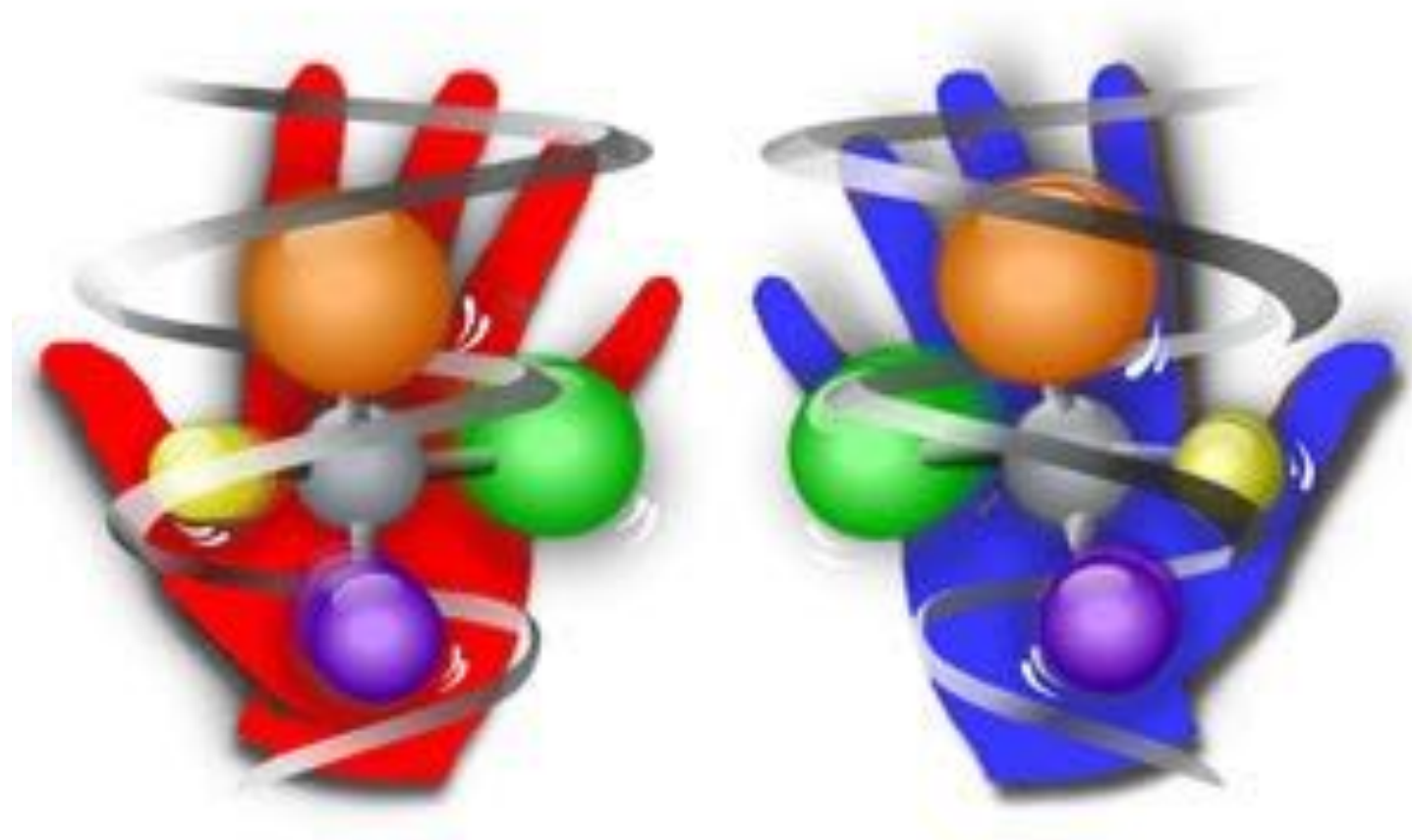


Exotic Mirror Fermions from Chiral Gauge Theories



This project starts from a technical issue:
how to simulate chiral gauge theories on
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...mirror fermions in the SM which
only interact with normal matter non-
locally through gauge field topology



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Devise a nonperturbative (lattice) regulator for chiral gauge theories.

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Why care? Who cares about 3 loops in a weakly coupled gauge theory?

Without understanding renormalization we do not understand how short distance physics can decouple from the theory

...maybe there is exotic physics needed to make the SM work that is partly hidden from us and unsuspected?



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What happens if you naively put a chiral gauge theory on the lattice?

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the continuum theory you simulate is vector-like (like QCD, QED)

What happens if you give the doublers large masses to decouple them?

You break the gauge symmetry.

Chiral symmetry — forbids mass
Renormalization — requires mass



Fundamental tension!

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Renormalization — requires mass  **Fundamental tension!**

The **anomaly** is key to understanding chiral symmetry on the lattice

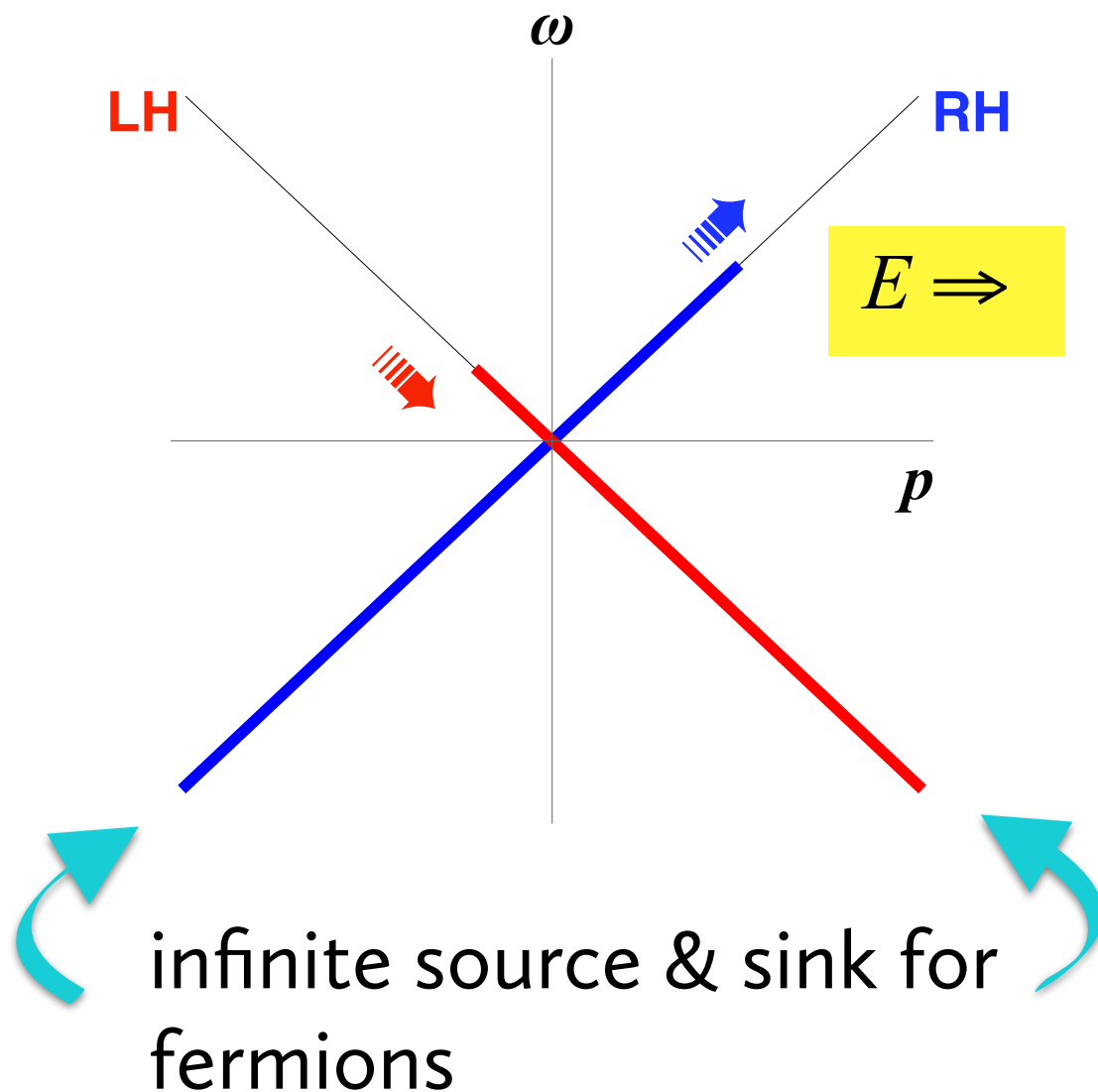
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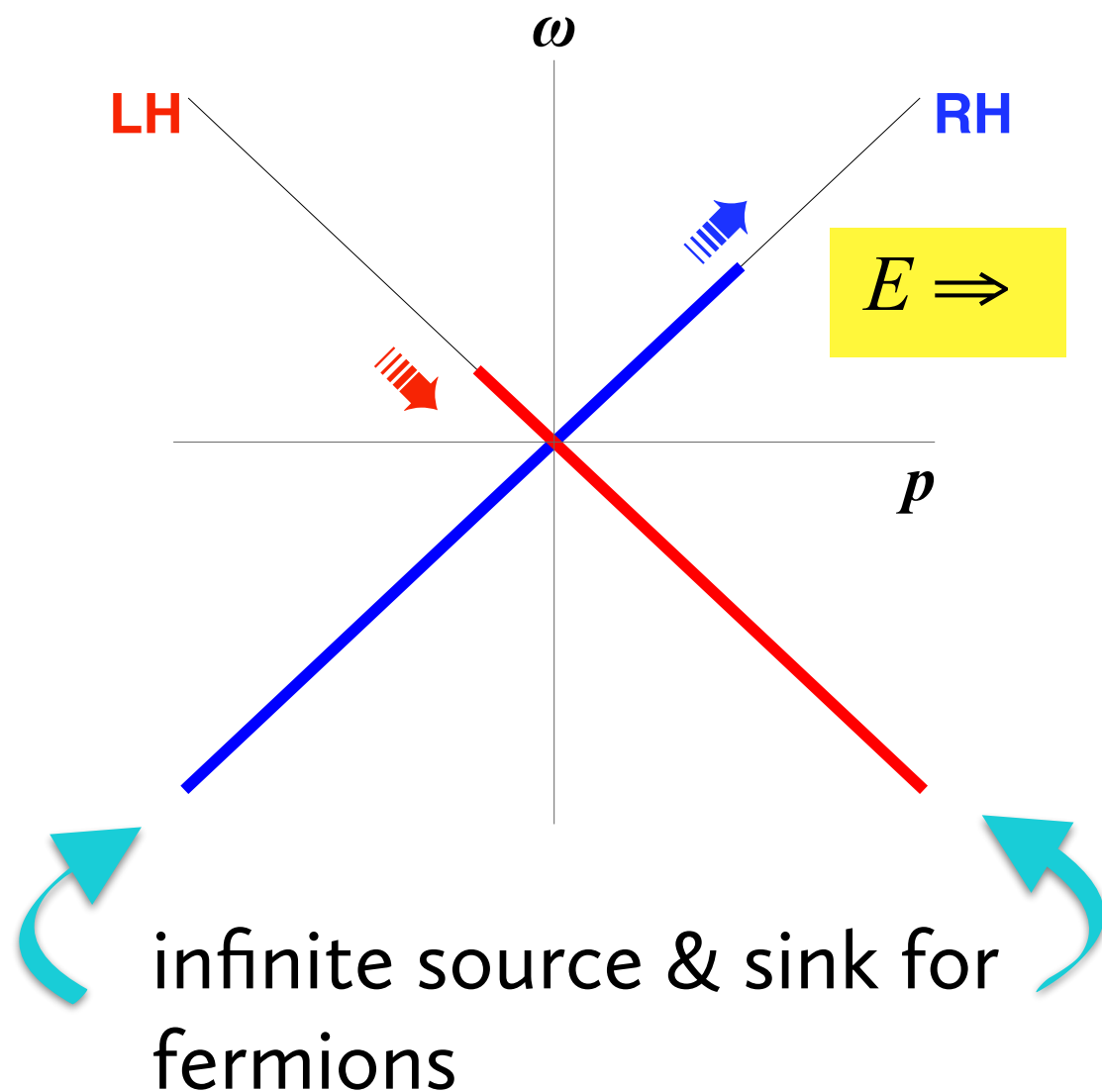
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$$dp = qE dt$$

$$dn_R = +\frac{dp}{2\pi}$$

$$dn_L = -\frac{dp}{2\pi}$$

$$\frac{dn_5}{dt} = \frac{qE}{\pi}$$



$$\partial_\mu j_5^\mu = \frac{q}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu}$$

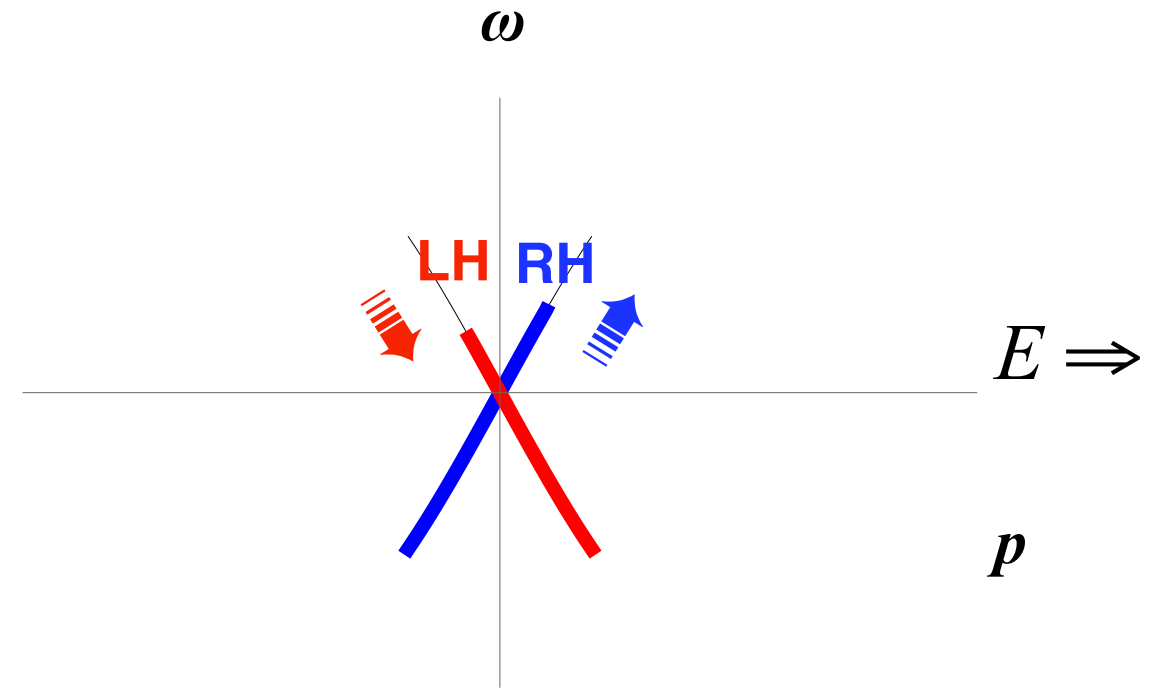
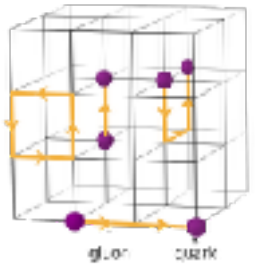
d=1+1 anomaly



In the continuum, the Dirac sea is filled...but is a Hilbert Hotel which always has room for more

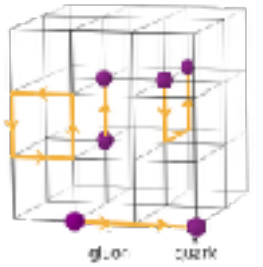
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Can reproduce continuum physics for long wavelength modes...

