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Multi-Higgs production at high energy

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- At very high energies, production of multiple Higgs and vector bosons becomes kinematically possible.

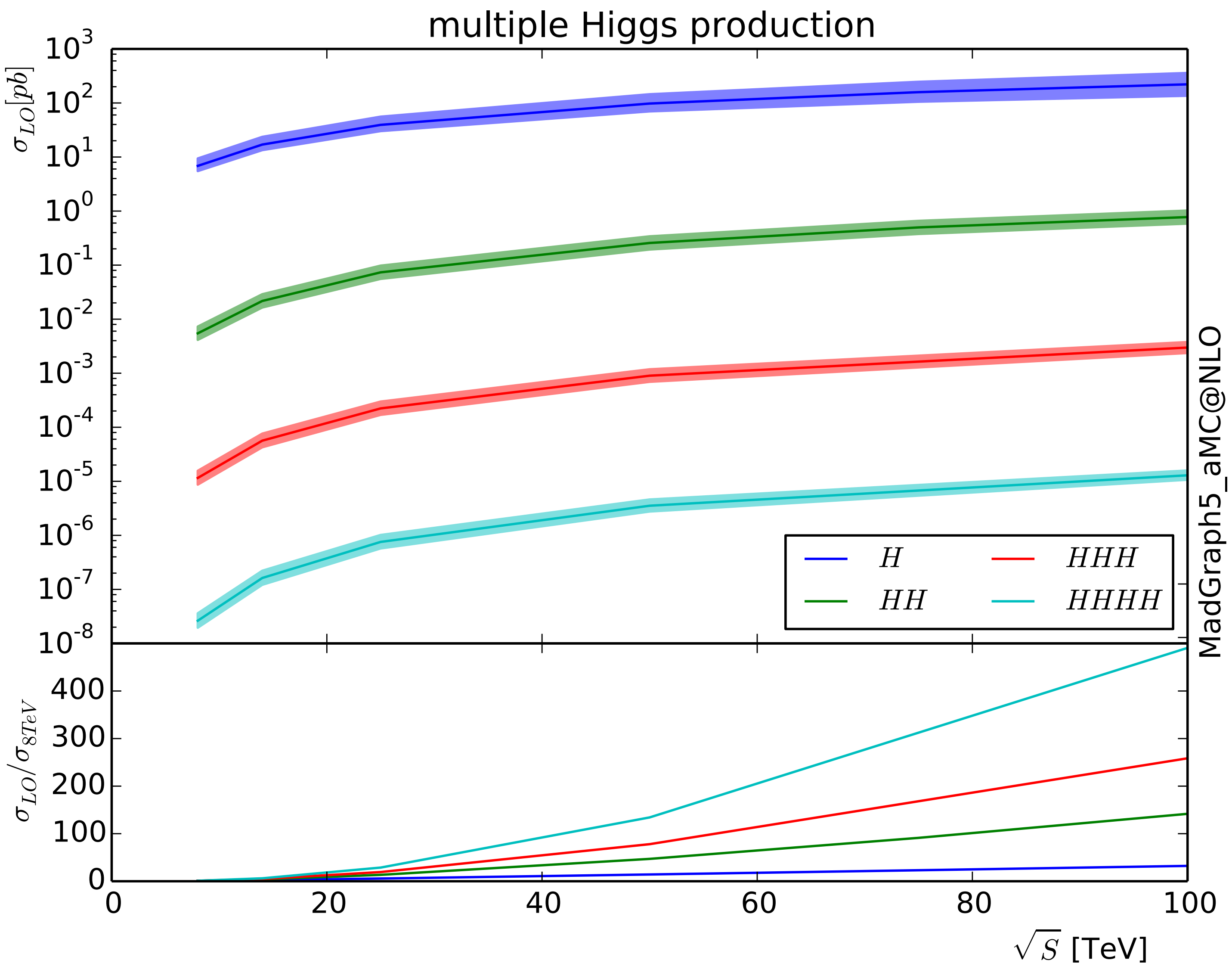
- In this talk: consider n~100s of Higgs bosons produced in the final state.   n x lambda >> 1

- Investigate scattering processes at ~ 50-100 TeV CoM energies E.

- The cross-sections computed in weakly-coupled perturbation theory become unsuppressed above certain critical values of n and E and continue to grow with energy eventually violating perturbative unitarity.

- [Conclusions will also apply to high-multiplicity longitudinal W and Z production.]

