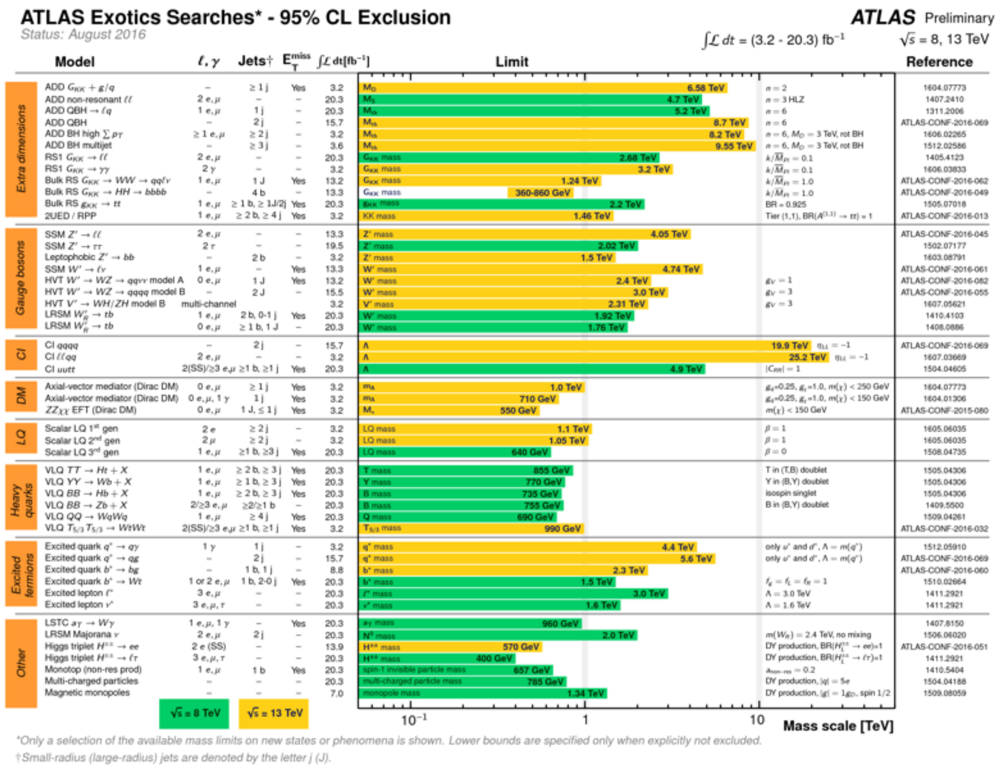
Aspen 2017: From the LHC to Dark Matter and Beyond

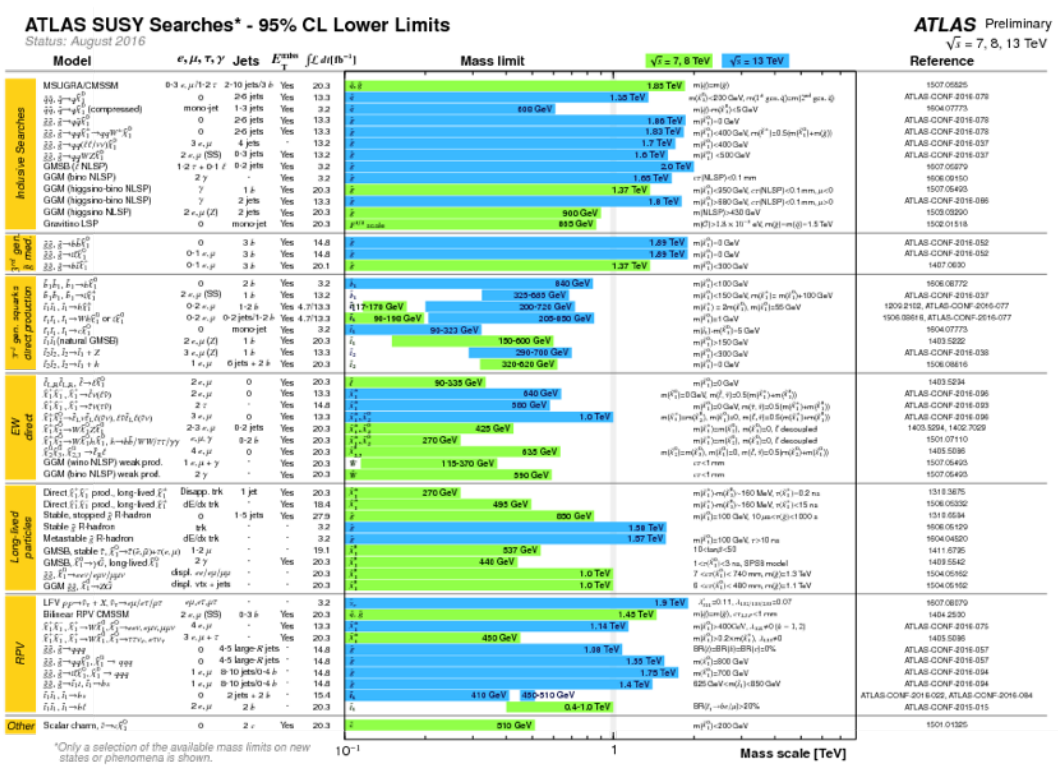
20 Increasingly Crazy Ideas About the Hierarchy Problem

