Neutrino Physics

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Neutrino Unbound: http://www.nu.to.infn.it

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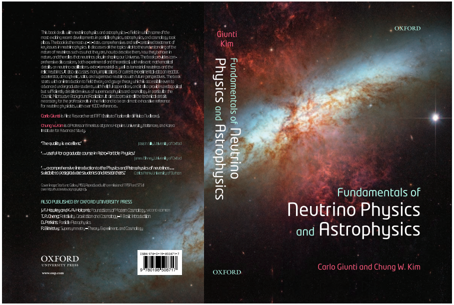


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics
and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term

Part II: Neutrino Oscillations in Vacuum and in Matter

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Mixing and Oscillations
- Neutrino Oscillations in Matter

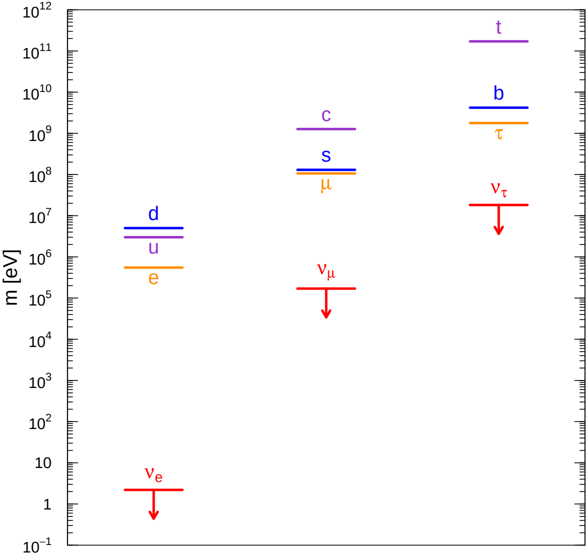
Part III: Phenomenology of Three-Neutrino Mixing

- Solar Neutrinos and KamLAND
- Atmospheric Neutrinos and LBL
- Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Experimental Neutrino Anomalies
- Conclusions

Part I

Theory of Neutrino Masses and Mixing

Fermion Mass Spectrum



Dirac Neutrino Masses and Mixing

Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0 \qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$

- Dirac Lagrangian: $\mathscr{L}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$

- Chiral decomposition: $\nu_L \equiv P_L \nu\,, \qquad \nu_R \equiv P_R \nu\,, \qquad \nu = \nu_L + \nu_R$

$$P_L \equiv \frac{1-\gamma^5}{2}\,, \quad P_R \equiv \frac{1+\gamma^5}{2}\,, \quad P_L^2 = P_R^2 = 1\,, \quad P_L P_R = P_R P_L = 0$$

$$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$$

- In SM only $\nu_L \Longrightarrow$ no Dirac mass

- Oscillation experiments have shown that neutrinos are massive

- Simplest extension of the SM: add ν_R

Higgs Mechanism in SM

- Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$ $\qquad |\Phi|^2 = \Phi^\dagger \Phi = \phi_+^\dagger \phi_+ + \phi_0^\dagger \phi_0$

- Higgs Lagrangian: $\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$

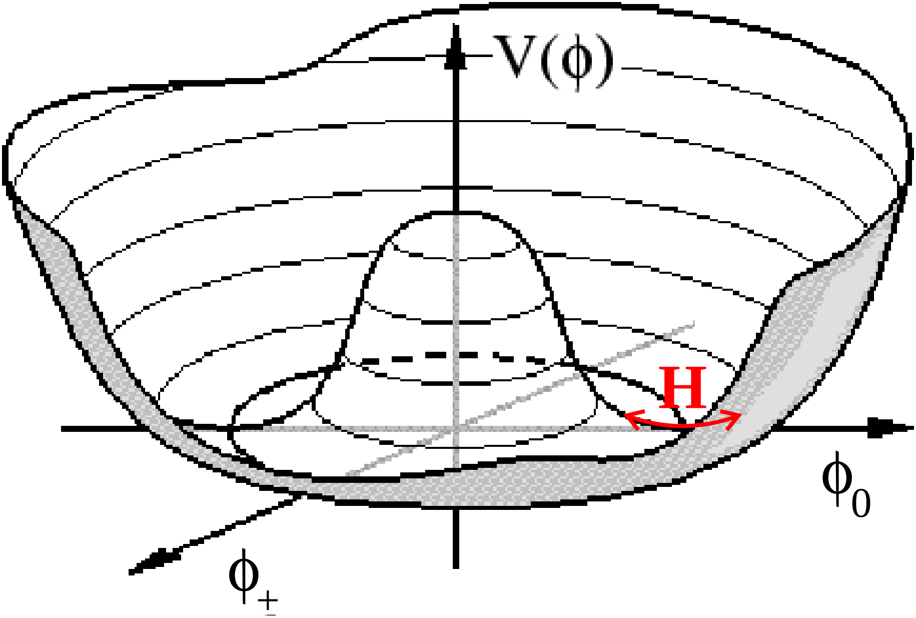
- Higgs Potential: $V(|\Phi|^2) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$

- $\mu^2 < 0$ and $\lambda > 0 \implies V(|\Phi|^2) = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2$, with $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$

- Vacuum: V_{min} for $|\Phi|^2 = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

- Spontaneous Symmetry Breaking: $SU(2)_L \times U(1)_Y \to U(1)_Q$

- Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$


V(ϕ)
H
ϕ0
ϕ±

Dirac Lepton Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -y^{\ell}\,\overline{L_L}\,\Phi\,\ell_R - y^{\nu}\,\overline{L_L}\,\widetilde{\Phi}\,\nu_R + \text{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2\,\Phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} = & -\frac{y^{\ell}}{\sqrt{2}}\begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}\ell_R \\ & -\frac{y^{\nu}}{\sqrt{2}}\begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}\nu_R + \text{H.c.} \end{aligned}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\, y^{\ell}\,\frac{v}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R - y^{\nu}\,\frac{v}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R$$

$$-\,\frac{y^{\ell}}{\sqrt{2}}\,\overline{\ell_L}\,\ell_R\,H - \frac{y^{\nu}}{\sqrt{2}}\,\overline{\nu_L}\,\nu_R\,H + \mathrm{H.c.}$$

$$m_{\ell} = y^{\ell}\,\frac{v}{\sqrt{2}} \qquad\qquad m_{\nu} = y^{\nu}\,\frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_{\ell}}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_{\nu}}{v}$$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -\sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \text{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta}\, \overline{\ell'_{\alpha L}}\, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta}\, \overline{\nu'_{\alpha L}}\, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \left[\overline{\boldsymbol{\ell}'_L}\, Y'^{\ell}\, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\, Y'^{\nu}\, \boldsymbol{\nu}'_R \right] + \mathrm{H.c.}$$

$$\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \boldsymbol{\ell}'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y'^{\ell} \equiv \begin{pmatrix} Y'^{\ell}_{ee} & Y'^{\ell}_{e\mu} & Y'^{\ell}_{e\tau} \\ Y'^{\ell}_{\mu e} & Y'^{\ell}_{\mu\mu} & Y'^{\ell}_{\mu\tau} \\ Y'^{\ell}_{\tau e} & Y'^{\ell}_{\tau\mu} & Y'^{\ell}_{\tau\tau} \end{pmatrix} \qquad Y'^{\nu} \equiv \begin{pmatrix} Y'^{\nu}_{ee} & Y'^{\nu}_{e\mu} & Y'^{\nu}_{e\tau} \\ Y'^{\nu}_{\mu e} & Y'^{\nu}_{\mu\mu} & Y'^{\nu}_{\mu\tau} \\ Y'^{\nu}_{\tau e} & Y'^{\nu}_{\tau\mu} & Y'^{\nu}_{\tau\tau} \end{pmatrix}$$

$$M'^{\ell} = \frac{v}{\sqrt{2}}\, Y'^{\ell} \qquad M'^{\nu} = \frac{v}{\sqrt{2}}\, Y'^{\nu}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\ell'_L}\,Y'^{\ell}\,\ell'_R + \overline{\nu'_L}\,Y'^{\nu}\,\nu'_R\right] + \mathrm{H.c.}$$

Diagonalization of Y'^{ℓ} and Y'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\ell'_L = V^{\ell}_L\,\ell_L \qquad \ell'_R = V^{\ell}_R\,\ell_R \qquad \nu'_L = V^{\nu}_L\,\mathbf{n}_L \qquad \nu'_R = V^{\nu}_R\,\mathbf{n}_R$$

Kinetic terms are invariant under unitary transformations of the fields

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\ell_L}V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R \ell_R + \overline{\nu_L}V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R \nu_R\right] + \mathrm{H.c.}$$

$$V^{\ell\dagger}_L\ Y'^{\ell}\ V^{\ell}_R = Y^{\ell} \qquad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha}\,\delta_{\alpha\beta} \qquad (\alpha,\beta = e,\mu,\tau)$$

$$V^{\nu\dagger}_L\ Y'^{\nu}\ V^{\nu}_R = Y^{\nu} \qquad Y^{\nu}_{kj} = y^{\nu}_{k}\,\delta_{kj} \qquad (k,j = 1,2,3)$$

Real and Positive y^{ℓ}_{α}, y^{ν}_{k}

Massive Chiral Lepton Fields

$\ell_L = V_L^{\ell\dagger}\,\ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\ell_R = V_R^{\ell\dagger}\,\ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger}\,\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger}\,\boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\ell_L}\,Y^{\ell}\,\ell_R + \overline{\mathbf{n}_L}\,Y^{\nu}\,n_R\right] + \mathrm{H.c.} \\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y_\alpha^{\ell}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y_k^{\nu}\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \mathrm{H.c.} \end{aligned}$$

