

Dark matter probes with FCC-ee

(based on preliminary results from two on-going projects)
with Lian-Tao Wang, Xiao-Ping Wang, Wei Xue, Felix Yu

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JGU Mainz

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Cluster of Excellence

PRISMA

Precision Physics,
Fundamental Interactions
and Structure of Matter

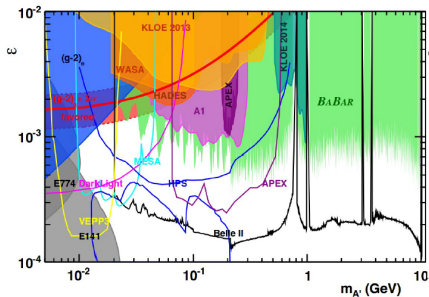
Motivation for first project

- Dark matter with two portals at FCC-ee (Xiao-Ping Wang, Felix Yu)
 - we consider $U(1)'$ dark photon UV model, with kinetic mixing portal ($\epsilon B_{\mu\nu} F'^{\mu\nu}$) and Higgs portal ($\Phi^\dagger \Phi H^\dagger H$)
 - LEP limits on Z' : for $m_{Z'} < 90\text{GeV}$, coupling (ϵe) $\lesssim 10^{-2}$
 - $A'\gamma$ searches (P.F. Yin et al 0904.4644; H.B Li et al 0911.2067; V. Prasad et al 1508.07659; D. Curtin et al 1412.0018; BABAR 1406.2980; M. Karliner et al 1503.07209; Y. Hochberg et al 1512.07917; Belle 1609.05599;)

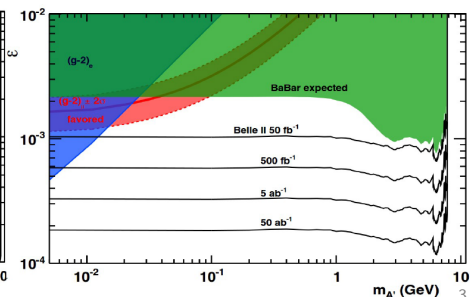
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 - $A'\gamma$ Belle projection ($< 10\text{GeV}$) (from Inguglia ICHEP2016)

$$e^+e^- \rightarrow \gamma A' \rightarrow \gamma e^+e^-, \gamma \mu^+\mu^-, \text{ prompt}$$

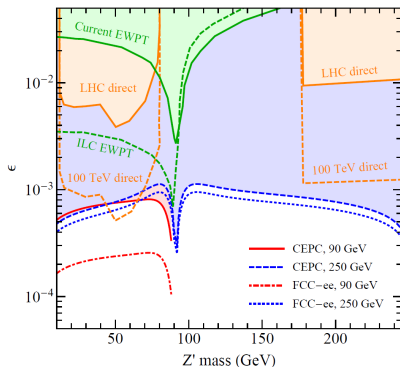


$$e^+e^- \rightarrow \gamma A' \rightarrow \gamma \chi \bar{\chi}$$



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 - $A'\gamma$ FCC-ee projection ($> 10\text{GeV}$, $A' \rightarrow \ell^+ \ell^-$) (M. Karliner et al 1503.07209)



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 - Electroweak precision observables at Giga Z $\Rightarrow \epsilon \sim 3 \times 10^{-3}$ Curtin et al 1412.0018
 - Higgs precision measurement FCC-ee projection $\frac{\Delta\sigma(ZH)}{\sigma(ZH)} \sim 0.5\%$
 $\Rightarrow \sin^2 \alpha \lesssim 0.005$
 - **New combinations:** explore all VV and VS possibilities, $V = \gamma, Z, A'$ and $S = H, H_d$ (HA' , SA' and ZS search by Gilad's talk and ; S. Biswas et al 1503.05836; Belle 1502.00084;)
 - $hZ \rightarrow (\bar{b}b/\tau^+\tau^-) + Z$ search at LEP hep-ph/0602042
 for $m_h < 112\text{GeV}$, $\sigma/\sigma_{\text{SM}} \sim \sin^2 \alpha \in [10^{-2}, 10^{-1}]$
 - $A', H_d \rightarrow$ invisible