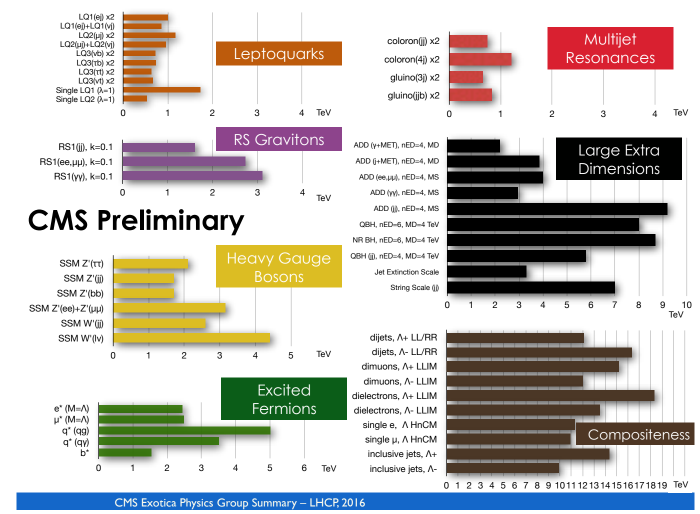
Nathaniel Craig
UC Santa Barbara

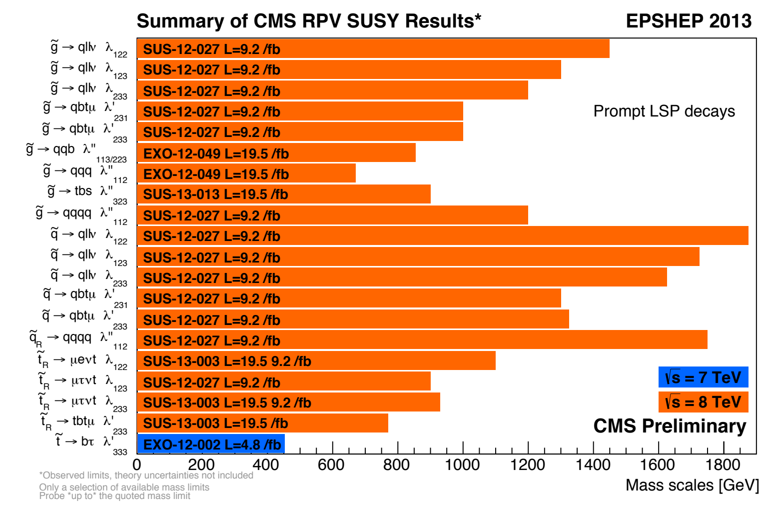
Aspen 2017: From the LHC to Dark Matter and Beyond

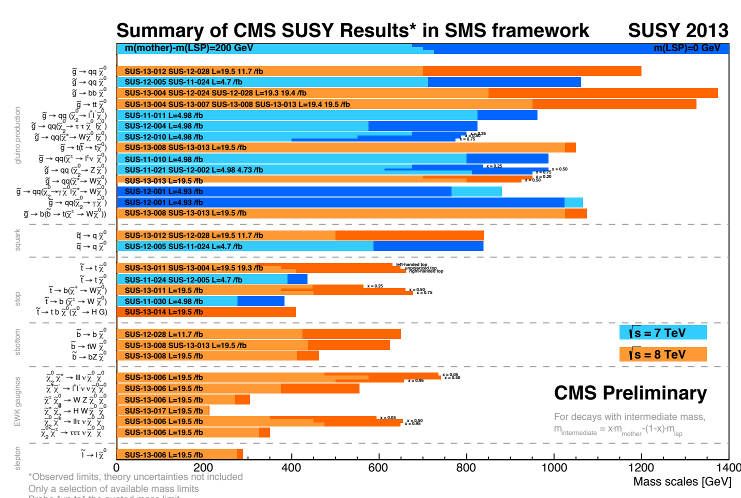
We (collectively) spend most of our time looking for solutions to the hierarchy problem