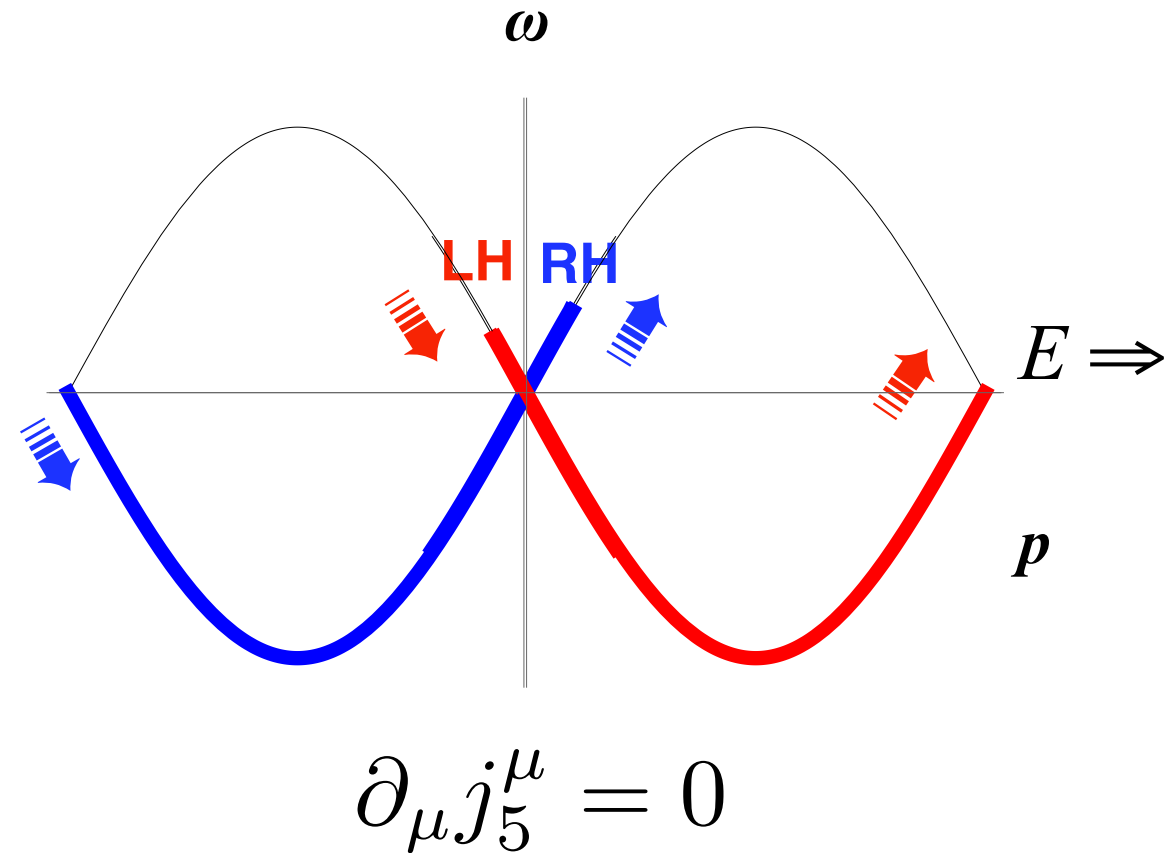


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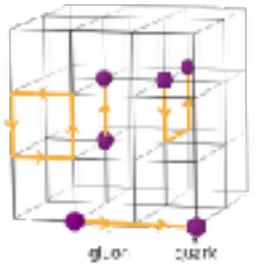


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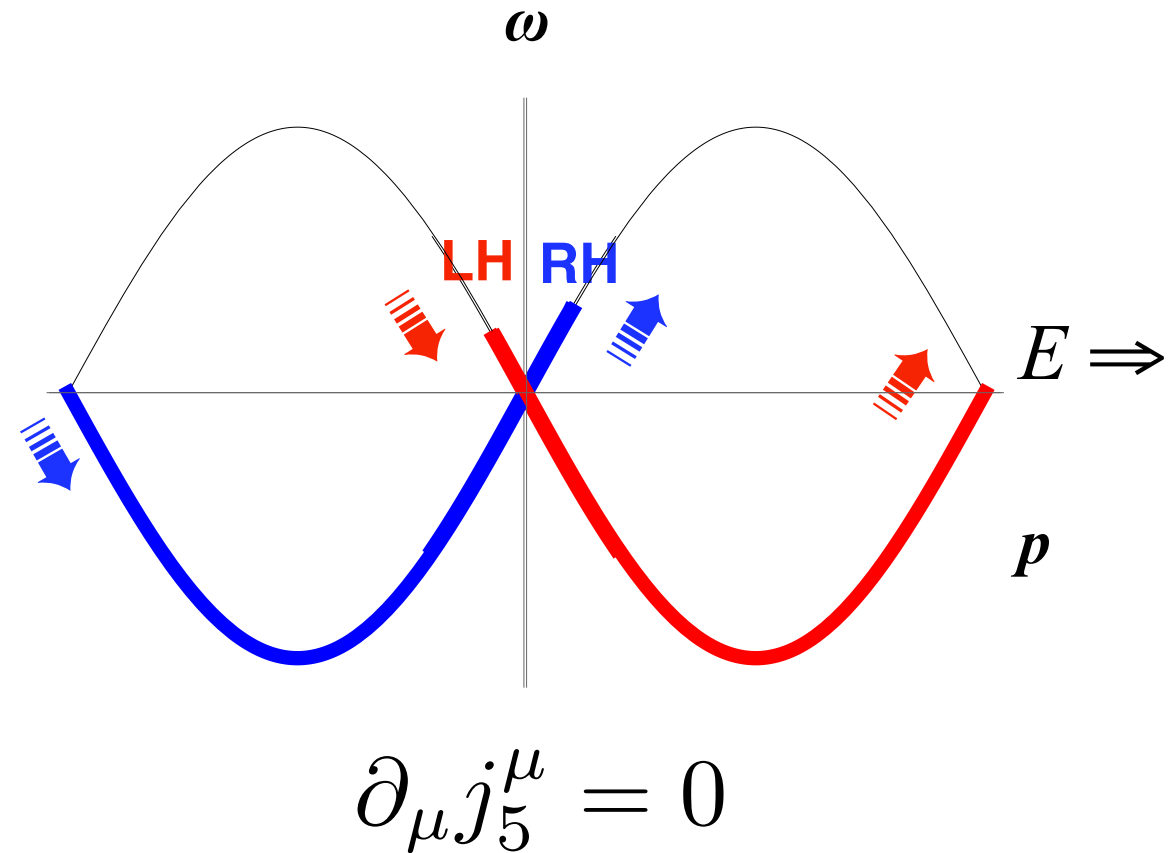


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anomalous symmetry in the continuum
must be
explicitly broken symmetry on the lattice

The Nielsen-Ninomiya Theorem:

The Euclidian fermion action:

$$S = \int_{-\pi/a}^{\pi/a} \frac{d^{2k}p}{(2\pi)^4} \bar{\Psi}_{-\mathbf{p}} \tilde{D}(\mathbf{p}) \Psi(\mathbf{p})$$

cannot have a kinetic operator D satisfying all four of the following properties simultaneously:

1. $\tilde{D}(\mathbf{p})$ is a periodic, analytic function of p_μ ;
2. $D(\mathbf{p}) \propto \gamma_\mu p_\mu$ for $a|p_\mu| \ll 1$;
3. $\tilde{D}(\mathbf{p})$ invertible everywhere except $p_\mu = 0$;
4. $\{\Gamma, \tilde{D}(\mathbf{p})\} = 0$. ($\Gamma = \gamma_5$)

⇐ regulated, local
⇐ Dirac @ long wavelength
⇐ No doubling of flavors
⇐ respects a chiral symmetry

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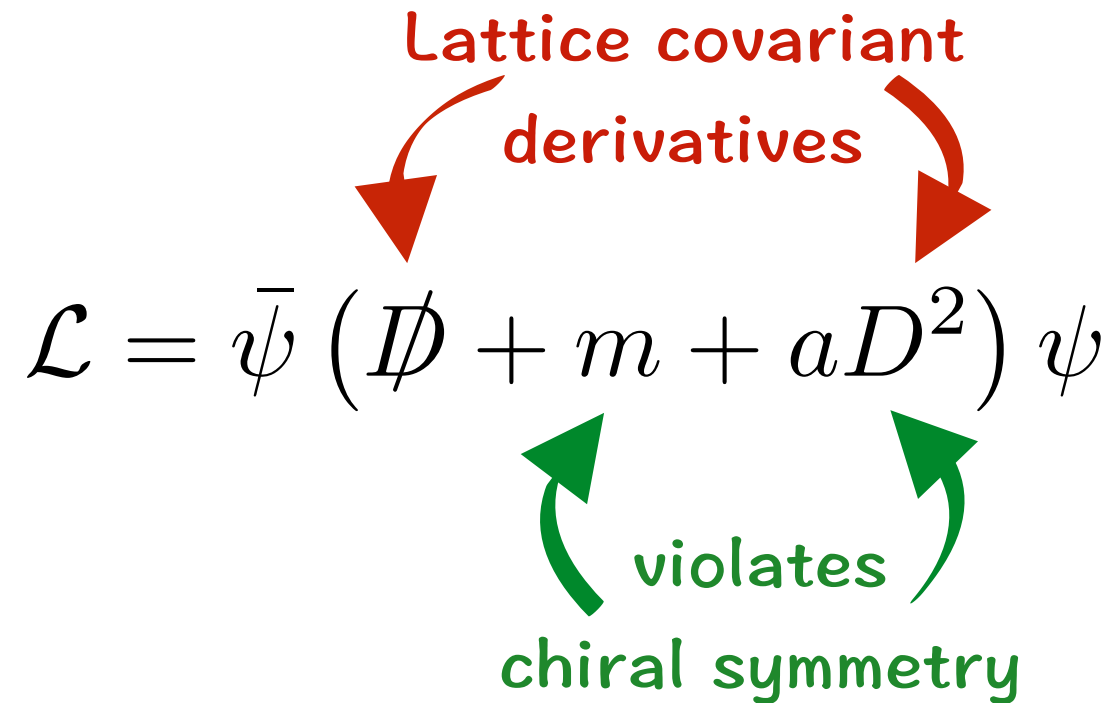
Advances in the 1990s showed us how to break global chiral symmetry in just the right way for QCD...but will be problematic when chiral symmetry is gauged!

How **Wilson fermions** reproduce the *global* chiral symmetries of QCD:

Lattice covariant
derivatives

$$\mathcal{L} = \bar{\psi} (\not{D} + m + aD^2) \psi$$

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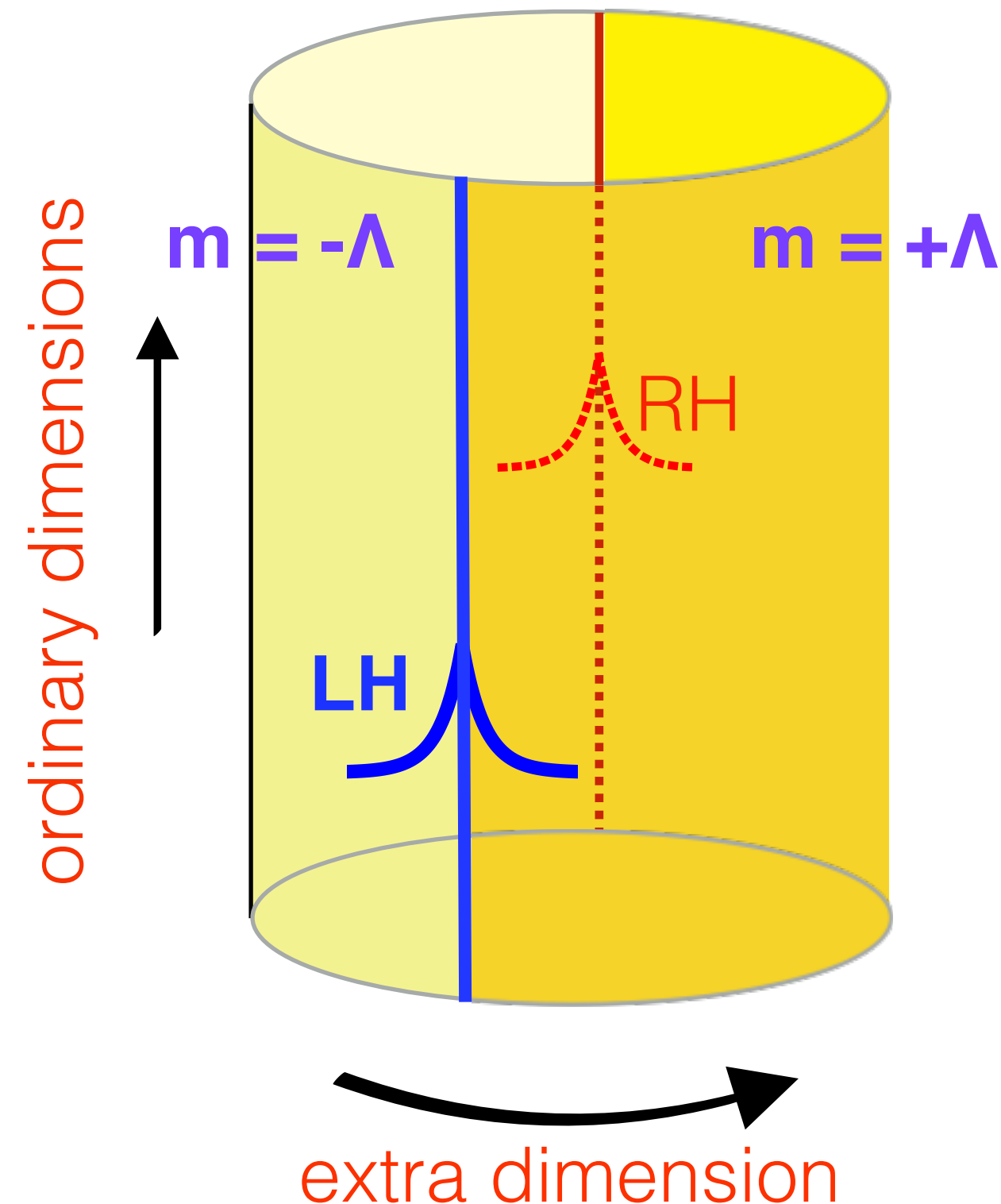
Lattice covariant derivatives