Warm-up: 1,2,3,4 Higgs bosons production
8 TeV < E < 100 TeV



Up to 4 Higgses with MSTW2008 pdf set & MadGraph5_aMC@NLO for different energies

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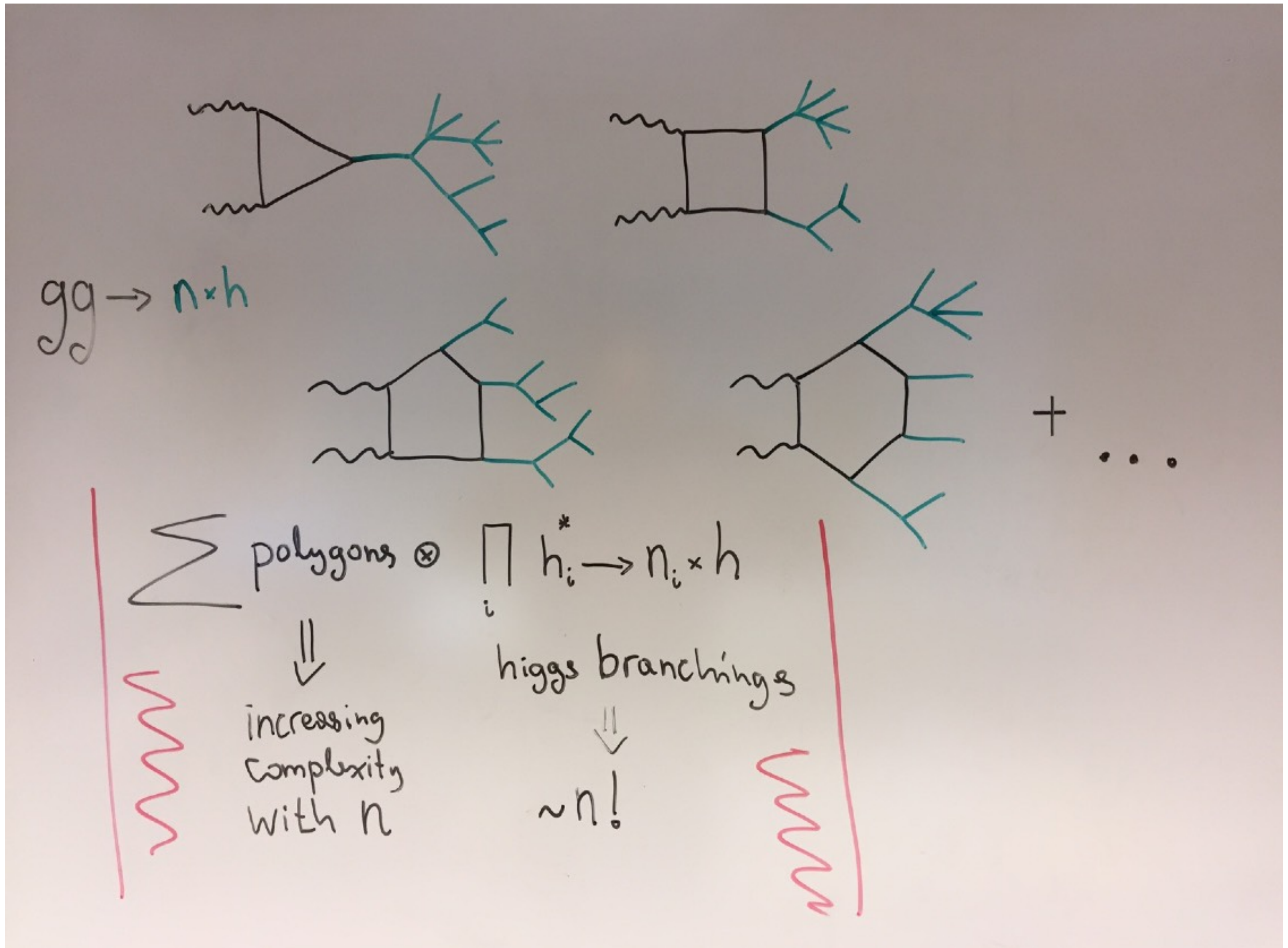
- What if ~100 Higgs bosons are produced in the final state at 100 TeV ?

Gluon fusion multi-Higgs production at large n



Two immediate problems to address:

1-loop polygons with up to n-2 edges increasing technical complexity

1-> n x h tree-level (& loop-corrected) Higgs branchings grow as n!

Will address this first

$$\mathcal{A}_{gg\to n\times h} = \sum_{\text{polygons}} \mathcal{A}^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\ldots+n_k=n} \prod_{i=1}^{k} \mathcal{A}_{h_i^*\to n_i\times h}$$

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Tree-level 1*->n Amplitudes on multi-particle mass thresholds determined by classical solutions that are uniform in space: h(x,t)=h(t)

Lagrangian for the scalar field:

$$\mathcal{L}(h) = \frac{1}{2}\left(\partial h\right)^2 - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 ,$$

prototype of the Higgs in the unitary gauge

The classical equation for the spatially uniform field $h(t)$,

$$d_t^2 h = -\lambda\, h^3 + \lambda v^2\, h ,$$

has a closed-form solution with correct initial conditions $h_{\rm cl} = v + z + \ldots$

$$h_{\rm cl}(t) = v\,\frac{1+\frac{z(t)}{2v}}{1-\frac{z(t)}{2v}} , \quad \text{where} \quad z(t) = z_0\, e^{iM_h t} = z_0\, e^{i\sqrt{2\lambda}\, v\, t}$$

$$h_{\rm cl}(t) = 2v \sum_{n=0}^{\infty} \left(\frac{z(t)}{2v}\right)^n d_n = v + 2v \sum_{n=1}^{\infty} \left(\frac{z(t)}{2v}\right)^n ,$$

i.e. with $d_0 = 1/2$ and all $d_{n\geq 1} = 1$.

$$\mathcal{A}_{1\to n} = \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} = \boxed{n!\,(2v)^{1-n}}$$

Factorial growth

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Factorial growth of large-n scalar amplitudes on mass thresholds: E=nm

- The n! growth of perturbative amplitudes is not entirely surprising: it reflects the large-n behaviour of perturbation theory:

- [Use of classical solutions is equivalent to summing over tree-level Feynman diagrams; the number of contributing Feynman diagrams is known to grow factorially with n]

- Important to distinguish between the two types of large-n corrections:

(a) *higher-order* perturbative corrections to some leading-order quantities

(b) our case where the *leading-order* tree-level contribution to the 1*->n Amplitude grows factorially with the particle multiplicity n of the final state.

- The n! growth of n-point perturbative Amplitudes persists also above the threshold => can integrate over n-particle phase space to obtain cross-sections

- This was studied in the 90s in scalar QFTs (Voloshin; Son; Libanov, Rubakov, Troitski; …)

- But now the FCC provides exciting challenges/opportunities to realise this in the context of the multi- Higgs and Massive Vector bosons production in the SM.

- Critical energy scale above which the production may be unsuppressed is ~ 50-100 TeV.