Massive Dirac Lepton Fields

$$\ell_\alpha \equiv \ell_{\alpha L} + \ell_{\alpha R} \qquad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \qquad (k = 1, 2, 3)$$

$$\mathscr{L}_{H,\mathrm{L}} = -\sum_{\alpha=e,\mu,\tau} \frac{y^\ell_\alpha\, v}{\sqrt{2}}\, \overline{\ell_\alpha}\, \ell_\alpha - \sum_{k=1}^{3} \frac{y^\nu_k\, v}{\sqrt{2}}\, \overline{\nu_k}\, \nu_k \qquad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y^\ell_\alpha}{\sqrt{2}}\, \overline{\ell_\alpha}\, \ell_\alpha\, H - \sum_{k=1}^{3} \frac{y^\nu_k}{\sqrt{2}}\, \overline{\nu_k}\, \nu_k\, H \qquad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y^\ell_\alpha\, v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \qquad\qquad m_k = \frac{y^\nu_k\, v}{\sqrt{2}} \qquad (k = 1, 2, 3)$$

Lepton-Higgs coupling $\propto$ Lepton Mass

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \mathrm{H.c.}$$

Weak Charged Current: $\qquad j_W^\rho = j_{W,\mathrm{L}}^\rho + j_{W,\mathrm{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\mathrm{L}}^\rho = \sum_{\alpha=e,\mu,\tau} \overline{\nu'_\alpha} \gamma^\rho \left(1-\gamma^5\right) \ell'_\alpha = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{\alpha L}} \gamma^\rho \ell'_{\alpha L} = 2 \overline{\boldsymbol{\nu}'_L} \gamma^\rho \boldsymbol{\ell}'_L$$

$$\underline{\boldsymbol{\ell}'_L = V_L^\ell \boldsymbol{\ell}_L} \qquad\qquad \underline{\boldsymbol{\nu}'_L = V_L^\nu \mathbf{n}_L}$$

$$j_{W,\mathrm{L}}^\rho = 2 \overline{\mathbf{n}_L} V_L^{\nu\dagger} \gamma^\rho V_L^\ell \boldsymbol{\ell}_L = 2 \overline{\mathbf{n}_L} V_L^{\nu\dagger} V_L^\ell \gamma^\rho \boldsymbol{\ell}_L = 2 \overline{\mathbf{n}_L} U^\dagger \gamma^\rho \boldsymbol{\ell}_L$$

Mixing Matrix

$$U^\dagger = V_L^{\nu\dagger} V_L^\ell \qquad\qquad \boxed{U = V_L^{\ell\dagger} V_L^\nu}$$

- Definition: Left-Handed Flavor Neutrino Fields

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton:

$$j^{\rho}_{W,\mathrm{L}} = 2\left(\overline{\nu_{eL}}\,\gamma^{\rho}\,e_L + \overline{\nu_{\mu L}}\,\gamma^{\rho}\,\mu_L + \overline{\nu_{\tau L}}\,\gamma^{\rho}\,\tau_L\right)$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Standard Model: Lepton numbers are conserved

$$\mathscr{L}_{\text{mass}}^{\text{D}} = - \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^{\text{D}} & m_{e\mu}^{\text{D}} & m_{e\tau}^{\text{D}} \\ m_{\mu e}^{\text{D}} & m_{\mu\mu}^{\text{D}} & m_{\mu\tau}^{\text{D}} \\ m_{\tau e}^{\text{D}} & m_{\tau\mu}^{\text{D}} & m_{\tau\tau}^{\text{D}} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

L_e, L_μ, L_τ are not conserved

L is conserved: $\quad L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Mixing Matrix

- Leptonic Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\boldsymbol{\ell}_L$

- $$U = V_L^{\ell\dagger}\,V_L^{\nu} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \quad \Longrightarrow \quad \begin{array}{ll} \dfrac{N\,(N-1)}{2} = 3 & \text{Mixing Angles} \\ \dfrac{N\,(N+1)}{2} = 6 & \text{Phases} \end{array}$$

- Not all phases are physical observables

- Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, U^{*}_{\alpha k}\, \gamma^{\rho}\, \ell_{\alpha L}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)
$$\nu_k \to e^{i\varphi_k}\, \nu_k \quad (k = 1, 2, 3)\,, \qquad \ell_\alpha \to e^{i\varphi_\alpha}\, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$
- Performing this transformation, the Charged Current becomes
$$j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, e^{-i\varphi_k}\, U^{*}_{\alpha k}\, e^{i\varphi_\alpha}\, \gamma^{\rho}\, \ell_{\alpha L}$$
$$j^{\rho}_{W,\mathrm{L}} = 2\, \underbrace{e^{-i(\varphi_1 - \varphi_e)}}_{1} \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}}\, \underbrace{e^{-i(\varphi_k - \varphi_1)}}_{2}\, U^{*}_{\alpha k}\, \underbrace{e^{i(\varphi_\alpha - \varphi_e)}}_{2}\, \gamma^{\rho}\, \ell_{\alpha L}$$
- There are 5 arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant.

- The mixing matrix contains 1 Physical Phase.
- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\qquad$ $s_{ab} \equiv \sin\vartheta_{ab}$ $\qquad$ $0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $\qquad$ $0 \leq \delta_{13} \leq 2\pi$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

Standard Parameterization

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Phase Convention

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} e^{i\delta_{23}} \\ 0 & -s_{23} e^{-i\delta_{13}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example of Different Parameterization

$$U = \begin{pmatrix} c'_{12} & s'_{12} e^{-i\delta'_{12}} & 0 \\ -s'_{12} e^{i\delta'_{12}} & c'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13} \\ 0 & 1 & 0 \\ -s'_{13} & 0 & c'_{13} \end{pmatrix}$$

CP Violation

$$U \neq U^* \quad \Longrightarrow \quad \text{CP Violation}$$

Jarlskog Rephasing Invariant

$$\boxed{J = \Im\mathfrak{m}\left[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}\right]}$$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunietz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

Jarlskog Rephasing Invariant

- Simplest rephasing invariants: $|U_{\alpha k}| = U_{\alpha k} U_{\alpha k}^{*}\,, \quad U_{\alpha k} U_{\alpha j}^{*} U_{\beta k}^{*} U_{\beta j}$

$$\Im\mathfrak{m}\left[U_{\alpha k} U_{\alpha j}^{*} U_{\beta k}^{*} U_{\beta j}\right] = \pm J$$

$$J = \Im\mathfrak{m}\left[U_{e2} U_{e3}^{*} U_{\mu 2}^{*} U_{\mu 3}\right] = \Im\mathfrak{m} \begin{pmatrix} \cdot & \circ & \times \\ \cdot & \times & \circ \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- In standard parameterization:

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^{2} s_{13} \sin\delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \, \sin 2\vartheta_{23} \, \cos\vartheta_{13} \, \sin 2\vartheta_{13} \, \sin\delta_{13} \end{aligned}$$

- Jarlskog invariant is useful for quantifying CP violation in a parameterization-independent way
- All measurable CP-violation effects depend on J.

Maximal CP Violation

- Maximal CP violation is defined as the case in which $|J|$ has its maximum possible value

$$|J|_{\text{max}} = \frac{1}{6\sqrt{3}}$$

- In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4\,, \quad s_{13} = 1/\sqrt{3}\,, \quad \sin\delta_{13} = \pm 1$$

- This case is called Trimaximal Mixing. All the absolute values of the elements of the mixing matrix are equal to $1/\sqrt{3}$:

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp\frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

Lepton Numbers Violating Processes

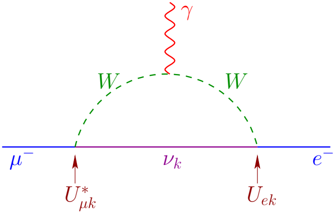
Dirac mass term allows L_e, L_μ, L_τ violating processes

Example: $\mu^\pm \to e^\pm + \gamma\,, \qquad \mu^\pm \to e^\pm + e^+ + e^-$

$$\boxed{\mu^- \to e^- + \gamma}$$

$\sum_k U_{\mu k}^* U_{ek} = 0 \Longrightarrow$ only part of ν_k propagator $\propto m_k$ contributes

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi} \left| \sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$



Suppression factor: $\frac{m_k}{m_W} \lesssim 10^{-11}$ for $m_k \lesssim 1\,\text{eV}$

$(\text{BR})_{\text{the}} \lesssim 10^{-47} \qquad (\text{BR})_{\text{exp}} \lesssim 10^{-11}$

Majorana Neutrino Masses and Mixing

Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- Dirac Equation: $(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$
- Chiral decomposition of a Fermion Field: $\psi = \psi_L + \psi_R$
- Equations for the Chiral components are coupled by mass:

$$i\gamma^{\mu}\partial_{\mu}\psi_L = m\,\psi_R$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = m\,\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^{\mu}\partial_{\mu}\psi_L = 0$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = 0$$

- A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor).