Dark matter with two portals

► Lagrangian

$$\mathcal{L}_{\text{DP}} \equiv \bar{\chi}(i\not{\partial} - m_{\chi} + ig_d \not{A}')\chi - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\epsilon}{2c_W}F'_{\mu\nu}B^{\mu\nu} \\ + (D_{\mu}\Phi)^{\dagger}(D_{\mu}\Phi) - \mu_d^2\Phi^{\dagger}\Phi + \frac{\lambda_d}{2}(\Phi^{\dagger}\Phi)^2 + \lambda_{HS}(\Phi^{\dagger}\Phi)(H^{\dagger}H)$$

► Mixing between $A' - Z$ and $\phi - H$

$$\begin{pmatrix} Z_{\text{SM}} \\ A_{\text{SM}} \\ A' \end{pmatrix} = \begin{pmatrix} 1 + \mathcal{O}(\epsilon^2) & 0 & -\epsilon t_W \frac{m_{A'}^2}{m_{A'}^2 - m_Z^2} \\ \mathcal{O}(\epsilon^2) & 1 & \epsilon \\ \epsilon t_W \frac{m_Z^2}{m_{A'}^2 - m_Z^2} & 0 & 1 + \mathcal{O}(\epsilon^2) \end{pmatrix} \cdot \begin{pmatrix} \tilde{Z} \\ \tilde{A} \\ \tilde{A}' \end{pmatrix} \quad A' \text{ search}$$

$$\begin{pmatrix} S \\ h \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \Phi \\ h_0 \end{pmatrix} \quad \text{Higgs precision measurement}$$

$$\tan 2\alpha = \frac{\lambda_{HS} v_h v_s}{\lambda_d v_s^2 - \lambda v_h^2}$$

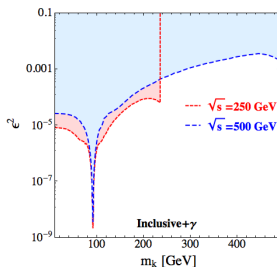
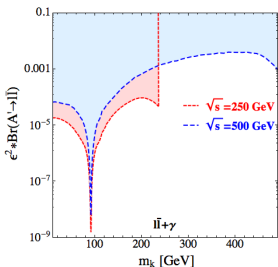
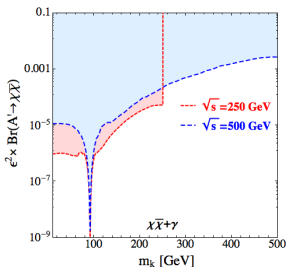
Phenomenology

- ▶ $A' \rightarrow \bar{\chi}\chi$ $\mathcal{O}(1)$, $A' \rightarrow \bar{f}f$ $\mathcal{O}(\epsilon^2)$
- ▶ $S \rightarrow A'A'$ $\mathcal{O}(1)$, $S \rightarrow \bar{b}b$ $\mathcal{O}(\sin^2 \alpha)$

	process	suppression	BKG
V+V	<u>$A'\gamma \rightarrow (\chi\bar{\chi})\gamma$</u>	ϵ^2	$\nu\bar{\nu}\gamma$
	$Z\gamma \rightarrow (SA')\gamma \rightarrow 3A'\gamma \rightarrow 3(\bar{\chi}\chi)\gamma$		
	$ZA' \rightarrow Z(\chi\bar{\chi})$		$l^+l^-\nu\bar{\nu}$
V+S	$Zh \rightarrow Z(2S) \rightarrow Z(4A') \rightarrow Z4(\bar{\chi}\chi)$	$\sin^2 \alpha$	$l^+l^-\nu\bar{\nu}$
	<u>$ZS \rightarrow Z(2A') \rightarrow Z2(\bar{\chi}\chi)$</u>		
	$A'h \rightarrow (\chi\bar{\chi})(\bar{b}b)$	ϵ^2	$\nu\bar{\nu}\bar{b}b$
	$Zh \rightarrow Z(A'Z) \rightarrow 2Z(\chi\bar{\chi})$		$2(l^+l^-\nu\bar{\nu})$
	$A'S \rightarrow (\chi\bar{\chi})(SM\bar{S}\bar{M})$		$\epsilon^2 \sin^2 \alpha$

