“tune”

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outline

introduction

- tune, coherent & incoherent tune, detectors

integer betatron tune

fractional betatron tune

- precision measurement, tune tracking, multiple bunches

modifications of tune signal

- damping, filamentation, chromaticity, linear coupling

some “complications”

- colliding beams, space charge, measuring incoherent tune

applications

- tune shift with amplitude,high-order resonances,tune scans, β function measurement, nonlinear field errors

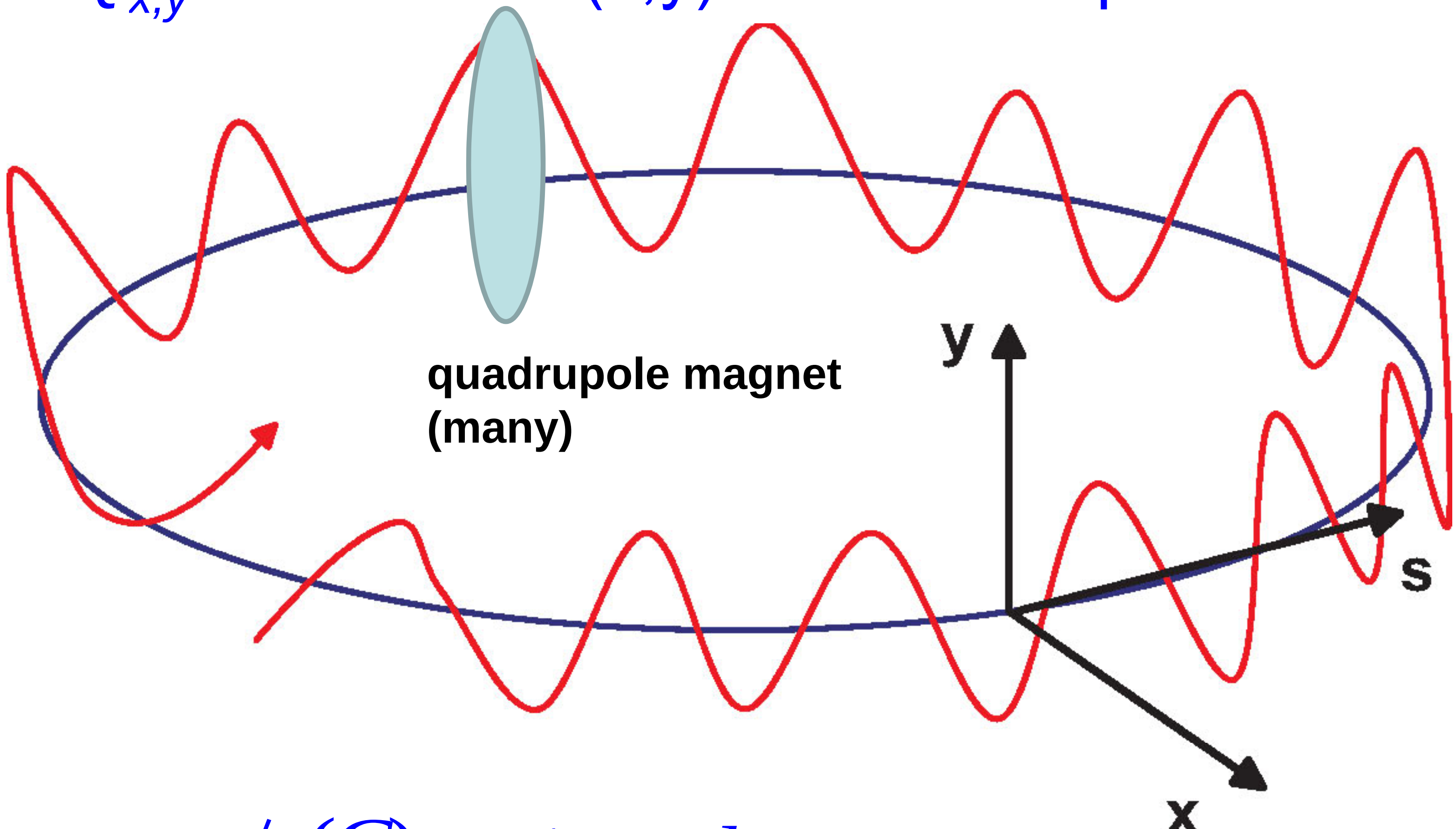
synchrotron tune

display of complex tune signals

introduction