- ψ_L and ψ_R have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \qquad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to parity violation ($\psi_L \overset{P}{\rightleftharpoons} \psi_R$)

- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields $\Longrightarrow$ Two-component Theory of a Massless Neutrino (1957)

- $V - A$ Charged-Current Weak Interactions $\Longrightarrow$ ν_L

- In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of ν_R

Majorana Equation

- Can a two-component spinor describe a massive fermion? Yes! (E. Majorana, 1937)
- Trick: ψ_R and ψ_L are not independent: $\boxed{\psi_R = \mathcal{C}\,\overline{\psi_L}^T}$
- $\mathcal{C}\,\overline{\psi_L}^T$ is right-handed: $P_R \mathcal{C}\,\overline{\psi_L}^T = \mathcal{C}\,\overline{\psi_L}^T$ $\qquad (\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu)$
- Majorana Equation: $\boxed{i\gamma^\mu \partial_\mu \psi_L = m\,\mathcal{C}\,\overline{\psi_L}^T}$
- Majorana Field: $\psi = \psi_L + \psi_R = \psi_L + \mathcal{C}\,\overline{\psi_L}^T$
- Majorana Condition: $\boxed{\psi = \mathcal{C}\,\overline{\psi}^T = \psi^C}$
- Only two independent components: $\psi = \begin{pmatrix} i\sigma^2 \chi_L^* \\ \chi_L \end{pmatrix} = \begin{pmatrix} \chi_{L2}^* \\ -\chi_{L1}^* \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix}$

- $\psi = \psi^C$ implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi^C}\gamma^\mu\psi^C = -\psi^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\overline{\psi}^T = \overline{\psi}\mathcal{C}\gamma^{\mu\,T}\mathcal{C}^\dagger\psi = -\overline{\psi}\gamma^\mu\psi = 0$$

Majorana Lagrangian

Dirac Lagrangian

$$\begin{aligned} \mathscr{L}^{\mathrm{D}} &= \overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu \\ &= \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R) \end{aligned}$$

$$\nu_R \quad \to \quad \nu_L^C = \mathcal{C}\,\overline{\nu_L}^T$$

$$\frac{1}{2}\mathscr{L}^{\mathrm{D}} \quad \to \quad \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right)$$

Majorana Lagrangian

$$\begin{aligned} \mathscr{L}^{\mathrm{M}} &= \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right) \\ &= \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(\overline{\nu_L^C}\,\nu_L + \overline{\nu_L}\,\nu_L^C\right) \end{aligned}$$

Lepton Number

$$\cancel{L=+1} \quad \longleftarrow \quad \boxed{\nu = \nu^C} \quad \longrightarrow \quad \cancel{L=-1}$$

$$\nu_L \quad \Longrightarrow \quad L=+1 \qquad\qquad \nu_L^C \quad \Longrightarrow \quad L=-1$$

$$\mathscr{L}^{\mathrm{M}} = \overline{\nu_L}\, i\partial\!\!\!/\, \nu_L - \frac{m}{2}\left(\overline{\nu_L^C}\,\nu_L + \overline{\nu_L}\,\nu_L^C\right)$$

Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

Best process to find violation of Total Lepton Number:

Neutrinoless Double-β Decay

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2e^- + \cancel{2\bar{\nu}_e} \qquad (\beta\beta^-_{0\nu})$$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z-2) + 2e^+ + \cancel{2\nu_e} \qquad (\beta\beta^+_{0\nu})$$

No Majorana Neutrino Mass in the SM

- Majorana Mass Term $\propto \left[\nu_L^T \, \mathcal{C}^\dagger \, \nu_L - \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM (one for each lepton generation)
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed Higgs triplet with $Y = 2$

Effective Majorana Mass

- Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- Dimensionless action: $I = \int \mathrm{d}^4x \, \mathscr{L}(x) \Longrightarrow \mathscr{L}(x) \sim [E]^4$
- Kinetic terms: $\overline{\psi} i \partial\!\!\!/ \psi \sim [E]^4, \quad (\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- Mass terms: $m \overline{\psi} \psi \sim [E]^4, \quad m^2 \phi^\dagger \phi \sim [E]^4$
- CC weak interaction: $g \, \overline{\nu_L} \gamma^\rho \ell_L W_\rho \sim [E]^4$
- Yukawa couplings: $y \, \overline{L_L} \Phi \ell_R \sim [E]^4$
- Product of fields $\mathscr{O}_d$ with energy dimension $d \equiv$ dim-d operator
- $\mathscr{L}_{(\mathscr{O}_d)} = C_{(\mathscr{O}_d)} \mathscr{O}_d \quad \Longrightarrow \quad C_{(\mathscr{O}_d)} \sim [E]^{4-d}$
- $\mathscr{O}_{d>4}$ are not renormalizable

- SM Lagrangian includes all $\mathscr{O}_{d \leq 4}$ invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- SM cannot be considered as the final theory of everything
- SM is an effective low-energy theory
- It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- It is plausible that at low-energy there are effective non-renormalizable $\mathscr{O}_{d>4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- All $\mathscr{O}_d$ must respect $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies

- $\mathscr{O}_{d>4}$ is suppressed by a coefficient $\mathcal{M}^{4-d}$, where $\mathcal{M}$ is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$

- Analogy with $\quad \mathscr{L}_{\text{eff}}^{(\text{CC})} \propto G_{\text{F}}\,(\overline{\nu_{eL}}\gamma^{\rho}e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots$

$$\mathscr{O}_6 \to (\overline{\nu_{eL}}\gamma^{\rho}e_L)\,(\overline{e_L}\gamma_{\rho}\nu_{eL}) + \ldots \qquad\qquad \frac{g_6}{\mathcal{M}^2} \to \frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

- $\mathcal{M}^{4-d}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM

- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality

- $\mathscr{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)

- $\mathscr{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)

- Only one dim-5 operator:

$$\mathscr{O}_5 = (L_L^T \, \sigma_2 \, \Phi) \, \mathcal{C}^\dagger \, (\Phi^T \, \sigma_2 \, L_L) + \text{H.c.}$$
$$= \frac{1}{2} \, (L_L^T \, \mathcal{C}^\dagger \, \sigma_2 \, \vec{\tau} \, L_L) \cdot (\Phi^T \, \sigma_2 \, \vec{\tau} \, \Phi) + \text{H.c.}$$

$$\mathscr{L}_5 = \frac{g_5}{2\mathcal{M}} \, (L_L^T \, \mathcal{C}^\dagger \, \sigma_2 \, \vec{\tau} \, L_L) \cdot (\Phi^T \, \sigma_2 \, \vec{\tau} \, \Phi) + \text{H.c.}$$

- Electroweak Symmetry Breaking: $\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathscr{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \, \frac{g_5 \, v^2}{\mathcal{M}} \, \nu_L^T \, \mathcal{C}^\dagger \, \nu_L + \text{H.c.} \quad \Longrightarrow \quad \boxed{m = \frac{g_5 \, v^2}{\mathcal{M}}}$

- The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM

- $m \propto \dfrac{v^2}{\mathcal{M}} \propto \dfrac{m_{\text{D}}^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses (special case: See-Saw Mechanism)

- Example: $m_{\text{D}} \sim v \sim 10^2\,\text{GeV}$ and $\mathcal{M} \sim 10^{15}\,\text{GeV} \implies m \sim 10^{-2}\,\text{eV}$

Mixing of Three Majorana Neutrinos

- $$\nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$$

$$\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \nu'^{T}_{L} \, \mathcal{C}^{\dagger} \, M^{L} \, \nu'_{L} + \text{H.c.}$$

$$= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} + \text{H.c.}$$

- In general, the matrix M^L is a complex symmetric matrix

$$\sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} = - \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, M^{L}_{\alpha\beta} \, (\mathcal{C}^{\dagger})^{T} \, \nu'_{\alpha L}$$

$$= \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\beta\alpha} \, \nu'_{\beta L}$$

$$M^{L}_{\alpha\beta} = M^{L}_{\beta\alpha} \quad \Longleftrightarrow \quad M^{L} = M^{L\,T}$$

- $\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^{T}_{L} \, \mathcal{C}^{\dagger} \, M^{L} \, \boldsymbol{\nu}'_{L} + \text{H.c.}$

- $\boldsymbol{\nu}'_{L} = V^{\nu}_{L} \, \mathbf{n}_{L} \qquad \Longrightarrow \qquad \mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2} \boldsymbol{\nu}'^{T}_{L} \, (V^{\nu}_{L})^{T} \, \mathcal{C}^{\dagger} \, M^{L} \, V^{\nu}_{L} \, \boldsymbol{\nu}'_{L} + \text{H.c.}$

- $(V^{\nu}_{L})^{T} \, M^{L} \, V^{\nu}_{L} = M \, , \qquad M_{kj} = m_{k} \, \delta_{kj} \qquad (k, j = 1, 2, 3)$

- Left-handed chiral fields with definite mass: $\mathbf{n}_{L} = V^{\nu \dagger}_{L} \, \boldsymbol{\nu}'_{L} = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\begin{aligned} \mathscr{L}^{\text{M}}_{\text{mass}} &= \frac{1}{2} \left(\mathbf{n}^{T}_{L} \, \mathcal{C}^{\dagger} \, M \, \mathbf{n}_{L} - \overline{\mathbf{n}_{L}} \, M \, \mathcal{C} \, \mathbf{n}^{T}_{L} \right) \\ &= \frac{1}{2} \sum_{k=1}^{3} m_{k} \left(\nu^{T}_{kL} \, \mathcal{C}^{\dagger} \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu^{T}_{kL} \right) \end{aligned}$$

- Majorana fields of massive neutrinos: $\nu_{k} = \nu_{kL} + \nu^{C}_{kL} \qquad \boxed{\nu^{C}_{k} = \nu_{k}}$

- $\mathbf{n} = \begin{pmatrix} \nu_{1} \\ \nu_{2} \\ \nu_{3} \end{pmatrix} \Longrightarrow \mathscr{L}^{\text{M}} = \frac{1}{2} \sum_{k=1}^{3} \overline{\nu_{k}} \, (i \partial\!\!\!/ - m_{k}) \, \nu_{k} = \frac{1}{2} \, \overline{\mathbf{n}} \, (i \partial\!\!\!/ - M) \, \mathbf{n}$

Mixing Matrix

- Leptonic Weak Charged Current:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L \qquad \text{with} \qquad U = V_L^{\ell\dagger}\,V_L^{\nu}$$

- Definition of the left-handed flavor neutrino fields:

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- Leptonic Weak Charged Current has the SM form

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2\sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- Majorana Mass Term $\mathscr{L}^{\mathrm{M}}_{\mathrm{mass}} = \frac{1}{2} \sum_{k=1}^{3} m_k \, \nu_{kL}^{T} \, \mathcal{C}^{\dagger} \, \nu_{kL} + \mathrm{H.c.}$ is not invariant under the global $\mathrm{U}(1)$ gauge transformations

$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \qquad (k = 1, 2, 3)$$

- Left-handed massive neutrino fields cannot be rephased in order to eliminate two Majorana phases factorized on the right of mixing matrix:

$$\boxed{U = U^{\mathrm{D}} \, D^{\mathrm{M}}} \qquad D^{\mathrm{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- U^{D} is analogous to a Dirac mixing matrix, with one Dirac phase
- Standard parameterization:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- Jarlskog rephasing invariant: $J = c_{12} s_{12} c_{23} s_{23} c_{13}^{2} s_{13} \sin\delta_{13}$

Dirac-Majorana Mass Term

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
  - One Generation
  - See-Saw Mechanism
  - Majorana Neutrino Mass?
  - Number of Massive Neutrinos?