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- Break the symmetries explicitly at the lattice cutoff ($m \sim 1/a$) !
- Fine tune the hell out of the theory to find chiral symmetry in the continuum...magically, the anomaly is reproduced as a byproduct
- **Lost:** the benefits of chiral symmetry - multiplicative mass renormalization, non-mixing of operators...
- ...and a particularly disturbing way to deal with **gauged** chiral symmetries

Domain Wall Fermions solved the problem of global chiral symmetry on the lattice (1992): \Rightarrow anomalies are the *only* breaking of chiral symmetry

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- Introduce a compact extra dimension
- 5d fermion has heavy positive mass on one side, negative on the other
- Chiral massless states appear at the mass defects; other modes are heavy
- Gauge fields do not depend on extra dimension
- Low energy theory looks like a single massless fermion with chiral symmetry

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They decouple completely except for induced Chern-Simons term:

(For now: 5d background gauge fields)

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Differentiate w.r.t. A_5 to get J_5

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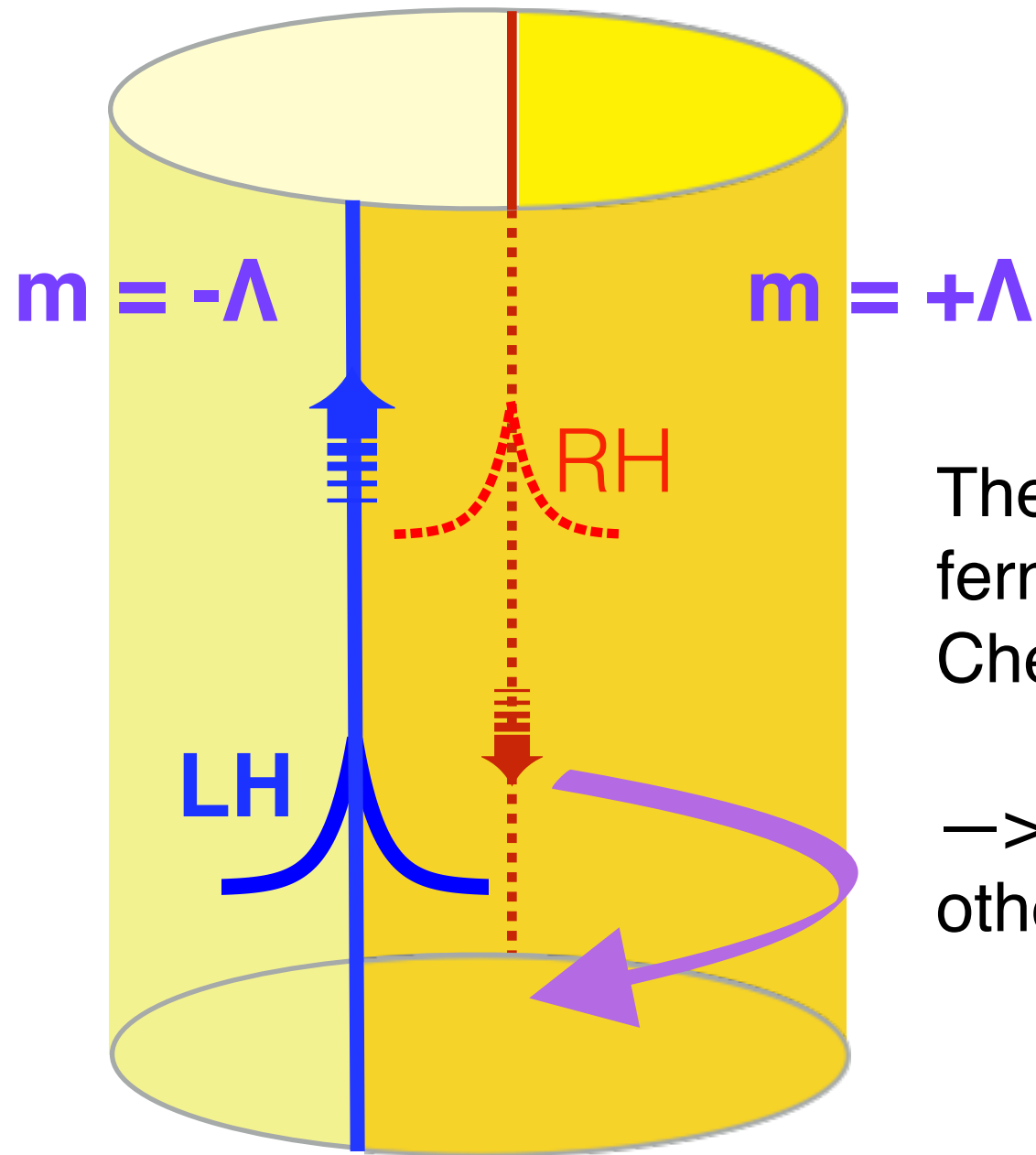
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Bulk current explains anomalous disappearance of charge on one defect and reappearance on the other
= anomalous chiral symmetry violation

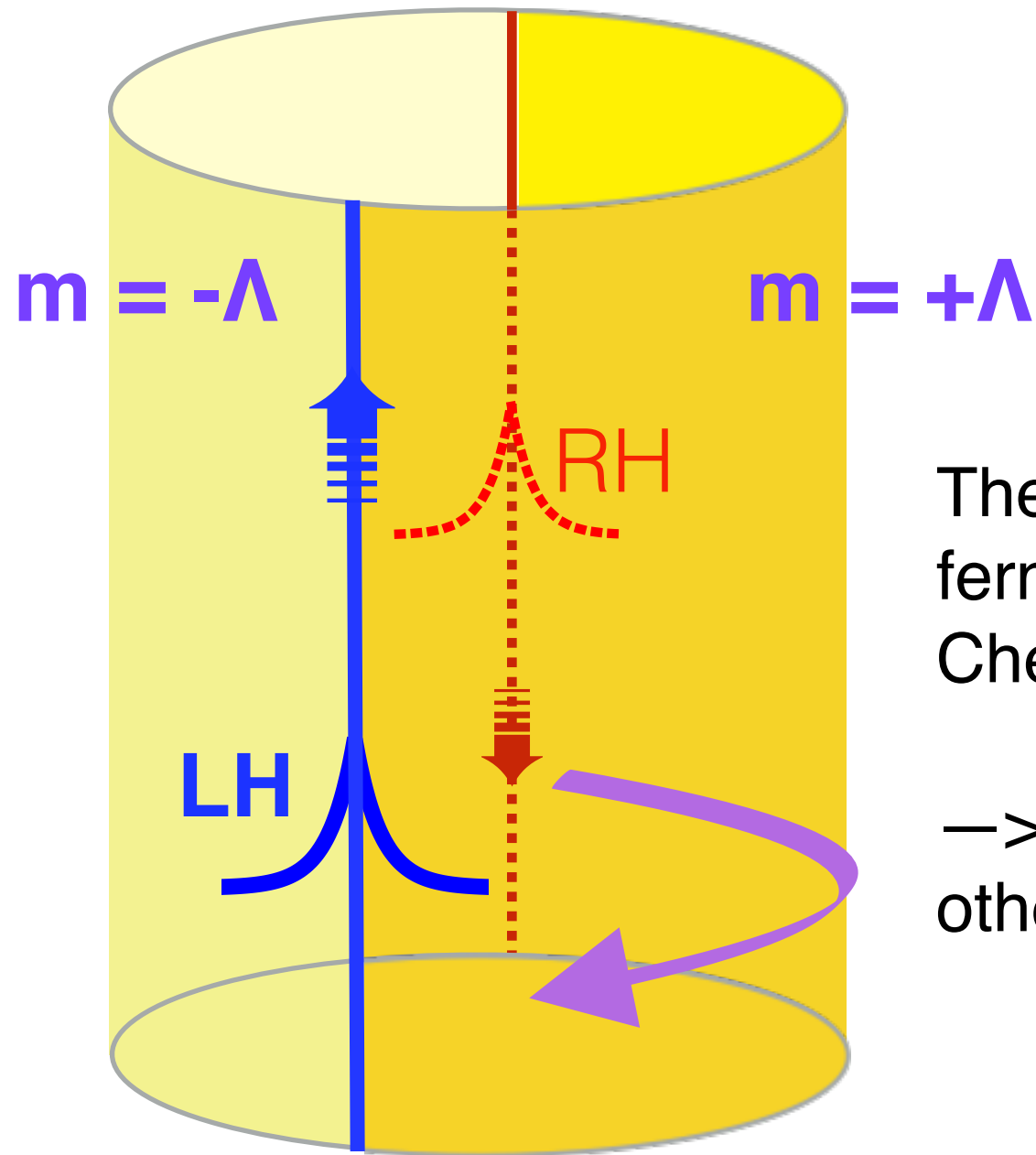
This is what is called a topological insulator
these days by condensed matter theorists





The only relevant effect of the heavy bulk fermions on low energy physics is the Chern-Simons current proportional to $F F^*$

—> gives the correct anomaly without any other feature of chiral symmetry breaking

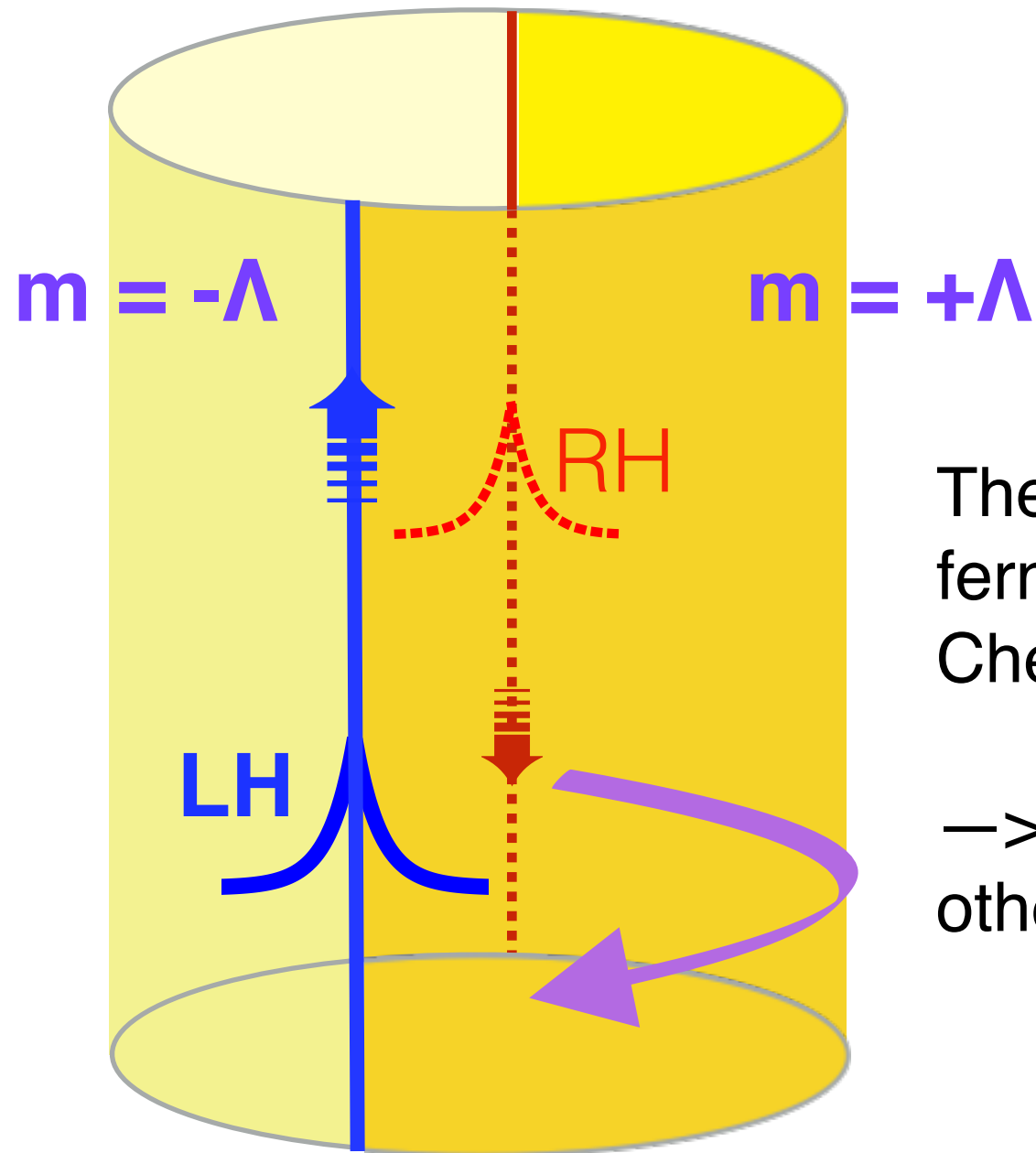


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Can construct the exact 4d effective theory in this limit (the “overlap operator”).