schematic of betatron oscillation around storage ring

tune $Q_{x,y}$= number of (x,y) oscillations per turn

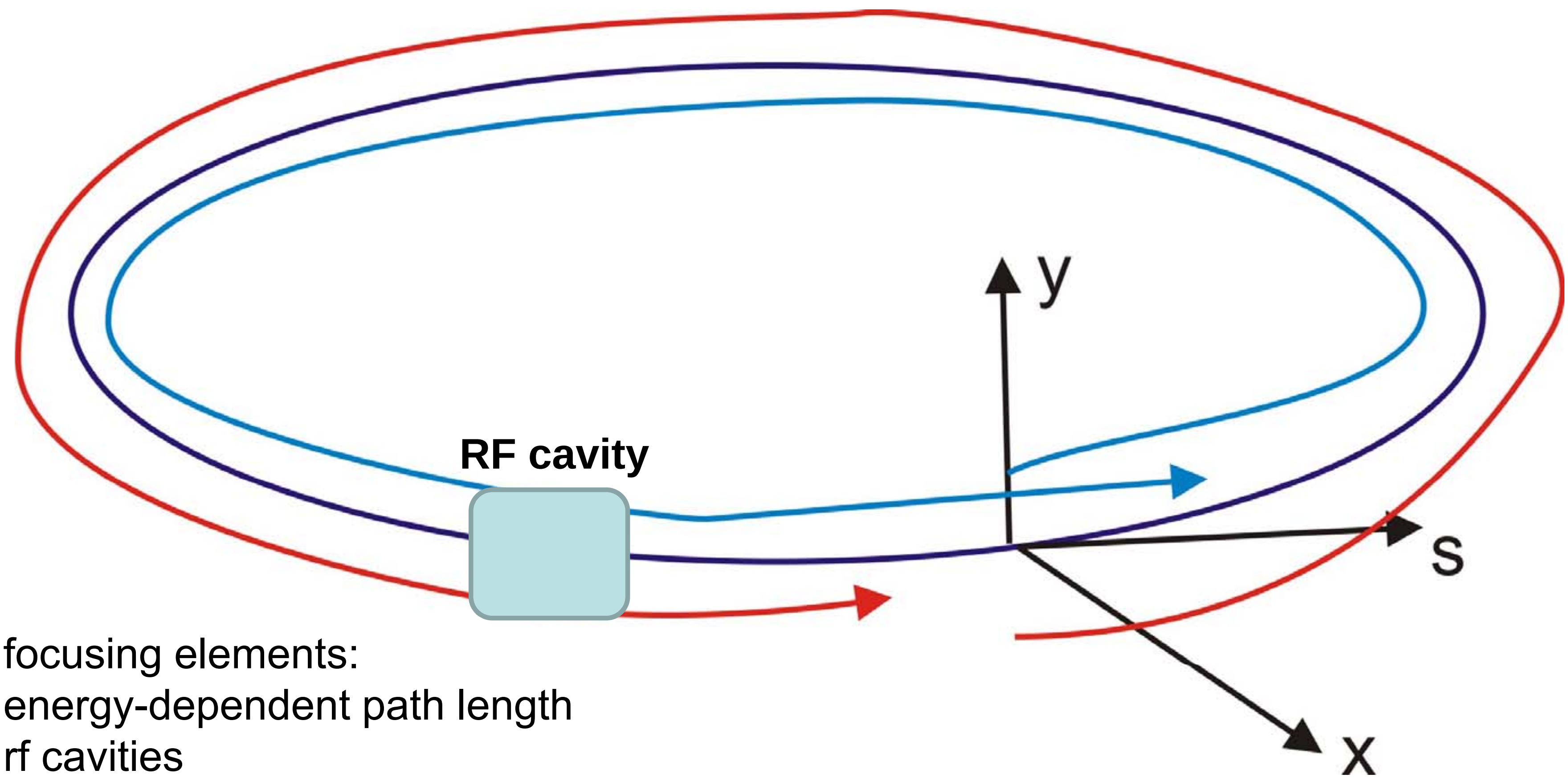


$$Q = \frac{\phi_\beta(C)}{2\pi} = \frac{1}{2\pi}\oint_C \frac{ds}{\beta(s)}$$

focusing elements:
quadrupole magnets

schematic of “longitudinal oscillation” around storage ring

tune Q_s = number of synchrotron oscillations per turn



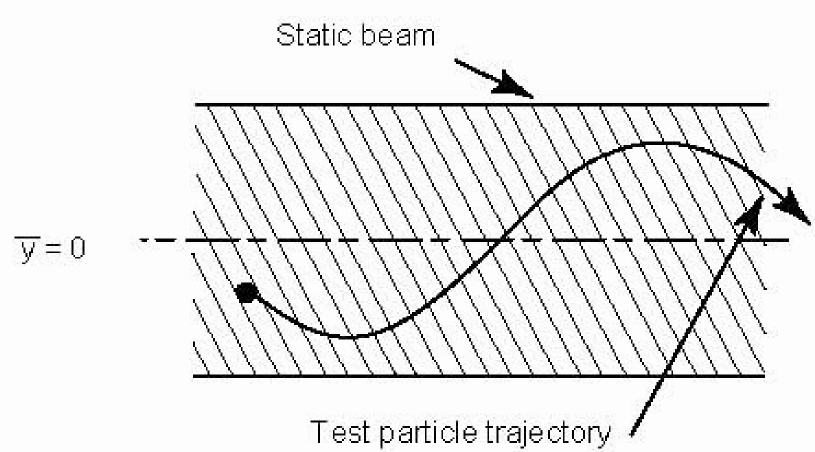
focusing elements:
energy-dependent path length
rf cavities

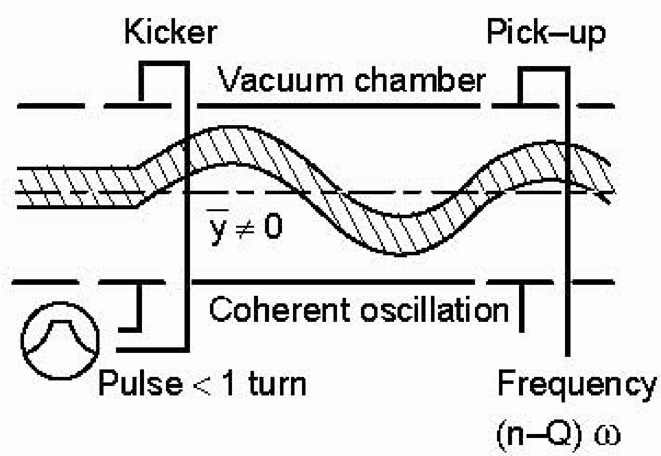
synchrotron tune $Q_s << 1$, typically $Q_s \sim 0.01\text{-}0.0001$

betatron tune $Q_{x,y} > 1$, typically $Q_{x,y} \sim 2 - 70$

incoherent and coherent tune

K.H. Schindl





incoherent betatron motion of a particle inside a static beam with its center of mass at rest

amplitude and phase are distributed at random over all particles

coherent motion of the whole beam after a transverse kick

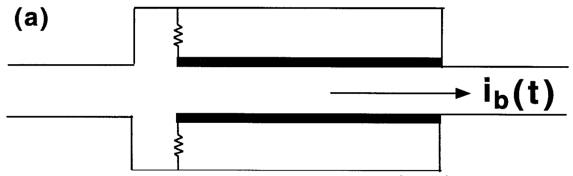
the **source of the direct space charge is now moving**, individual particles still continue incoherent motion around the common coherent trajectory

at low beam intensity these two tunes should be about the same

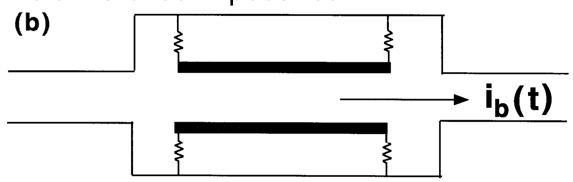
detectors to measure the coherent beam oscillations