One Generation

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \mathscr{L}_{\text{mass}}^{\text{D}} + \mathscr{L}_{\text{mass}}^{L} + \mathscr{L}_{\text{mass}}^{R}$$

$$\mathscr{L}_{\text{mass}}^{\text{D}} = -m_{\text{D}} \, \overline{\nu_R} \, \nu_L + \text{H.c.}$$ Dirac Mass Term

$$\mathscr{L}_{\text{mass}}^{L} = \frac{1}{2} \, m_L \, \nu_L^T \, \mathcal{C}^\dagger \, \nu_L + \text{H.c.}$$ Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{R} = \frac{1}{2} \, m_R \, \nu_R^T \, \mathcal{C}^\dagger \, \nu_R + \text{H.c.}$$ New Majorana Mass Term!

- Column matrix of left-handed chiral fields: $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\,\overline{\nu_R}^T \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2}\, N_L^T\, \mathcal{C}^\dagger\, M\, N_L + \text{H.c.} \qquad M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$$

- The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- Diagonalization: $n_L = U^\dagger\, N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T\, M\, U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \text{Real } m_k \geq 0$$

- $\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \sum_{k=1,2} m_k\, \nu_{kL}^T\, \mathcal{C}^\dagger\, \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k\, \overline{\nu_k}\, \nu_k$

$$\nu_k = \nu_{kL} + \nu_{kL}^C$$

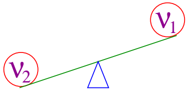
- Massive neutrinos are Majorana! $\qquad \nu_k = \nu_k^C$

See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

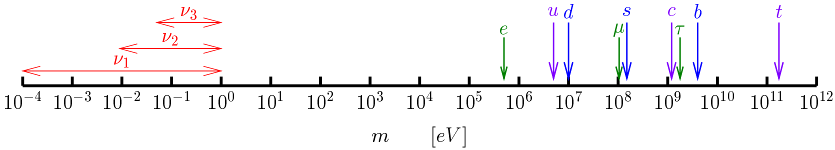
$$\boxed{m_L = 0 \quad m_{\mathrm{D}} \ll m_R}$$

- $\mathscr{L}^L_{\mathrm{mass}}$ is forbidden by SM symmetries $\Longrightarrow m_L = 0$
- $m_{\mathrm{D}} \lesssim v \sim 100\,\mathrm{GeV}$ is generated by SM Higgs Mechanism (protected by SM symmetries)
- m_R is not protected by SM symmetries $\Longrightarrow m_R \sim \mathcal{M}_{\mathrm{GUT}} \gg v$
- $m_1 \simeq \dfrac{m_{\mathrm{D}}^2}{m_R} \qquad m_2 \simeq m_R$



- Natural explanation of smallness of neutrino masses
- Mixing angle is very small: $\tan 2\vartheta = 2\,\dfrac{m_{\mathrm{D}}}{m_R} \ll 1$
- ν_1 is composed mainly of active ν_L: $\nu_{1L} \simeq \nu_L$
- ν_2 is composed mainly of sterile ν_R: $\nu_{2L} \simeq \nu_R^C$

Majorana Neutrino Mass?



known natural explanation of smallness of ν masses

New High Energy Scale $\mathcal{M}$ $\Rightarrow$ { See-Saw Mechanism (if ν_R's exist) / 5-D Non-Renormaliz. Eff. Operator }

both imply { Majorana ν masses $\Longleftrightarrow$ $|\Delta L| = 2$ $\Longleftrightarrow$ $\beta\beta_{0\nu}$ decay / see-saw type relation $m_\nu \sim \dfrac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}}$ }

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Number of Massive Neutrinos?

$$Z \to \nu\bar{\nu} \quad \Rightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$$ active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

STERILE NEUTRINOS

singlets of SM $\Longrightarrow$ no interactions!

active $\to$ sterile transitions are possible if $\nu_4, \ldots$ are light (no see-saw)

$\Downarrow$

disappearance of active neutrinos

Part II

Neutrino Oscillations in Vacuum and in Matter

Neutrino Oscillations in Vacuum

Neutrino Oscillations

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{CC} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\quad \nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad$ States

initial flavor: $\alpha = e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U_{\alpha k}^* \, e^{-iE_k t + i p_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0,0) = \sum_k U_{\alpha k}^* U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t>0, x>0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha \to \nu_\beta}(t,x) = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x) \right|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L,E) &= \left| \sum_k U_{\alpha k}^* \, e^{-im_k^2 L/2E} \, U_{\beta k} \right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathrm{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\mathrm{CP}} \nu_{\alpha L}^{\mathrm{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathrm{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathrm{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $\quad U \leftrightarrows U^* \quad$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

CPT, CP and T Symmetries

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
  - CPT Symmetry
  - CP Symmetry
  - T Symmetry
- Two-Neutrino Mixing and Oscillations
- Neutrino Oscillations in Matter

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CPT}} \quad P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $\quad A^{\text{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\quad \Longrightarrow \quad A^{\text{CPT}}_{\alpha\beta} = 0 \quad$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E} \right)$$

is invariant under CPT: $\quad U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CP}} \quad P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A_{\alpha\beta}^{\text{CP}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$ $\boxed{\text{CPT} \quad \Rightarrow \quad A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}}$

$$\boxed{A_{\alpha\beta}^{\text{CP}}(L, E) = 4 \sum_{k>j} \operatorname{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)}$$

Jarlskog rephasing invariant: $\operatorname{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$$

violation of CP in neutrino oscillations is proportional to

$|U_{e3}| = \sin\vartheta_{13}$ and $\sin\delta_{13}$

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $\quad A^{\text{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

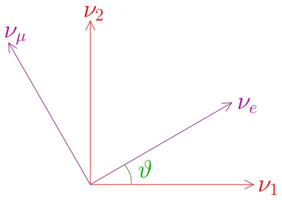
$$\begin{aligned} \text{CPT} \quad \Longrightarrow \quad 0 = A^{\text{CPT}}_{\alpha\beta} &= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} + P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\ &= A^{\text{T}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} = A^{\text{T}}_{\alpha\beta} - A^{\text{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\text{T}}_{\alpha\beta} = A^{\text{CP}}_{\alpha\beta}} \end{aligned}$$

$$\boxed{A^{\text{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\quad \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m^2_2 - m^2_1$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

Types of Experiments

Two-Neutrino Mixing

$$\boxed{P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)}$$

observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL

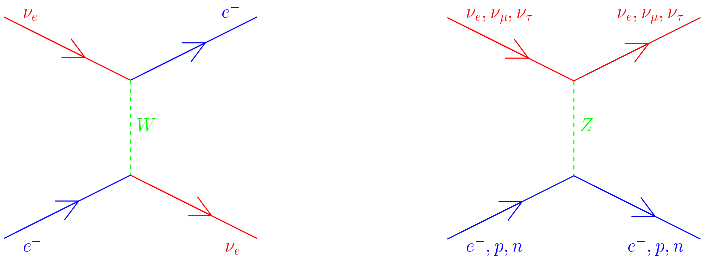
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$

KamLAND

Neutrino Oscillations in Matter

Effective Potentials in Matter



$$\boxed{V_{\mathrm{CC}} = \sqrt{2} G_{\mathrm{F}} N_e} \qquad V_{\mathrm{NC}}^{(e^-)} = -V_{\mathrm{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\mathrm{NC}} = V_{\mathrm{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\mathrm{F}} N_n}$$

$$V_e = V_{\mathrm{CC}} + V_{\mathrm{NC}} \qquad V_\mu = V_\tau = V_{\mathrm{NC}}$$

only $V_{\mathrm{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\mathrm{CC}} = -V_{\mathrm{CC}}$ $\qquad \overline{V}_{\mathrm{NC}} = -V_{\mathrm{NC}}$

Evolution of Flavor Transition Amplitudes

$$\boxed{i \, \frac{\mathrm{d}}{\mathrm{d}x} \, \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2 E V_{\mathrm{CC}} = 2\sqrt{2} E G_{\mathrm{F}} N_e$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

potential due to coherent forward elastic scattering

Two-Neutrino Mixing

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

$$\text{initial } \nu_e \quad \Longrightarrow \quad \begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\boxed{\begin{array}{l} P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 \\ P_{\nu_e\to\nu_e}(x) = |\psi_e(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x) \end{array}}$$

Constant Matter Density

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x} = 0$$

Diagonalization of Effective Hamiltonian

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2_{\mathrm{M}} & 0 \\ 0 & \Delta m^2_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\mathrm{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A^{\mathrm{R}}_{\mathrm{CC}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N^{\mathrm{R}}_e = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\mathrm{F}}}$$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}$$

$$\nu_e \to \nu_\mu \quad \Longrightarrow \quad \begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}\psi_1(0)\\ \psi_2(0)\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}}\end{pmatrix}$$

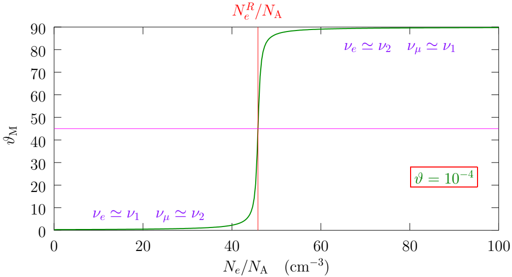
$$\psi_1(x) = \cos\vartheta_{\mathrm{M}} \exp\left(i\frac{\Delta m^2_{\mathrm{M}} x}{4E}\right)$$

