Electroweak Precision Observables: Theory Future

Sven Heinemeyer, IFT/IFCA (CSIC, Madrid/Santander)

CERN, 01/2017

FCC-ee physics WG2: Precision EW Calculations

Co-convenor/editor: *Ayres Freitas*

Contributors (so far): *M. Beneke, A. Blondel, A. Hoang, P. Janot,*
J. Reuter, C. Schwinn, S. Weinzierl, . . .

1. Introduction
2. Electroweak Precision Observables
3. What is needed to match the FCC-ee precision
4. Conclusions

1. Introduction

Experimental situation:

LHC/ILC/FCC-ee/CEPC/. . . will provide (high!) accuracy measurements!

Theory situation:

– Measurements are performed using theory predictions

– measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, . . .)

Full uncertainty is given by the (linear) sum of
experimental and theoretical uncertainties!

1. Introduction

Experimental situation:

LHC/ILC/FCC-ee/CEPC/. . . will provide (high!) accuracy measurements!

Theory situation:

– Measurements are performed using theory predictions

– measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, . . .)

Full uncertainty is given by the (linear) sum of experimental and theoretical uncertainties!

⇒ not so much an issue for today's precision (see previous talk)

1. Introduction

Experimental situation:

LHC/ILC/FCC-ee/CEPC/. . . will provide (high!) accuracy measurements!

Theory situation:

– Measurements are performed using theory predictions

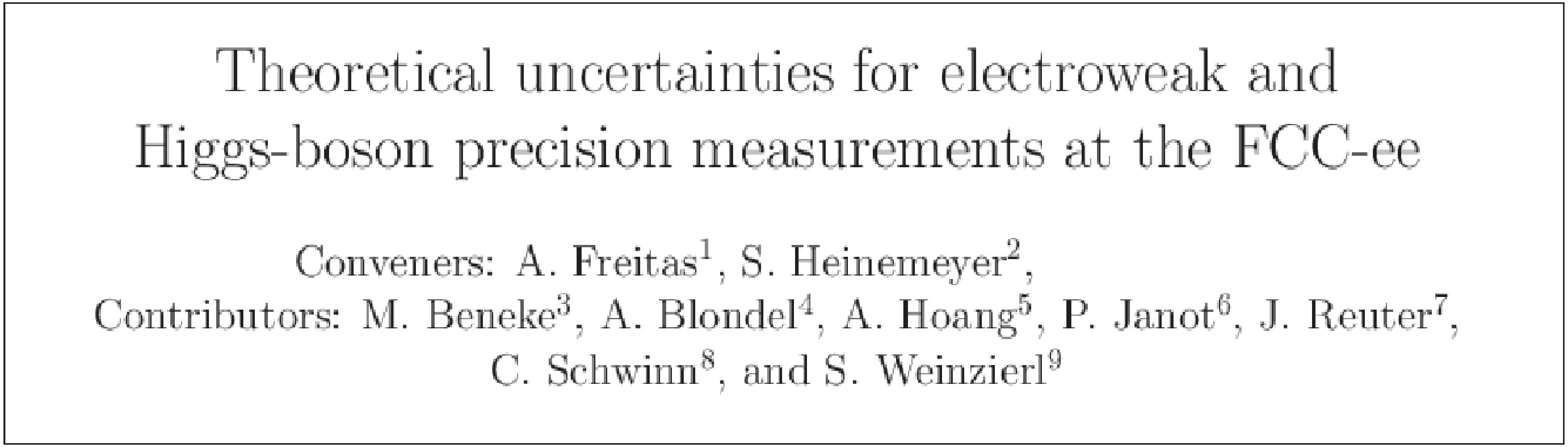
– measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, . . .)

Full uncertainty is given by the (linear) sum of
experimental and theoretical uncertainties!

⇒ not so much an issue for today's precision (see previous talk)

⇒ What about the anticipated FCC-ee precision?

FCC-ee physics WG2 – Precision EW Calculations: Write-up



Theoretical uncertainties for electroweak and
Higgs-boson precision measurements at the FCC-ee

Conveners: A. Freitas[1], S. Heinemeyer[2],
Contributors: M. Beneke[3], A. Blondel[4], A. Hoang[5], P. Janot[6], J. Reuter[7],
C. Schwinn[8], and S. Weinzierl[9]

⇒ will go into CDR!

