Transplanckian Cosmic Strings

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Work with M. Dolan, J. Kozaczuk, & H. Patel

$$\epsilon, \eta \ll 1$$

The flatness of the inflaton potential is another hierarchy problem. It has been difficult to solve.

1990: Natural Inflation (Freese, Frieman, & Olinto). Inflaton = axion.

NI and its descendants are "large field" models of inflation.

Transplanckian excursions of the inflaton in time

Transplanckian flat directions in field space

The study of models with long flat directions has provided a useful probe of effective field theories coupled to gravity.

- Constrained by consistency criteria based on the WGC
- Associated with "censorship" properties in some cases

Arkani-Hamed et al 2007
Nicolis 2008

A censorship example

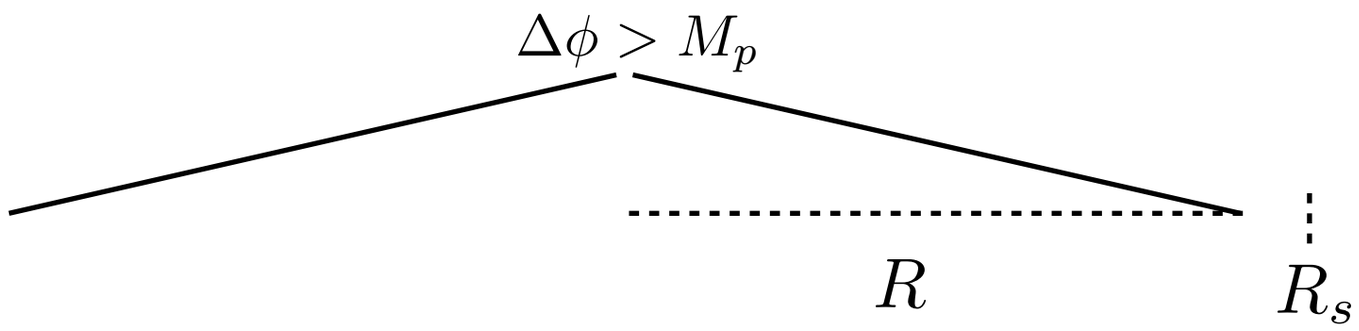
Nicolis 2008

Take a static, spherically symmetric source for a scalar field with a superplanckian field range and zero potential.

Couple to gravity in asymptotically flat space.

Can an observer set up an experiment to probe $\Delta\phi > M_p$?

Tension between setting up large excursions and preventing collapse into a black hole.



A censorship example

Nicolis 2008

Newtonian analysis:

$$G \cdot (\Delta\phi)^2_{out} \leq \frac{\Phi_{max}}{\pi} \ll 1 \qquad \Phi_{max} \equiv 2\pi G \int_0^{r_*} dr r(\phi'(r))^2$$

$$G \cdot (\Delta\phi)^2_{in} = \frac{\Phi_{max}}{2\pi} \log(r_*/\ell_{UV})$$

Controlling backreaction and sourcing a large excursion requires an exponentially large source.

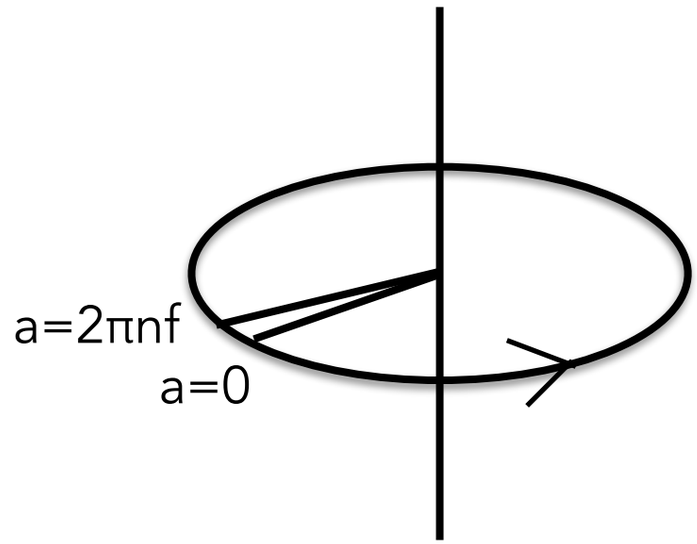
In GR, again a bound on $\Delta\phi_{out}$

Detailed description of the source required to study $\Delta\phi_{in}$

Might it eliminate the log(r)?

In a similar vein, axion models suggest another natural context in which to study large field excursions.

Axions $\Longleftrightarrow$ discrete gauge symmetries $a \to a+2\pi f$ $\Longleftrightarrow$ global cosmic strings:



The axion may trace out a large excursion around the string.
"supercritical" or "supermassive"

Complementary to the previous example, this realization of transplanckian fields is cylindrically symmetric, has a specified source, and will turn out to be non-static.

Cosmic strings realize in **space** the trajectory taken by the axion-inflaton in **time**.

Heuristically, different string types reflect different classes of models:

$$\Delta a \sim f > M_p, \;\; f = \langle \phi \rangle \qquad \Longleftrightarrow \text{natural inflation}$$

$$\Delta a \sim f > M_p, \;\; f = (gR)^{-1} \qquad \Longleftrightarrow \text{extranatural inflation}$$

$$\Delta a \sim nf > M_p > f, \qquad \Longleftrightarrow \text{monodromy inflation}$$
$$f = \langle \phi \rangle \text{ or } f = (gR)^{-1}$$

To get at the question of censorship, we need the geometry.

Complications:

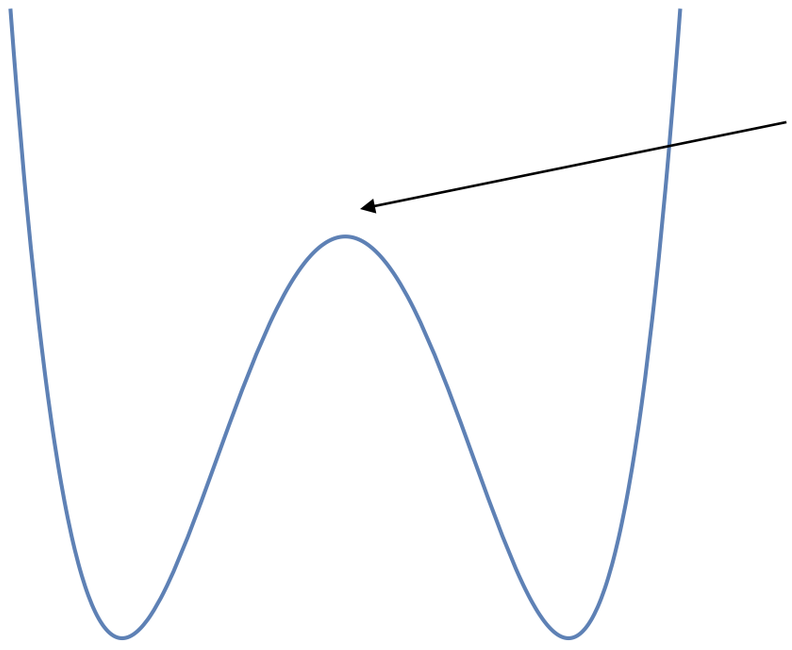
*Axion potentials induce domain walls around the string. Will set V(a)→0.