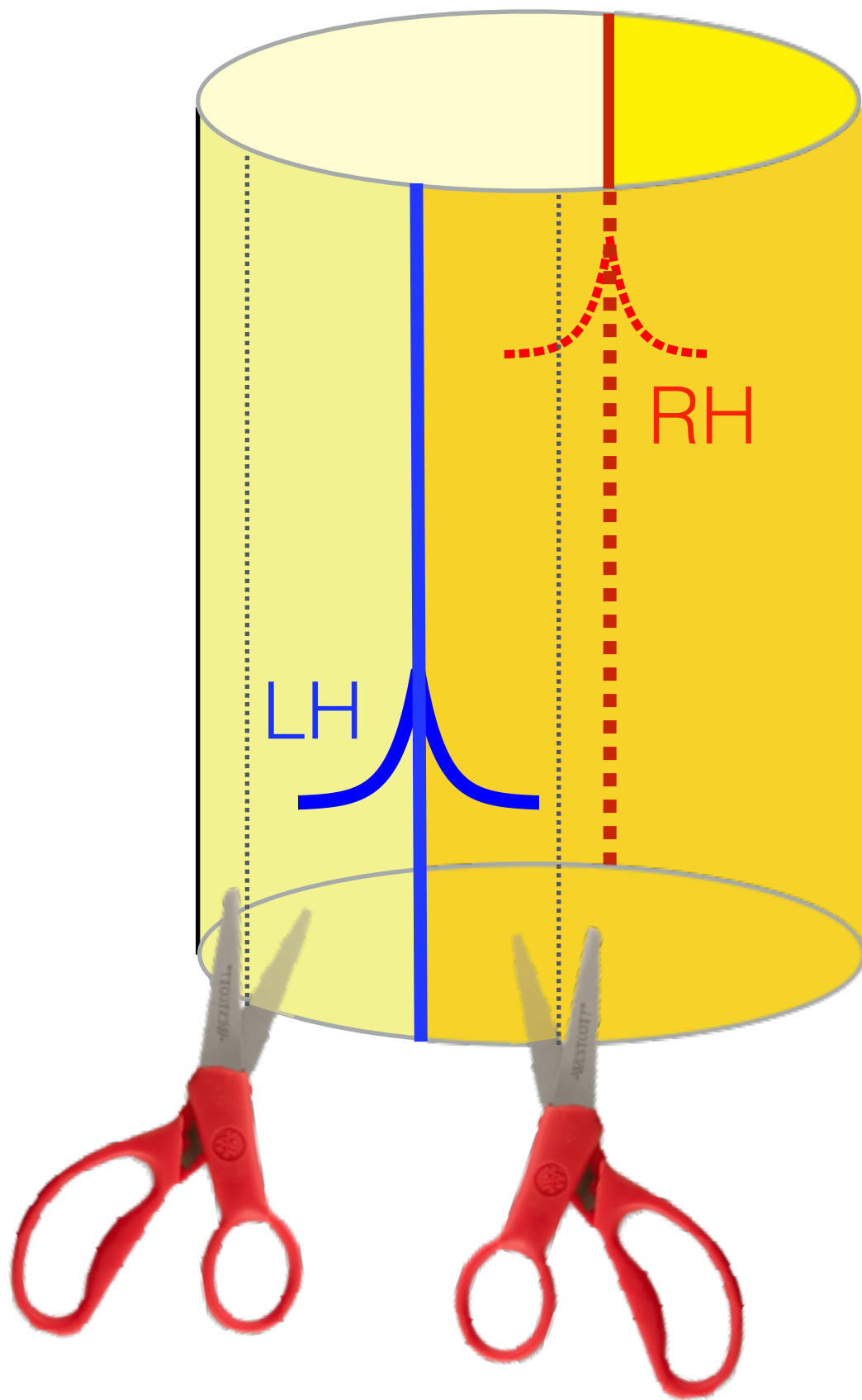
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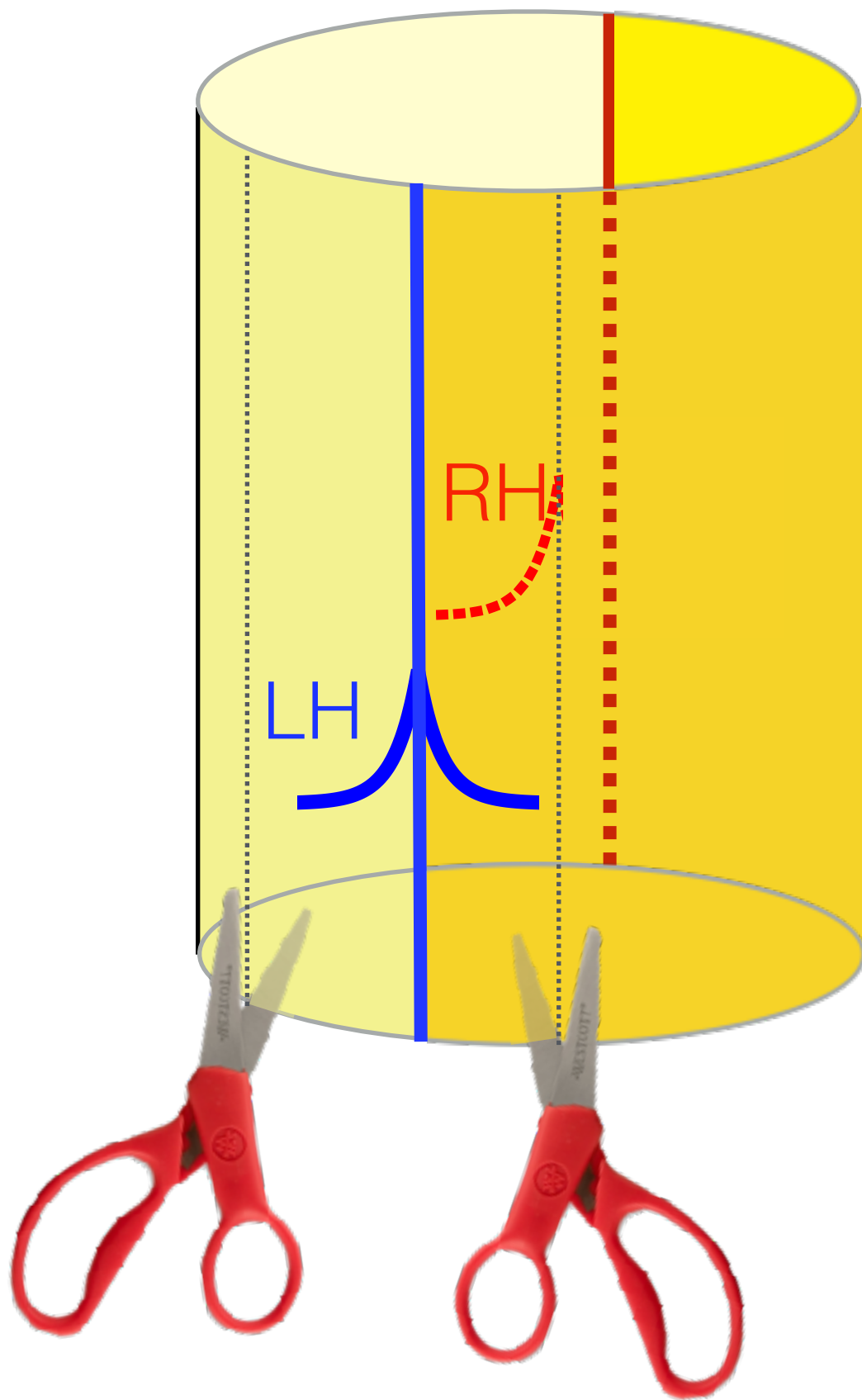
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Can this construction be used for chiral gauge theories?

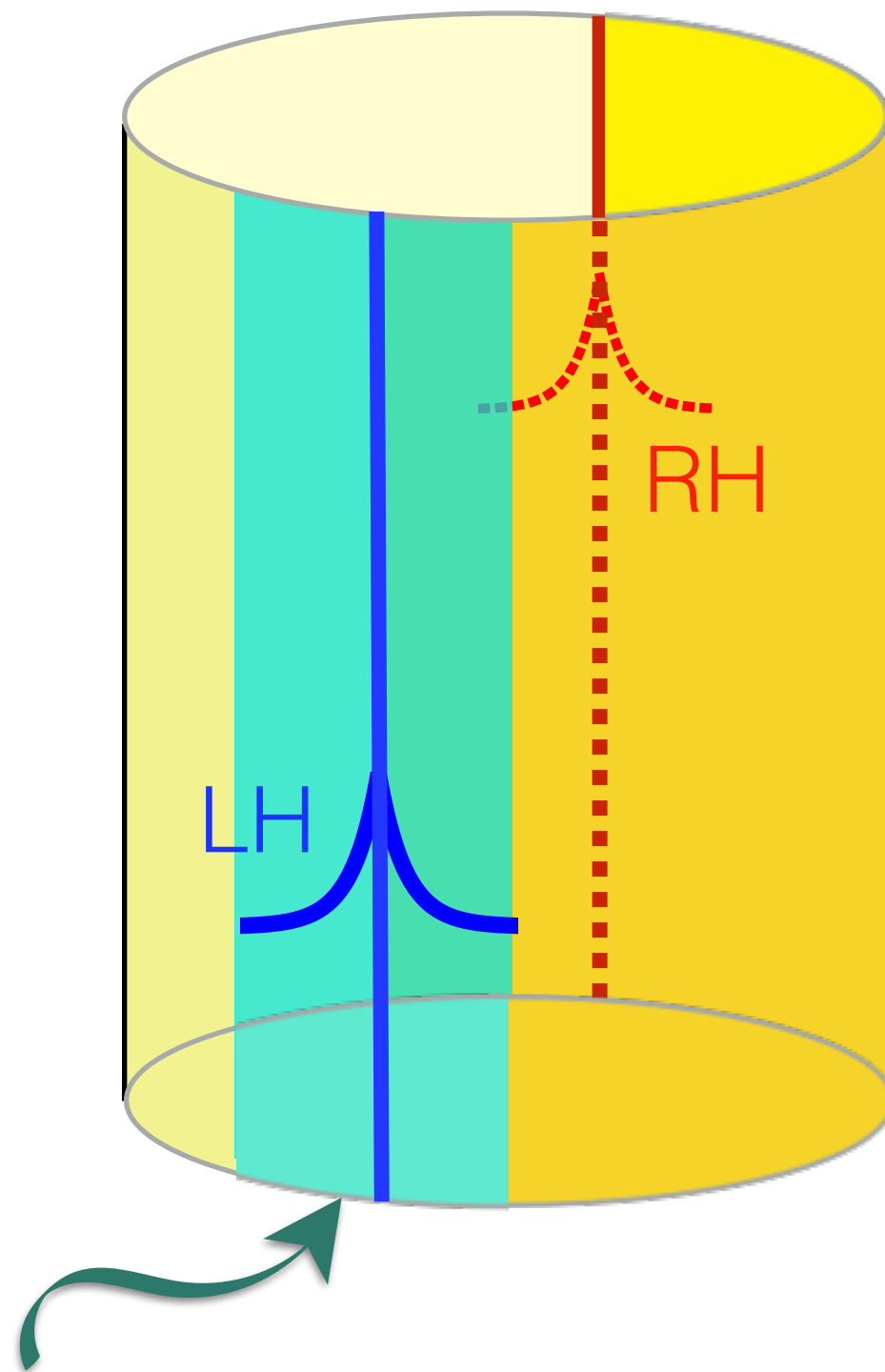


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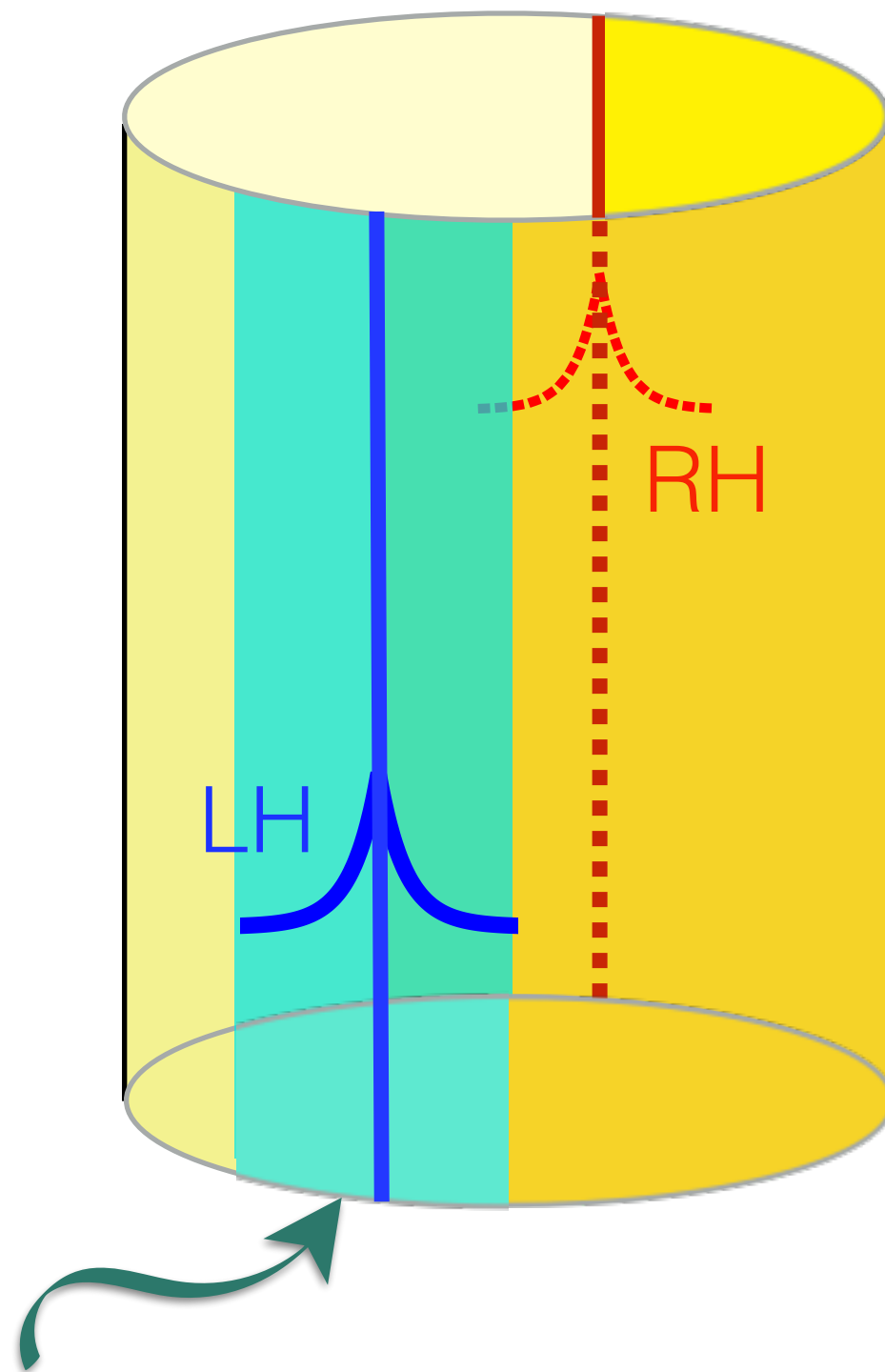


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No: The 5d kinetic term allows fermions to "hop" in the extra dimension; localizing the gauge field would explicitly break gauge symmetry.

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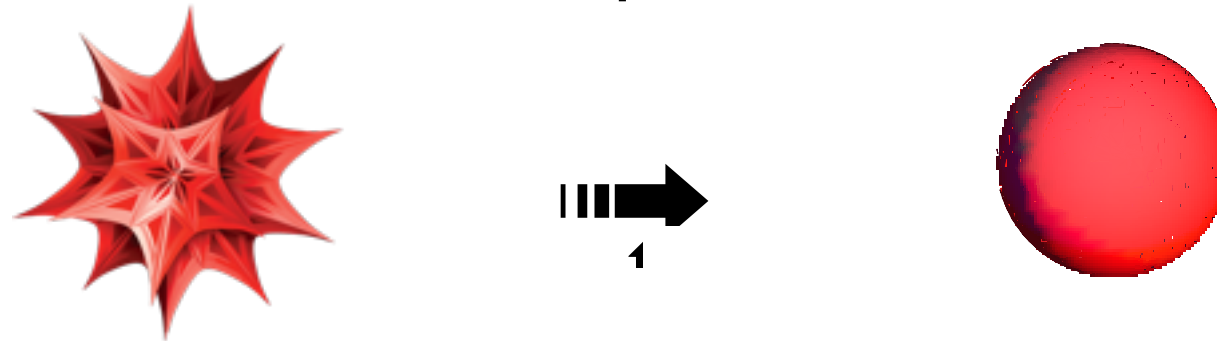
Make mirror fermions decouple in a gauge invariant way

New proposal: “localize” gauge fields using gradient flow

Dorota Grabowska, D.B.K.