Tree-level Amplitudes *above mass thresholds* are determined by recursive solutions to classical equations — now include the kinematic dependence

$$-\,(\partial^\mu\partial_\mu + M_h^2)\,\varphi \;=\; 3\lambda v\,\varphi^2 \;+\; \lambda\,\varphi^3$$

This classical equation for $\varphi(x) = h(x) - v$ determines directly the structure of the recursion relation for tree-level scattering amplitudes:

$$(P_{\rm in}^2 - M_h^2)\,\mathcal{A}_n(p_1\dots p_n) \;=\; 3\lambda v \sum_{n_1,n_2}^{n} \delta^n_{n_1+n_2} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)},\dots,p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\dots p_{n_2}^{(2)})$$

$$+\lambda \sum_{n_1,n_2,n_3}^{n} \delta^n_{n_1+n_2+n_3} \sum_{\mathcal{P}} \mathcal{A}_{n_1}(p_1^{(1)}\dots p_{n_1}^{(1)})\,\mathcal{A}_{n_2}(p_1^{(2)}\dots p_{n_2}^{(2)})\,\mathcal{A}_{n_3}(p_1^{(3)}\dots p_{n_2}^{(3)})$$

Away from the multi-particle threshold, the external particles 3-momenta $\vec{p}_i$ are non-vanishing. In the non-relativistic limit, the leading momentum-dependent contribution to the amplitudes is proportional to $E_n^{\rm kin}$ (Galilean Symmetry),

$$\mathcal{A}_n(p_1\dots p_n) \;=\; \mathcal{A}_n + \mathcal{M}_n\,E_n^{\rm kin} \;:=\; \mathcal{A}_n + \mathcal{M}_n\,n\,\varepsilon\,,$$

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\rm kin} \;=\; \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n}\vec{p}_i^{\,2}\,.$$

In the non-relativistic limit we have $\varepsilon \ll 1$.

Above the n-particle thresholds: solution of the recursion relations

$$\varepsilon = \frac{1}{n\, M_h}\, E_n^{\rm kin} \;=\; \frac{1}{n}\,\frac{1}{2M_h^2}\,\sum_{i=1}^{n} \vec{p}_i^{\;2}$$

$$\mathcal{A}_n(p_1 \ldots p_n) \;=\; n!\,(2v)^{1-n}\left(1 - \frac{7}{6}\,n\,\varepsilon \;-\; \frac{1}{6}\,\frac{n}{n-1}\,\varepsilon + \mathcal{O}(\varepsilon^2)\right).$$

An important observation is that by exponentiating the order-$n\varepsilon$ contribution, one obtains the expression for the amplitude which solves the original recursion relation to all orders in $(n\varepsilon)^m$ in the large-n non-relativistic limit,

$$\mathcal{A}_n(p_1 \ldots p_n) \;=\; n!\,(2v)^{1-n}\,\exp\left[-\frac{7}{6}\,n\,\varepsilon\right], \quad n \to \infty\,, \quad \varepsilon \to 0\,, \quad n\varepsilon = {\rm fixed}\,.$$

Simple corrections of order ε, with coefficients that are not-enhanced by n are expected, but the expression is correct to all orders $n\varepsilon$ in the double scaling large-n limit. The exponential factor can be absorbed into the z variable so that

$$\varphi(z) = \sum_{n=1}^{\infty} d_n\, \left(z\, e^{-\frac{7}{6}\,\varepsilon}\right)^{n}\,,$$

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remains a solution to the classical equation and the original recursion relations.

Can now integrate over the phase-space

In general: Methods based on classical solutions result in the exponential form for the n-particle cross-section: exp[F_holy_grail]

- Libanov, Rubakov, Son, Troitsky; M Voloshin; …

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

$$\sigma_n \propto \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) + \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\, n\,\varepsilon\right]$$

More generally, in the large-n limit with λn = fixed and ε = fixed, one expects

$$\sigma_n \propto \exp\left[\frac{1}{\lambda}\, F_{\rm h.g.}(\lambda n, \varepsilon)\right]$$

[e.g. Libanov, Rubakov, Troitsky review 1997]

where the *holy grail* function $F_{\rm h.g.}$ is of the form,

known function at tree level

$$\frac{1}{\lambda}\, F_{\rm h.g.}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda}\,(f_0(\lambda n) + f(\varepsilon))$$

known at eps<<1

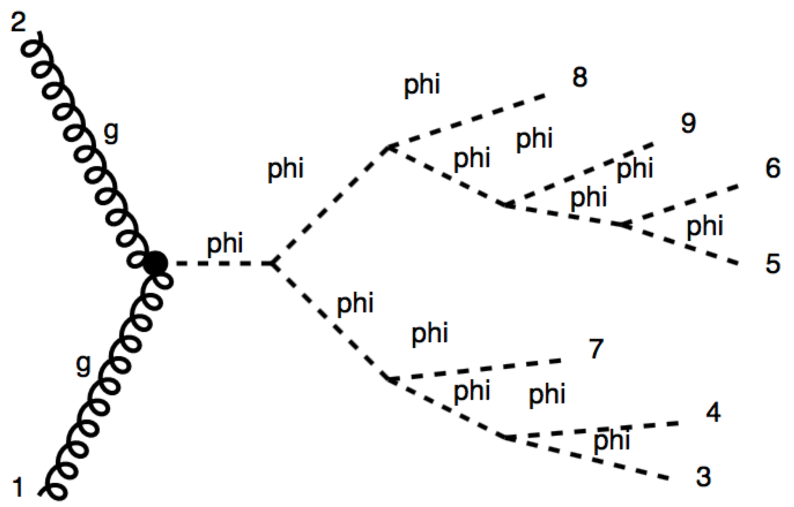
In our higgs model, i.e. the scalar theory with SSB,