*To cut off the winding singularity, string cores sample the UV completion of the axion. Model dependent. Will assume either dilatonic or ϕ^4

Cohen & Kaplan: first complete solution to Einstein+global cosmic string. Metric is **static and singular** a certain distance from the core. Singularity moves inside the core for $f>M_p$.

Gregory: **add t-dependence** in g_{zz}, remove singularity. String retains fixed proper radius, but inflates longitudinally. However, still requires $f<M_p$.

Vilenkin; Linde: Topological defects with large symmetry breaking scales may inflate in their cores: "**Topological inflation**"



inflation:

$$r_{core} > H^{-1}$$

Cho: Numerical study of ϕ^4 string exhibits topological inflation for $f>M_p$, nonsingular exterior.

What if $f<M_p<n*f$?

What is the analytic form of the exterior metric?

What happens for dilatonic strings?

Are large axion excursions around a string censored in some way?

Progress on these questions, but not complete resolution.

$$V(\phi) = \frac{\lambda}{4}(|\Phi|^2 - f^2)^2, \quad \Phi = \phi e^{ia/f}, \quad a = n\theta f, \quad nf \gtrsim M_p > f$$

There is a physical argument that the cores of these strings can also topologically inflate.

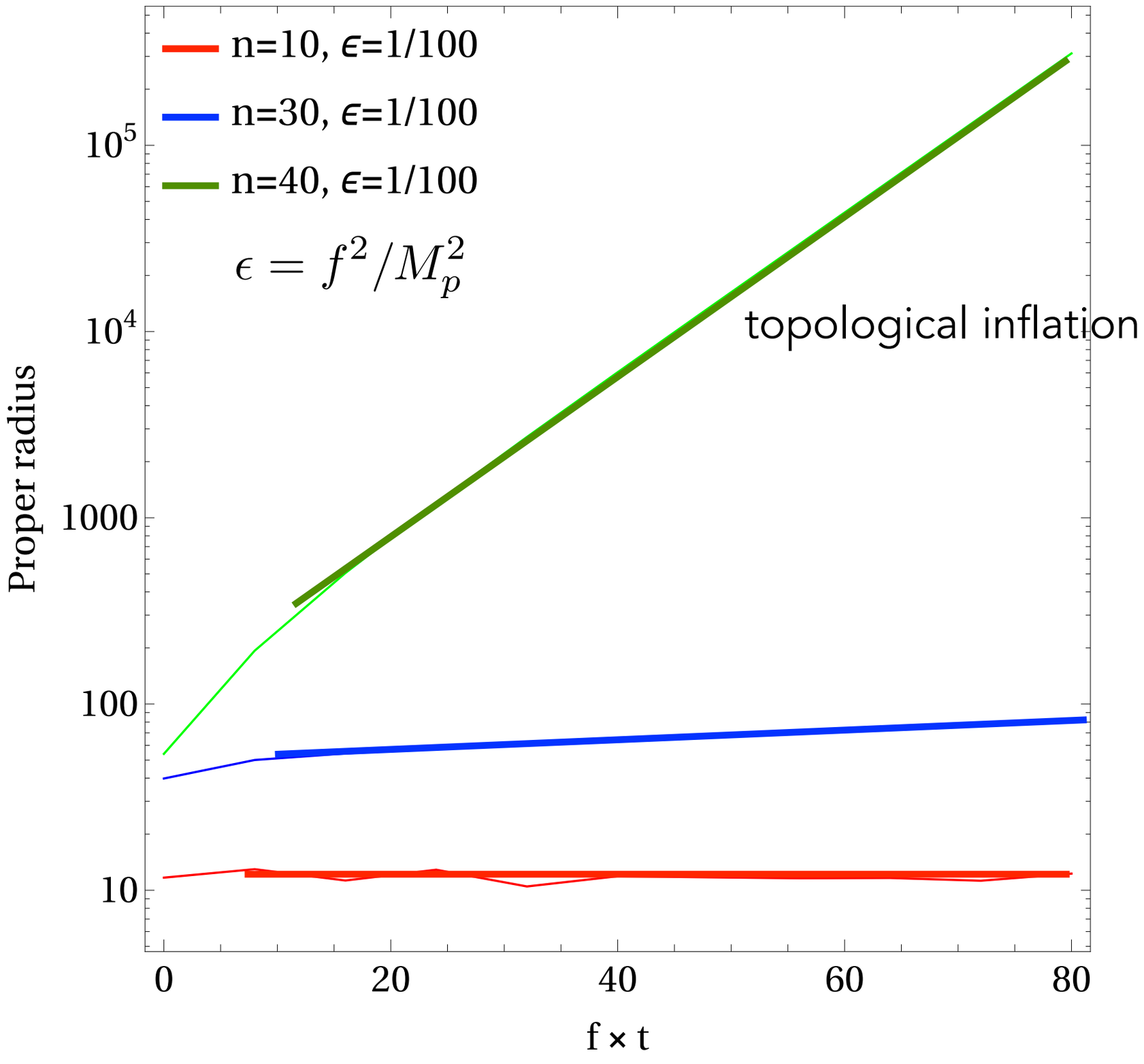
Define the boundary of the core by $\phi(t, r(t)) = kf$ for some small const k. If the metric in the core grows, the boundary will no longer be supported by gradient energy, and will collapse.

The proper distance to r(t) evolves as $d(r) \simeq e^{\left(H - \frac{\sqrt{V''(f)}}{n}\right)t}$.
Incorporates expansion metric evolution in the core and collapse of the core.