⇒ should be taken into account by other (exp) groups!

⇒ Here: anticipated accuracy of EWPO TH calculations in $\mathcal{O}(20)$ years

Where we need theory prediction:

1. Prediction of the measured quantity
   Example: M_W
   → at the same level or better as the experimental precision

2. Prediction of the measured process to extract the quantity
   Example: $e^+e^- \to W^+W^-$
   → better than then “pure” experimental precision

Two types of theory uncertainties:

1. intrinsic: missing higher orders

2. parametric: uncertainty due to exp. uncertainty in SM input parameters
   Example: m_t, m_b, α_s, $\Delta\alpha_{\text{had}}$, . . .

Options for the evaluation of intrinsic uncertainties:

1. Determine all prefactors of a certain diagram class (couplings, group factors, multiplicities, mass ratios) and assume the loop is $\mathcal{O}(1)$
2. Take the known contribution at n-loop and $(n-1)$-loop and thus estimate the $n+1$-loop contribution:

$$\frac{(n+1)^{(\text{estimated})}}{n^{(\text{known})}} \approx \frac{n^{(\text{known})}}{(n-1)^{(\text{known})}}$$

   ⇒ simplified example! Has to be done
   "coupling constant by coupling constant"

3. Variation of $\mu^{\overline{\text{MS}}}$ (QCD!, EW?)
4. Compare different renormalizations

⇒ Mostly used here: 1 & 2

Our future estimates:

- assume to go substantially beyond what is known now
- assume that many theorists will put many^2 hours of work into it (motivation?)
- do not assume that magically new calculational methods are invented
- are overall optimistic

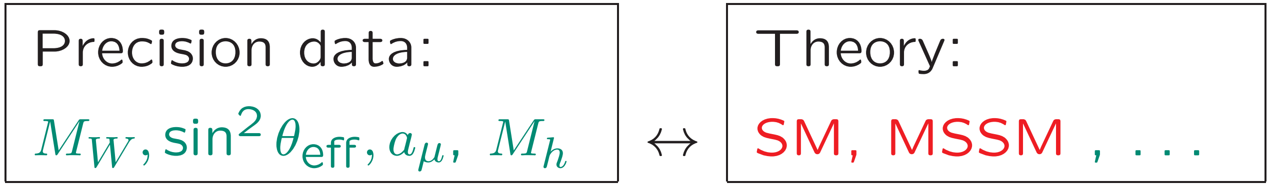
⇒ they should be taken seriously!

Saying “Ah, theorists will have to work a bit harder and solve this”
is not a realistic option!

⇒ An honest evaluation of theory uncertainties will increase the robustness of the FCC-ee physics case!

2. Electroweak Precision Observables

Comparison of observables with theory:



Precision data:

$M_W, \sin^2\theta_{\rm eff}, a_\mu,\ M_h$

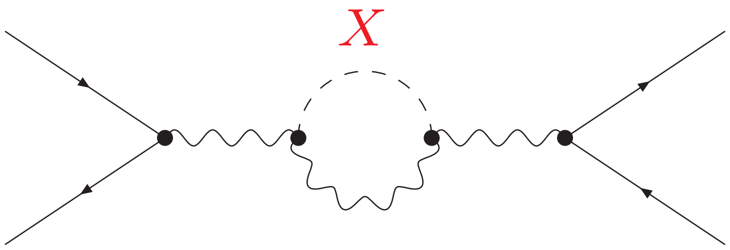
$\leftrightarrow$

Theory:

SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. X



$\Downarrow$

SM: limits on M_H, BSM: limits on M_X

Very high accuracy of measurements and theoretical predictions needed

$\Rightarrow$ only models "ready" so far: SM, MSSM

The EWPO:

M_W (best from threshold scan)

$$\sigma^0_{\text{had}} = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}],$$ (from a fit to $\sigma_f(s)$ at various values of s)

$$R_\ell = \left[\textstyle\sum_q \sigma_q(M_Z^2)\right] / \sigma_\ell(M_Z^2),$$ $(\ell = e, \mu, \tau)$

$$R_q = \sigma_q(M_Z^2) / \left[\textstyle\sum_q \sigma_q(M_Z^2)\right],$$ $(q = b, c)$

$$A^f_{\text{FB}} = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \frac{3}{4}\mathcal{A}_e\mathcal{A}_f,$$

$$A^f_{\text{LR}} = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e|P_e|$$

$$\mathcal{A}_f = 2\frac{g_{V_f}/g_{A_f}}{1 + (g_{V_f}/g_{A_f})^2} = \frac{1 - 4|Q_f|\sin^2\theta^f_{\text{eff}}}{1 - 4|Q_f|\sin^2\theta^f_{\text{eff}} + 8(|Q_f|\sin^2\theta^f_{\text{eff}})^2}$$ $(f = \ell, b, \ldots)$

3. What is needed to match the FCC-ee precision

Compare:

1. FCC-ee (pure) experimental (anticipated) precision
2. Intrinsic uncertainties
3. Parametric uncertainties
   → taking into account the improved precision of SM paramters at FCC-ee

Combined uncertainty:

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

Intrinsic uncertainties: ⇒ always a limiting factor!

Quantity	FCC-ee	Current intrinsic unc.		Projected unc.
M_W [MeV]	1	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\text{eff}}$ $[10^{-5}]$	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.5	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.2
R_b $[10^{-5}]$	6	15	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s)$	7
R_l $[10^{-3}]$	1	5	$(\alpha^2_{\text{bos}}, \alpha^3, \alpha^2\alpha_s)$	1.5

These calculations are required for the projection:

- complete $\mathcal{O}\left(\alpha\alpha_s^2\right)$ corrections
- fermionic $\mathcal{O}\left(\alpha^2\alpha_s\right)$ corrections
- double-fermionic $\mathcal{O}\left(\alpha^3\right)$ corrections
- leading four-loop corrections enhanced by the top Yukawa coupling
- the $\mathcal{O}\left(\alpha^2_{\text{bos}}\right)$ corrections are not the leading uncertainties now

For these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift.

Parametric uncertainties:

1. M_H: better than 50 MeV $\Rightarrow$ negligible

2. M_Z: $\sim$ 0.1 MeV with negligible theory uncertainties $\Rightarrow$ negligible

3. $\alpha_s(M_Z)$: from (mainly) R_ℓ
   $\delta\alpha_s^{\text{exp}} \sim 10^{-4}$, $\delta\alpha_s^{\text{theo}} \sim 1.5 \times 10^{-4}$

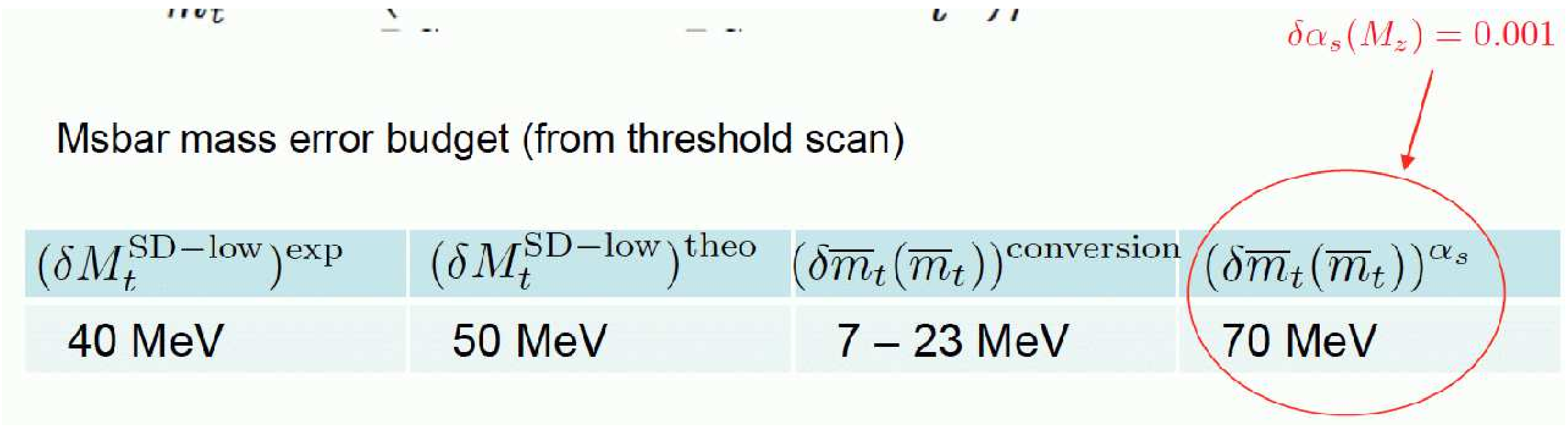
4. m_t: from threshold scan
   $\delta m_t^{\text{exp}} \sim \mathcal{O}(10\ \text{MeV})$
   $\delta m_t^{\text{theo}} \sim 50\ \text{MeV}\ (\text{NNNLO/NNLL} \oplus 1S \to \overline{\text{MS}} \oplus \delta\alpha_s)$