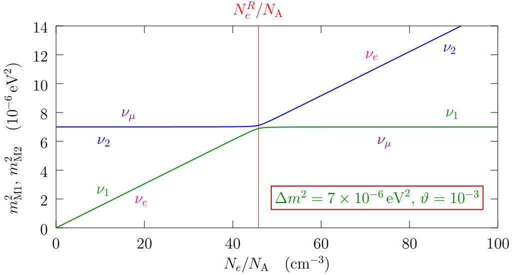
$$\psi_2(x) = \sin\vartheta_{\mathrm{M}} \exp\left(-i\frac{\Delta m^2_{\mathrm{M}} x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 = |-\sin\vartheta_{\mathrm{M}}\psi_1(x) + \cos\vartheta_{\mathrm{M}}\psi_2(x)|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\vartheta_{\mathrm{M}}\ \sin^2\left(\frac{\Delta m^2_{\mathrm{M}} x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)





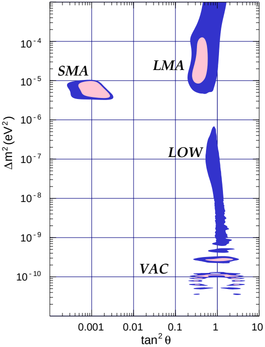
$$\nu_e = \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

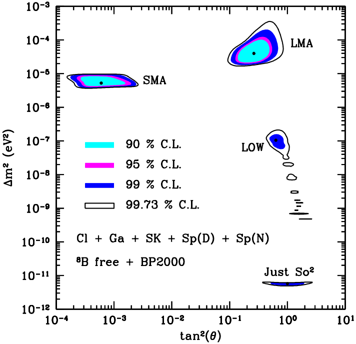
$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

Phenomenology of Solar Neutrinos

LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\text{eV}^2$,	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]

In Neutrino Oscillations Dirac = Majorana

Evolution of Amplitudes: $$\frac{\mathrm{d}\nu_\alpha}{\mathrm{d}t} = \frac{1}{2E} \sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \nu_\beta$$

difference: $$\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})} D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

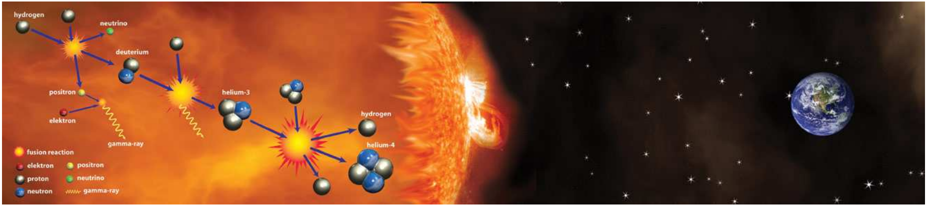
$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2 D \quad \Longrightarrow \quad DM^2 D^\dagger = M^2$$

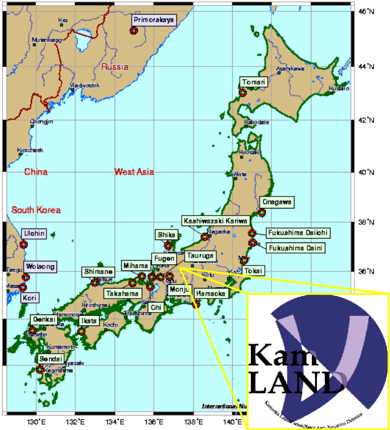
$$U^{(\mathrm{M})} M^2 (U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})} D M^2 D^\dagger (U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})} M^2 (U^{(\mathrm{D})})^\dagger$$

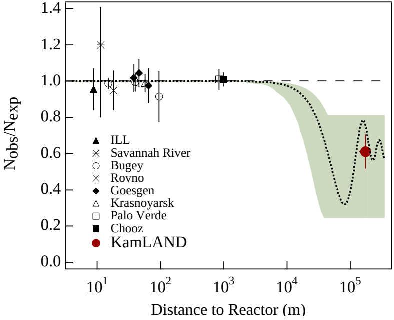
Part III

Phenomenology of Three-Neutrino Mixing

Solar Neutrinos and KamLAND





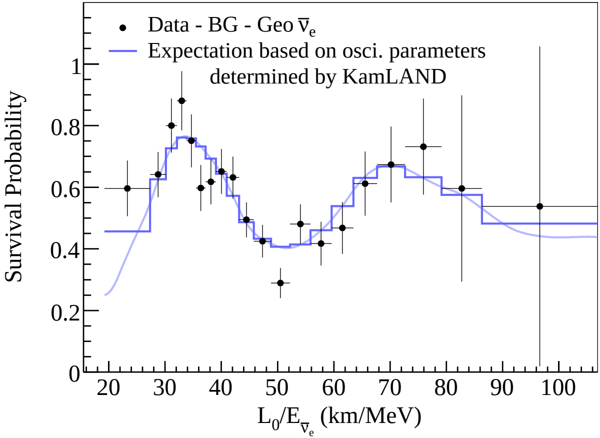


KamLAND

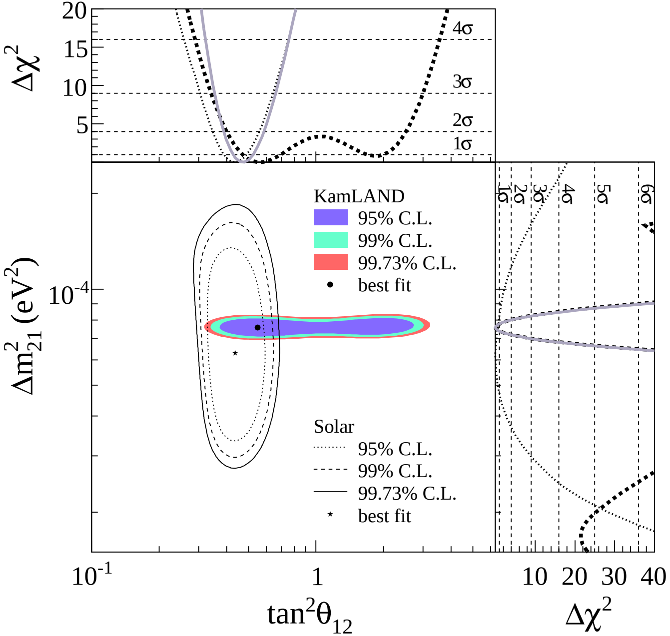
Reactor $\bar{\nu}_e \to \bar{\nu}_e$ confirmation of LMA (December 2002)

53 nuclear power reactors in Japan and Korea $\to$ Kamioka Mine

$\langle L \rangle \simeq 180\,\text{km}$ $\langle E \rangle \simeq 4\,\text{MeV}$

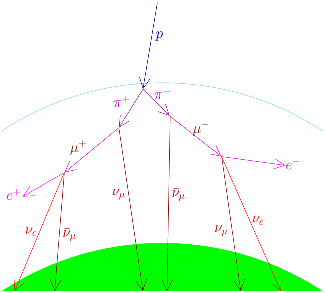


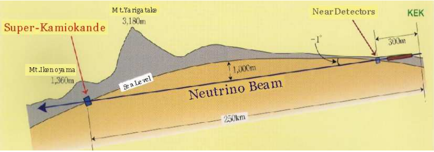
[KamLAND, PRL 100 (2008) 221803]

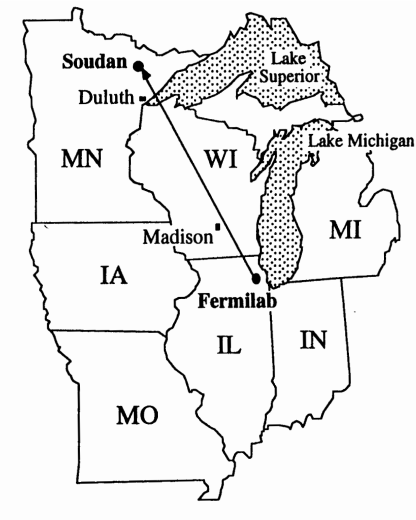


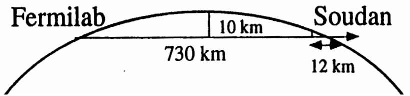
[KamLAND, PRL 100 (2008) 221803]

Atmospheric Neutrinos and LBL

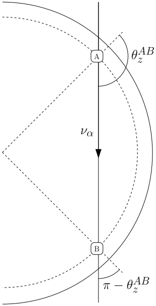








Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1\,\text{GeV} \Rightarrow$ isotropic flux of cosmic rays

$$\phi_{\nu_\alpha}^{(A)}(\theta_z^{AB}) = \phi_{\nu_\alpha}^{(B)}(\pi - \theta_z^{AB}) \qquad \phi_{\nu_\alpha}^{(A)}(\theta_z^{AB}) = \phi_{\nu_\alpha}^{(B)}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi_{\nu_\alpha}^{(A)}(\theta_z) = \phi_{\nu_\alpha}^{(A)}(\pi - \theta_z)$$

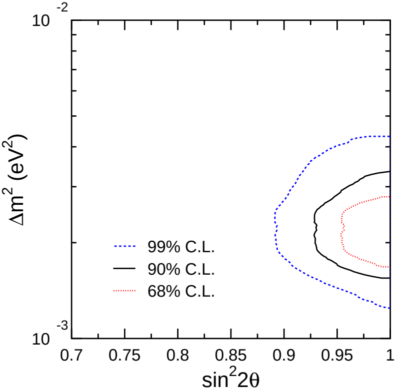
(December 1998)

$$\boxed{A_{\nu_\mu}^{\text{up-down}}(\text{SK}) = \left(\frac{N_{\nu_\mu}^{\text{up}} - N_{\nu_\mu}^{\text{down}}}{N_{\nu_\mu}^{\text{up}} + N_{\nu_\mu}^{\text{down}}}\right) = -0.296 \pm 0.048 \pm 0.01}$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6σ MODEL INDEPENDENT EVIDENCE OF ν_μ DISAPPEARANCE!