A positive exponent for d(r) signals inflation, and corresponds to **$n*f>M_p$.**

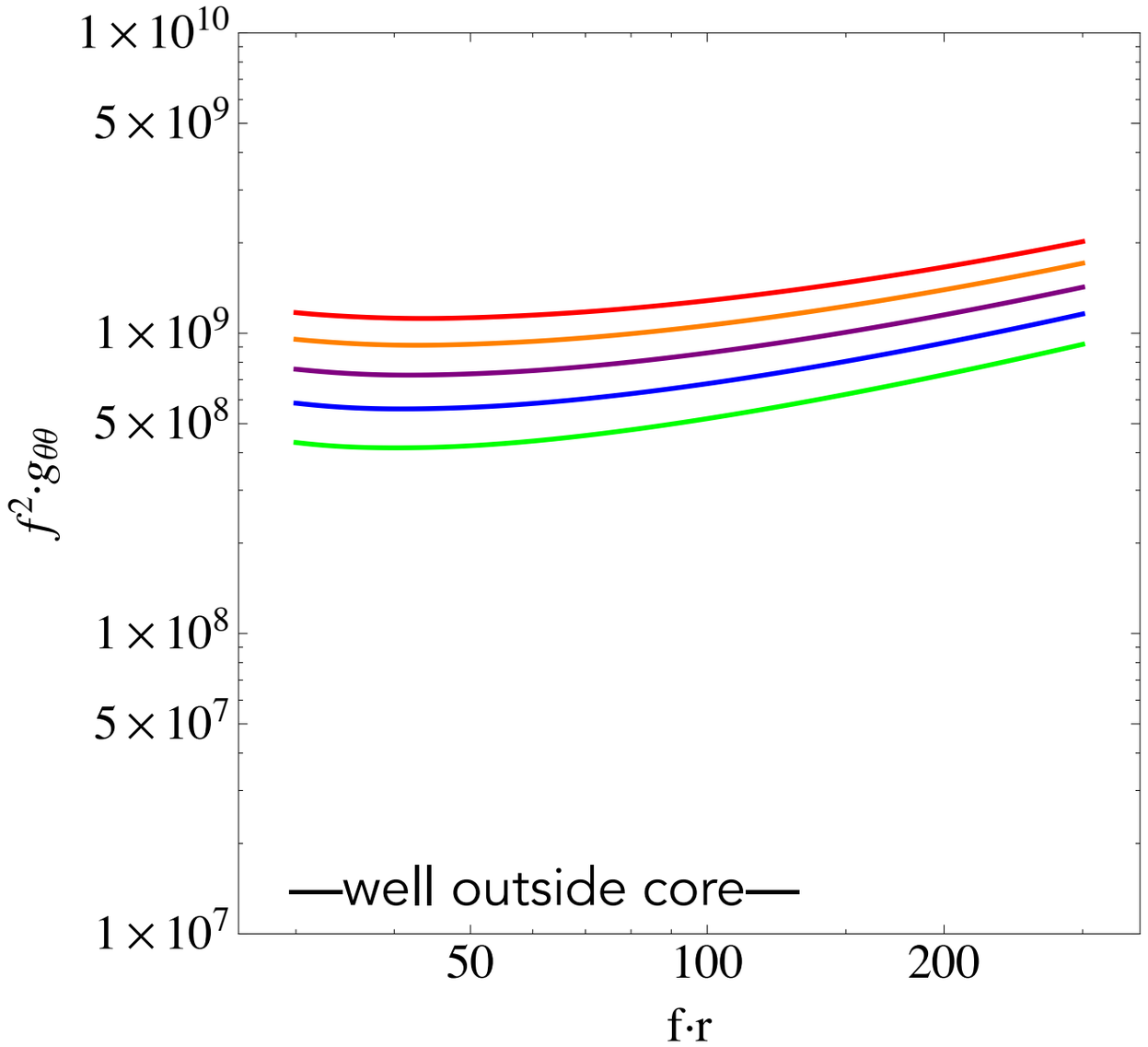
(negative exponents correspond to incorrect assumptions leading to this form of d(r))

Numerical analysis of Einstein+cosmic string:



We observe exponential growth of the core for n*f sufficiently larger than M_p.

What happens in the exterior? The $\theta\theta$ component of the metric in the numerical analysis offers a clue.



flat: r^2

Instead, only weak r-dependence.

Numerics suggest a cylindrical geometry exterior to the core.

With this ansatz, the string exterior may be reinterpreted as the geometry of a radion rolling in a potential generated by axion flux.

$$ds^2_{(4)} = \frac{1}{R^2(t)} ds^2_{(3)} - R^2(t) d\theta^2$$

$$S_{(3)} = \int d^3x \sqrt{\det g_{(3)}} \left(\frac{f^2}{2\epsilon} \mathcal{R}_{(3)} + \frac{f^2}{\epsilon R(t)^2} g^{ab}_{(3)} \nabla_a R \nabla_b R - \frac{n^2 f^2}{R(t)^4} \right)$$

With the ansatz $ds^2_{(3)} = dt^2 - a(t)^2 \left(dr^2 + g(r)^2 dz^2 \right)$

the equations of motion become

$$R\dot{R} + R^3 \frac{\ddot{a}}{a} - \frac{n^2 \epsilon}{R} = 0$$

$$-2\frac{\dot{a}}{a}\dot{R} + 2\frac{n^2\epsilon}{R^3} + \frac{\dot{R}^2}{R} - \ddot{R} = 0$$

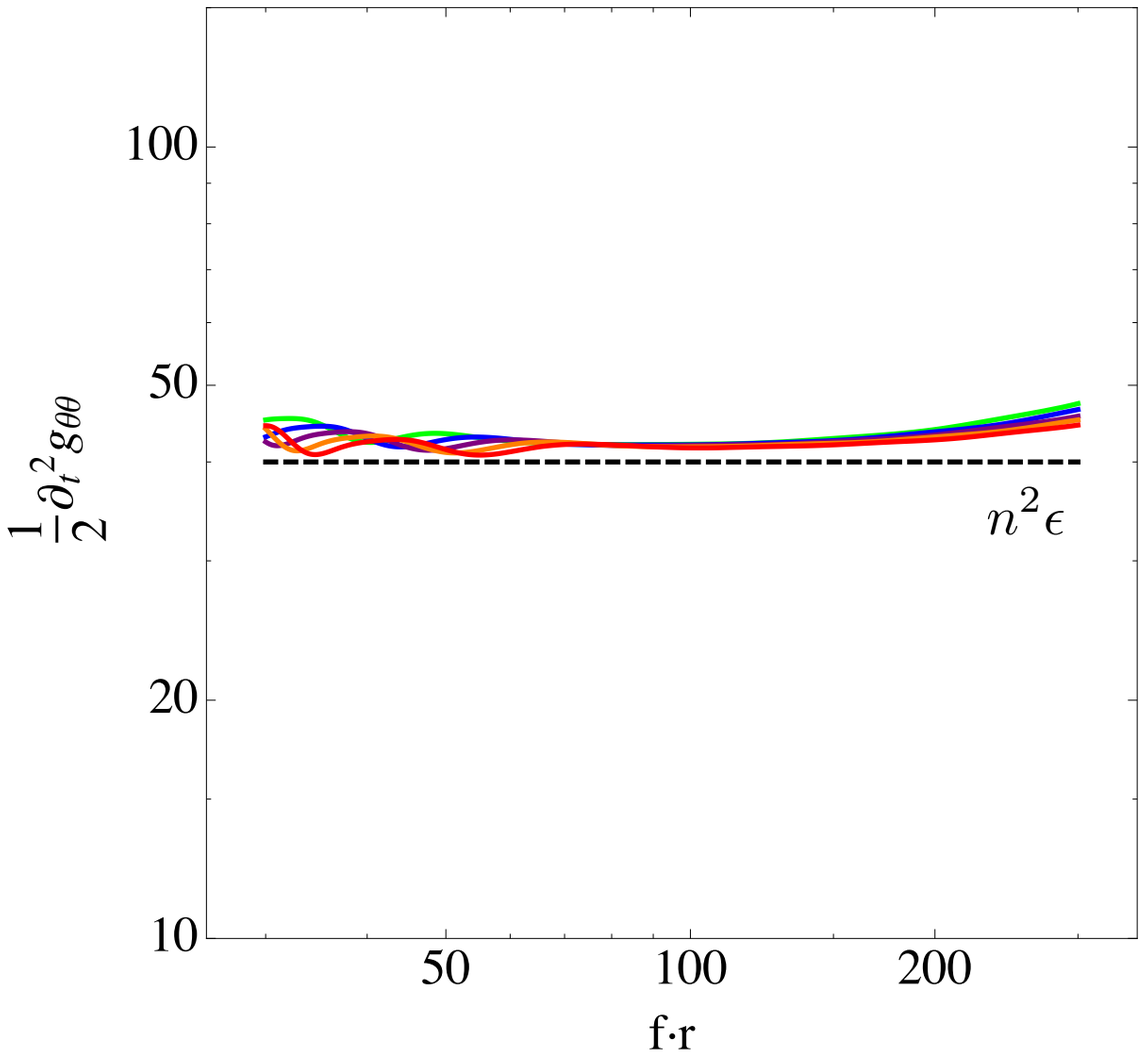
$$-2\frac{n^2 \epsilon a^2}{R^4} + \dot{a}^2 + a\ddot{a} - \frac{g''}{g} = 0,$$