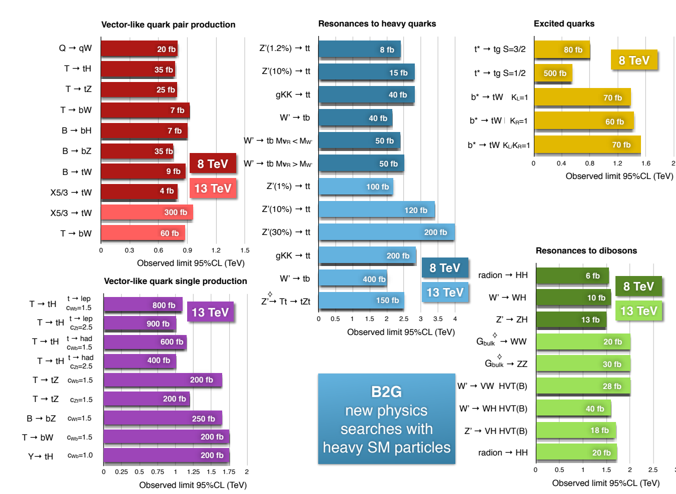










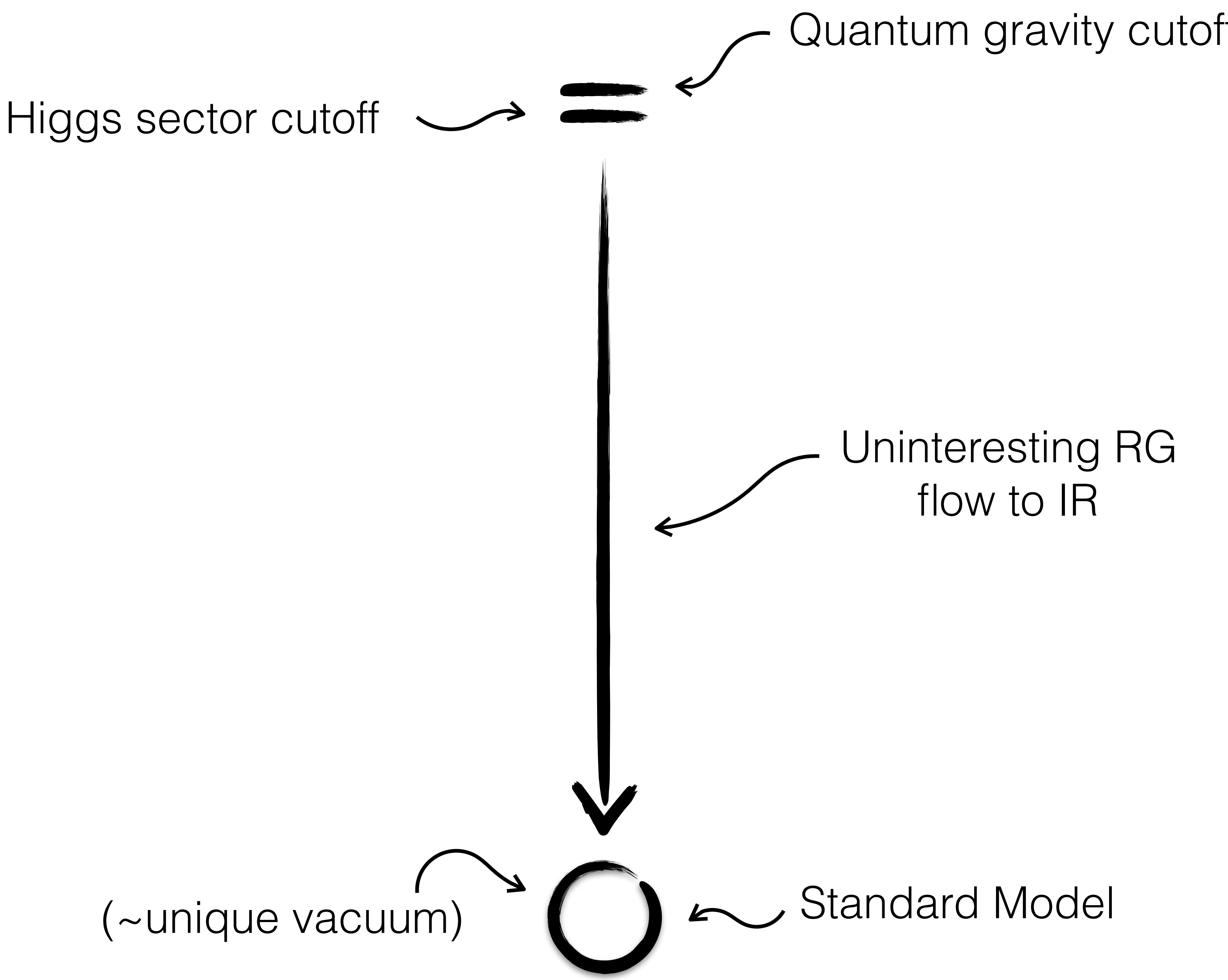


We have yet to find evidence for these solutions.