Fit of Super-Kamiokande Atmospheric Data



$\nu_\mu \to \nu_\tau$

Best Fit: $\begin{cases} \Delta m^2 = 2.1 \times 10^{-3}\,\mathrm{eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of ν_τ CC Int. is Difficult:

- $E_{\mathrm{th}} = 3.5\,\mathrm{GeV} \Longrightarrow \sim 20$events/yr
- τ-Decay $\Longrightarrow$ Many Final States

ν_τ-Enriched Sample

$N_{\nu_\tau}^{\mathrm{the}} = 78 \pm 26$ @ $\Delta m^2 = 2.4 \times 10^{-3}\,\mathrm{eV}^2$

$\boxed{N_{\nu_\tau}^{\mathrm{exp}} = 138^{+50}_{-58}}$

$N_{\nu_\tau} > 0$ @ 2.4σ

[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

Check: OPERA ($\nu_\mu \to \nu_\tau$)
CERN to Gran Sasso (CNGS)
$L \simeq 732\,\mathrm{km}$ $\langle E \rangle \simeq 18\,\mathrm{GeV}$

[NJP 8 (2006) 303, hep-ex/0611023]

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Reactor
$\bar{\nu}_e$ disappearance

$$\left.\begin{array}{c}\left(\begin{array}{c}\text{Homestake}\\ \text{Kamiokande}\\ \text{GALLEX/GNO \& SAGE}\\ \text{Super-Kamiokande}\\ \text{SNO}\\ \text{BOREXino}\end{array}\right)\\ \\ \text{(KamLAND)}\end{array}\right\} \to \left\{\begin{array}{c}\Delta m^2_{\text{SOL}} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2\\ \tan^2\vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06\end{array}\right.$$

Atmospheric
$\nu_\mu \to \nu_\tau$

Accelerator
ν_μ disappearance

$$\left.\begin{array}{c}\left(\begin{array}{c}\text{Kamiokande}\\ \text{IMB}\\ \text{Super-Kamiokande}\\ \text{MACRO}\\ \text{Soudan-2}\end{array}\right)\\ \\ \text{(K2K \& MINOS)}\end{array}\right\} \to \left\{\begin{array}{c}\Delta m^2_{\text{ATM}} \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2\\ \sin^2\vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07\end{array}\right.$$

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

three flavor fields: ν_e, ν_μ, ν_τ

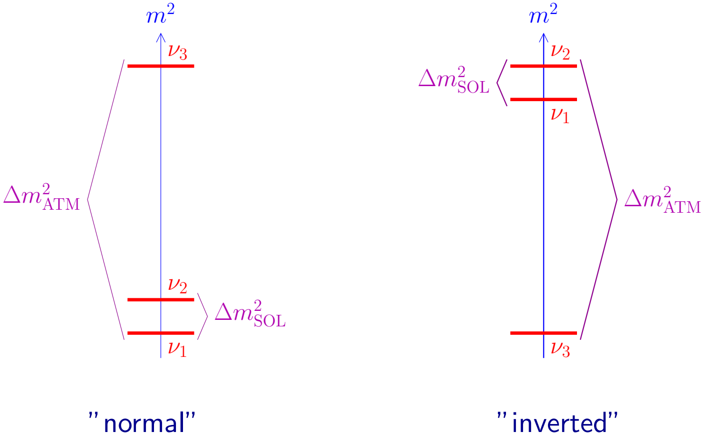
three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{13} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

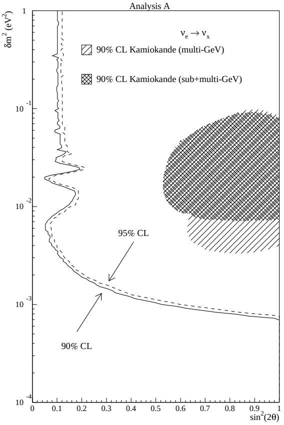
$$\begin{array}{l} \text{SOL} \rightarrow \\ \\ \end{array} U = \begin{pmatrix} \boxed{U_{e1} \quad U_{e2}} & \boxed{U_{e3}} \\ U_{\mu 1} \quad U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} \quad U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$\uparrow \text{ATM}$$

CHOZ: $\left\{ \begin{array}{l} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{array} \right.$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau 3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

Bilarge Mixing

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\sin^2 \vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2 \vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

$$\sin^2 \vartheta_{13} < 0.035 \quad \text{(90\% C.L.)}$$

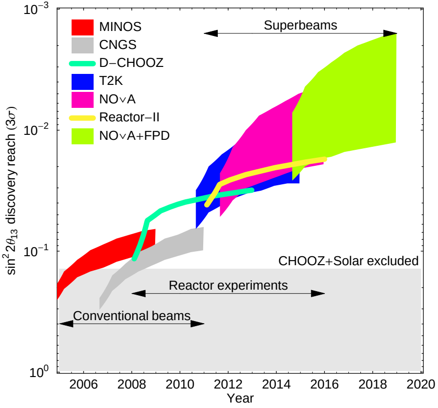
[Schwetz, Tortola, Valle, New J. Phys. 10 (2008) 113011]

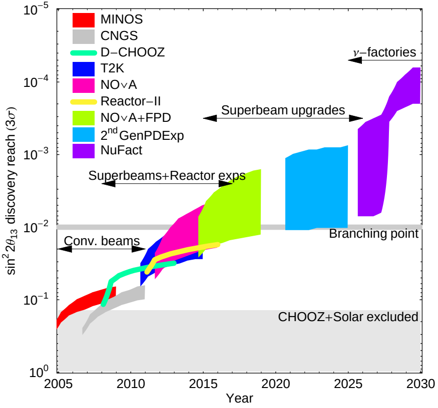
Hint of $\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$\sin^2 \vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]

The Hunt for ϑ_{13}





3σ sensitivities. Bands reflect dependence of sensitivity on the CP violating phase δ_{13}.

"Branching point" refers to the decision between an upgraded superbeam and/or detector and a neutrino factory program. Neutrino factory is assumed to switch polarity after 2.5 years.

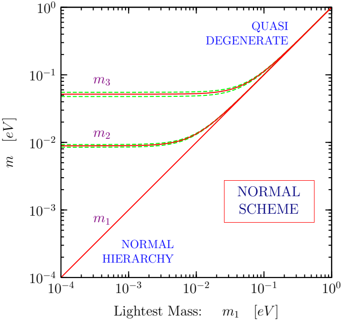
[Physics at a Fermilab Proton Driver, Albrow et al, hep-ex/0509019]

Absolute Scale of Neutrino Masses

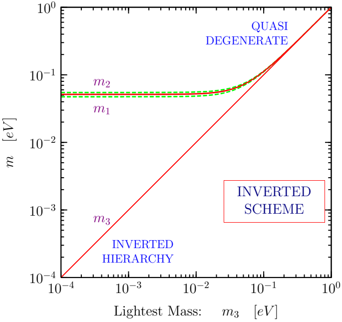
- Solar Neutrinos and KamLAND
- Atmospheric Neutrinos and LBL
- Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
  - Mass Hierarchy or Degeneracy?
  - Tritium Beta-Decay
  - Neutrinoless Double-Beta Decay
  - Cosmological Bound on Neutrino Masses
- Experimental Neutrino Anomalies
- Conclusions

Mass Hierarchy or Degeneracy?

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium Beta-Decay

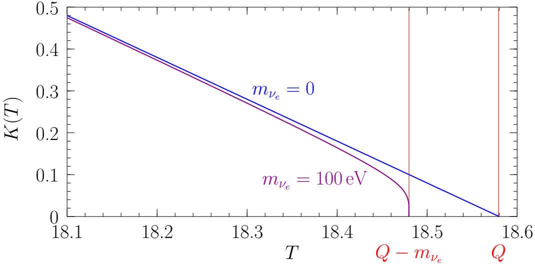
$^{3}\text{H} \to {}^{3}\text{He} + e^{-} + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C\, G_F)^2}{2\pi^3} |\mathcal{M}|^2\, F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\text{H}} - M_{^3\text{He}} - m_e = 18.58\,\text{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C\, G_F)^2}{2\pi^3} |\mathcal{M}|^2\, F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\text{eV} \quad (95\%\ \text{C.L.})}$

Mainz & Troitsk

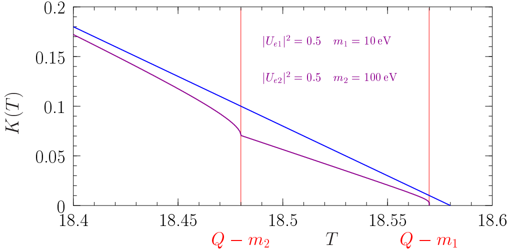
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2012)

[arXiv:0810.3281]

sensitivity: $m_{\nu_e} \simeq 0.2\,\text{eV}$ (3y)

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N - 1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

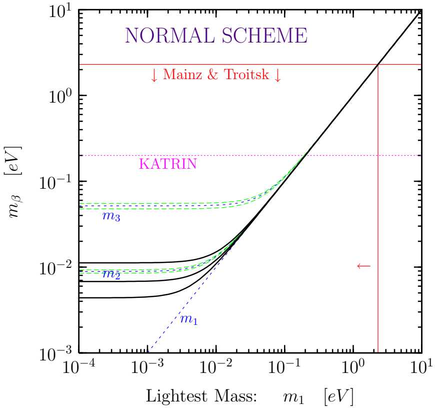
if experiment is not sensitive to masses ($m_k \ll Q - T$)

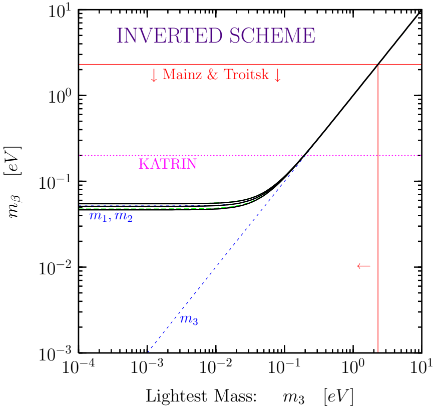
effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$