$A'\gamma$ sensitivity to ϵ

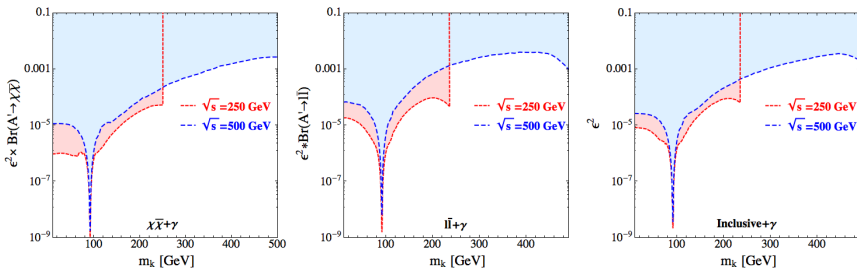
- ▶ $A' \rightarrow \bar{\chi}\chi$ $\mathcal{O}(1)$, $\ell^+\ell^-$ $\mathcal{O}(\epsilon^2)$
- ▶ Cuts: $E_\gamma^{\text{recoil}} = \frac{s-m_{A'}^2}{2\sqrt{s}}$, $|E_\gamma - E_\gamma^{\text{recoil}}| < 2.5\text{GeV}$ ($\frac{\delta E_\gamma}{E_\gamma} = \frac{0.16}{\sqrt{E/\text{GeV}}} \oplus 0.01$)
- ▶ preliminary results



- ▶ $m_{A'} = m_k(1 + \mathcal{O}(\epsilon^2))$
- ▶ for visible $A' \rightarrow \ell^+\ell^-$: adding $m_{\ell^+\ell^-}$ cut can improve $\epsilon \sim [10^{-4}, 10^{-3}]$ (see M. Karliner et al 1503.07209)

$A'\gamma$ sensitivity to ϵ

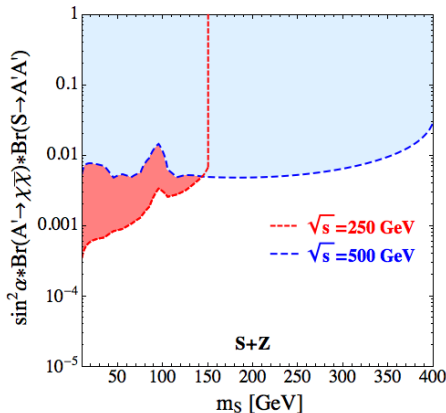
- ▶ $A' \rightarrow \bar{\chi}\chi$ $\mathcal{O}(1)$, l^+l^- $\mathcal{O}(\epsilon^2)$
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- ▶ preliminary results



- ▶ Comparing with DM relic abundance requirement:
 - ▶ DM relic abundance: $\sigma(\bar{\chi}\chi \rightarrow A'^*/Z^* \rightarrow \bar{f}f) \sim \epsilon^2 \alpha_{\text{EM}} \alpha_d \frac{1}{m_{A'}^2}$
 - ▶ Assuming $\alpha_d \sim \alpha_{\text{EM}}$, $m_{A'} \sim \mathcal{O}(10)\text{GeV}$, $\epsilon \sim \mathcal{O}(0.1 \sim 1)$

ZS sensitivity to $\sin \alpha$

- ▶ $Z \rightarrow \ell^+ \ell^-$, $S \rightarrow A' A'$ $\mathcal{O}(1)$ and $A' \rightarrow \bar{\chi} \chi$ $\mathcal{O}(1)$
- ▶ $|m_{\ell^+ \ell^-} - m_Z| < 10 \text{ GeV}$ with $|m_{\text{inv}} - m_S| < 2 \text{ GeV}$
- ▶ preliminary results



- ▶ Higgs precision measurement $\frac{\Delta\sigma(ZH)}{\sigma(ZH)} \sim 0.5\% \implies \sin^2 \alpha \lesssim 0.005$

Summary for the first project

► Dark matter with two portals

► ϵ sensitivity

► $A'\gamma$ channel sensitivity $\epsilon \sim \mathcal{O}(10^{-3})$

■ DM relic abundance: $\sigma(\bar{\chi}\chi \rightarrow \bar{f}f) \sim \epsilon^2 \alpha_{\text{EM}} \alpha_d \frac{1}{m_{A'}^2}$