- Phys.Rev.Lett. 116 211602 (2016) [arXiv:1511.03649]
- Phys.Rev. D94 (2016) no.11, 114504 [arXiv:1610.02151]

Gradient flow smooths out fields by evolving them classically in an extra dimension via a heat equation

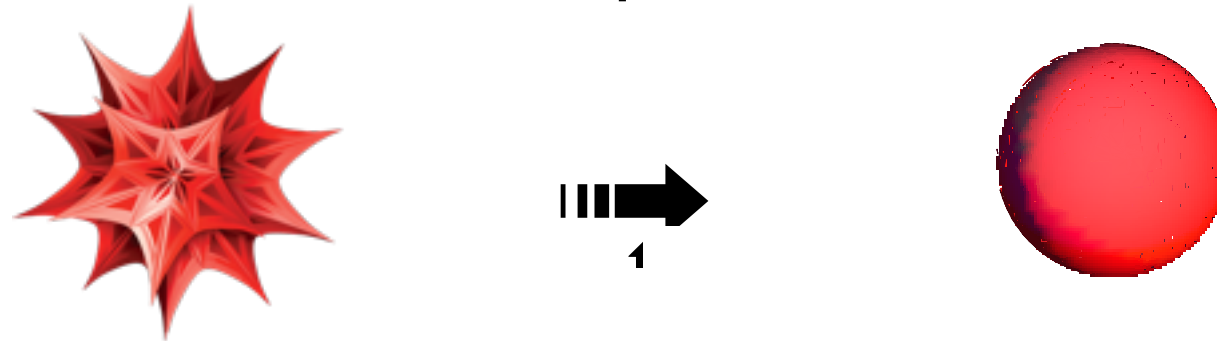


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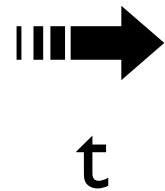
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covariant flow eq.

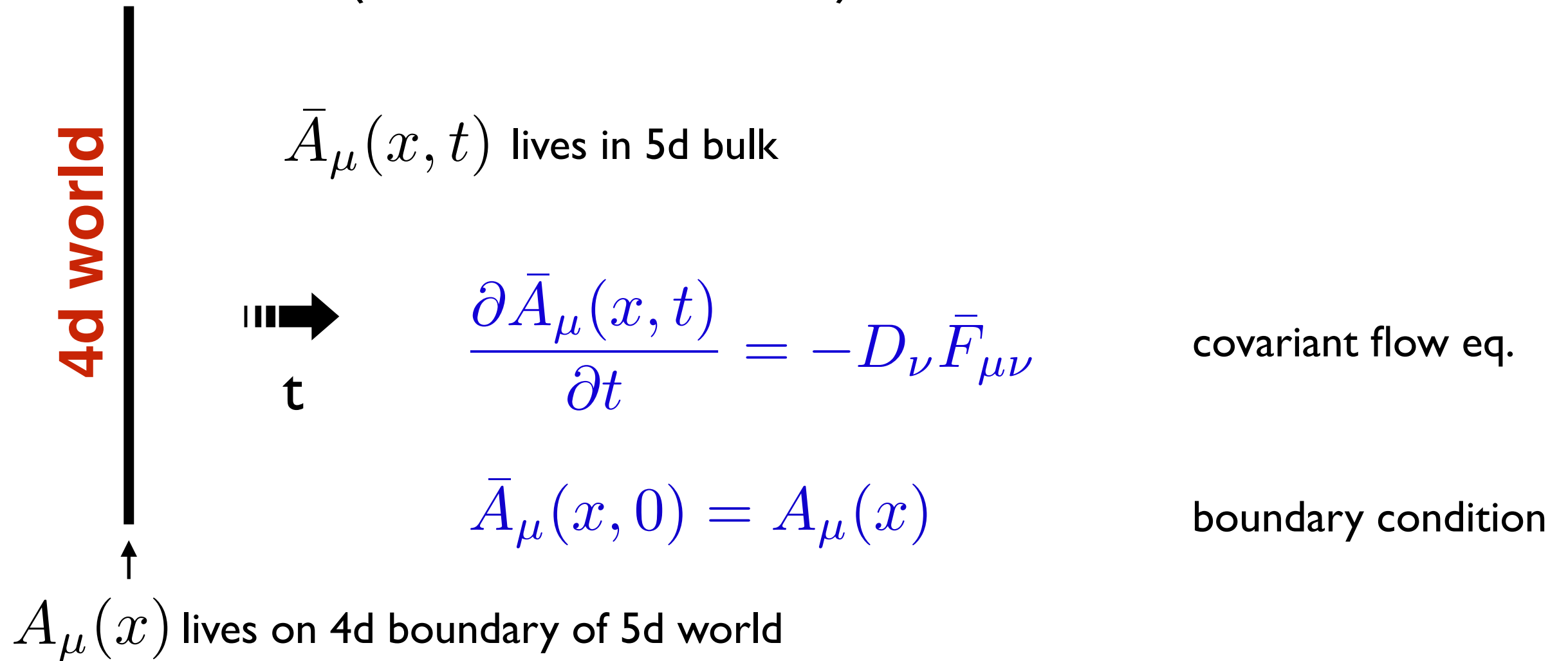
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boundary condition

4d world

$A_\mu(x)$ lives on 4d boundary of 5d world

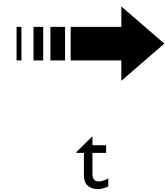
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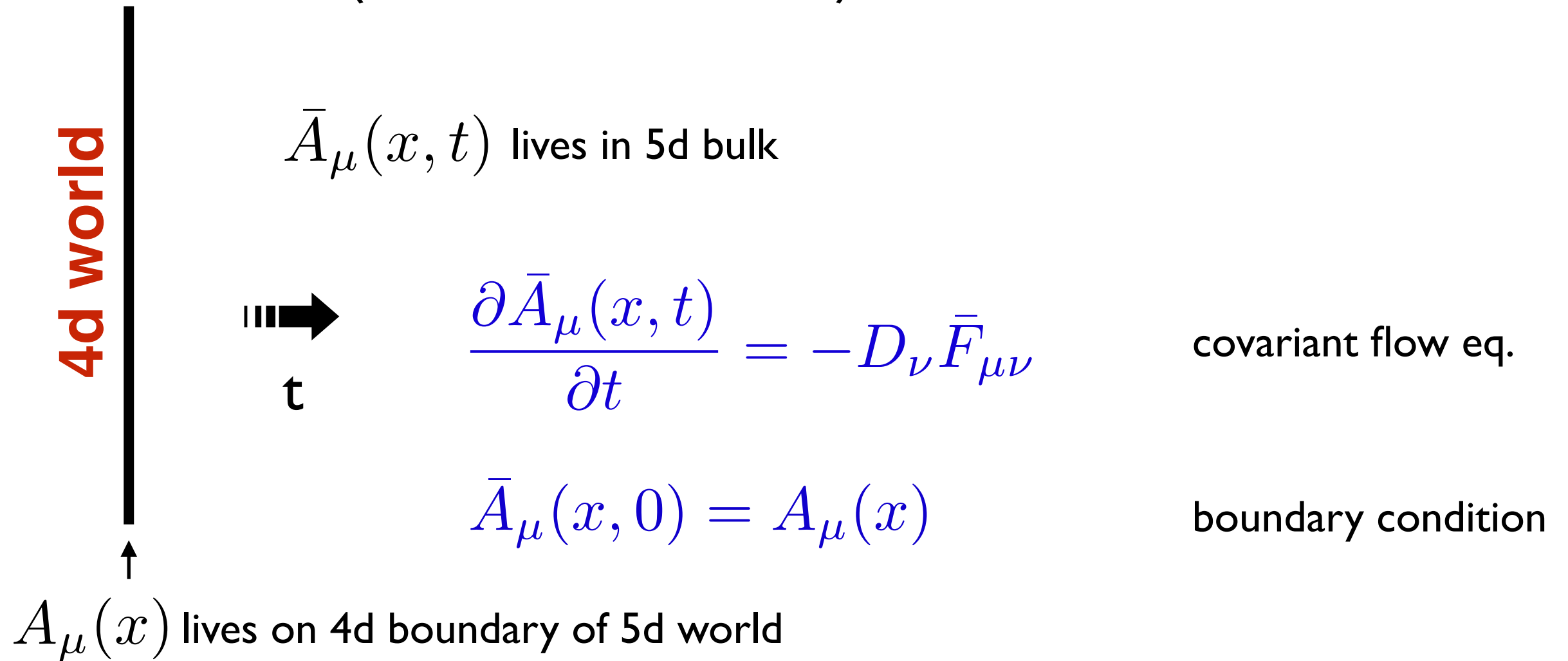
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2d/3d U(1)
example:

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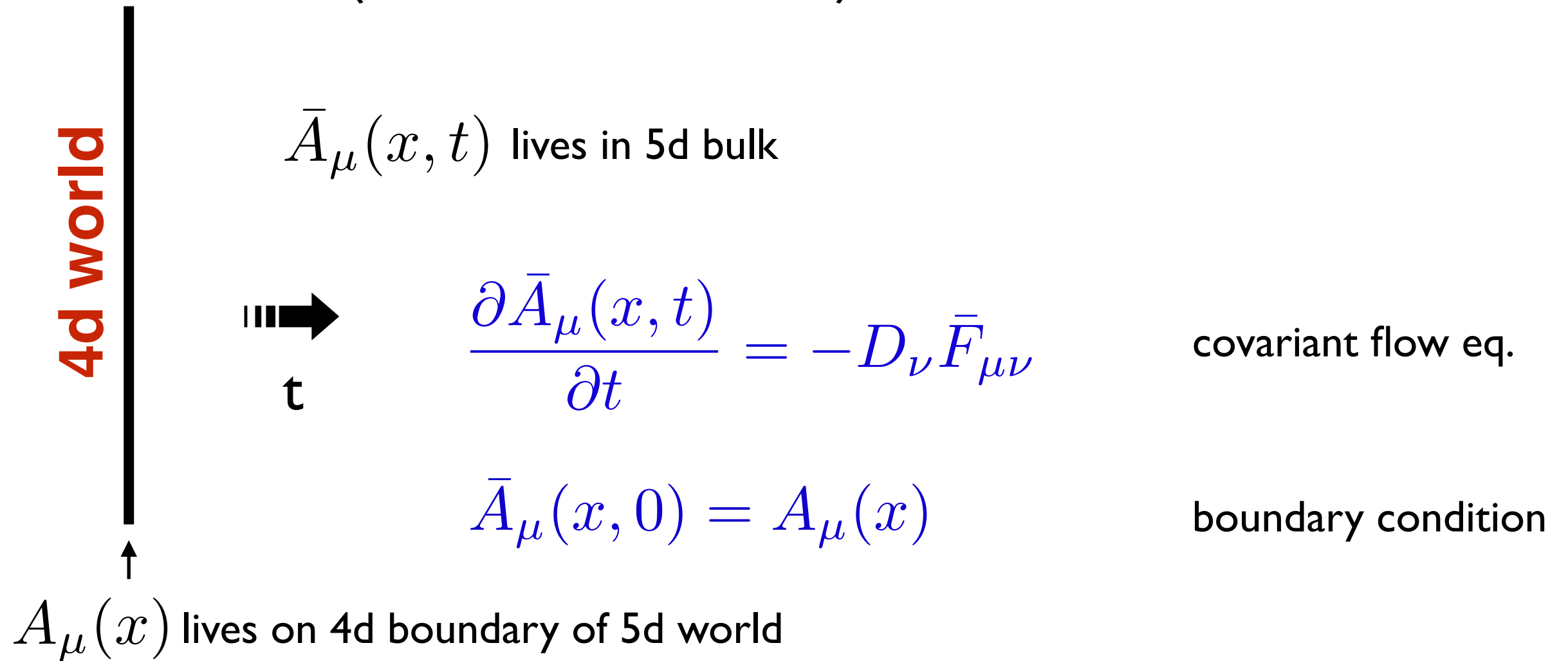
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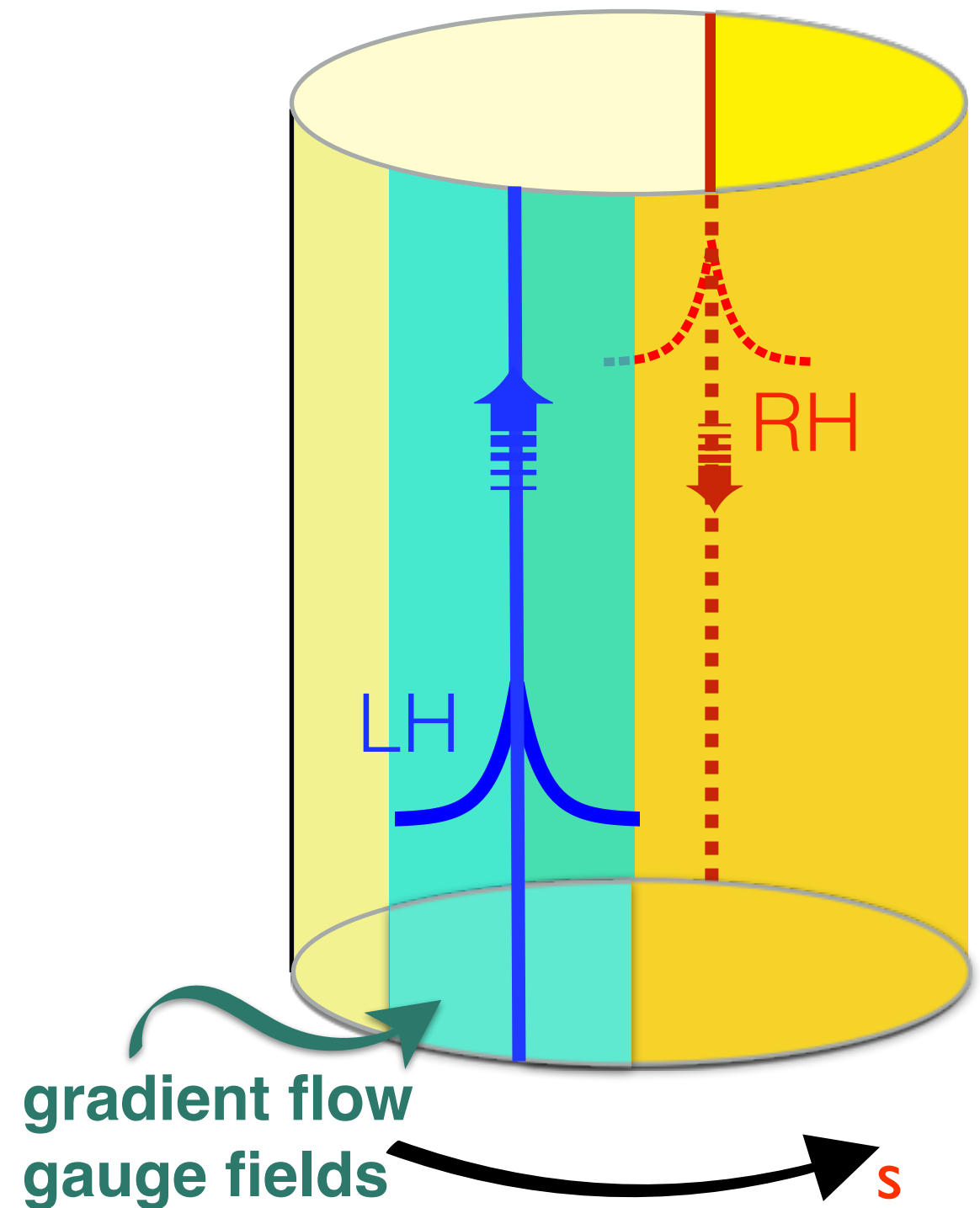
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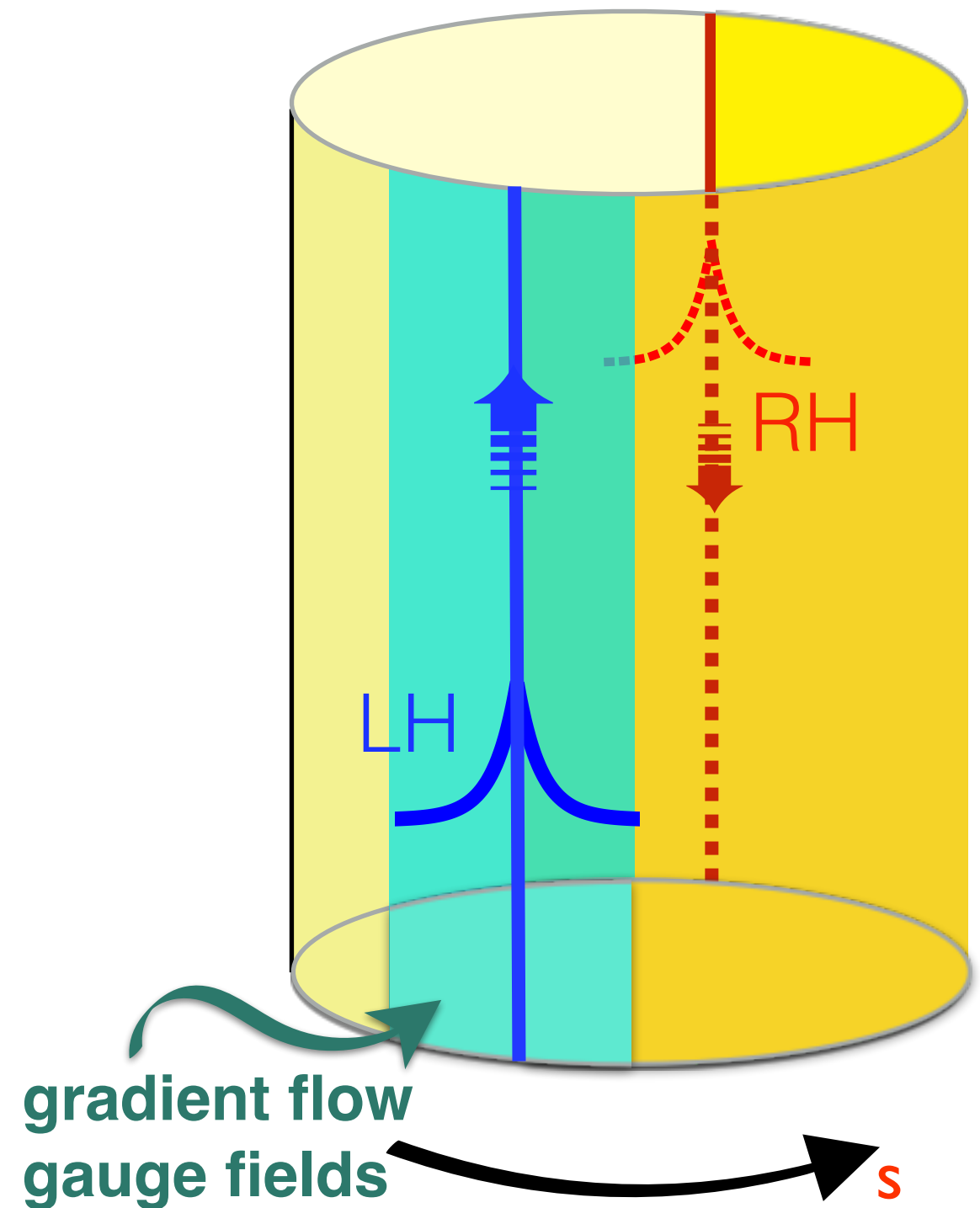
This will allow $\lambda(p)$ to be localized near $t=0$ while maintaining gauge invariance