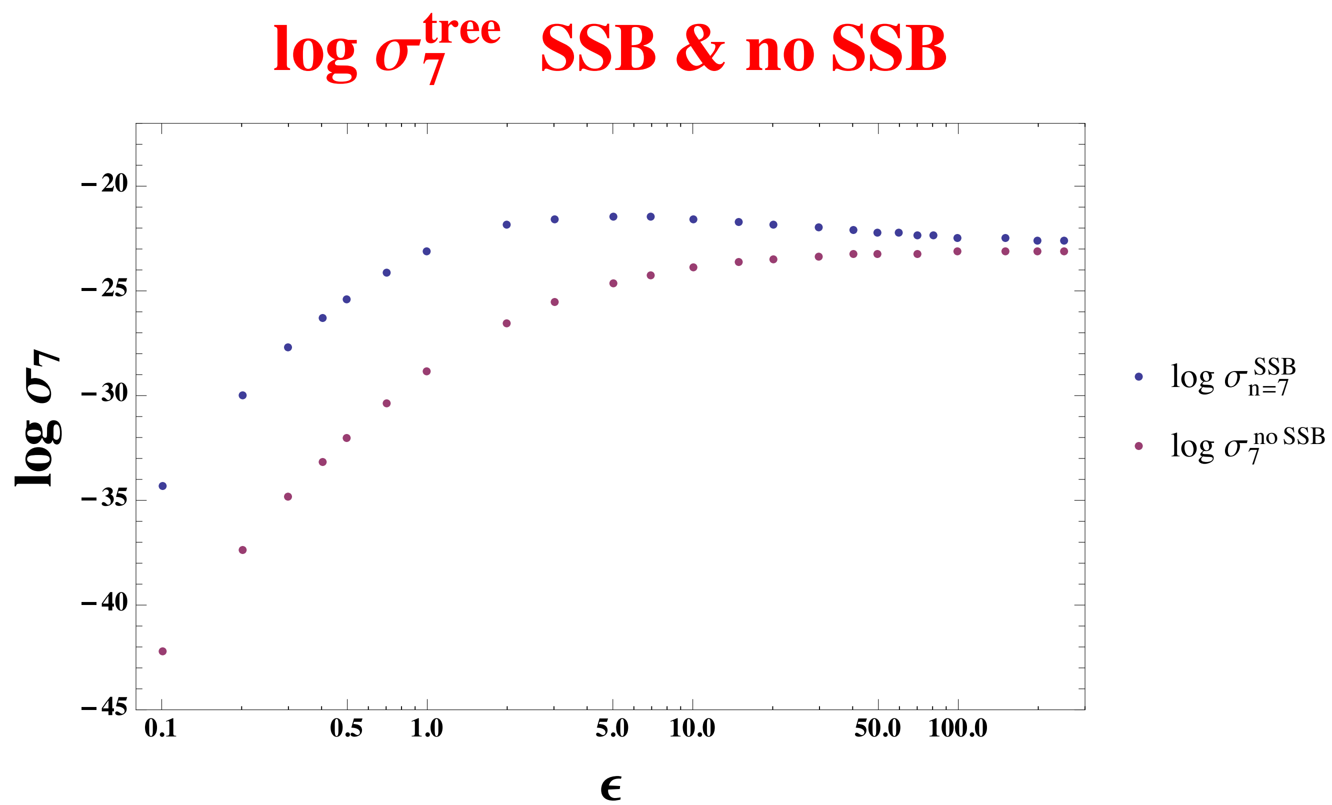
$$\begin{aligned} f_0(\lambda n) &= \log\frac{\lambda n}{4} - 1 && \text{at tree level} \\ f(\varepsilon) &\to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\,\varepsilon && \text{for } \varepsilon \ll 1 \end{aligned}$$

Next step:

compute $f(\varepsilon)$ for any epsilon

1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)





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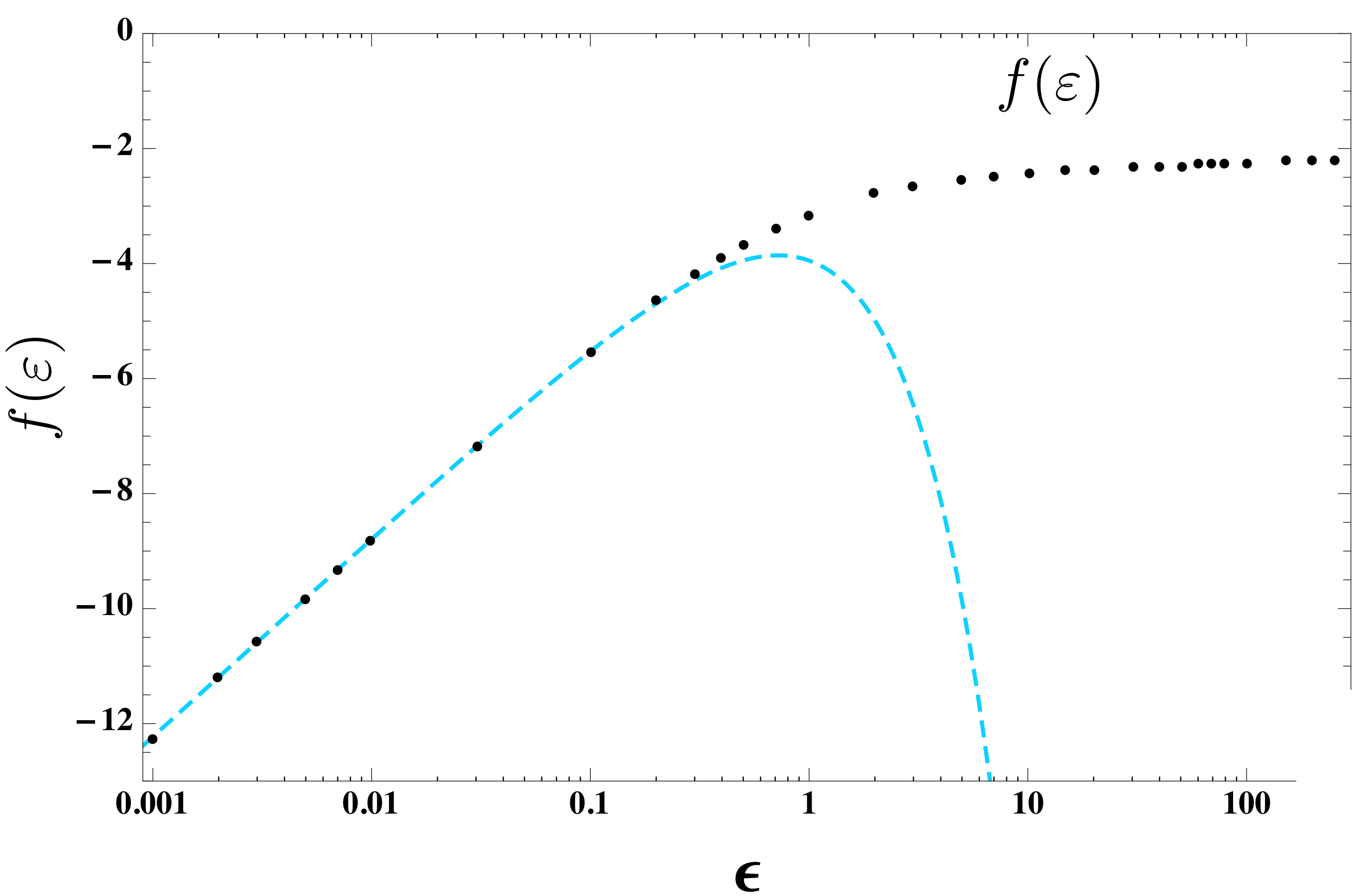