button pick ups

unterminated transmission line

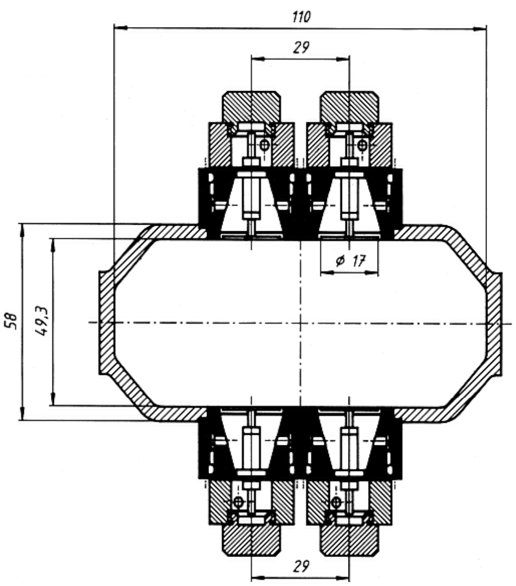


transmission line terminated (rhs) to a matched impedance



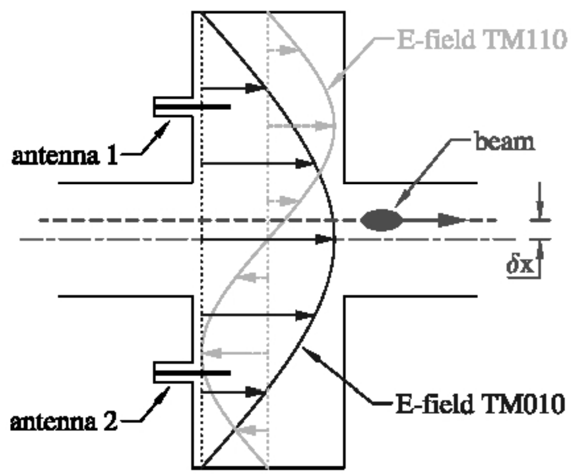
strip line pick ups

the LEUTL at Argonne shorted S-band quarter-wave four-plate stripline BPM (courtesy R.M. Lill, 2003)



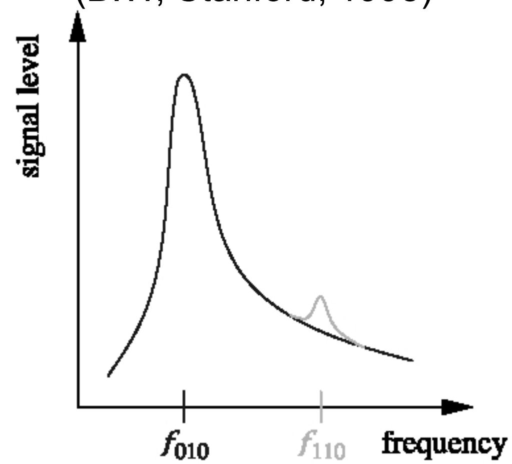
button electrode for use between the undulators of the TTF II SASE FEL (courtesy D. Noelle and M. Wendt, 2003)

cavity BPMs



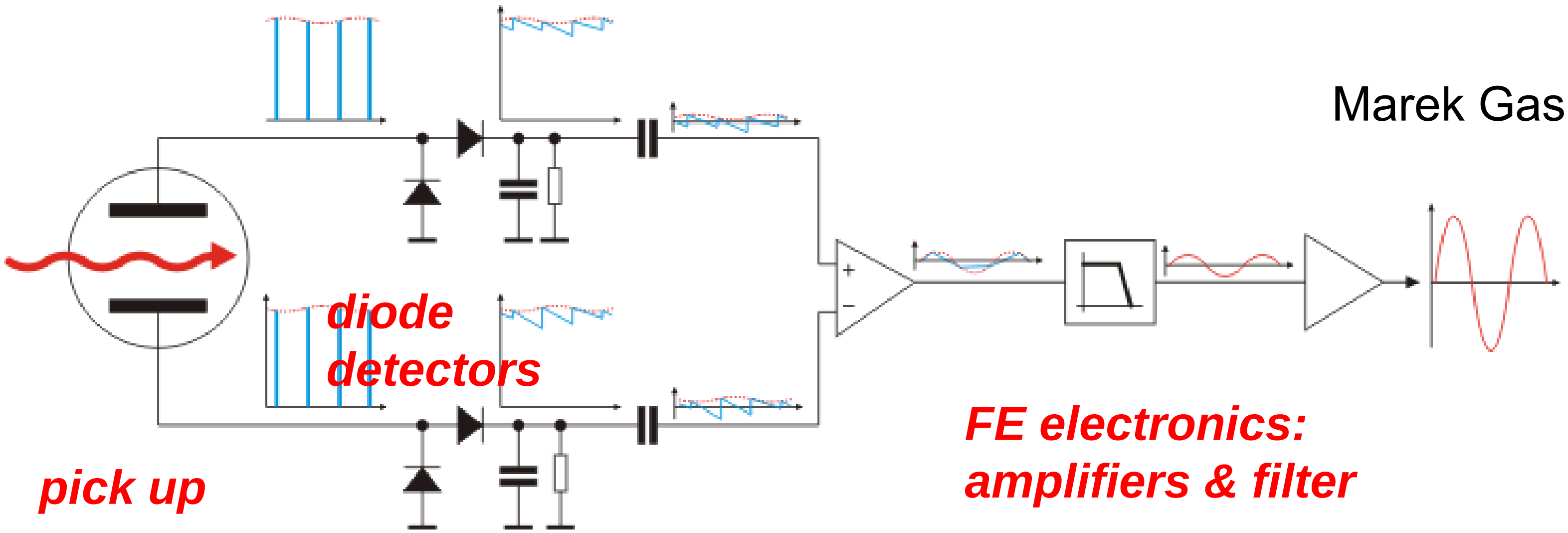
TM_{010}, "common mode" ($\propto$ I)
TM_{110}, dipole mode of interest