(not for lack of outstanding experimental effort)

Natural question: *have we exhausted the solutions?*

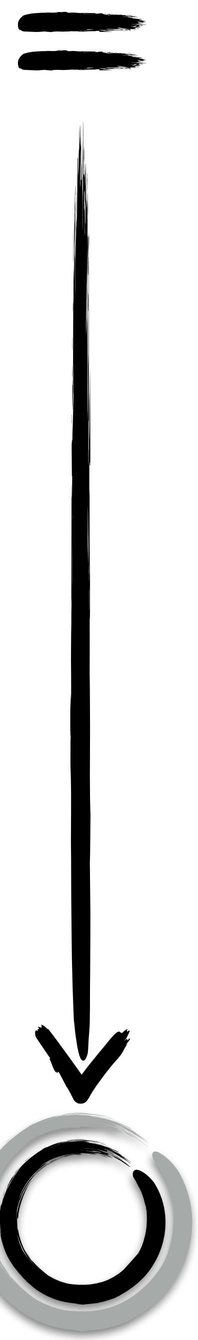
The Hierarchy Problem



m_H is not technically natural ⇒ hierarchy problem

Adding a symmetry

…and breaking it softly



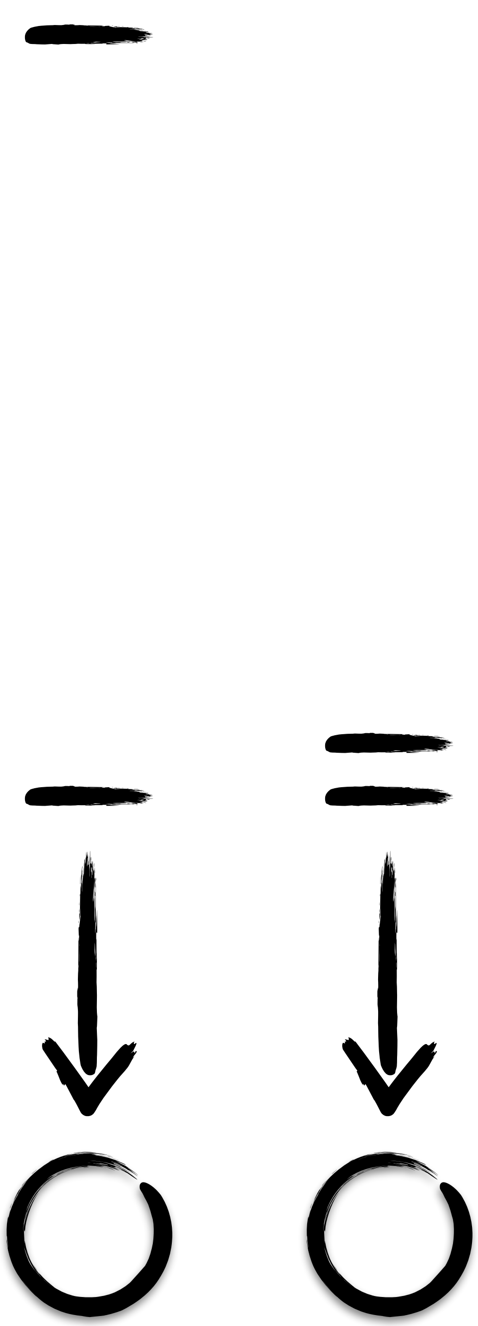
1. Supersymmetry
2. Global symmetry
3. Discrete subgroups thereof (“neutral naturalness”)

Experimental signals: partner particles

- The familiar host of prompt signals (with or without missing energy)
- Rich variety of displaced decays (RPV, fraternal twin higgs, folded SUSY, …)