2. Assume large n, subtract $f_0(\lambda n)$ and extract $f(\varepsilon)$ using

$$\frac{1}{n}\log\sigma_n = \frac{1}{\lambda n} F_{\rm h.g.}(\lambda n, \varepsilon) = f_0(\lambda n) + f(\varepsilon)$$

$$f_0(\lambda n) = \log\frac{\lambda n}{16} - 1$$

$f(\varepsilon)$ asymptotes to a const at large eps (highly relativistic final state)

Can also include *loop corrections* to amplitudes on thresholds:

The 1-loop corrected threshold amplitude for the pure n Higgs production:

$$\phi^4 \text{ with SSB}: \quad \mathcal{A}_{1\to n}^{\text{tree+1loop}} = n!\,(2v)^{1-n}\left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$\mathcal{A}_{1\to n} = \mathcal{A}_{1\to n}^{\text{tree}} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right]$$

in the limit $\lambda \to 0$, $n \to \infty$ with λn^2 fixed. Here B is determined from the 1-loop calculation (as above) – *Smith; Voloshin 1992*): $B = +\lambda n \frac{\sqrt{3}}{4\pi}$

$$f_0(\lambda n) = \log\frac{\lambda n}{4} - 1 + \lambda n \frac{\sqrt{3}}{4\pi}$$

a significant enhancement (but higher orders unknown)

$$f(\varepsilon) \to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\varepsilon \qquad \text{for } \varepsilon \ll 1$$

In Summary:

1. Compute cross-sections with MadGraph 2 -> 5,6,7 at all energies (i.e. arbitrary epsilon)
2. Scale to large n using the known n-dependence in the holy grail including the leading-loop factor to the exponent $+\lambda n \frac{\sqrt{3}}{4\pi}$

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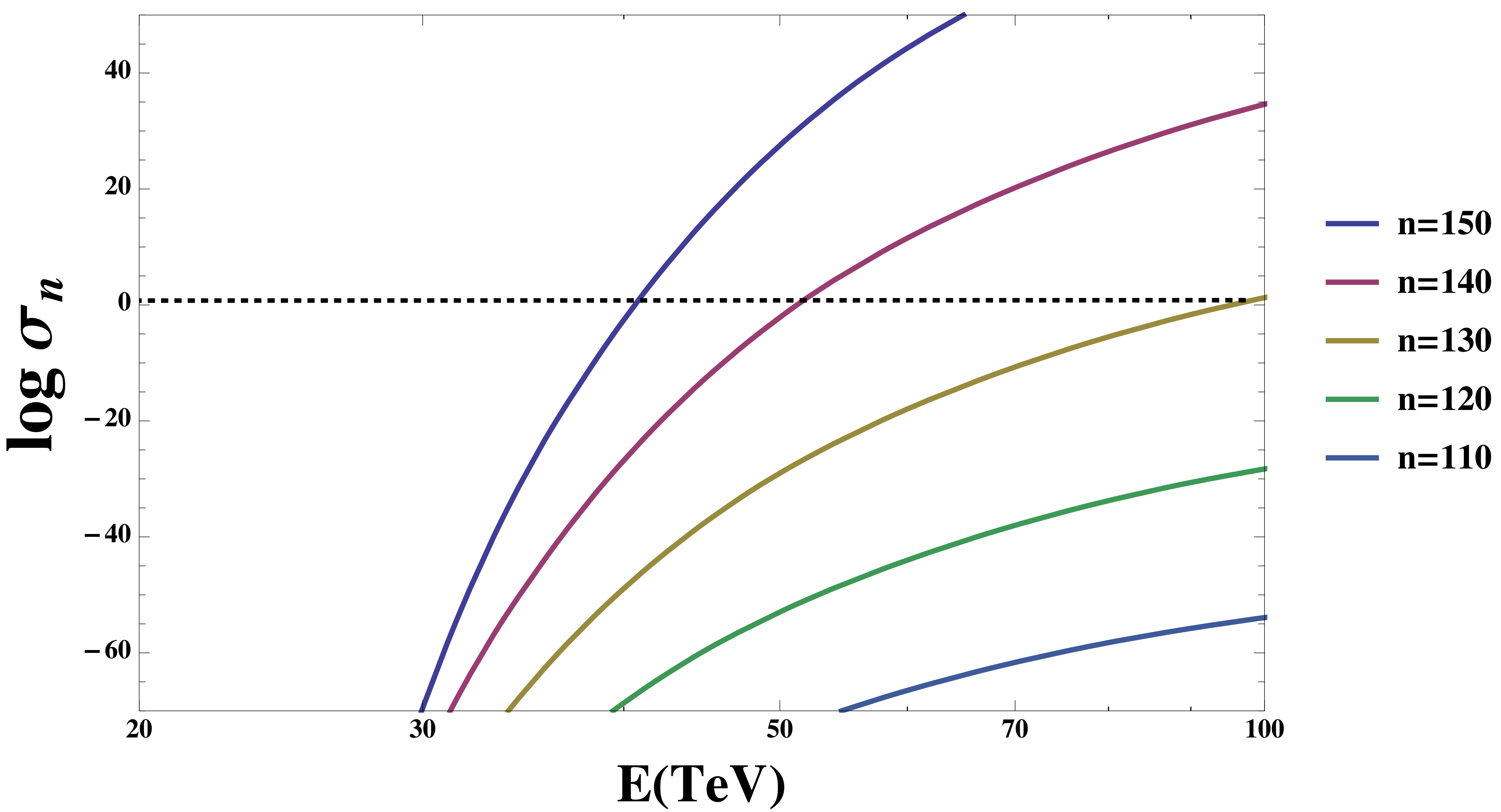
$$\log \sigma_n^{\text{loop}}$$