reference:
"Cavity BPMs", R. Lorentz
(BIW, Stanford, 1998)

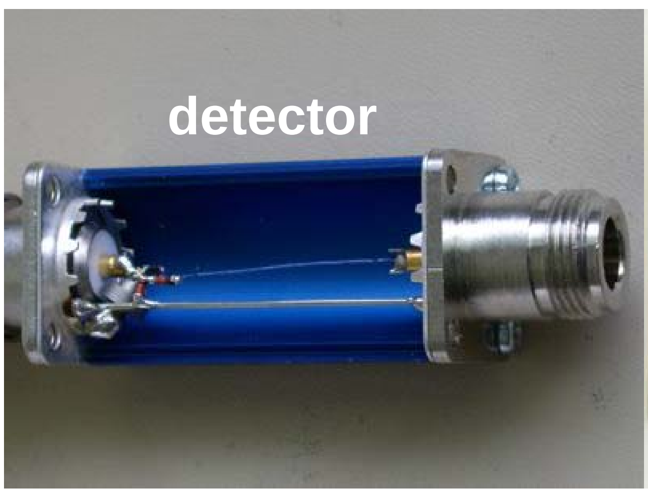


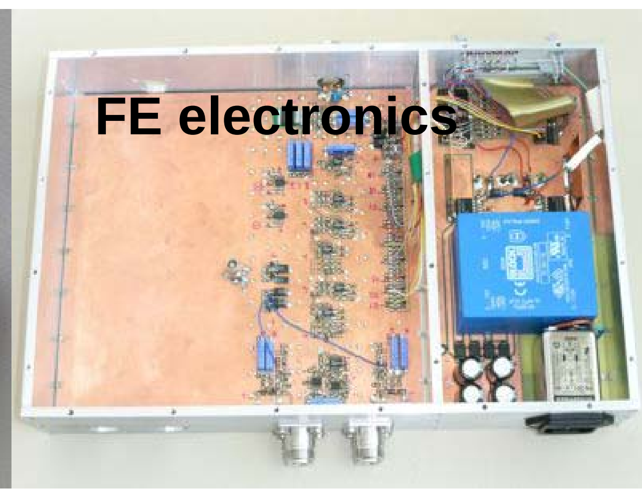
Direct Diode Detection Base-Band Q (3D-BBQ) Measurement in CERN Accelerators - Principle

Marek Gasior



Apart from detectors, the **filter is most important element** of the system. It attenuates revolution frequency and its harmonics, as well as low frequencies.