Lowering the cutoff

…in diverse dimensions

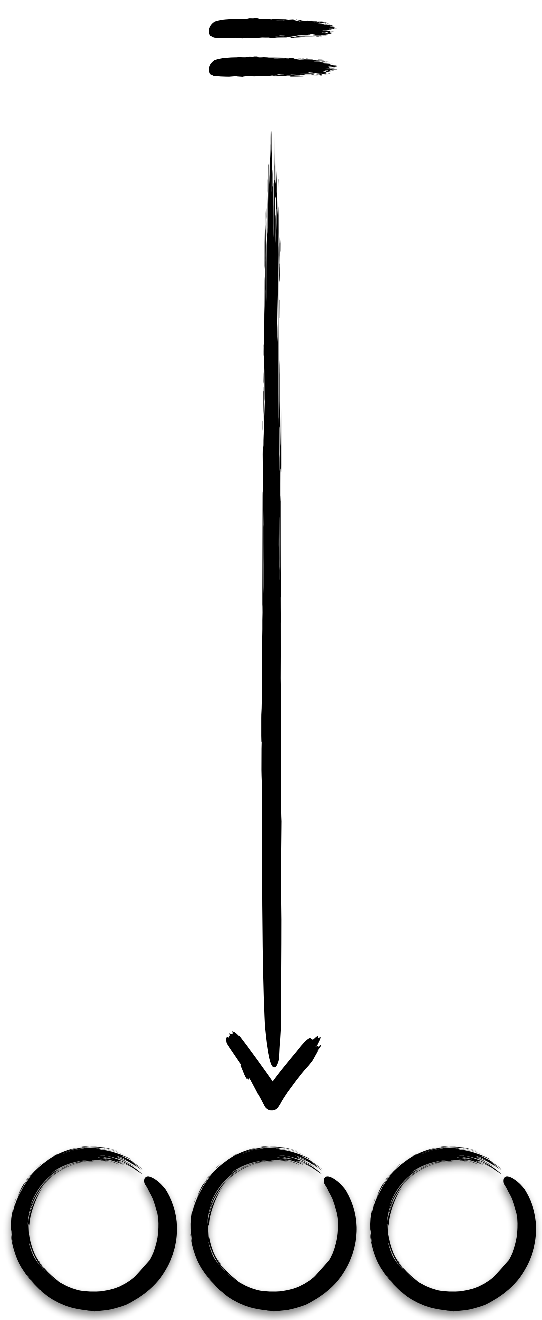


4. RS / Technicolor
5. LED / 10^{32} x SM
6. LST / Clockwork
7. Classicalization

Experimental signals: resonances

- Primary distinctions are in spacing & coupling of resonances
- Potential goldmine of unexplored signals for LST — e.g. perturbative string excitations

Selecting a vacuum



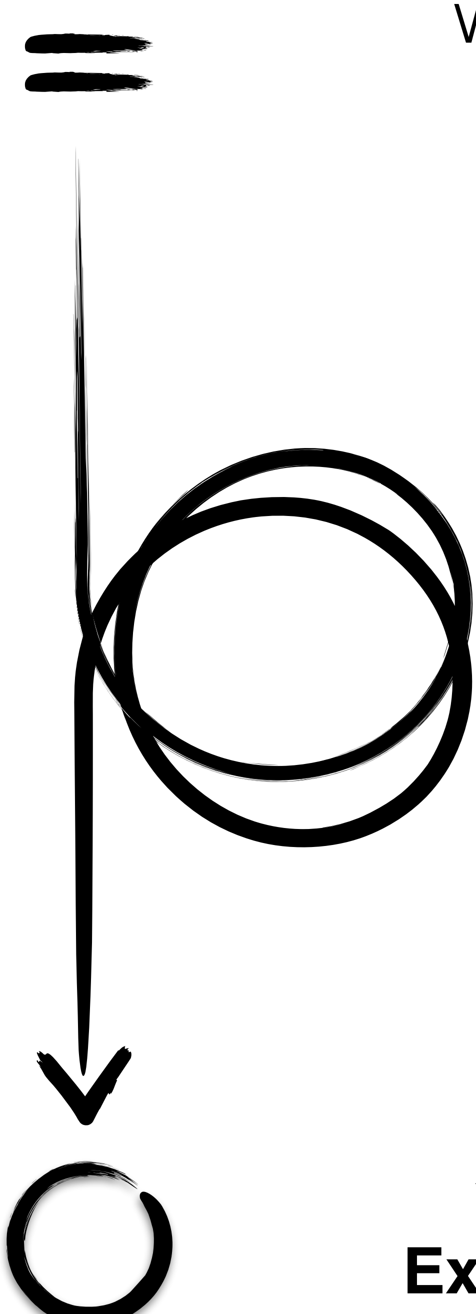
Vacuum is one of many; end up in observed vacuum through dynamical process or anthropic constraint.

8. Anthropics (pressure)
9. Relaxation (dynamics)
10. NNaturalness (reheating)

Experimental signals: Diverse, but typically

- Cosmology (Bubble collisions; axions; contributions to N_{eff} and $\sum m_\nu$)
- Exotic LHC signals (displaced decays, hidden sector confinement, …)

Complicating the flow



SM is reached from some intermediate fixed point
where, say, a generalized Veltman condition is satisfied

$$\delta m_H^2 = \sum_i c_i \frac{g_{i,\star}^2}{16\pi^2} \Lambda_i^2 = 0$$

This is the sense in which

11. Conformal symmetry

could address the hierarchy problem

Top-down: Embed SM in orbifold of N=4 SYM
[Frampton, Vafa '99; Csaki, Skiba, Terning '99]

Bottom-up: "Little conformal symmetry"
[Houtz, Colwell, Terning '16]

A challenge: how do fixed point couplings know about UV scale?

Experimental signals: Not fully understood, but expect new particles w/ SM quantum numbers around the TeV scale. Novelty is that their statistics, representations & couplings differ from more familiar solutions.

Exploding the cutoff ◆◆

Gravity doesn't provide a UV scale & the SM takes care of itself

12. Asymptotic fragility
[Dubovsky, Gorbenko, Mirbabayi '13]

13. Agravity [Salvio, Strumia '14]

Scale M_{Pl} not associated with relevant operator becoming strong, not "felt" by non-grav physics.