■ Assuming $\alpha_d \sim \alpha_{\text{EM}}$, $m_{A'} \sim \mathcal{O}(10)\text{GeV}$, $\epsilon \sim \mathcal{O}(0.1 \sim 1)$

■ DM relic abundance region well explored

► $\sin \alpha$ sensitivity

► ZS channel sensitivity $\sin^2 \alpha \sim \mathcal{O}(10^{-3} \sim 10^{-2})$

■ Higgs precision measurement $\frac{\Delta\sigma(ZH)}{\sigma(ZH)} \sim 0.5\% \implies \sin^2 \alpha \lesssim 0.005$

■ New combinations of VV and VS can give better sensitivity

Motivation for the second project



- What can we do for dark matter probes at FCC-ee?
 - LEP Shines Light on Dark Matter Fox et al 1103.0240
 - missing energy + X

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- What can we do for dark matter probes at FCC-ee?
 - LEP Shines Light on Dark Matter Fox et al 1103.0240
 - missing energy + X
- Giga(Tera) Z as **intensity frontier** (Lian-Tao Wang, Xiao-Ping Wang, Wei Xue)
 - We consider exotic Z decay via dark matter or dark mediators
 - indirectly constrain the model via Electroweak Precision Observables e.g. CMSSM O. Buchmueller et al 1505.04702; Linear Collider report 1504.01726;
 - Z invisible width M. Carena et al hep-ph/0308053;
 - sterile neutrino from Z pole Blondel's talk and 1411.5230;
 - $Z \rightarrow \phi \bar{f} f$ Gilad's talk ;
 - Z decay to hidden valley Verhaaren's talk
 - $e^+ e^- \rightarrow Z \rightarrow Z^* h$ from LEP PLB 385, 454; also see Flacke et al 1610.02025
 $m_h < 60.2 \text{ GeV}$ from visible, $m_h < 66.7 \text{ GeV}$ from invisible
 - $Z \rightarrow H_1 H_2$ from LEP hep-ph/0602042
 - rare Z decays Durieux's talk 1512.03071
 - Z lepton flavor violating decay Dam's talk
- Emphasize on channels with missing energy

Exotic Z decay at FCC-ee (Giga Z)

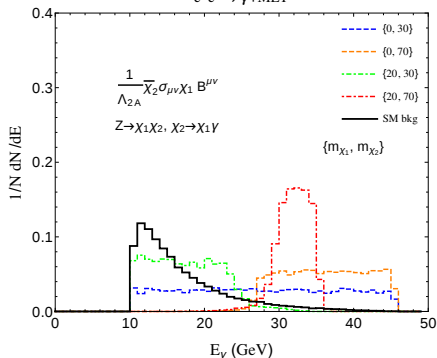
- ▶ Exotic Z decay classification by final state
 - ▶ FCC-ee speciality: missing energy and clean environment
 - ▶ final state
 - ▶ $\gamma + \cancel{E}$
 - ▶ $2\gamma + \cancel{E}$
 - ▶ $l^+l^- + \cancel{E}$
 - ▶ $JJ + \cancel{E}$
 - ▶ JJJ
 - ▶ 3γ
 - ▶ ...
 - ▶ motivation for event topologies
 - ▶ dark matter UV models
 - ▶ higher dimensional operators