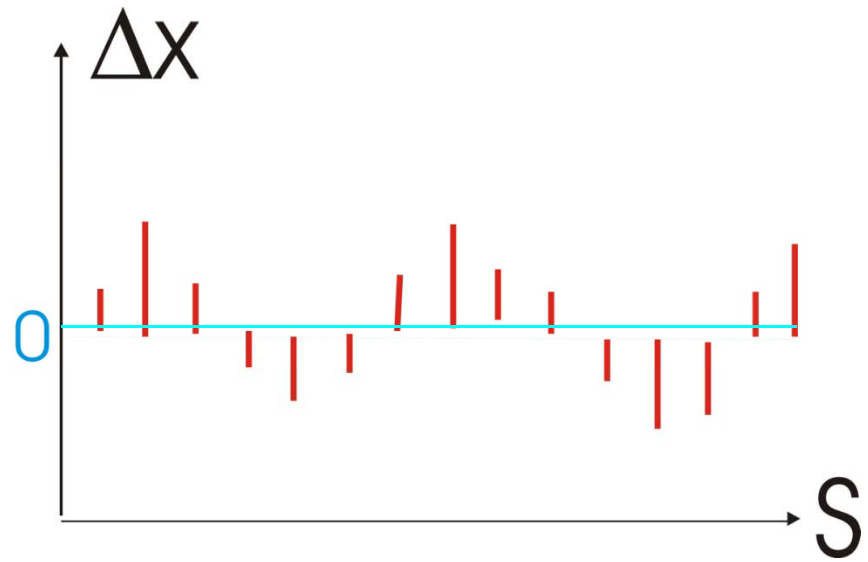




integer betatron tune

integer part of betatron tune

- **first turn injection oscillation**
- or **difference orbit** after exciting a single steering corrector



oscillation

$$x \approx A \cos \phi_\beta(s)$$

$$Q = \frac{\phi_\beta(C)}{2\pi} = \frac{1}{2\pi} \oint_C \frac{ds}{\beta(s)}$$

count number of oscillations (directly or via FFT)

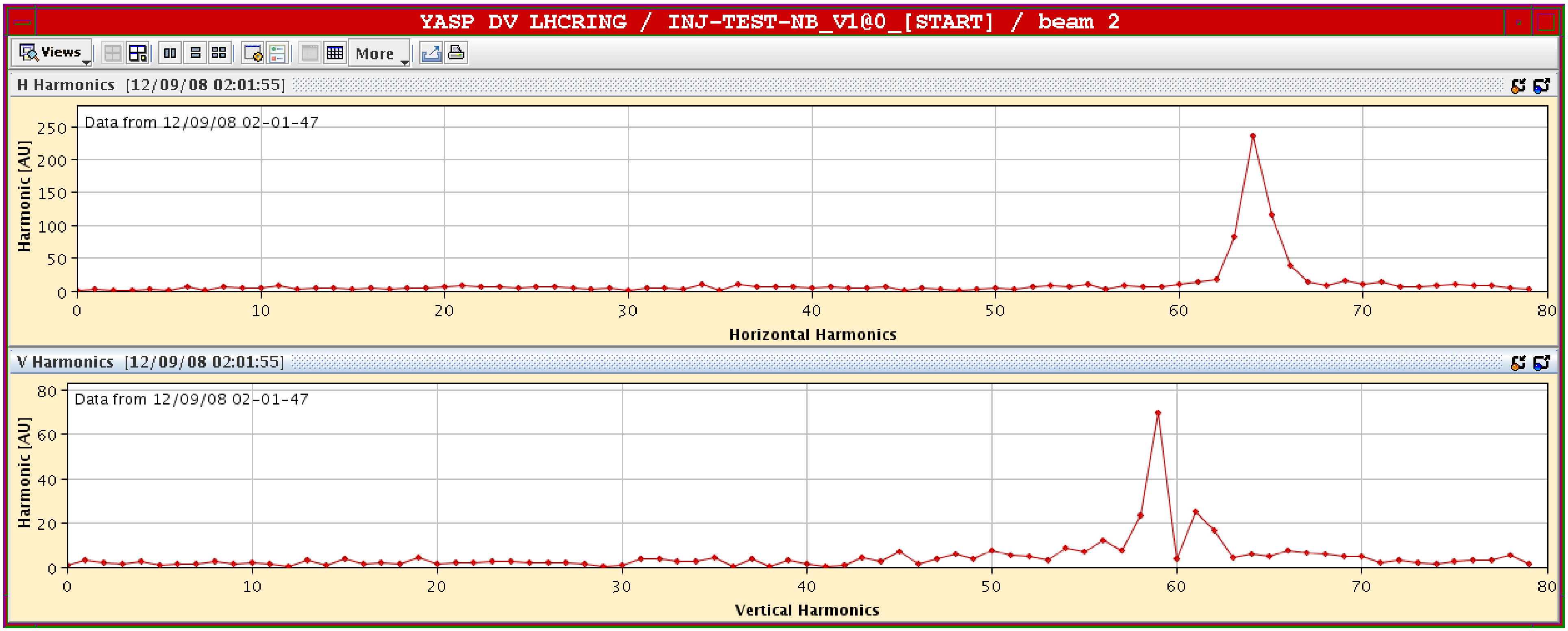
→ integer value of tune Q

more intricate method: use multi-turn BPM data to measure ϕ at each BPM; then find $\Delta\phi$ between BPMs