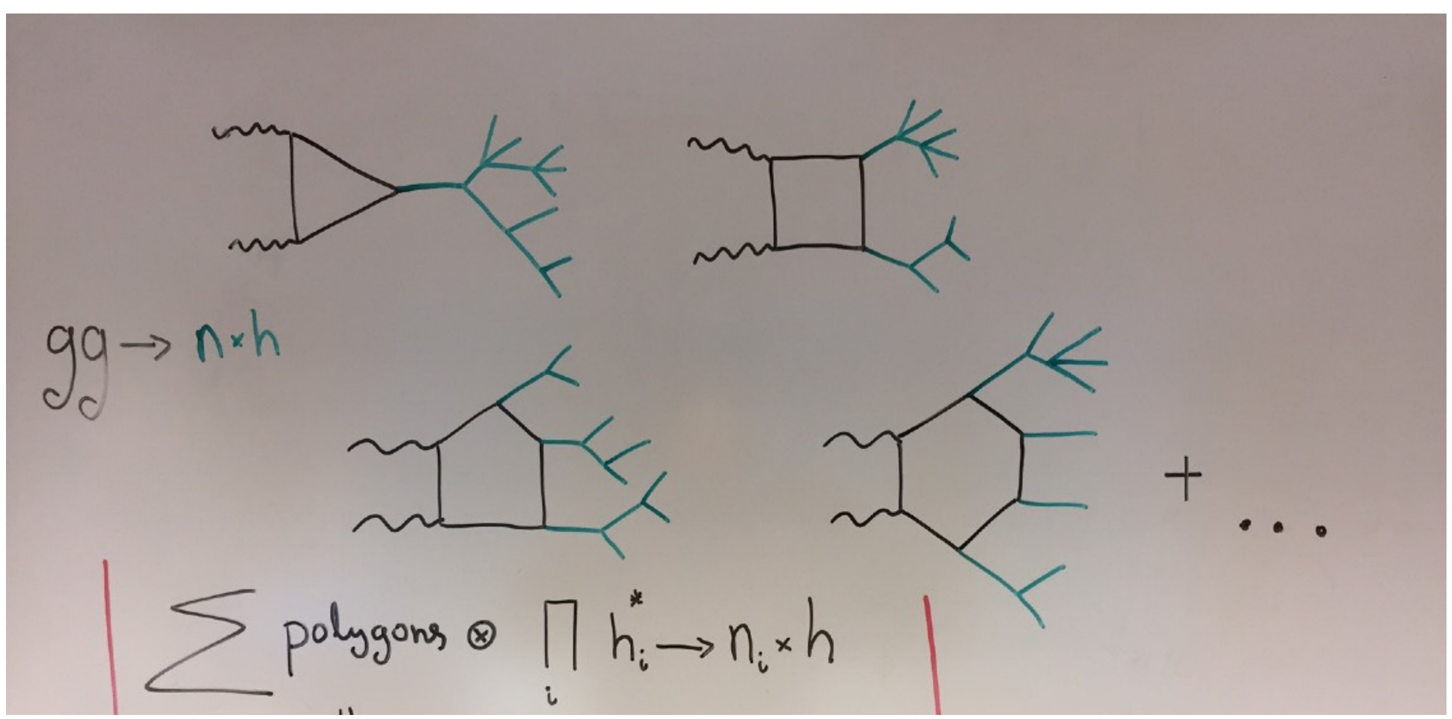
Now: full Gluon fusion process including polygons

$$\mathcal{A}_{gg\to n\times h} = \sum_{\text{polygons}} \mathcal{A}^{\text{polygons}}_{gg\to k\times h^*} \sum_{n_1+\ldots+n_k=n} \prod_{i=1}^{k} \mathcal{A}_{h_i^*\to n_i\times h}$$

1-loop polygons
Compute numerically
in the high-energy limit where E>> all kin scales