Exotic decay final states and topologies

exotic Z decay	topology	SM bkg (pb)
$\gamma + \cancel{E}$	$Z \rightarrow \chi_1 \chi_2, \chi_2 \rightarrow \chi_1 \gamma$ $Z \rightarrow \chi \bar{\chi} \gamma, {}_a B_{\mu\nu} \tilde{B}^{\mu\nu}, {}_a W_{\mu\nu} \tilde{W}^{\mu\nu}$	0.243
$2\gamma + \cancel{E}$	$Z \rightarrow \phi_d A', \phi_d \rightarrow (\gamma\gamma), A' \rightarrow (\bar{\chi}\chi)$ $Z \rightarrow \chi_2 \chi_2, \chi_2 \rightarrow \gamma \chi_1$ $Z \rightarrow \chi_2 \chi_1, \chi_2 \rightarrow \chi_1 \phi, \phi \rightarrow (\gamma\gamma)$	1.5×10^{-4}
$\ell^+ \ell^- + \cancel{E}$	$Z \rightarrow \phi_d A', A' \rightarrow (\ell^+ \ell^-), \phi_d \rightarrow (\bar{\chi}\chi)$ $Z \rightarrow \chi_2 \chi_1, \chi_2 \rightarrow \ell^+ \ell^- \chi_1$ $Z \rightarrow \bar{\chi} \chi \ell^+ \ell^-$ vector-like heavy lepton	1.7×10^{-3}
$JJ + \cancel{E}$	$Z \rightarrow \phi_d A', A' \rightarrow (jj), \phi_d \rightarrow (\bar{\chi}\chi)$ $Z \rightarrow \phi_d A', \phi_d \rightarrow (\bar{b}b), A' \rightarrow (\bar{\chi}\chi)$ $Z \rightarrow \chi_1 \chi_2, \chi_2 \rightarrow \bar{b}b \chi_1$	9.5×10^{-3}
$Z \rightarrow JJ + JJ$	$Z \rightarrow \phi_d A', \phi_d \rightarrow jj, A' \rightarrow jj$ $Z \rightarrow \phi_d A', \phi_d \rightarrow bb, A' \rightarrow jj$ $Z \rightarrow \phi_d A', \phi_d \rightarrow bb, A' \rightarrow bb (H_1 H_2, G_0 G_0)$	5450
$Z \rightarrow 3\gamma$	${}_a B_{\mu\nu} \tilde{B}^{\mu\nu}, {}_a W_{\mu\nu} \tilde{W}^{\mu\nu}$	

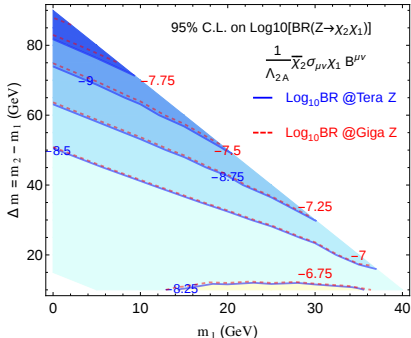
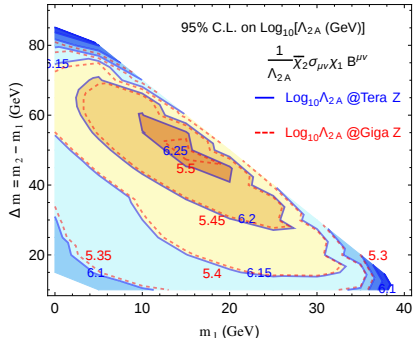
Background for $Z \rightarrow \chi_1 \chi_2, \chi_2 \rightarrow \chi_1 \gamma$

- ▶ SM background from γZ^* , for $\bar{\nu}_e \nu_e W^*$ also contributes
 - ▶ γ dominates at small energy due to radiative nature
- ▶ Signal topology from: $\frac{1}{\Lambda^2} \bar{\chi}_2 \sigma^{\mu\nu} \chi_1 B_{\mu\nu}$
 - ▶ E_γ has a box shape due to boost of χ_2
 - ▶ $E_\gamma^{max} = \frac{m_2^2 - m_1^2}{4m_2^2} \frac{s + m_2^2 - m_1^2 + \sqrt{s^2 + (m_2^2 - m_1^2)^2 - 2s(m_2^2 + m_1^2)}}{\sqrt{s}}$
 - ▶ $E_\gamma^{min} = \frac{m_2^2 - m_1^2}{4m_2^2} \frac{s + m_2^2 - m_1^2 - \sqrt{s^2 + (m_2^2 - m_1^2)^2 - 2s(m_2^2 + m_1^2)}}{e^+e^- \rightarrow \gamma + MET}$