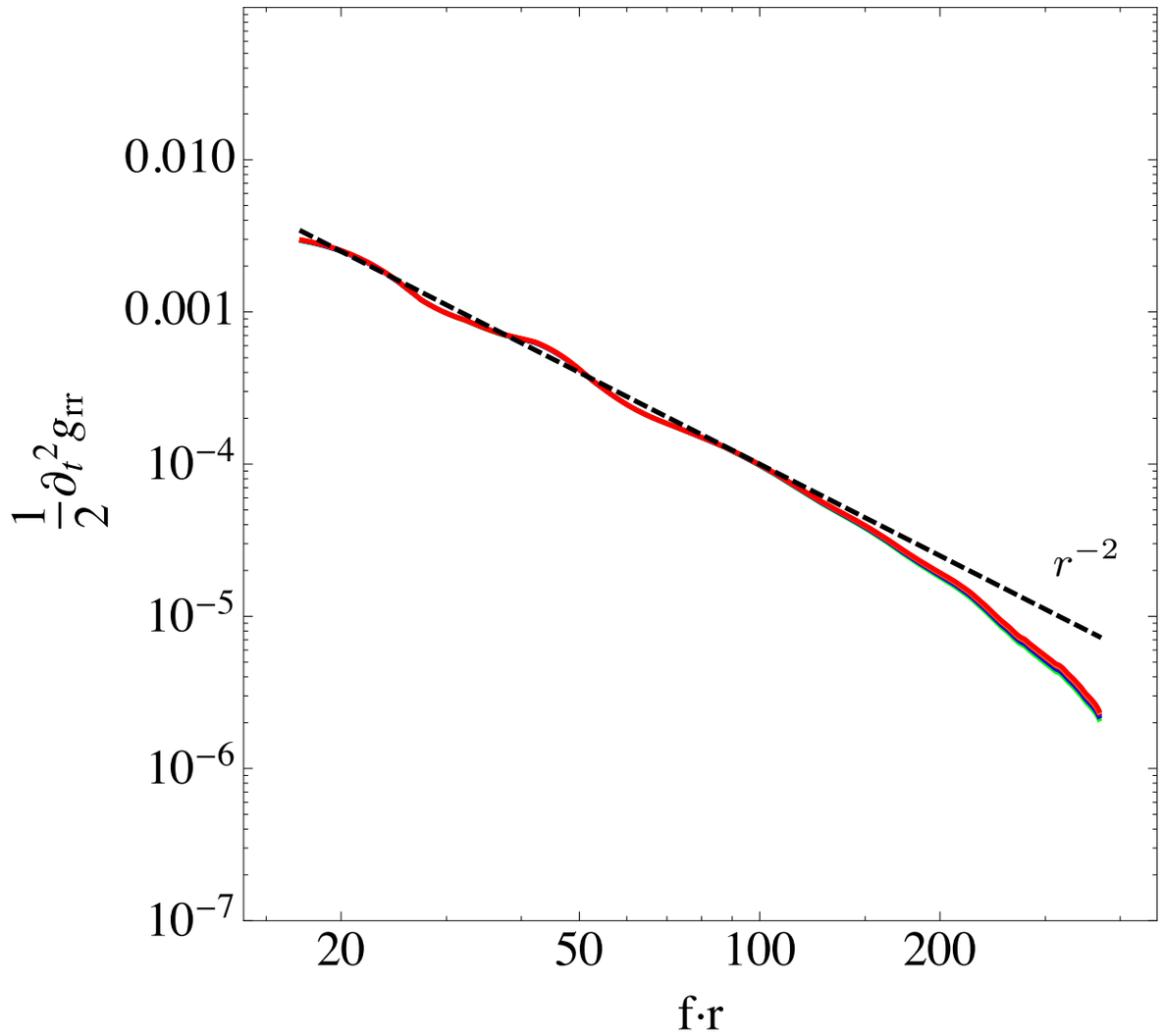
A solution is:

$$ds^2 = dt^2 - t^2\left(r^{-2}dr^2 + n^2\epsilon d\theta^2 + (fr)^{-2\sqrt{2}}f^2dz^2\right)$$

corresponding to a cylinder decompactifying as R ~ t. $\epsilon = f^2/M_p^2$

It provides a good fit to the numerical solution at late times:

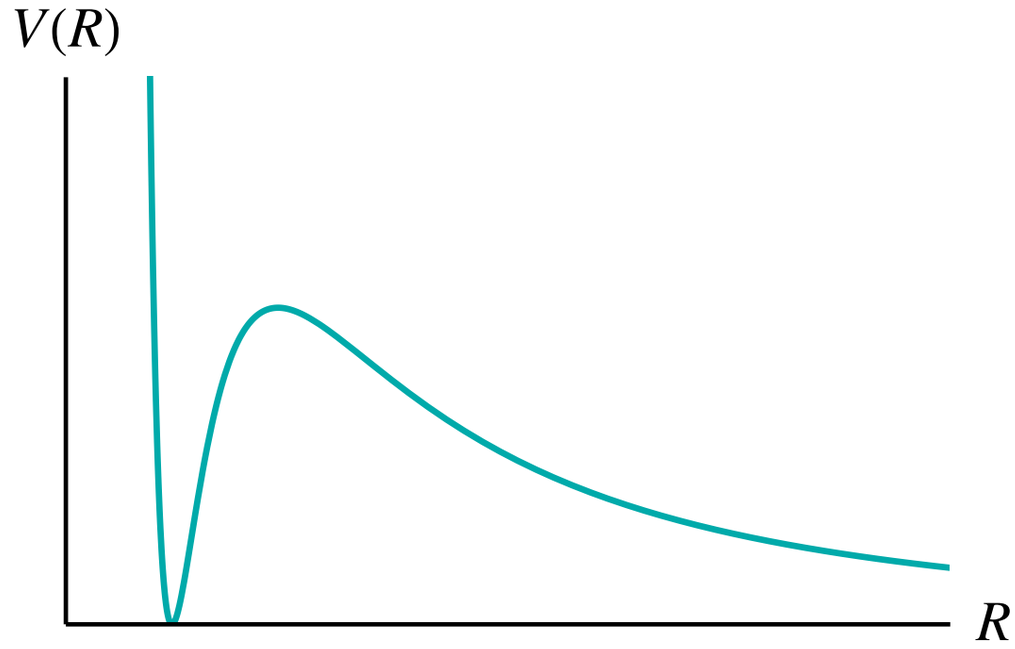




What about dilatonic axions? (Axions whose noncompact modulus partner is a dilaton/radion). For example, as in extranatural inflation.

The string is defined by $A^5 = n\theta/g\langle R\rangle$. It is supercritical for $n/g\langle R\rangle \gtrsim M_p$

A typical radion potential has the form:



In the string exterior, the radion is frozen in its vacuum at finite R.

The string core samples large R, and is separated from the exterior by a domain wall

Sitting in the degenerate vacuum at R=∞, the core geometry does **not** undergo exponential inflation.

However, the core may still inflate. If some expansion occurs, then in the core, the radion begins to roll in its asymptotic potential.

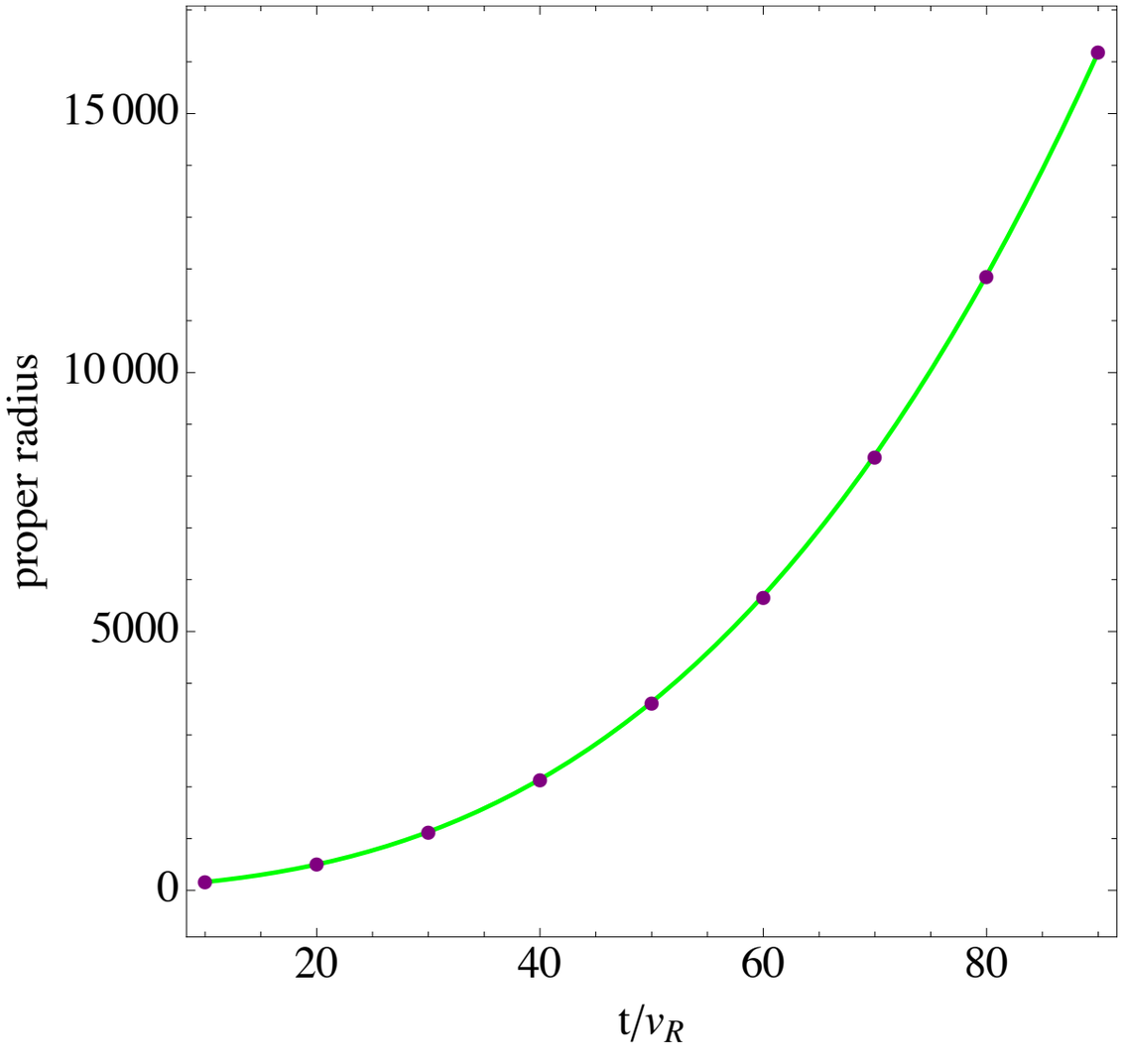
The dominant term at large R is the bulk cc, V(R)~1/R

There is a cosmological solution with scale factor a(t) ~ t^3 (w=-7/9)
Accelerated expansion.

Proper radius of the core:

numerical results vs t^3

$$n/g\langle R\rangle \sim 30 M_p$$



The stress tensor outside the core of the wilson loop axion string is the same as in the exterior of the ϕ^4 string — it *may* also take on cylindrical geometry.