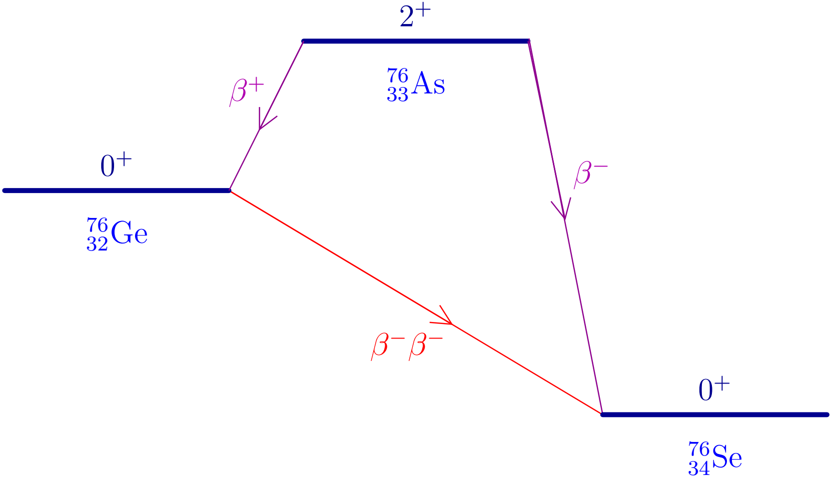




Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



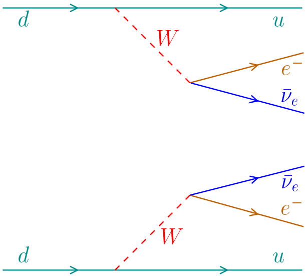
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

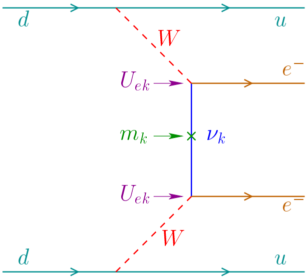


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$

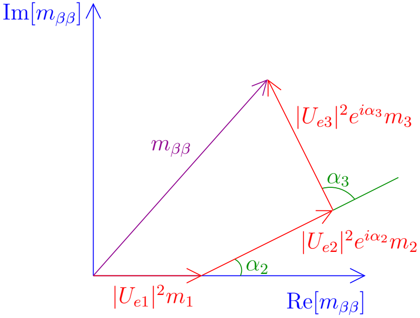


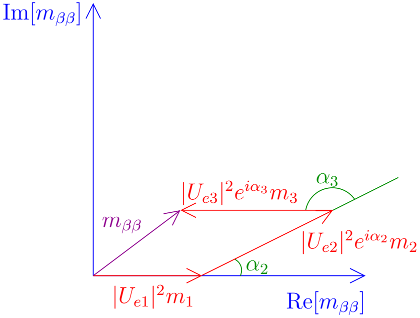
Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$





Experimental Bounds

CUORICINO (^{130}Te) [PRC 78 (2008) 035502]

$$\boxed{T^{0\nu}_{1/2} > 3 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.19 - 0.68\,\text{eV}}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$\boxed{T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

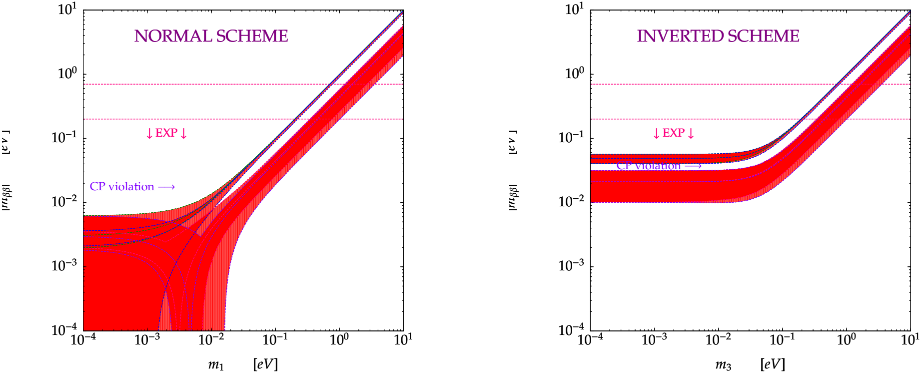
$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$

Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$



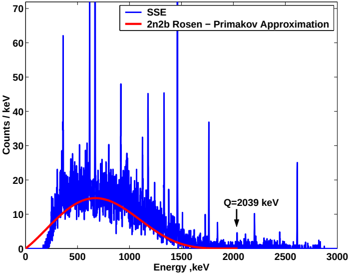
FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Experimental Positive Indication

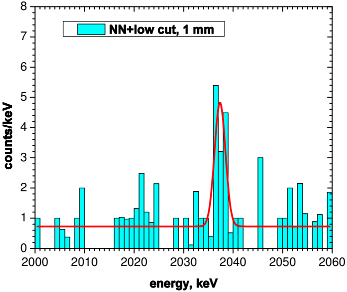
[Klapdor et al., MPLA 16 (2001) 2409]

$$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}\,\text{y}$$

6.5σ evidence [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



[MPLA 21 (2006) 1547]

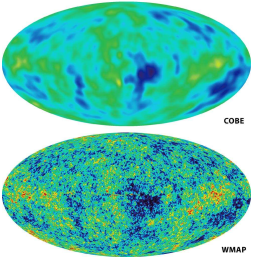
the indication must be checked by other experiments

$$|m_{\beta\beta}| = 0.32 \pm 0.03\,\text{eV}$$

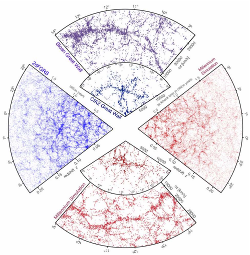
[MPLA 21 (2006) 1547]

if confirmed, very exciting (Majorana ν and large mass scale)

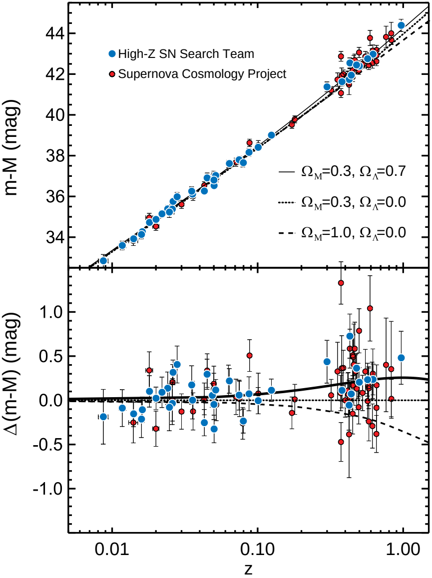
Cosmological Bound on Neutrino Masses



[WMAP, http://map.gsfc.nasa.gov]



[Springel, Frenk, White, Nature 440 (2006) 1137]



[http://cfa-www.harvard.edu/supernova/]

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$

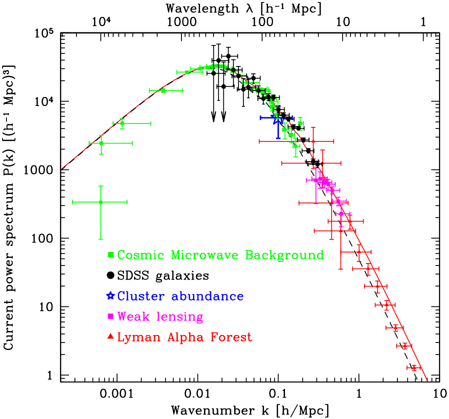
number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$

density contribution: $\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$ $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\, m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\, h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\mathrm{M}}^{0} = 0.28 \, , \; h = 0.72 \, , \; \Omega_{\mathrm{B}}^{0}/\Omega_{\mathrm{M}}^{0} = 0.16)$

Dashed Curve: $\displaystyle\sum_{k=1}^{3} m_k = 1\,\mathrm{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3k}{(2\pi)^3} \, e^{i\vec{k}\cdot\vec{x}} \, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\mathrm{eV}}\right)\left(\frac{0.1}{\Omega_m\,h^2}\right)$$

for

$$k \gtrsim k_{\mathrm{nr}} \approx 0.026\sqrt{\frac{m_\nu}{1\,\mathrm{eV}}}\sqrt{\Omega_m}\,h\,\mathrm{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP (First Year), AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia $\Longrightarrow$ Flat ΛCDM

$$T_0 = 13.7 \pm 0.2\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_b = 0.044 \pm 0.004 \qquad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad \text{(95\% conf.)} \quad \Longrightarrow \quad \sum_{k=1}^{3} m_k < 0.71\,\text{eV}$$

WMAP (Five Years), AJS 180 (2009) 330, astro-ph/0803.0547

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12\,\text{Gyr} \qquad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad \text{(95\% C.L.)}$$

$$\Omega_b = 0.0456 \pm 0.0015 \qquad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^{3} m_k < 0.67\,\text{eV} \quad \text{(95\% C.L.)} \qquad N_{\text{eff}} = 4.4 \pm 1.5$$

Fogli, Lisi, Marrone, Melchiorri, Palazzo, Rotunno, Serra, Silk, Slosar

[PRD 78 (2008) 033010, hep-ph/0805.2517]

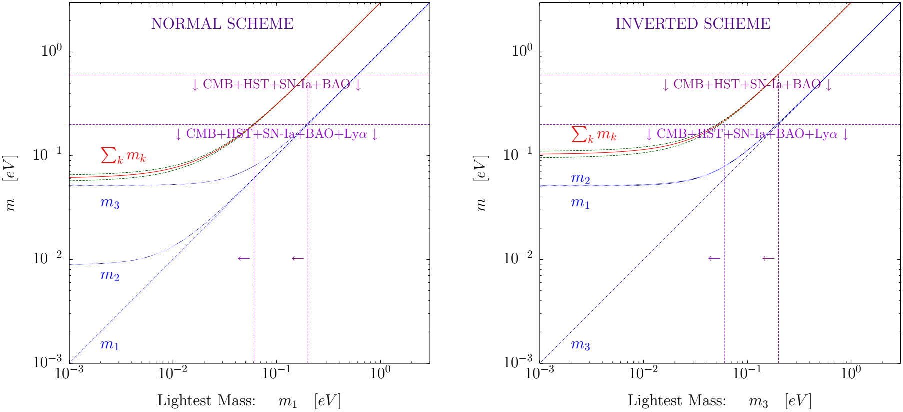
Flat ΛCDM

Case	Cosmological data set	Σ (at 2σ)
1	CMB	< 1.19 eV
2	CMB + LSS	< 0.71 eV
3	CMB + HST + SN-Ia	< 0.75 eV
4	CMB + HST + SN-Ia + BAO	< 0.60 eV
5	CMB + HST + SN-Ia + BAO + Lyα	< 0.19 eV

2σ (95% C.L.) constraints on the sum of ν masses Σ.