MadGraph5_aMC@NLO

1*->n multi-Higgs processes.
Already computed



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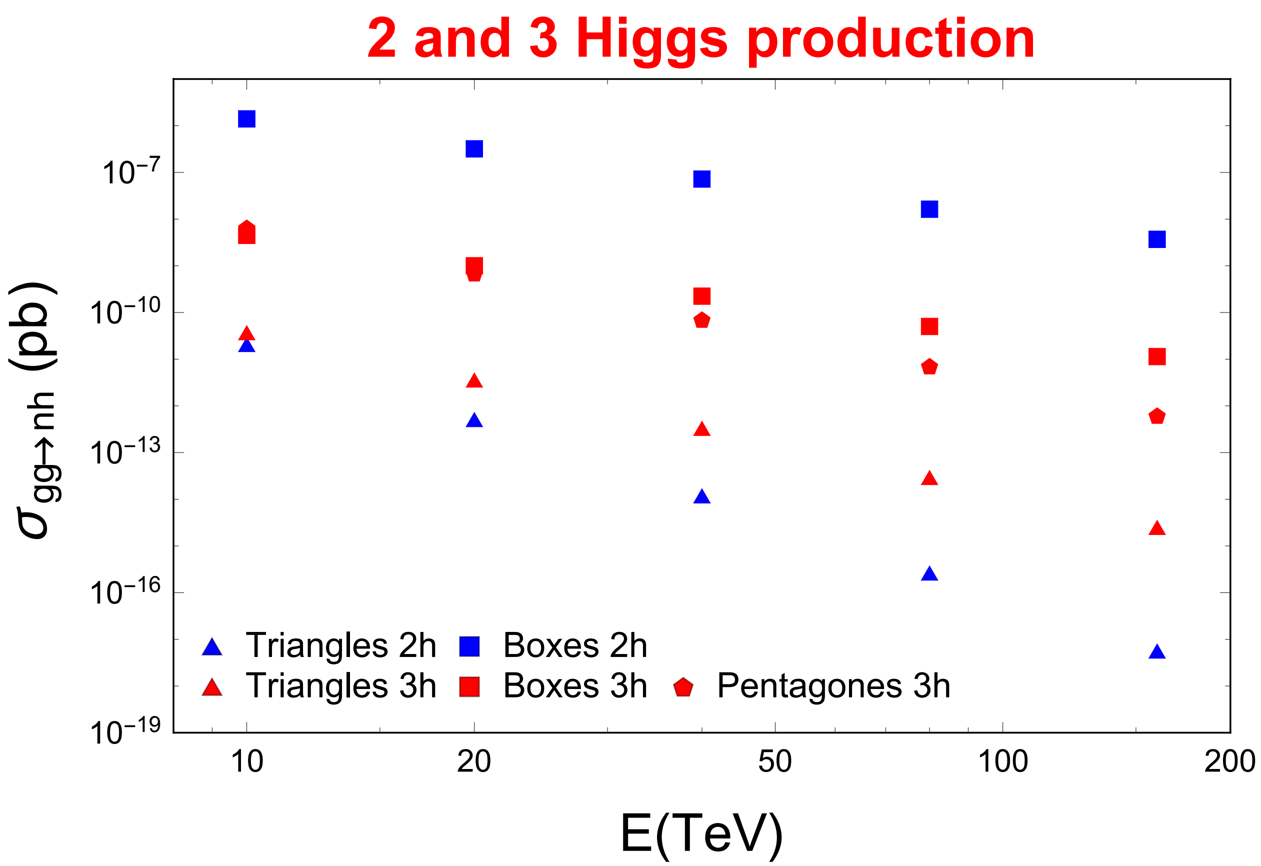
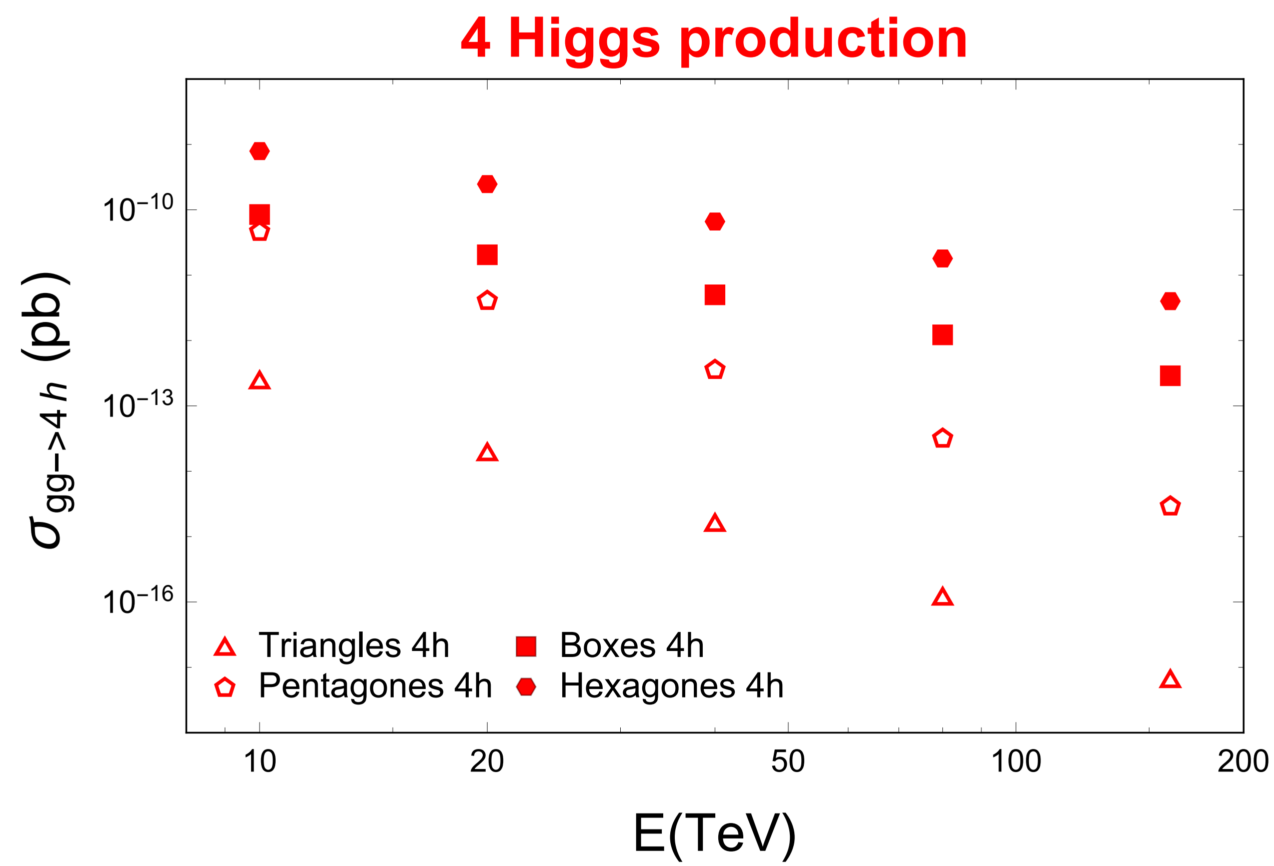
Polygon contributions:





$s \gg m_t\,,\ M_h$ limit

	$\sigma_{gg\to hh}$	$\sigma_{gg\to hhh}$	$\sigma_{gg\to hhhh}$
Triangles	$y_t^2 \frac{m_t^2 M_h^2}{s^3} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^4}{v^4}$	$y_t^2 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^6}{v^6}$
Boxes	$y_t^4 \frac{1}{s}$	$y_t^4 \frac{1}{s} \frac{M_h^2}{v^2}$	$y_t^4 \frac{1}{s} \frac{M_h^4}{v^4}$
Pentagons	–	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right)$	$y_t^6 \frac{m_t^2}{s^2} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \frac{M_h^2}{v^2}$
Hexagons	–	–	$y_t^8 \frac{1}{s}$

- Compare to the high-energy scaling behaviour resulting from the effective vertices:

	$\sigma^{\text{eft}}_{gg\to hh}$	$\sigma^{\text{eft}}_{gg\to hhh}$	$\sigma^{\text{eft}}_{gg\to hhhh}$
$\alpha_s \text{tr}(G_{\mu\nu}G^{\mu\nu})\, h^1$	$\frac{M_h^2}{v^2} s^0$	$\frac{M_h^4}{v^4} s^0$	$\frac{M_h^6}{v^6} s^0$
$\alpha_s \text{tr}(G_{\mu\nu}G^{\mu\nu})\, h^2$	s	$\frac{M_h^2}{v^2} s$	$\frac{M_h^4}{v^4} s$
$\alpha_s \text{tr}(G_{\mu\nu}G^{\mu\nu})\, h^3$	–	s^2	$\frac{M_h^2}{v^2} s^2$
$\alpha_s \text{tr}(G_{\mu\nu}G^{\mu\nu})\, h^4$	–	–	s^3