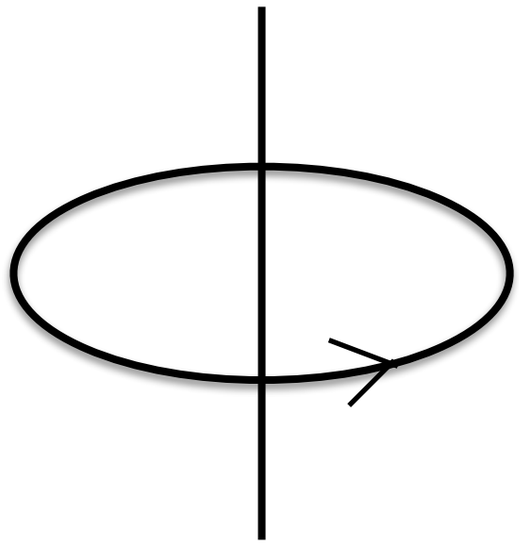
The exterior of this string is more difficult to simulate. It is nonsingular, but we do not yet have sufficiently late-time numerical solutions to test the cylindrical hypothesis — work in progress.

We have found that transplanckian excursions may be obtained around global cosmic strings, without introducing singularities in the geometry.

Is there a notion of censorship?

For a simple example, we can ask whether causal trajectories can circumnavigate the cylinder geometry and observe the excursion.

$$ds^2 = dt^2 - t^2 \left(r^{-2} dr^2 + n^2 \epsilon d\theta^2 + (fr)^{-2\sqrt{2}} f^2 dz^2 \right)$$



In fact, they can.

Null geodesics wrap the cylinder. An observer who emits a beam at t_0 receives it again at

$$t = t_0 e^{\frac{2\pi n f}{M_p}}$$

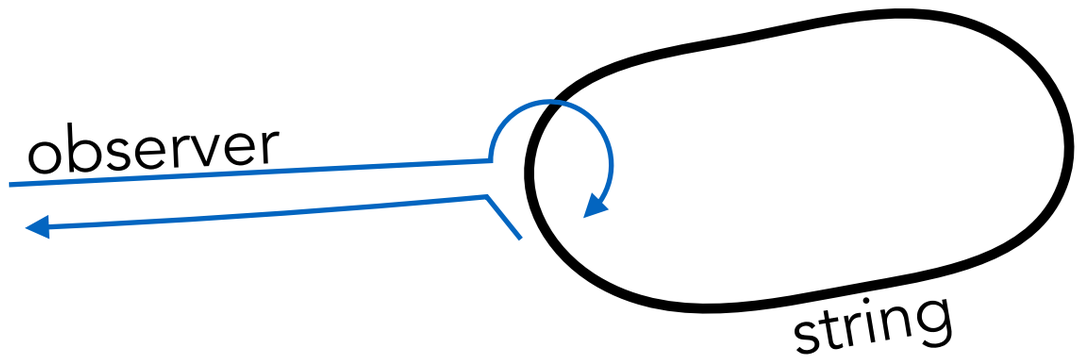
a **finite but exponentially long time** in the field excursion.

Reminiscent of, but not obviously related to, the exponentially large experiments found by Nicolis. (but not in asymptotically flat space)

Any instability of the string could compete with the exponential.

Obvious question:
What about large loops of supercritical string in asymptotically flat space?

An observer following a trajectory that threads the loop could access most of the axion excursion.



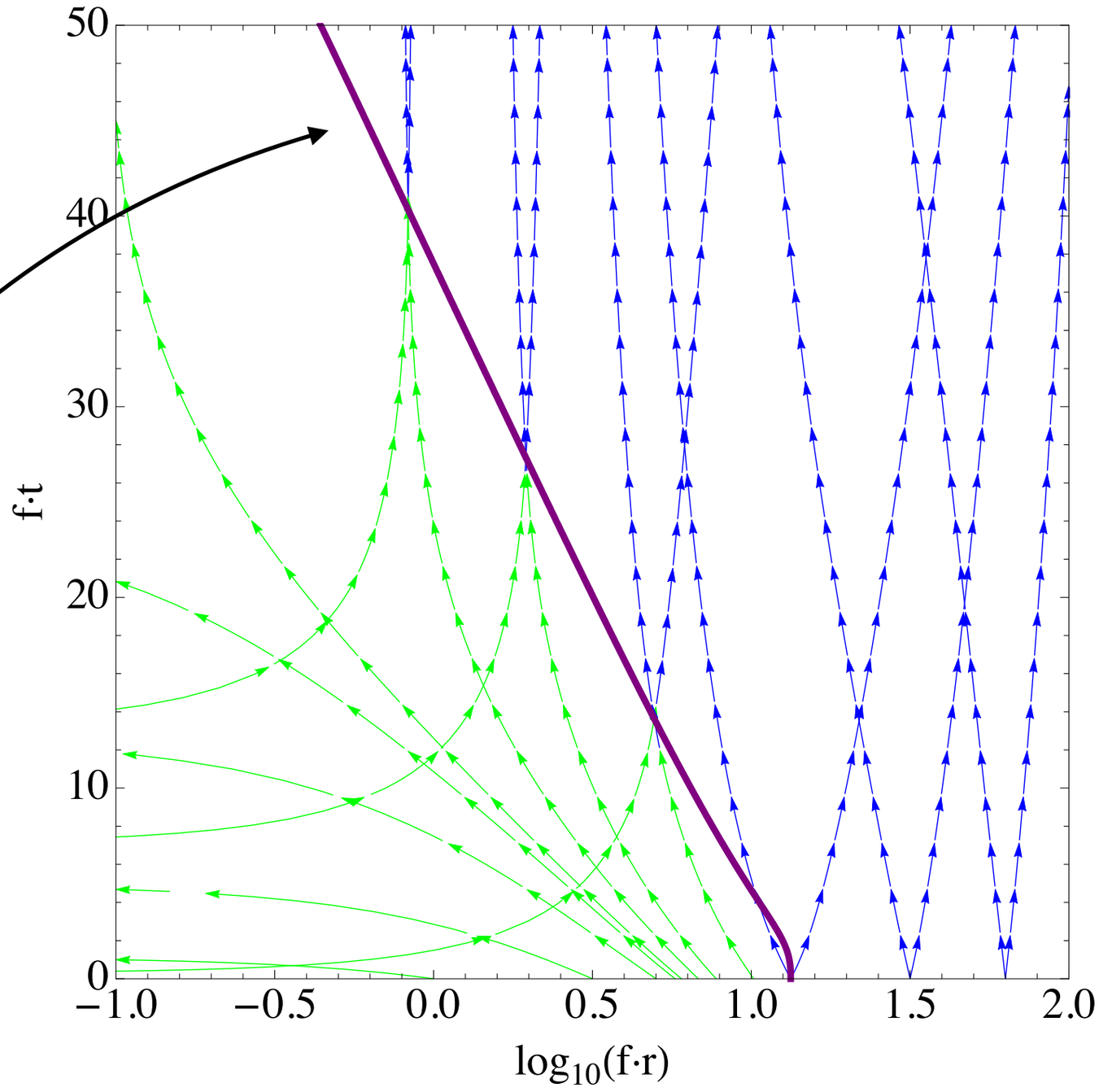
The dynamical evolution of the closed loop is much more complicated than the infinite strings we have studied.