5. m_b: from lattice calculations
   $\delta m_b \sim 10$ MeV (still under discussion, too optimistic?)

6. $\Delta\alpha_{\text{had}}$: BES III and Belle II: $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$
   better from measurements “around the Z pole?

Uncertainty budget for m_t:

[talk by A. Hoang '15]



$\delta\alpha_s(M_z) = 0.001$

Msbar mass error budget (from threshold scan)

$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{exp}}$	$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{theo}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\mathrm{conversion}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

$\Rightarrow$ improvement in α_s crucial

e^+e^- collider: precision measurement:

$$R_l := \frac{\Gamma(Z \to \text{ hadrons})}{\Gamma(Z \to l^+ l^-)}$$

Improvement down to $\delta^{\mathrm{exp}}\alpha_s \sim 0.001 - 0.0001$ possible?!

Note: TH uncertainty (assuming fermionic 3-loop corrections):

$\delta R_l^{\mathrm{theo}} \sim 0.0015 \Rightarrow \delta\alpha_s^{\mathrm{theo}} \sim 0.00015$ $\Rightarrow$ hard to beat . . .

SM input: $\Delta\alpha_{\rm had}$ $\Rightarrow$ could be limiting factor!

From $e^+e^- \to$ had. using dispersion relation

today: $\delta(\Delta\alpha_{\rm had}) \sim 10^{-4}$
possible improvement in the future: $\delta(\Delta\alpha_{\rm had}) \sim 5 \times 10^{-5}$

Direct determination at FCC-ee from $e^+e^- \to f\bar{f}$ off the Z peak

[*P. Janot '15*]
possible improvement in the future: $\delta(\Delta\alpha_{\rm had}) \sim 2 \times 10^{-5}$ $\Rightarrow$ TU neglected

Calculation of $e^+e^- \to f\bar{f}$ needed at 3-loop and beyond: [*A. Freitas '16*]
current techniques (2L/3L): corrections of $\sim 10^{-3}$
new calculation methods (2L/3L): corrections of $\sim 10^{-4}$
unknown methods 3L: $\lesssim 10^{-5}$
unknown methods 4L: $\sim 10^{-5}$
(+ higher-orders in real photon emission)
$\Rightarrow$ improvement unclear $\Rightarrow \delta(\Delta\alpha_{\rm had}) \sim 3 \times 10^{-5}$

Additional uncertainty for M_W from threshold scan:

Not only $e^+e^- \to W^{(*)}W^{(*)}$, but $e^+e^- \to WW \to$ 4f needed

Current status:

full one-loop for $2 \to 4$ process

[*A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02*]

⇒ extraction of M_W at the level of $\sim$ 6 MeV

Most recent improvement:

leading 2L corrections from EFT

[*Actis, Beneke, Falgari, Schwinn '08*]

⇒ impact on M_W at the level of $\sim$ 3 MeV

⇒ full 2L for $2 \to 4$ process not foreseeable

Potentially possible:

2L resummed higher-order terms for $e^+e^- \to WW$ and $W \to ff'$

⇒ extraction of M_W at $\sim$ 1 MeV?? $\oplus$ pure exp. uncertainty of $\sim$ 0.5 MeV

Summary of future parametric uncertainties:

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$1 - 1.5$	1 (0.6)	$\delta(\Delta\alpha_{\text{had}})$
$\sin^2\theta^{\ell}_{\text{eff}}$ $[10^{-5}]$	0.6	2 (1)	$\delta(\Delta\alpha_{\text{had}})$
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_s$
R_ℓ $[10^{-3}]$	1	1.3	$\delta\alpha_s$

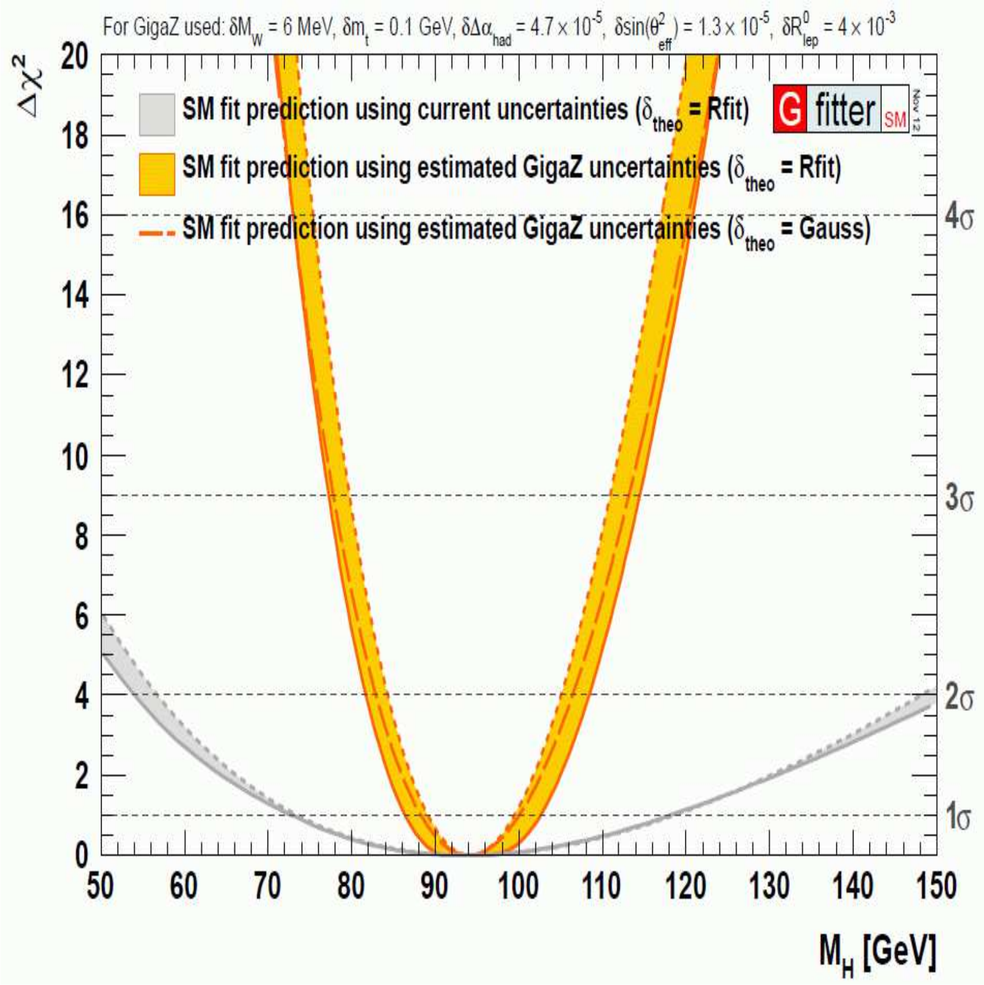
$\delta(\Delta\alpha_{\text{had}}) = 5(3) \times 10^{-5}$