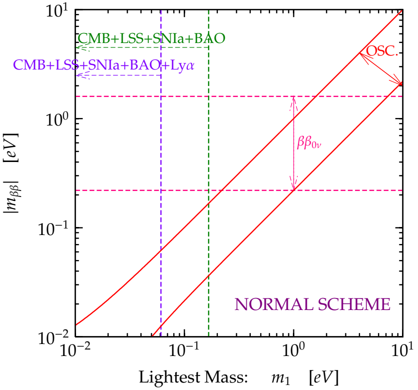
$$\sum_{k=1}^{3} m_k \lesssim 0.6\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO}$$

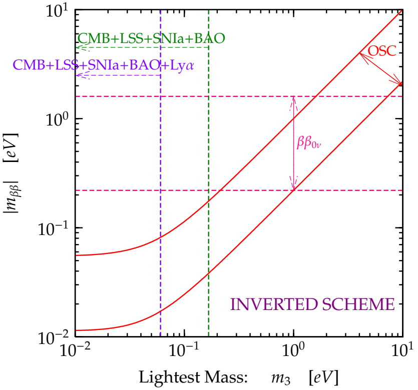
$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$



FUTURE: IF $\displaystyle\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\mathrm{eV} \Longrightarrow$ NORMAL HIERARCHY

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV}$ ($\sim 3\sigma$ range)





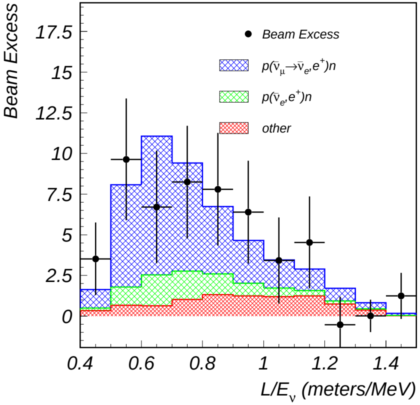
tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal

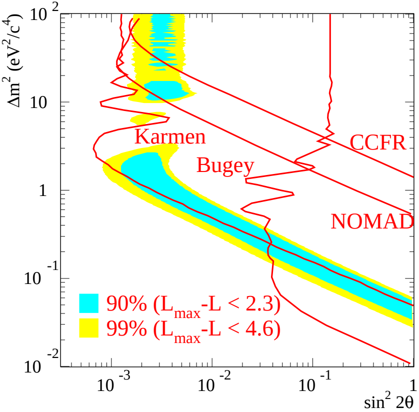
Experimental Neutrino Anomalies

LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 30\,\mathrm{m} \quad 20\,\mathrm{MeV} \leq E \leq 200\,\mathrm{MeV}$



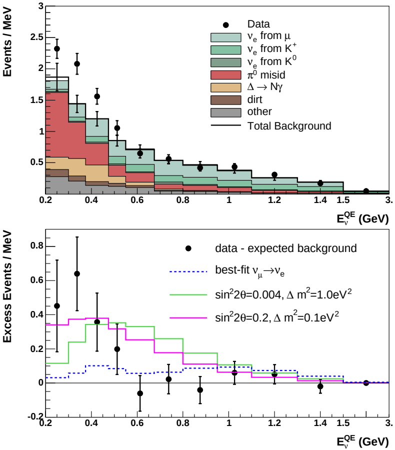


$$\Delta m^2_{\mathrm{LSND}} \gtrsim 0.2\,\mathrm{eV}^2 \qquad (\gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SOL}})$$

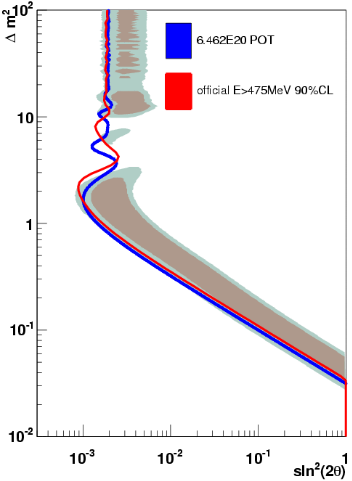
MiniBooNE

[PRL 98 (2007) 231801]

$\nu_\mu \to \nu_e$ $L \simeq 541$ m 475 MeV $\leq E \lesssim 3$ GeV



[arXiv:0812.2243]



[arXiv:0901.1648]

Low-Energy Anomaly!

Gallium Radioactive Source Experiments

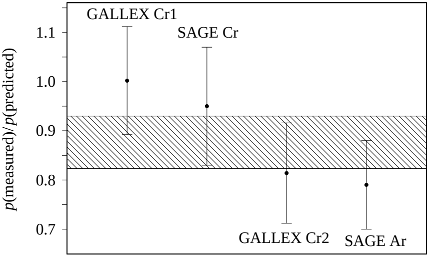
tests of solar neutrino detectors

GALLEX [PLB 342 (1995) 440; PLB 420 (1998) 114]

SAGE [PRL 77 (1996) 4708; PRC 59 (1999) 2246; PRC 73 (2006) 045805; arXiv:0901.2200]

Sources: $e^- + {}^{51}\text{Cr} \to {}^{51}\text{V} + \nu_e$ $\qquad e^- + {}^{37}\text{Ar} \to {}^{37}\text{Cl} + \nu_e$

Detector: $\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$



[SAGE, PRC 73 (2006) 045805]

$$\boxed{R_{\text{Ga}} = 0.87 \pm 0.05}$$

[SAGE, arXiv:0901.2200]

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (SOL, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (ATM, K2K, MINOS)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

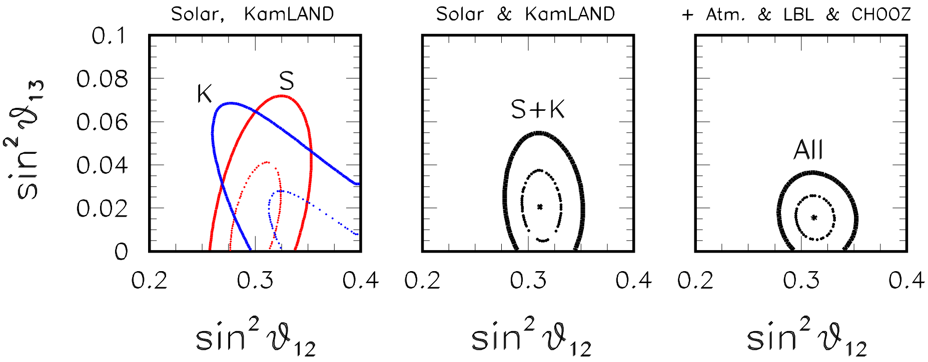
FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?
(Due to Majorana nature of ν's?)
Why only $|U_{e3}|^2 \ll 1$?
Explain experimental neutrino anomalies (sterile ν's?).

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy.
Check experimental neutrino anomalies.
Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale.
Improve β & $\beta\beta_{0\nu}$ Decay and Cosmology measurements.

Hint of $\vartheta_{13} > 0$

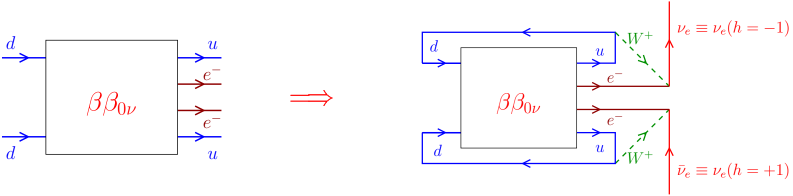
[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]



$\sin^2\vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]

$$P_{\overset{(-)}{\nu_e} \to \overset{(-)}{\nu_e}} \simeq \begin{cases} \left(1 - \sin^2\vartheta_{13}\right)^2 \left(1 - 0.5\sin^2\vartheta_{12}\right) & \text{SOL low-energy \& KamLAND} \\ \left(1 - \sin^2\vartheta_{13}\right)^2 \sin^2\vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

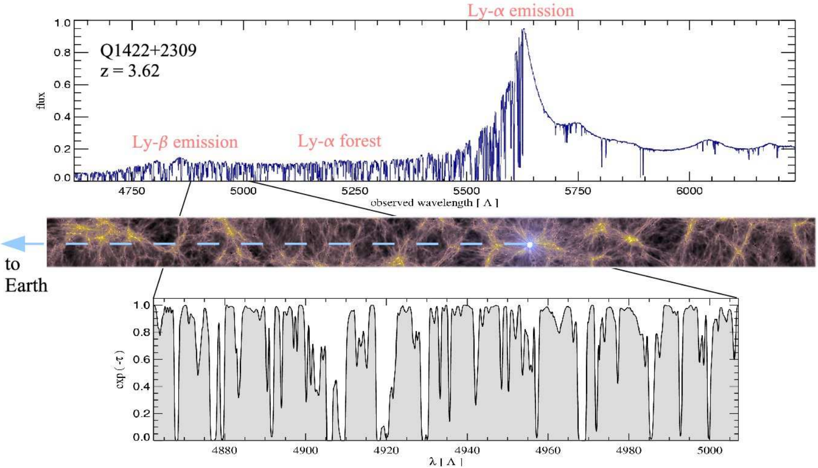


[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term

$$\boxed{\mathcal{L}_{eL}^{\mathrm{M}} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu_{eL}^{c}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu_{eL}^{c} \right)}$$

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67\ \AA$, $\lambda_\beta^0 = 1025.72\ \AA$, $\lambda_\gamma^0 = 972.54\ \AA$

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$