Without dedicated numerical simulation, it is not clear what happens. Maybe strings inflate. Maybe n>1 strings fragment. Maybe it all collapses to a BH.

However, I want to end with one curious observation about this system.

Accelerated expansion in the core ⇒ surfaces of constant phi are spacelike hypersurfaces in the causal past of exterior observers.

core “boundary”



Imagine that the core of the large closed loop of string topologically inflates.

If there is a causal trajectory T from I^- to I^+ that threads the loop, it **cannot be smoothly deformed** into another causal trajectory that does not thread the loop.

Any family of curves realizing such a deformation would have to cross the de Sitter horizon of the inflating region.

But in GR, all causal trajectories must be smoothly deformable into topologically trivial trajectories. (Topological censorship theorem; Friedman, Schleich, & Witt). So **T cannot exist.**

However, since we don't know the dynamical evolution of the closed loop, this is not yet a sharp statement of censorship.

Summary

Inflation models based on shift symmetries require large field excursions and are rather difficult to construct.

Supercritical cosmic strings are a natural probe of transplanckian excursions in axion models.

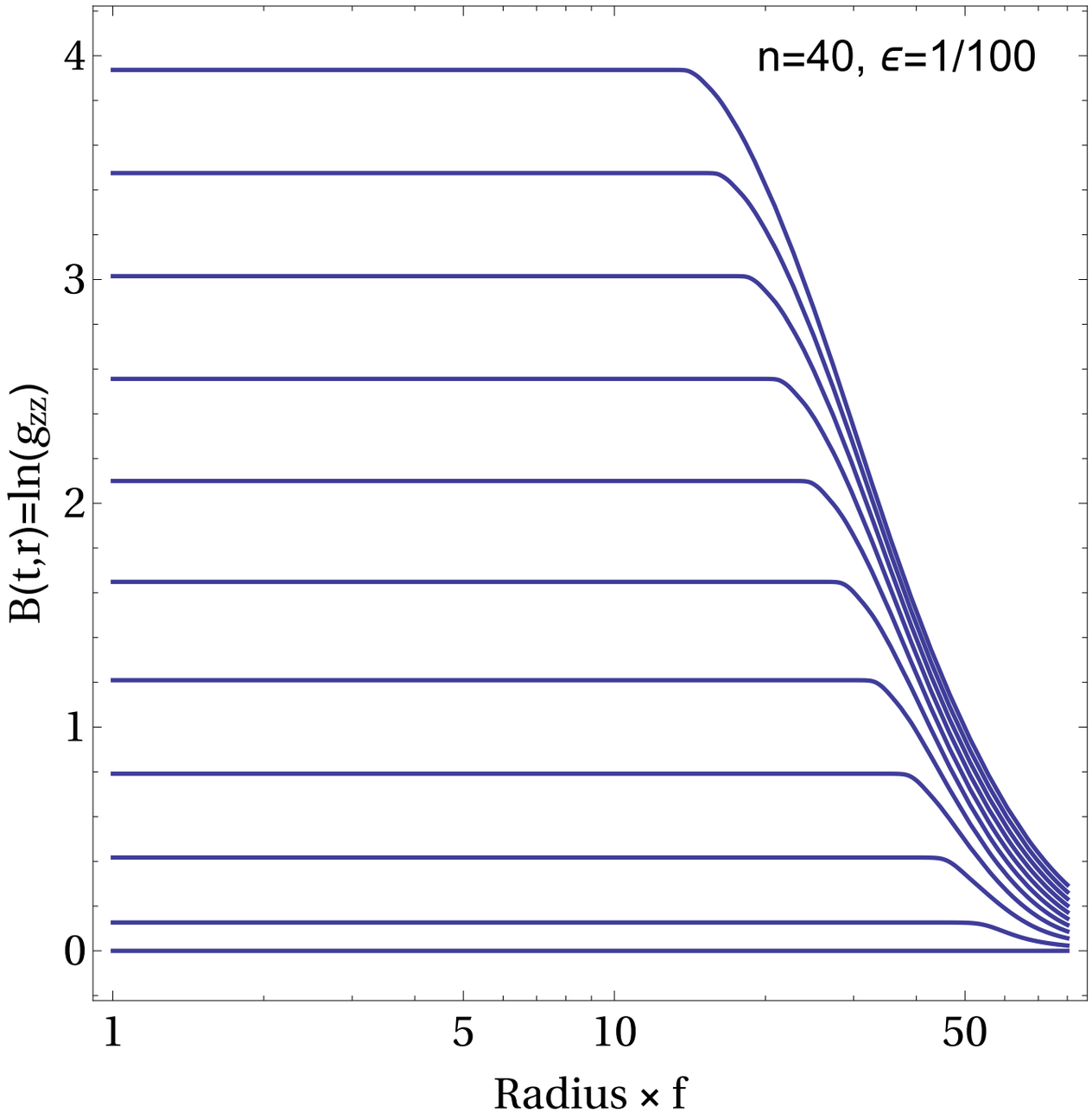
We find that they have nonsingular, non-static geometries and their cores realize different types of topological inflation.