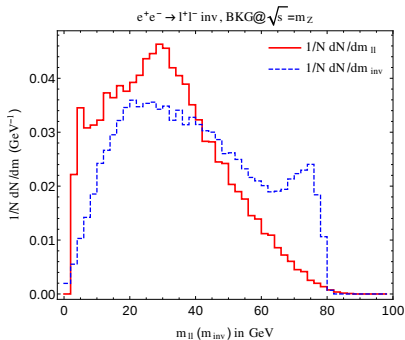
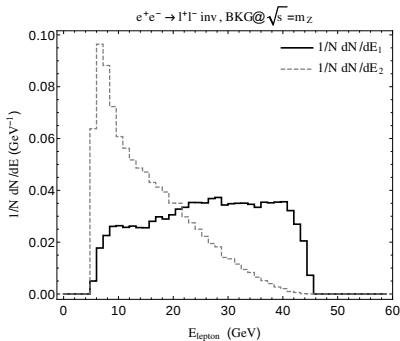
Sensitivity for $Z \rightarrow \chi_1 \chi_2, \chi_2 \rightarrow \chi_1 \gamma$

- ▶ pre-cuts: $E_\gamma > 10\text{GeV}, \eta < 2.3$
- ▶ cuts: $E_\gamma^{max} > E_\gamma > E_\gamma^{min}, m_{inv} > 2m_{\chi_1}$
 - ▶ BR sensitivity $\propto \sqrt{L}, BR \sim 10^{-7}, 10^{-8.5}$
 - ▶ Λ sensitivity $\propto L^{1/4}, \Lambda \sim 10^{5.5}\text{GeV}, 10^{6.25}\text{GeV}$
 - ▶ DM relic requires: $\Lambda \sim \mathcal{O}(10^2 \sim 10^3)\text{GeV}$
 - ▶ preliminary results

 $e^+e^- \rightarrow \gamma + inv$ @ $\sqrt{s} = m_Z$ @2A $e^+e^- \rightarrow \gamma + inv$ @ $\sqrt{s} = m_Z$ @2A

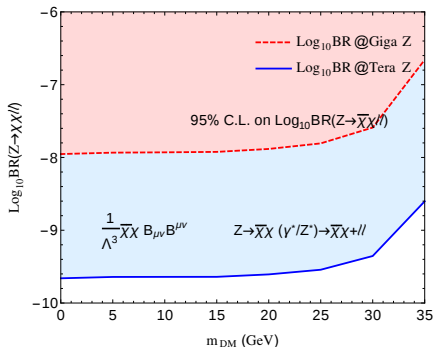
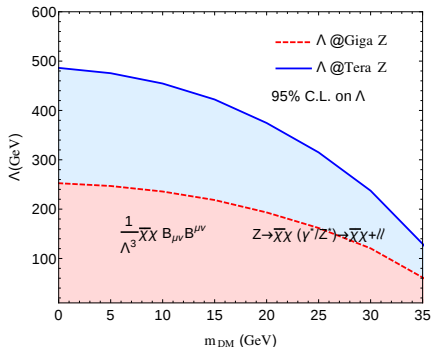
Background for $Z \rightarrow \bar{\chi}\chi Z^*/\gamma^*, Z^*/\gamma^* \rightarrow l^+l^-$

- ▶ SM background from γ^* , W^* and Z^*
- ▶ Signal topology from: $\frac{1}{\Lambda^3} \bar{\chi}\chi B_{\mu\nu} B^{\mu\nu}$



Sensitivity for $Z \rightarrow \bar{\chi}\chi Z^*/\gamma^*, Z^*/\gamma^* \rightarrow \ell^+\ell^-$

- ▶ pre-cuts: pre-cuts: $E_{\ell_1} > 10\text{GeV}$, $E_{\ell_2} > 5\text{GeV}$, $\eta < 2.3$
- ▶ cuts: $m_{\ell^+\ell^-} < 10\text{GeV}$, $2m_1 < m_{\text{inv}}$
 - ▶ BR sensitivity $\propto \sqrt{L}$, $BR \sim 10^{-8}$, $10^{-9.5}$
 - ▶ Λ sensitivity $\propto L^{1/6}$, $\Lambda \sim [50, 250]\text{GeV}$, $[100, 500]\text{GeV}$
 - ▶ DM relic requires: $\Lambda \sim \mathcal{O}(100 \sim 300)\text{GeV}$
 - ▶ preliminary results

95% C.L. on $\text{Log}_{10}\text{BR}(Z \rightarrow \bar{\chi}\chi/\ell)$ 95% C.L. on Λ 