Combining gradient flow gauge fields with domain wall fermions:



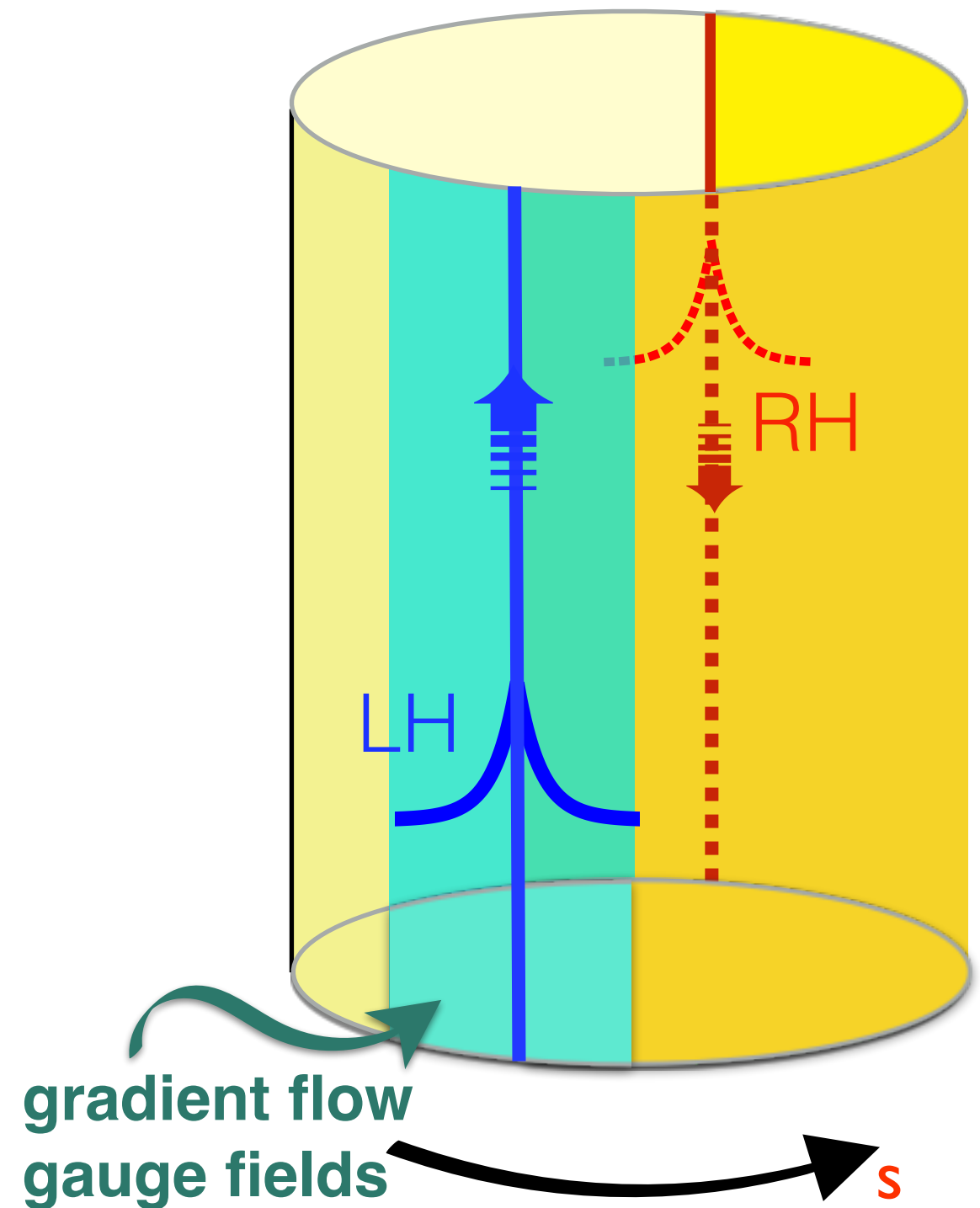
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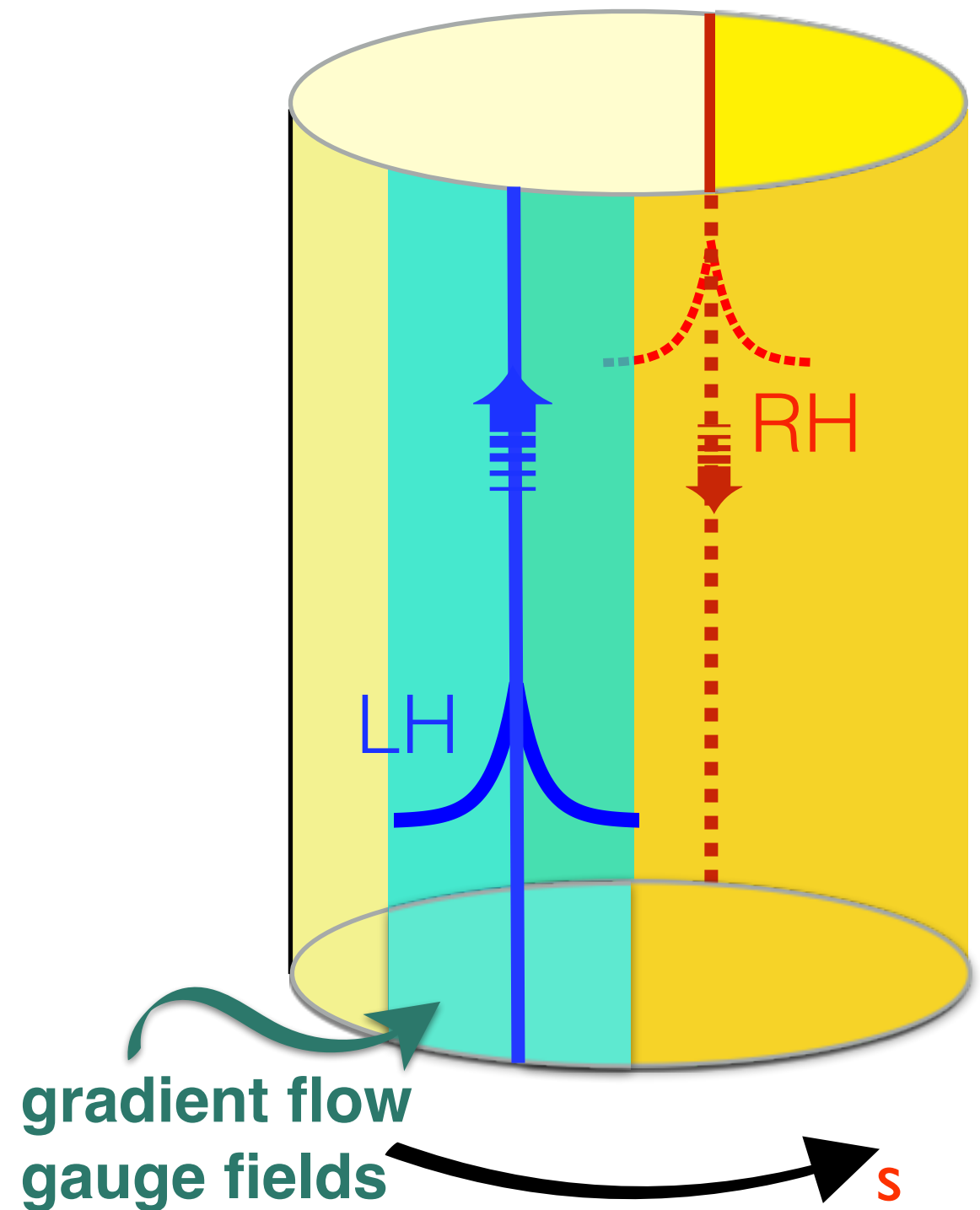
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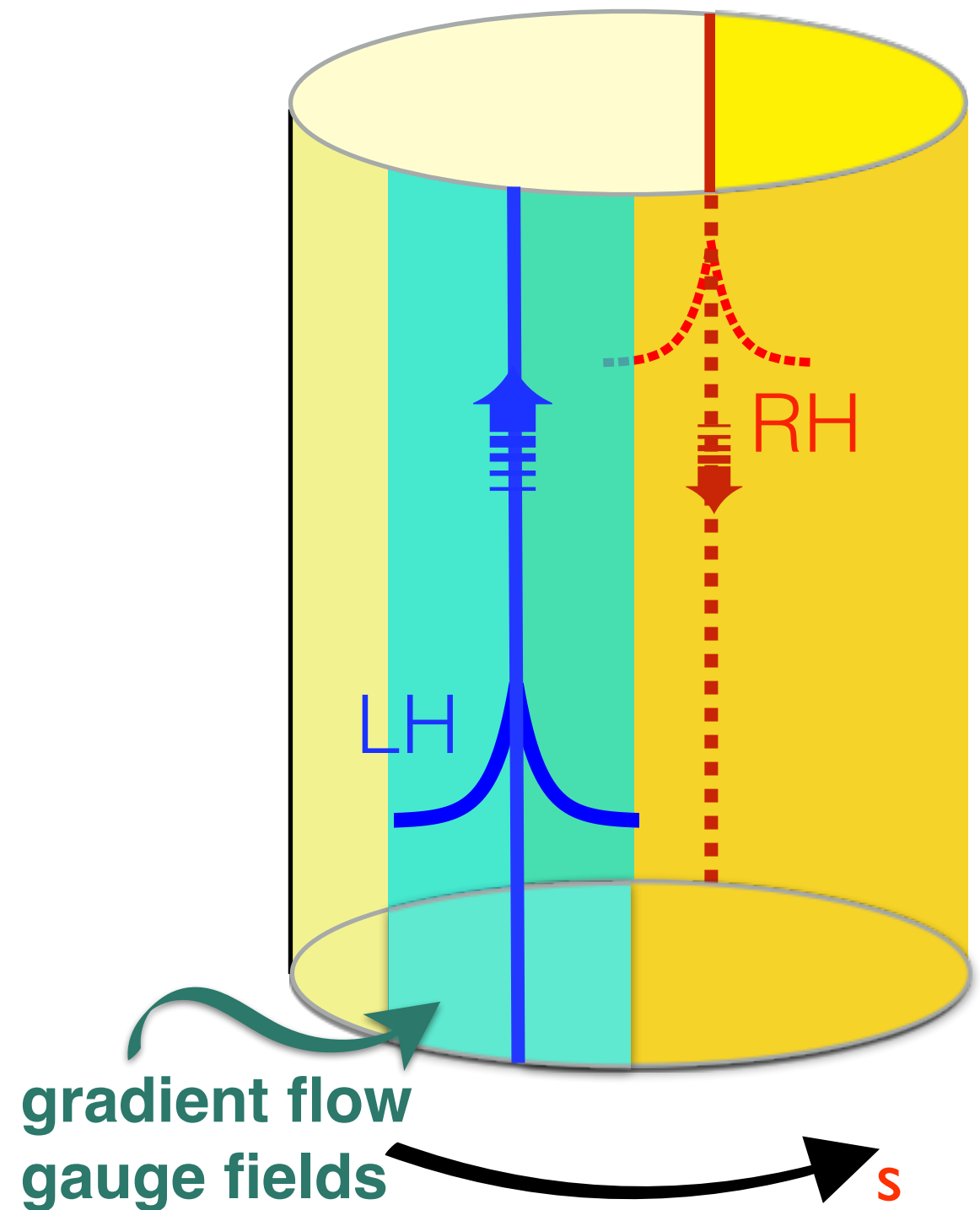
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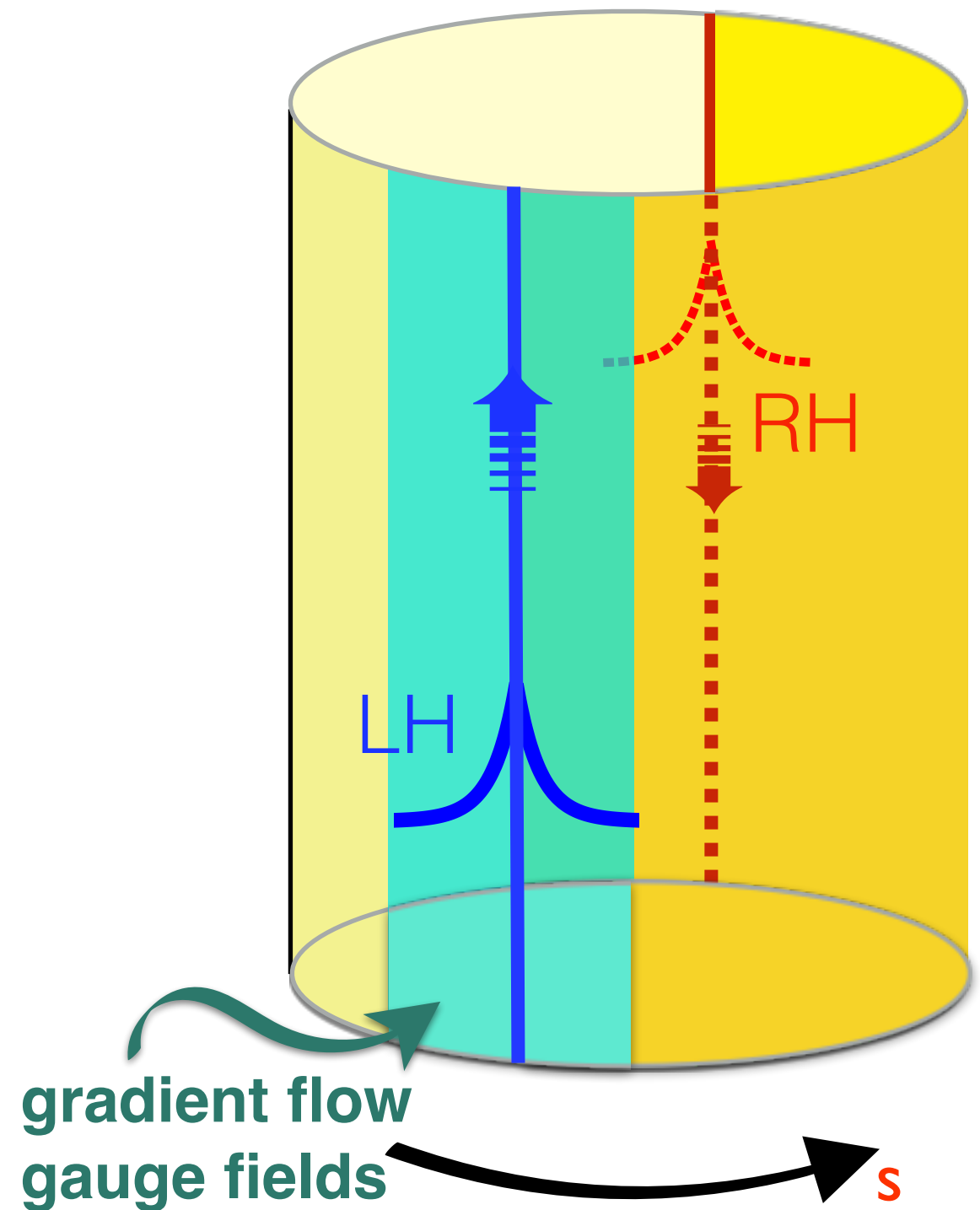
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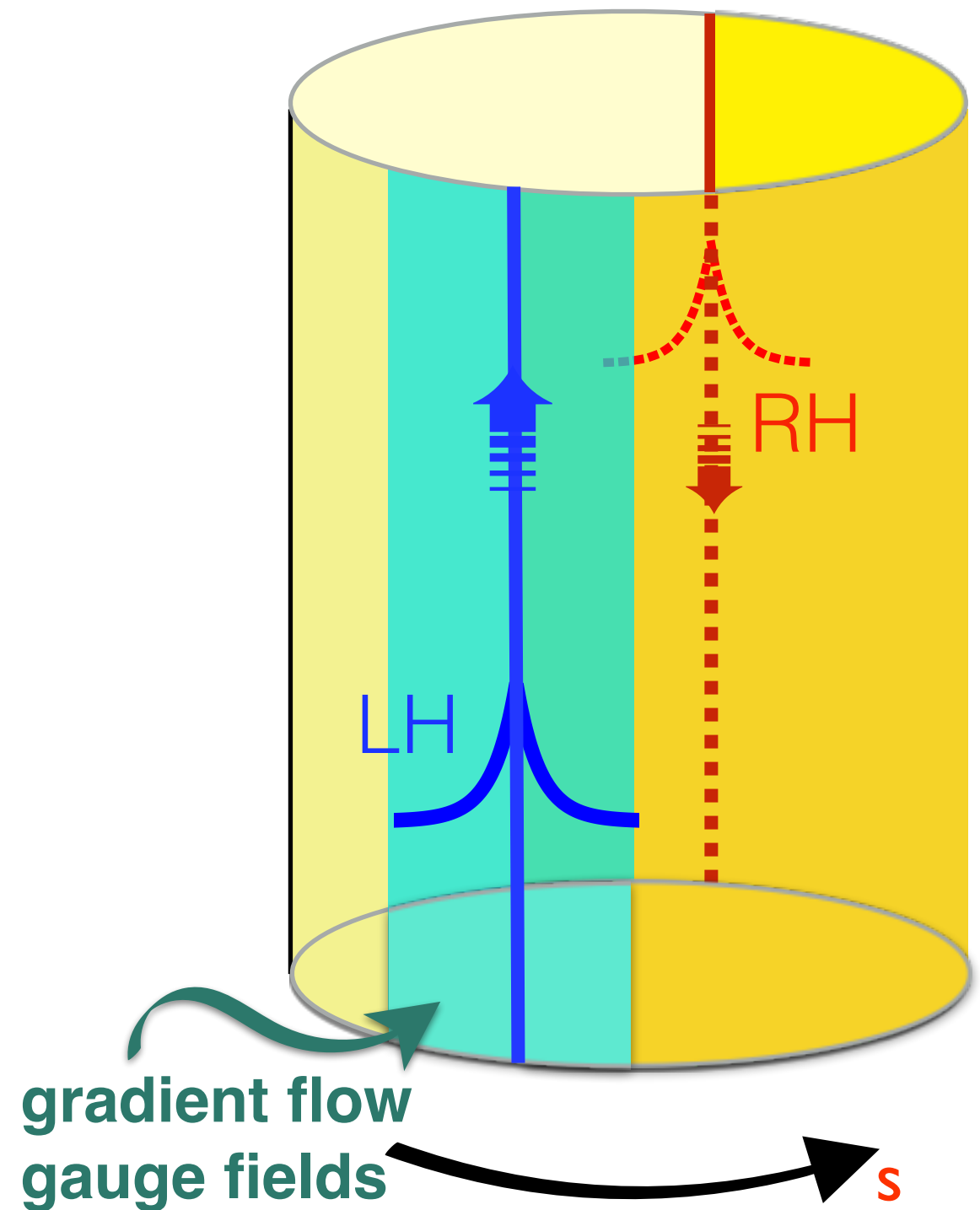
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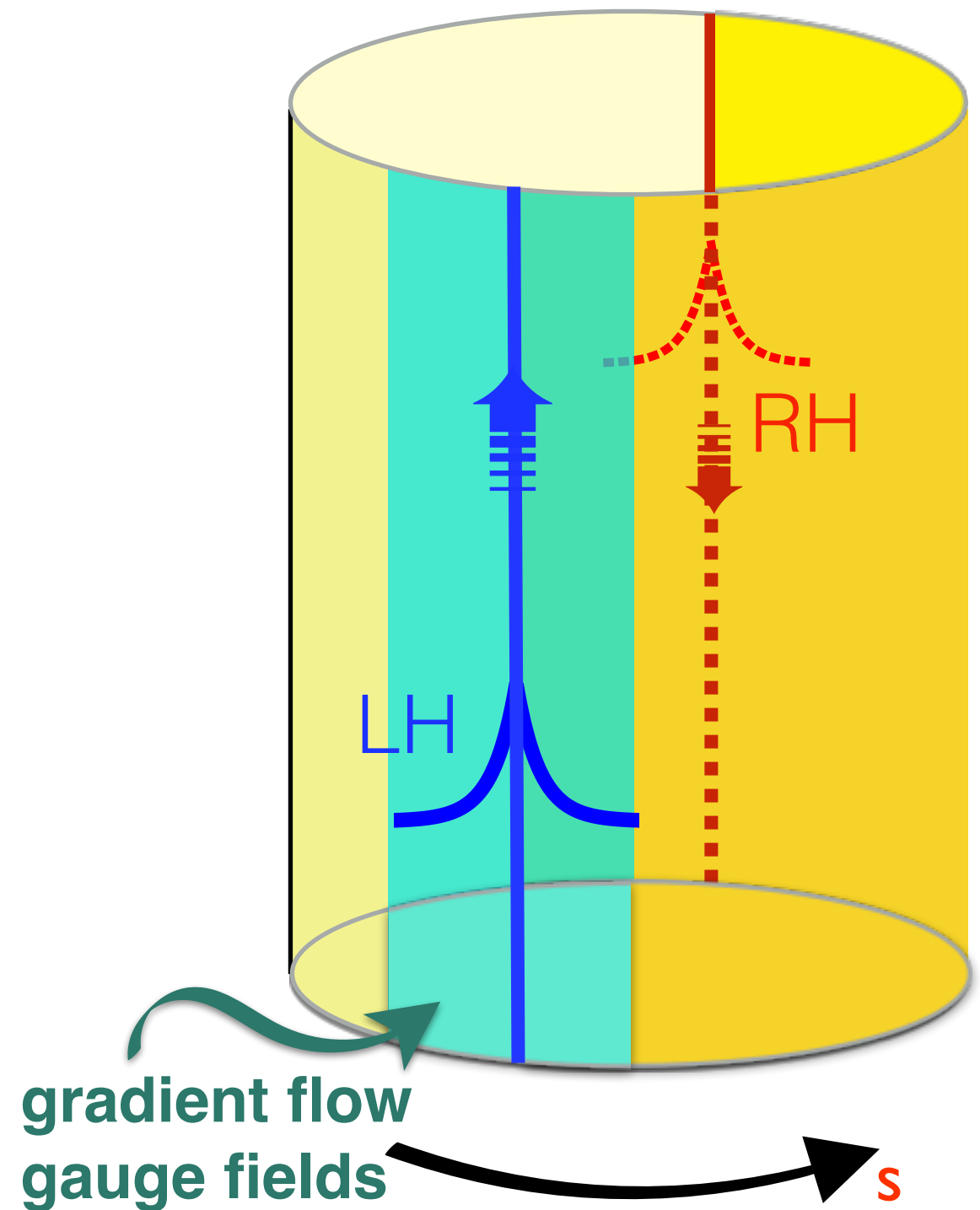
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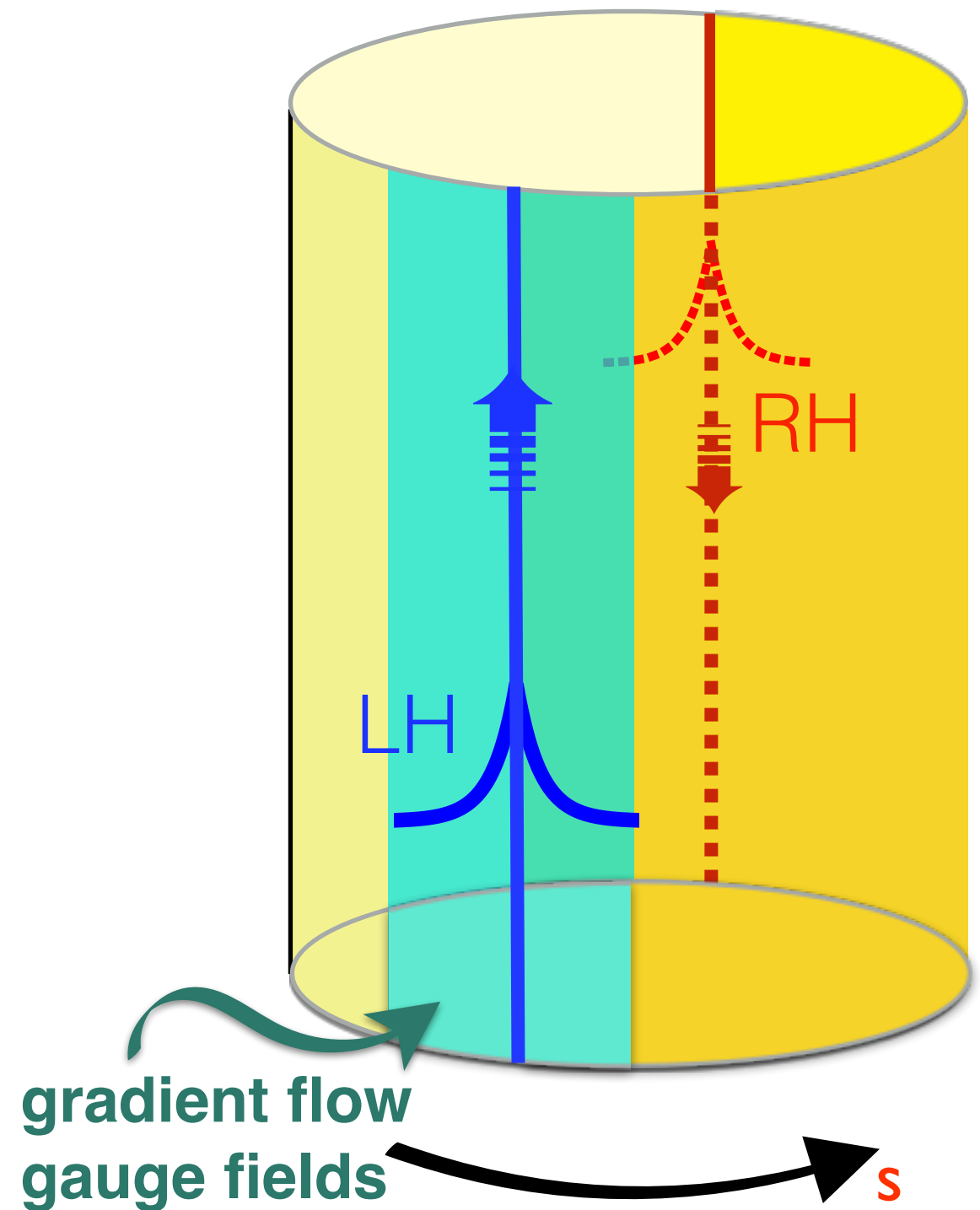
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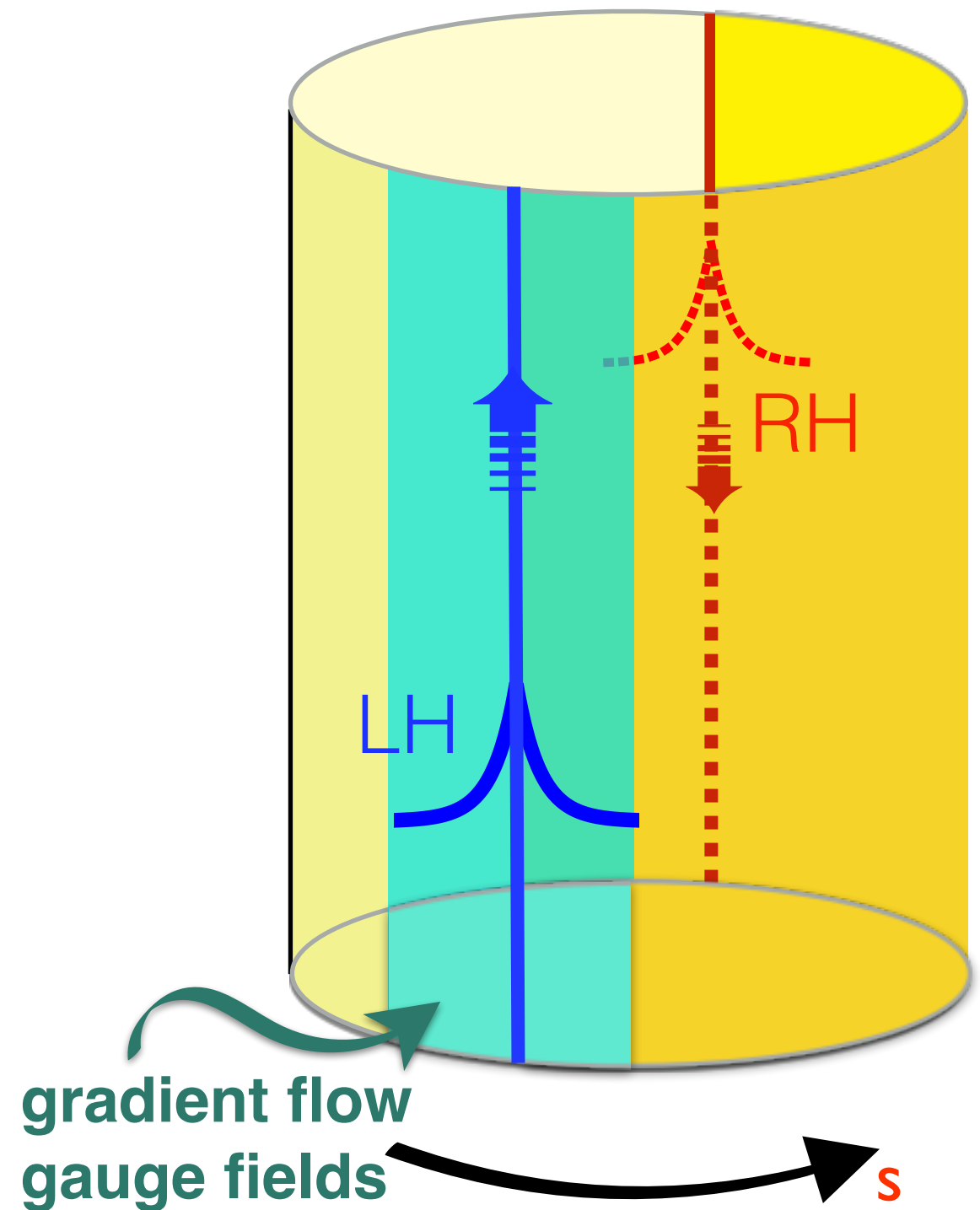