At low energies, looks like IR CFT perturbed by irrelevant operators.

At high energies there is no UV fixed point; cannot define local observables.

Example in 2d, no proposal for 4d.

Gravity has no intrinsic length scale and is "renormalizable"

$$S \sim \int d^4x \sqrt{g} \left(\frac{R^2}{f_1^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \dots \right)$$

(E-H term via vev of some field)

Can be re-written in terms of 2-deriv fields w/ ghosts. Like Lee-Wick (next slide) but **not obvious that ghosts are innocuous here**

Experimental signals: Details of gravity sector might be irrelevant. Crucially, must render SM couplings asymptotically free. Not a property of the SM itself, so entails low-scale unification (~10 TeV)

Not actually the SM

Maybe our IR theory is not actually the SM

Might help if: introduces states of non-positive norm

14. Lee-Wick (higher derivative scalar)

[Grinstein, O'Connell, Wise '06]

15. Non-compact gauge group?

[Please give me a hat tip if it works '17]

Higher-derivative theory,

$$\sim \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2M^2}(\partial^2 \phi)^2 + \ldots$$

improves UV convergence of diagrams

Can write in terms of a normal scalar plus a new field with wrong-sign quadratic action

$$-\frac{1}{2}\partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \frac{1}{2}M^2 \tilde{\phi}^2 + \ldots$$

Can be defined in a unitary, Lorentz-invariant manner with only microscopic acausality

Cosmology may be a bit wacky.

Usually restrict to compact simple subalgebras & U(1)'s to guarantee positive-norm states.

Then EWK group definitely SU(2)xU(1)

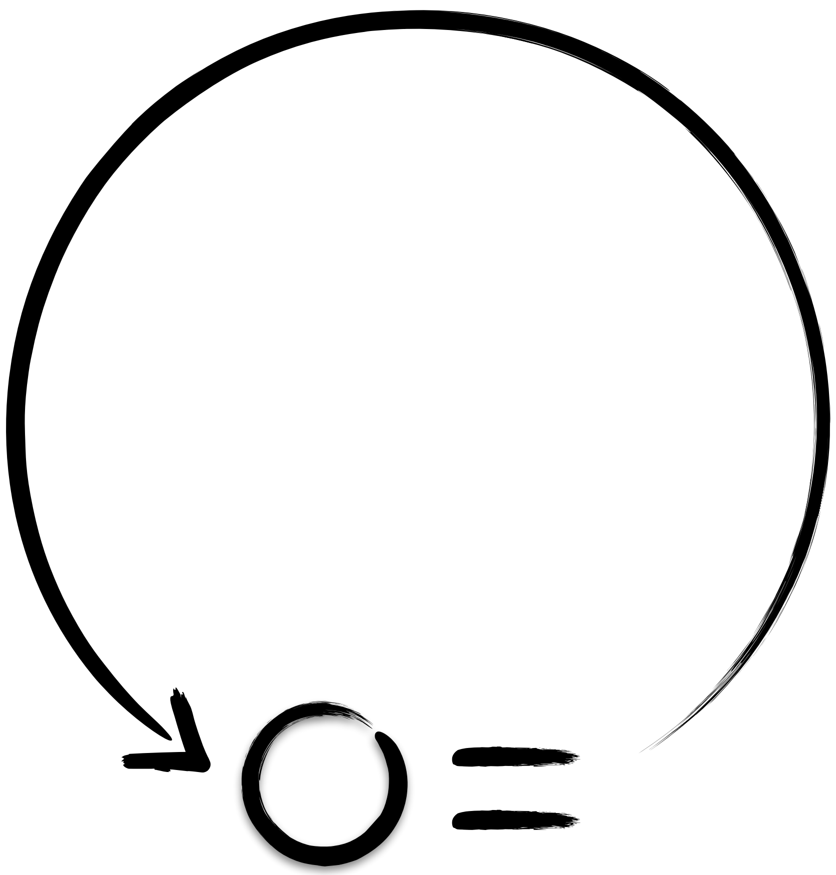
But [Tseytlin '95] a 4d gauge theory of a non-semisimple Lie algebra can be fully renormalized at 1 loop, finite S-matrix; negative-norm state factorizes

E.g. gauge theory based on E_2^C

$$[e_3, e_i] = \epsilon_{ij} e_j \qquad [e_i, e_j] = \epsilon_{ij} e_4$$
$$[e_4, e_i] = [e_4, e_3] = 0 \qquad i, j = 1, 2$$

Special limit of SU(2) x [U(1) ghost factor]

Connecting UV & IR



Essential feature of the hierarchy problem is that the UV doesn't know about the IR… unless it does?

Two "theories" exhibiting UV/IR mixing:
Quantum gravity & non-commutative field theory

QG (cartoon version): probe spacetime with sufficiently energetic particles, make a black hole. More energetic particles → bigger black hole.