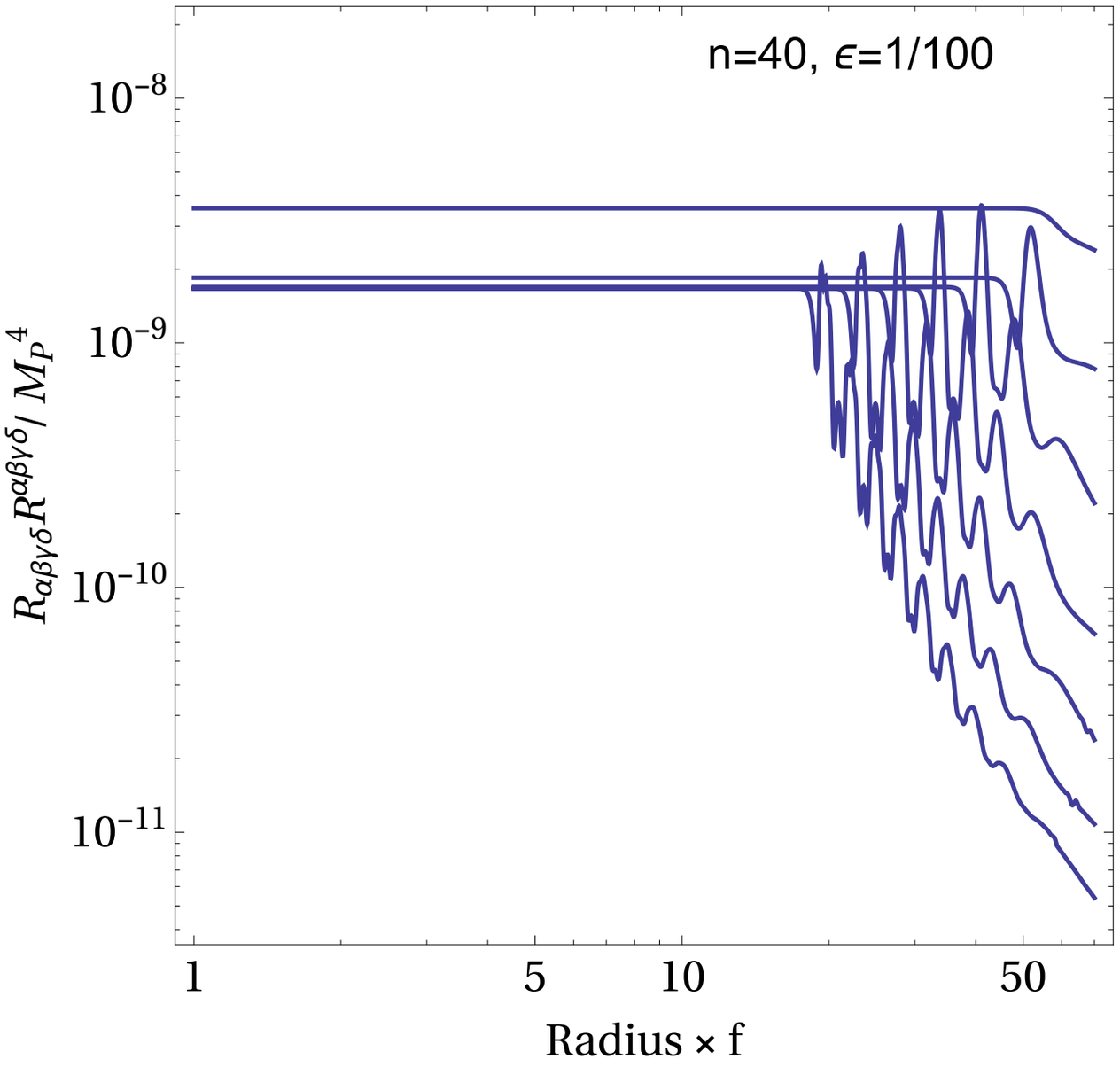
An exterior observer may be able to access the full field excursion, but it is difficult, requiring exponentially long times if it is possible at all (finite string loops, quantum effects may make impossible.)

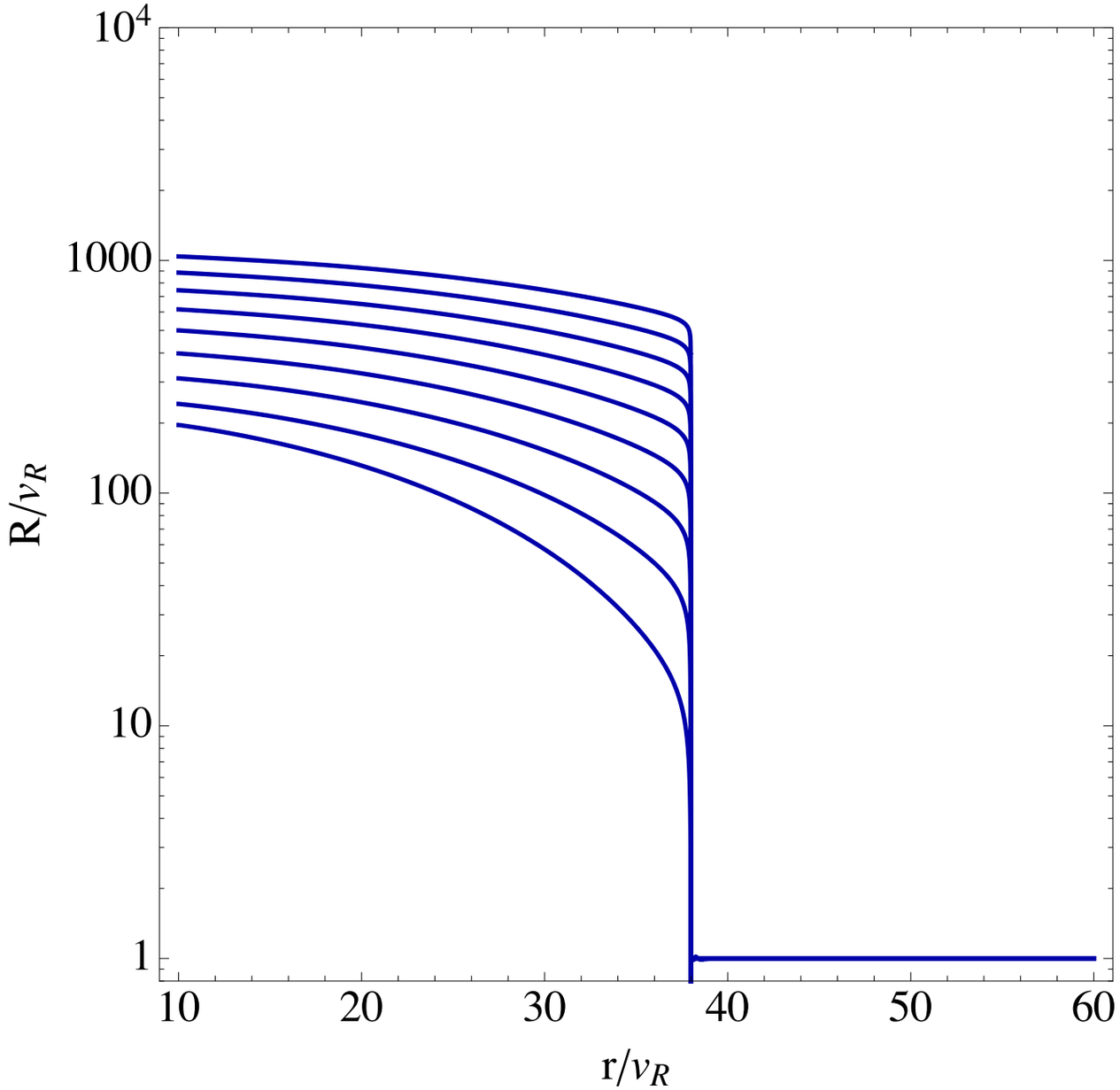
Reflective of a more general censorship theorem at work? So far, not well-understood. What would it imply for large-field inflation models? Relation to WGC?

Backup

Numerical analysis of Einstein+cosmic string:




n=40, ϵ=1/100
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} / M_P{}^4$
1
5
10
50
Radius × f


10^4
1000
100
10
1
R/v_R
10
20
30
40
50
60
r/v_R