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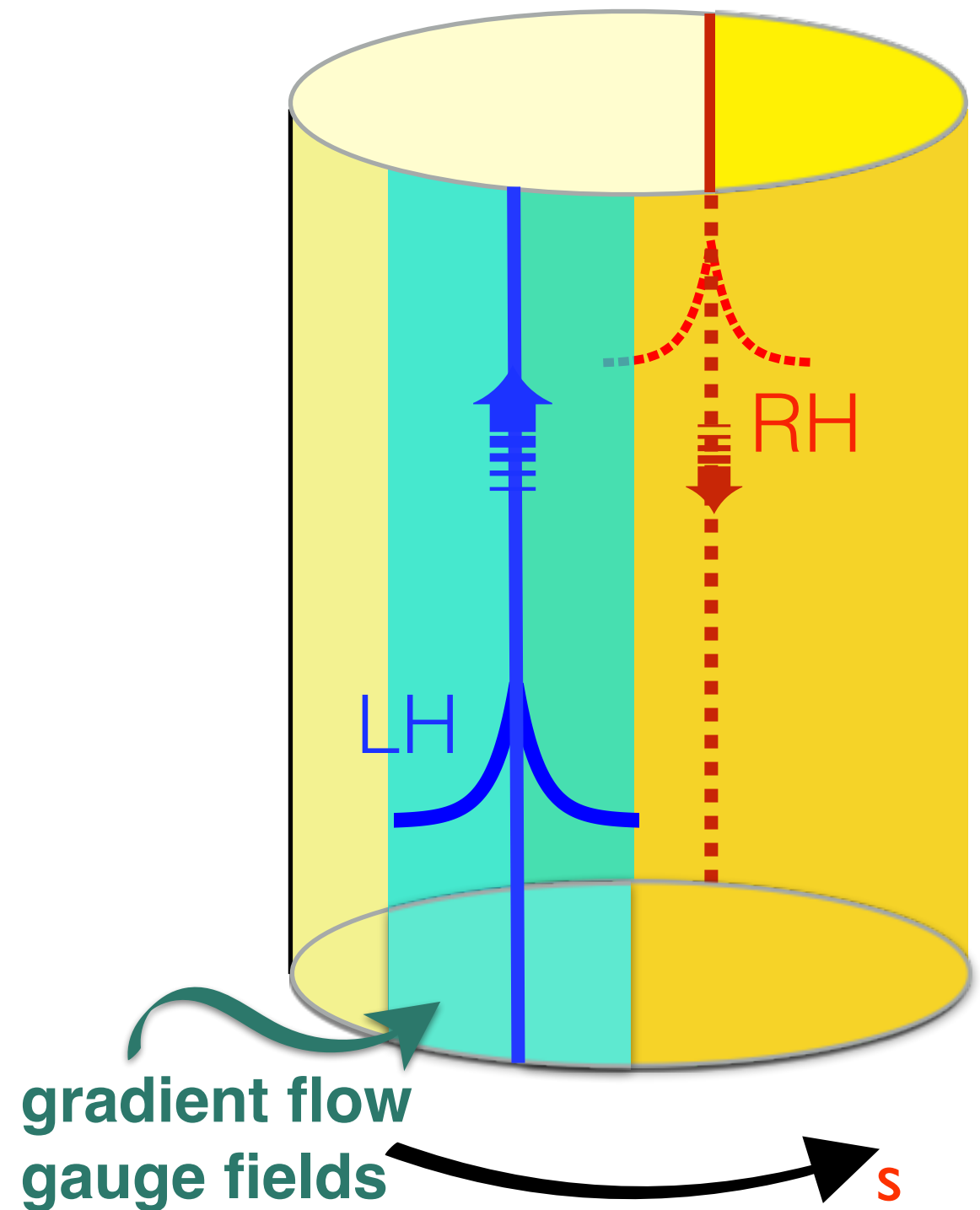
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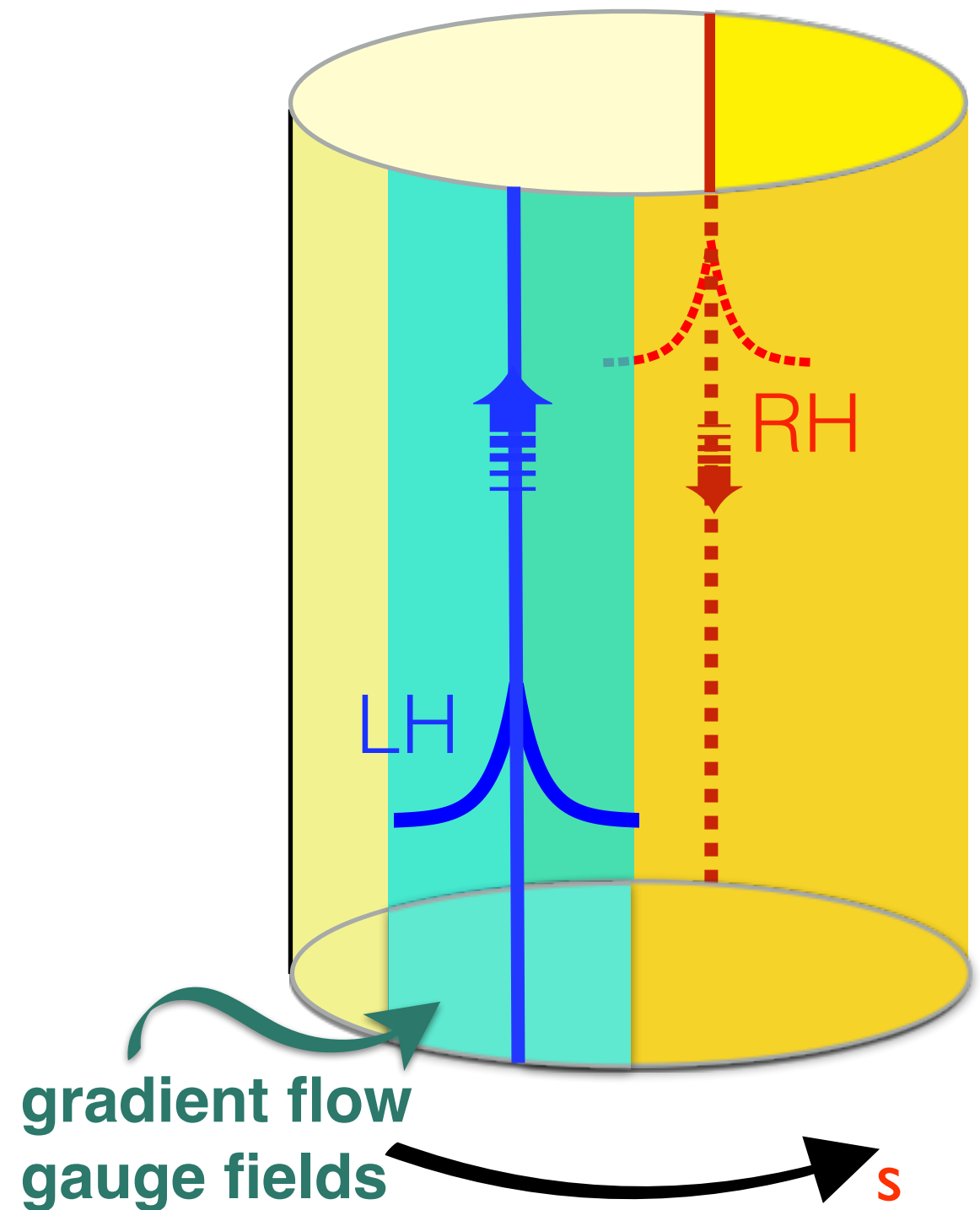
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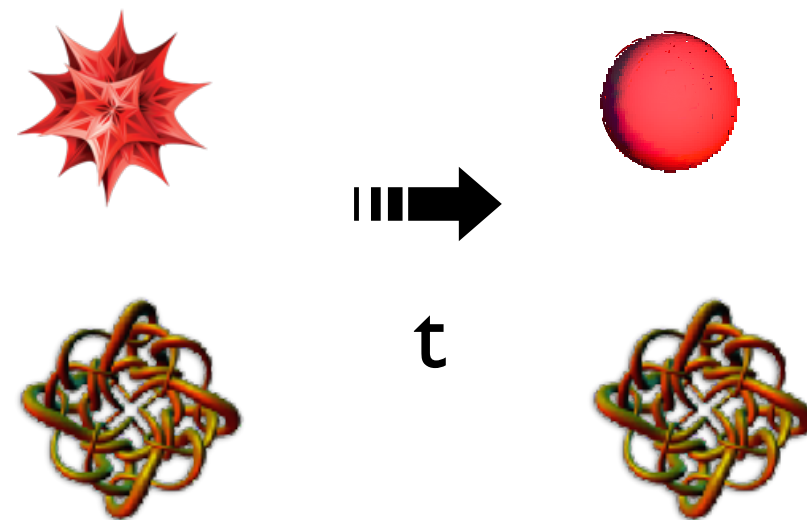
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- method fails when theory is anomalous (a good thing)



Decoupling mirror fermions as soft fluff in a gauge invariant way:

- 😊 Can show that this could only lead to a local 4d quantum field theory if the fermion representation has no gauge anomalies
- 🤔 ...exp(-p²s) form factors are a problem in Minkowski spacetime!...but can take $s \rightarrow \infty$ limit first before lattice spacing $a \rightarrow 0$ using the overlap operator method. OK theory?
- 😬 gradient flow doesn't damp out instantons, which can induce interactions with fluff

$$\frac{\partial \bar{A}_\mu(x, t)}{\partial t} = -D_\nu \bar{F}_{\mu\nu}$$



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Should we take Fluff seriously? What is its phenomenology? Can it explain the strong CP problem or have other effects? Does it have similarly soft gravitational interactions?

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- A possible lattice regularization that does not break gauge invariance implies the existence of mirror matter, Fluff, with bizarre nonlocal interactions but potentially quite hidden
- Implications/viability not well understood!
- **What could go wrong throwing away locality?**