NCQFT (cartoon version): non-commutativity of the form $[x^\mu, x^\nu] = i\Theta^{\mu\nu}$, qualitatively a space-space uncertainty principle.

Two ways to put this to work for hierarchy problem:

16. Weak gravity conjecture

17. Non-commutative SM

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

A UV/IR "Solution": Weak gravity

Weak gravity conjecture: an abelian gauge theory must contain a state of charge q and mass m satisfying

$$q > \frac{m}{M_{Pl}}$$

Justification: consider BH of charge Q, mass M decaying to this particle

particles produced = Q/q

Conservation of energy: $mQ/q < M$

Then BH satisfies $Z = Q\, M_{Pl}/M < z = q\, M_{Pl}/m$

Extremal BH (Z=1) stable unless there exists a state with z > 1

⇒ $q > m/M_{Pl}$ to avoid BH remnants, in conflict w/ holography

Connection to the weak scale [Cheung, Remmen '14]

Charge SM fermions under weakly gauged (unbroken) $U(1)_{B-L}$ (bounds currently $q \lesssim 10^{-24}$). Cancel anomalies with RHN ν_R

Neutrino mass is $y_\nu H \bar{L} \nu_R \to m_\nu \sim y_\nu v$ so $m_\nu \sim 0.1$ eV, $q \gtrsim 10^{-29}$

For fixed yukawa, if v were any larger, WGC would be violated
Physics in the UV needs to know about the IR scale v.

A UV/IR "Solution": NCQFT

Extensive literature starting with [Minwalla, Seiberg, Van Raamsdonk '99]

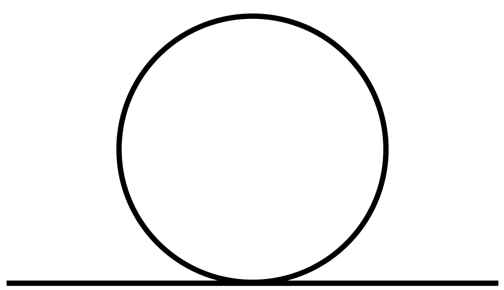
Noncommutativity manifested by star product

$$(\phi_1 \star \phi_2)(0) = e^{i\Theta^{\mu\nu}\partial^y_\mu \partial^z_\nu} \phi_1(y)\phi_2(z)\Big|_{y=z=0}$$

Consider just ϕ^4 in d=4:

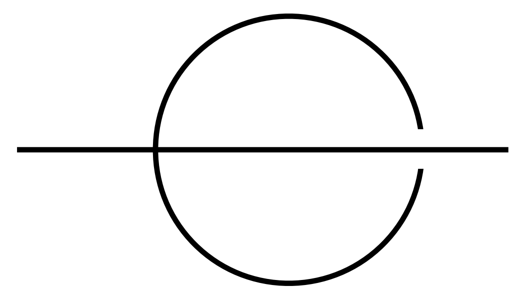
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} g^2 \phi \star \phi \star \phi \star \phi$$

Now there are "planar" and "non-planar" diagrams.
E.g. at one loop



$$\sim \int \frac{d^4 k}{k^2}$$

UV divergent as usual



$$\sim \int \frac{d^4 k}{k^2} e^{ip\Theta k} \sim \frac{1}{\Theta^2 p^2}$$

IR divergence!

Can define a suitable noncommutative SM [Calmet et al '01]

Strongly constrained by Lorentz violation

Far from an actual proposal to solve the hierarchy problem, but…

?????????????

Things I can't (yet) cleanly compartmentalize

18. Tune the CC to set the weak scale
    [Arvanitaki, Dimopoulos, Gorbenko, Huang, Van Tilburg '16]

19. Massless moduli from explicitly broken SUSY
    [Dong, Freedman, Zhao '14, '15]

20. Self-organized criticality

Example: explicit marginal SUSY breaking involving $U(1)_R$ gauge fields on bdy of AdS_3

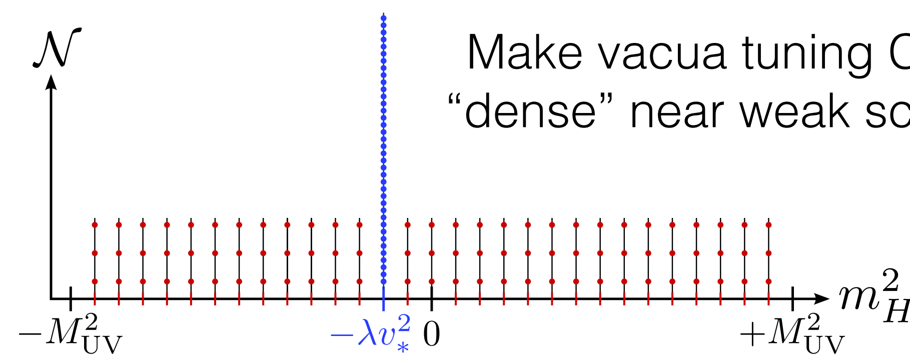
$$\delta S \sim \int_{bdy} A \wedge \tilde{A} \sim \int d^2 z J(z) \tilde{J}(\bar{z})$$