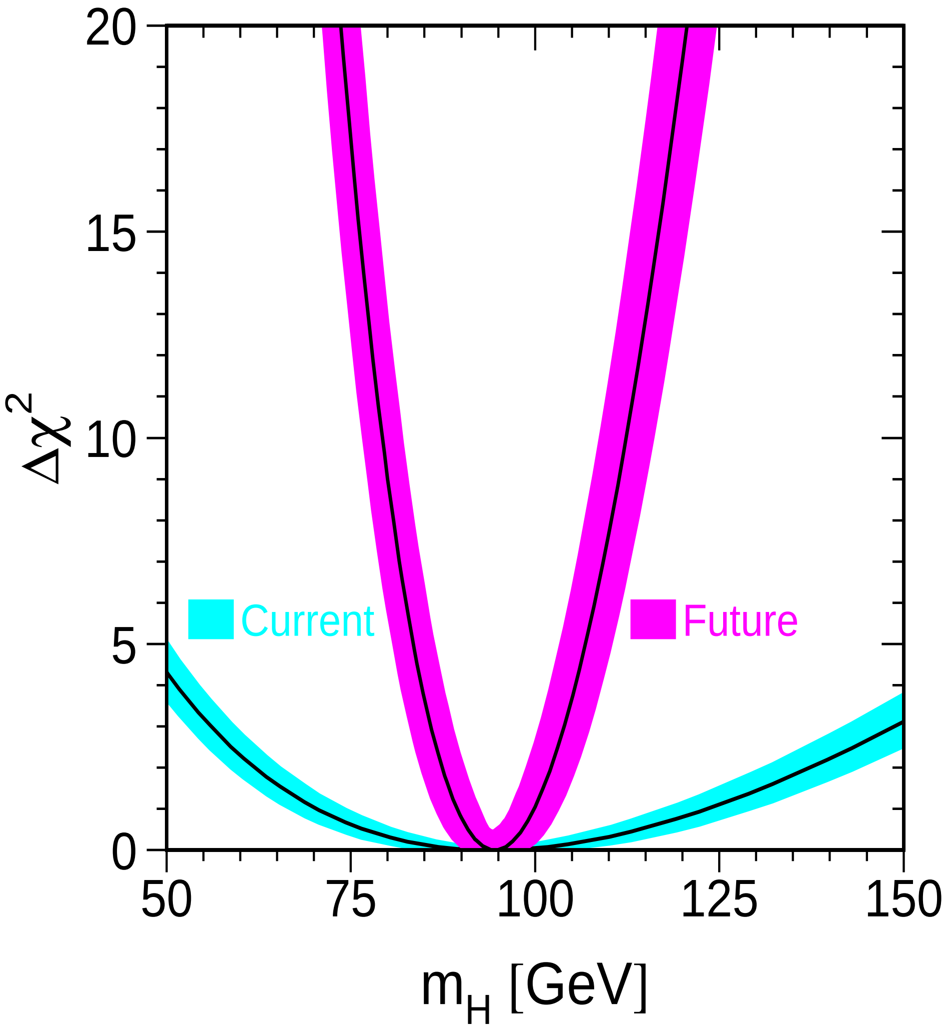
⇒ add quadratic to experimental uncertainties!

⇒ add linearly to intrinsic uncertainties!

Precise M_H test with the ILC precision:

[*GFitter '13*] [*LEPEWWG '13*]





⇒ $\delta M_H^{\text{ind}} \lesssim 6$ GeV ⇐ only ILC analysis available so far

⇒ extremely sensitive test of SM (and BSM) possible

One more word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

We know that BSM physics must exist! (DM, gravity, . . .)

As soon as BSM physics will be discovered, an evaluation of the EWPO in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

4. Conclusions

- High anticipated experimental precision for EWPO at FCC-ee
- Crucial: theory uncertainties: intrinsic and parametric

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

- We give (realistic/optimistic) estimates for future intrinsic and parametric uncertainties
- intrinsic unc. larger than anticipated experimental unc.
  parametric unc. often larger than experimental uncertainties
  ⇒ particularly true for M_W and $\sin^2\theta_{\text{eff}}$
- Write-up is available, will go into CDR
  Uncertainties should be taken into account by other (exp) groups!


Further Questions?