Exotic decay final states and preliminary results

exotic Z decay	topology	preliminary BR at Giga(Tera)Z
$\gamma + \cancel{E}$	$Z \rightarrow \chi_1 \chi_2, \chi_2 \rightarrow \chi_1 \gamma$	$10^{-6.5} \sim 10^{-7.5} (10^{-8} \sim 10^{-9})$
	$Z \rightarrow \chi \bar{\chi} \gamma$	$10^{-6.5} \sim 10^{-7.5} (10^{-8} \sim 10^{-9})$
$2\gamma + \cancel{E}$	$Z \rightarrow \phi_d A', \phi_d \rightarrow (\gamma\gamma), A' \rightarrow (\bar{\chi}\chi)$	$10^{-8.1} \sim 10^{-8.4} (10^{-10.5} \sim 10^{-11.5})$
	$Z \rightarrow \chi_2 \chi_2, \chi_2 \rightarrow \gamma \chi_1$	$10^{-7.5} \sim 10^{-8.5} (10^{-9} \sim 10^{-10})$
	$Z \rightarrow \chi_2 \chi_1, \chi_2 \rightarrow \chi_1 \phi, \phi \rightarrow (\gamma\gamma)$	$10^{-8.1} \sim 10^{-8.4} (10^{-10} \sim 10^{-11})$
$l^+ l^- + \cancel{E}$	$Z \rightarrow \phi_d A', A' \rightarrow (l^+ l^-), \phi_d \rightarrow (\bar{\chi}\chi)$	$\sim 10^{-8.3} (\sim 10^{-10})$
	$Z \rightarrow \chi_2 \chi_1, \chi_2 \rightarrow l^+ l^- \chi_1$	$\sim 10^{-8} (\sim 10^{-9.5})$
	$Z \rightarrow \bar{\chi} \chi l^+ l^-$	$\sim 10^{-8} (\sim 10^{-9.5})$
$JJ + \cancel{E}$	$Z \rightarrow \phi_d A', A' \rightarrow (jj), \phi_d \rightarrow (\bar{\chi}\chi)$	$\sim 10^{-8.5} (\sim 10^{-10})$
	$Z \rightarrow \phi_d A', \phi_d \rightarrow (\bar{b}b), A' \rightarrow (\bar{\chi}\chi)$	
	$Z \rightarrow \chi_1 \chi_2, \chi_2 \rightarrow \bar{b}b \chi_1$	$\sim 10^{-8.5} (\sim 10^{-9.6})$

Summary for the second project

- ▶ Exotic Z decay
 - ▶ BR sensitivity
 - ▶ $10^{-6.5} \sim 10^{-9.5}$ ($10^{-8} \sim 10^{-11}$) for Giga(Tera) Z
 - ▶ usually scale with \sqrt{L}
 - ▶ scale sensitivity
 - ▶ extremely good $10^5 \sim 10^6 \text{ GeV}$ for $\frac{1}{\Lambda} \bar{\chi} \sigma^{\mu\nu} \chi_1 B_{\mu\nu}$
 - ▶ $100 \sim 500 \text{ GeV}$ for $\frac{1}{\Lambda^3} \bar{\chi} \chi B^{\mu\nu} B_{\mu\nu}$
 - ▶ Giga Z already provides great sensitivity on exotic Z decay
 - ▶ Tera Z might be helpful for high dimensional operators to explore relic abundance parameter region

Thank you!

Pre-selection and smearing

▶ Pre-selection

- ▶ $E_\gamma > 10\text{GeV}, |\eta| < 2.3$
- ▶ $E_\ell > 5\text{GeV}, |\eta| < 2.3$
- ▶ $E_j > 5\text{GeV}, |\eta| < 2.3$

▶ Detector resolution by Gaussian smearing

- ▶ $\frac{\delta E_\gamma}{E_\gamma} = \frac{0.16}{\sqrt{E/\text{GeV}}} \oplus 0.01$
- ▶ $\frac{\delta E_j}{E_j} = \frac{0.3}{\sqrt{E/\text{GeV}}} \oplus 0.02$
- ▶ $\Delta\left(\frac{1}{p_T^\ell/\text{GeV}}\right) = \frac{10^{-3}\text{GeV}}{p_T \sin\theta} \oplus 2 \times 10^{-5}$