Induces splitting in R-charged multiplets. Feed to R-neutral multiplets w/ yukawa

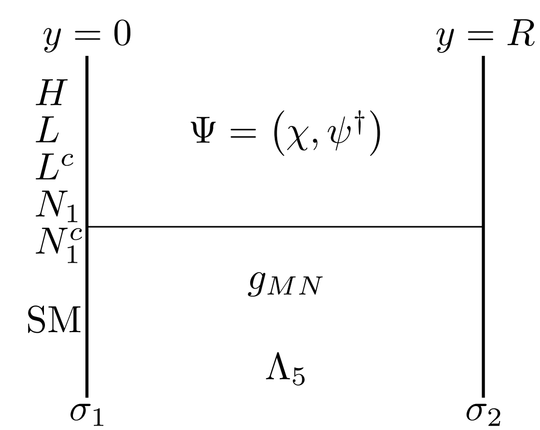
$$\lambda \phi_N \phi_R^\dagger \phi_R$$

R-neutral scalars are massless to all orders

Analogous to $y_t^2 m_t^2 - y_{\tilde{t}}^2 m_{\tilde{t}}^2 = 0$

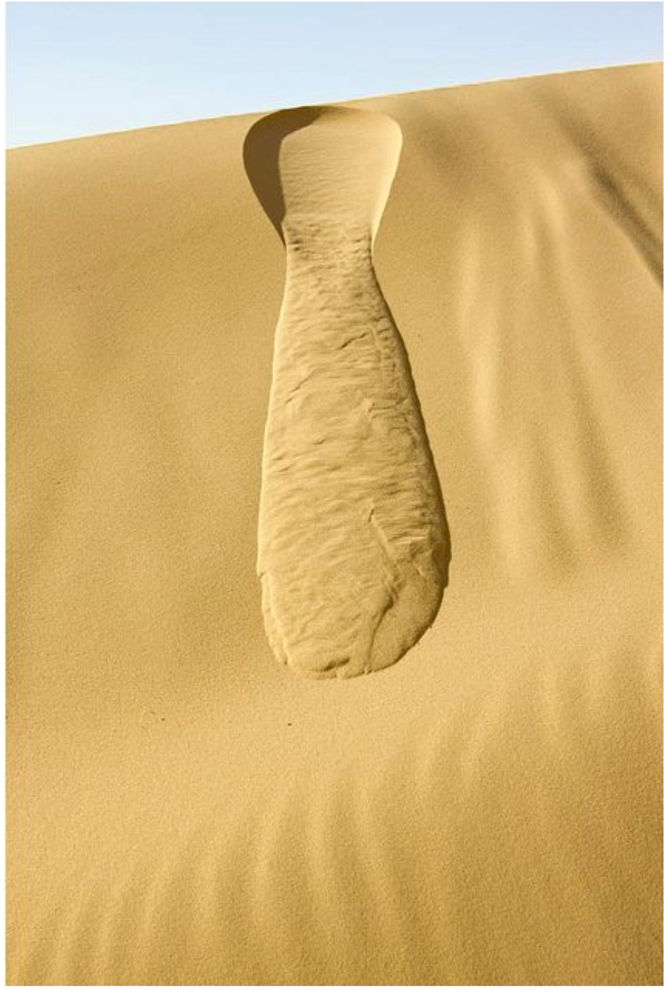


Make vacua tuning CC "dense" near weak scale



Signals

- Vector-like leptons (direct search, Higgs invisible width, precision electroweak)
- Super-light radion ($O(10^{-10}$ eV$)$)



A ??? Solution: Self-Organized Criticality

Some systems evolve into critical states on their own. Wouldn't that be nice?

Canonical example: Sandpile. Initially dynamics of individual grains. Critical slope → one grain causes avalanche; correlations far larger than individual grains.

The QFT analog of SOC has been called:

- A free scalar field
- The (2,0) theory in 6d
- A classical FT w/ dissipation
- Soft gluons
- The relaxion
- "A terrible idea"

All of these in some sense true, but it's time to figure out which senses give novel, functional solutions to the hierarchy problem

Conclusions

- Electroweak hierarchy problem remains one of the biggest motivations for physics beyond the SM.
- Close to comprehensively understanding what conventional solutions look like & searching accordingly. Should obviously keep searching for these as hard as possible, but…
- …at some point **data** tells us that we should look more closely at truly unconventional solutions. Most of these are a way of making sense of the failure of Wilsonian EFT.
- Promising places to look: conformal symmetry; naive IR pathologies; UV/IR mixing. But who am I to say? Lots to explore.
- Experimental possibilities potentially vast.

Thank you!