- The pattern established for polygons with 2+k edges:

$$(2+k)\text{−polygons} : \qquad \sigma_{gg\to n\times h} \propto \frac{1}{s}\, y_t^{2k} \left(\frac{M_h}{v}\right)^{2(n-k)} \times \begin{cases} 1 & : \ k = \text{even} \\ \frac{m_t^2}{s} \log^4\left(\frac{m_t}{\sqrt{s}}\right) & : \ k = \text{odd}\,. \end{cases}$$

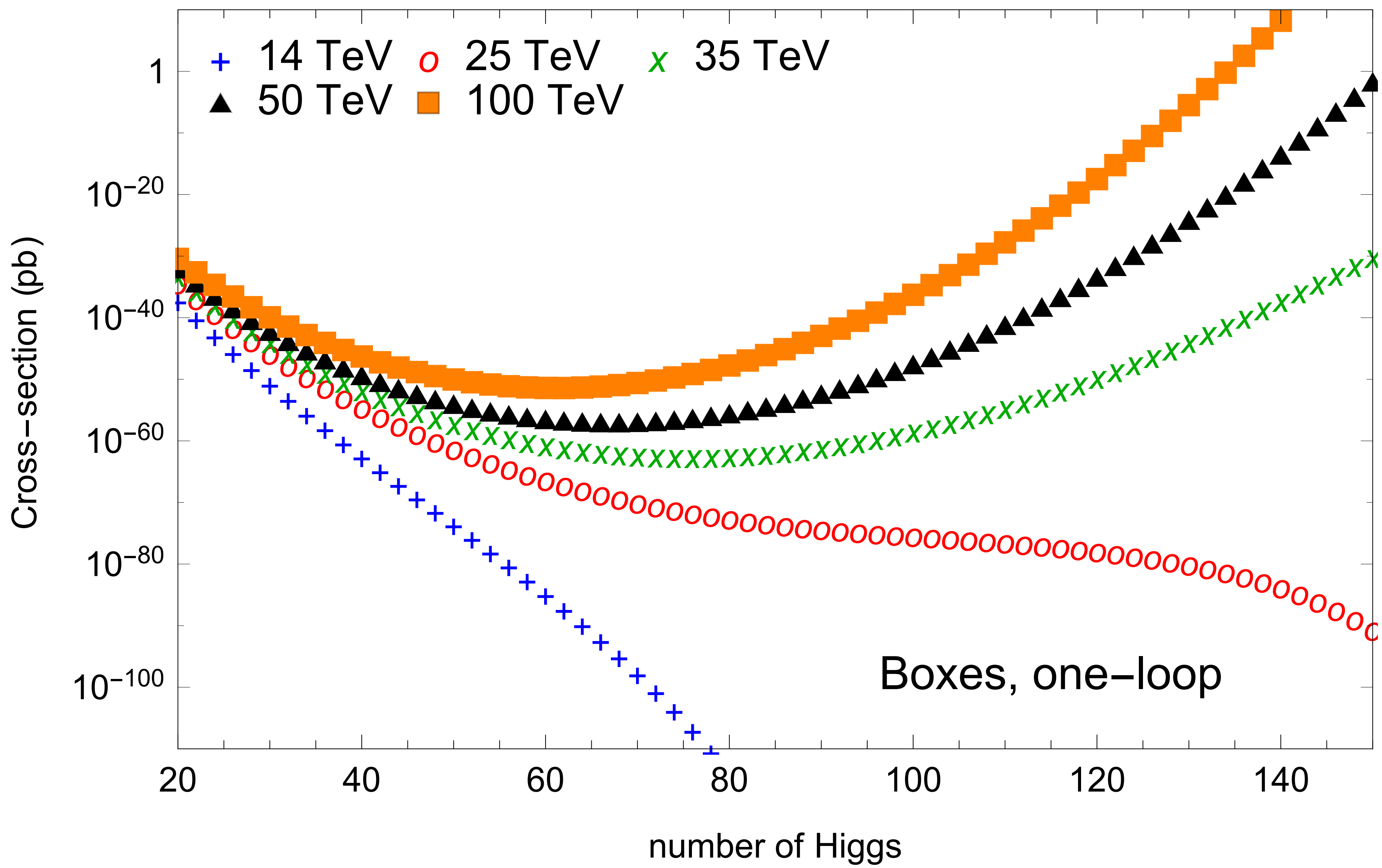
- allows to associate the full 1-loop result from rank-(2+k) polygons in the high E limit to the effective vertex - now including the form-factors - via:

$$\mathcal{V}_k = C_k \frac{\alpha_s(\sqrt{s})}{\pi} \text{tr}(G_{\mu\nu}G^{\mu\nu}) \left(\frac{y_t\, h}{\sqrt{s}}\right)^k \times \begin{cases} 1 & : \ k = \text{even} \geq 2 \\ \frac{m_t}{\sqrt{s}} \log^2\left(\frac{m_t}{\sqrt{s}}\right) & : \ k = \text{odd} \geq 3\,. \end{cases}$$

- For h substitute the classical solution generating functional to represent subsequent Higgs branchings. Ck constants are known (computed). [More detail in the paper.]

Finally: combine with the multi-Higgs branchings & convolute with gluon PDFs

<u>Main results:</u> 20 to 150 Higgs bosons @ different collider energies



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Summary: multi-Higgs production

- At (not too high) high energies perturbative Standard Model exhibits a formal breakdown. Perturbative unitarity is broken.

- OPTIONS:

A. At high energies (multiplicities) the Standard Model is fundamentally non-perturbative (?)

B. The theory classicalizes: the ultra-high multiplicity processes will completely dominate everything else (?)

C. New physics beyond the Standard Model has to set in before the cross-sections become large, i.e. as early as at ~50 TeV (?)

- New theoretical approaches & computational techniques are needed to go beyond (or better than) perturbation theory.