example - checking the integer tune

LHC beam commissioning 12 September 2008

J. Wenninger



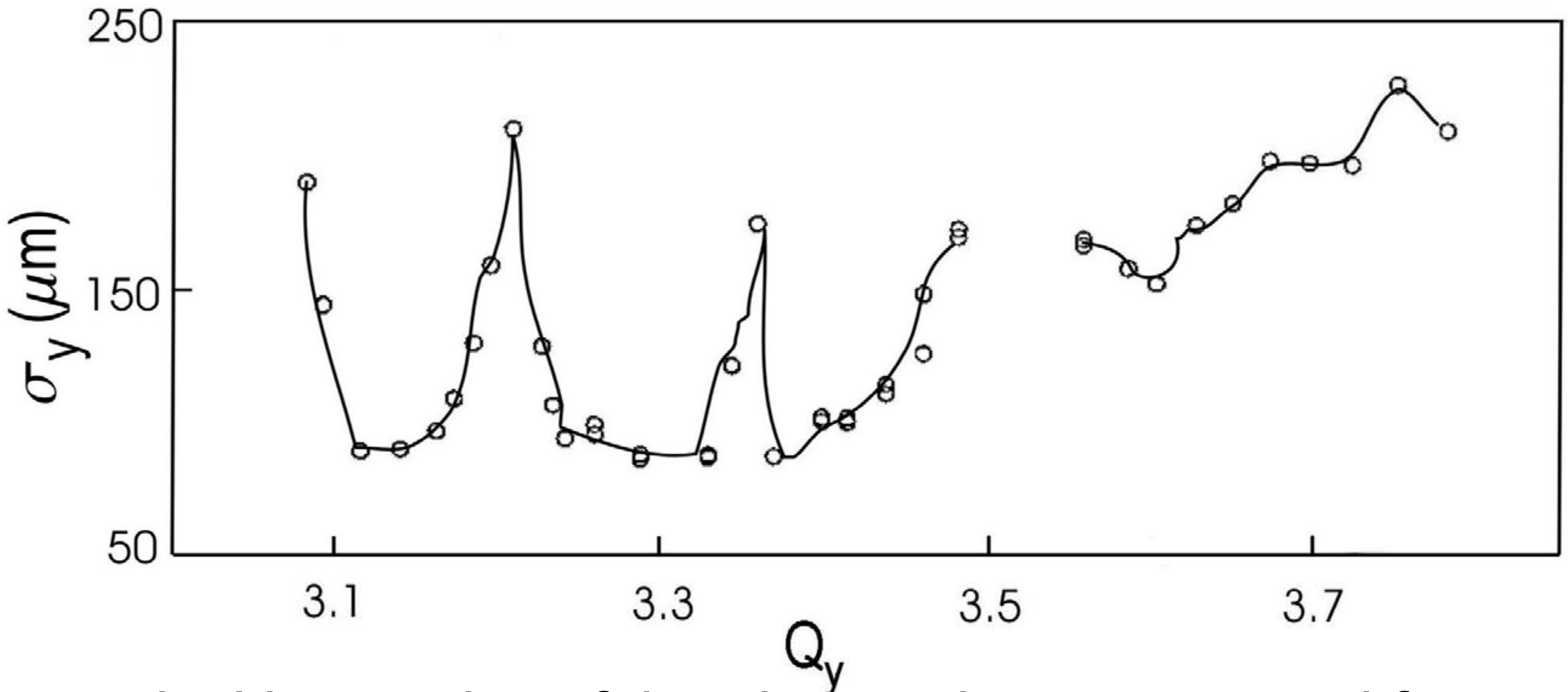
integer tunes 64 and 59 equal to their design values!
(vertical FFT has second peak!?) – basic check of optics

fractional betatron tune

- precision measurement
- tune tracking
- multiple bunches

fractional part of the tune – why is it important?

one example



rms vertical beam size of the electron beam extracted from the SLC damping ring as a function of the vertical betatron tune, under unusually poor vacuum conditions.

all nonlinear and high-intensity effects are very sensitive to the fractional tune - best performance requires optimum tune!

two categories:

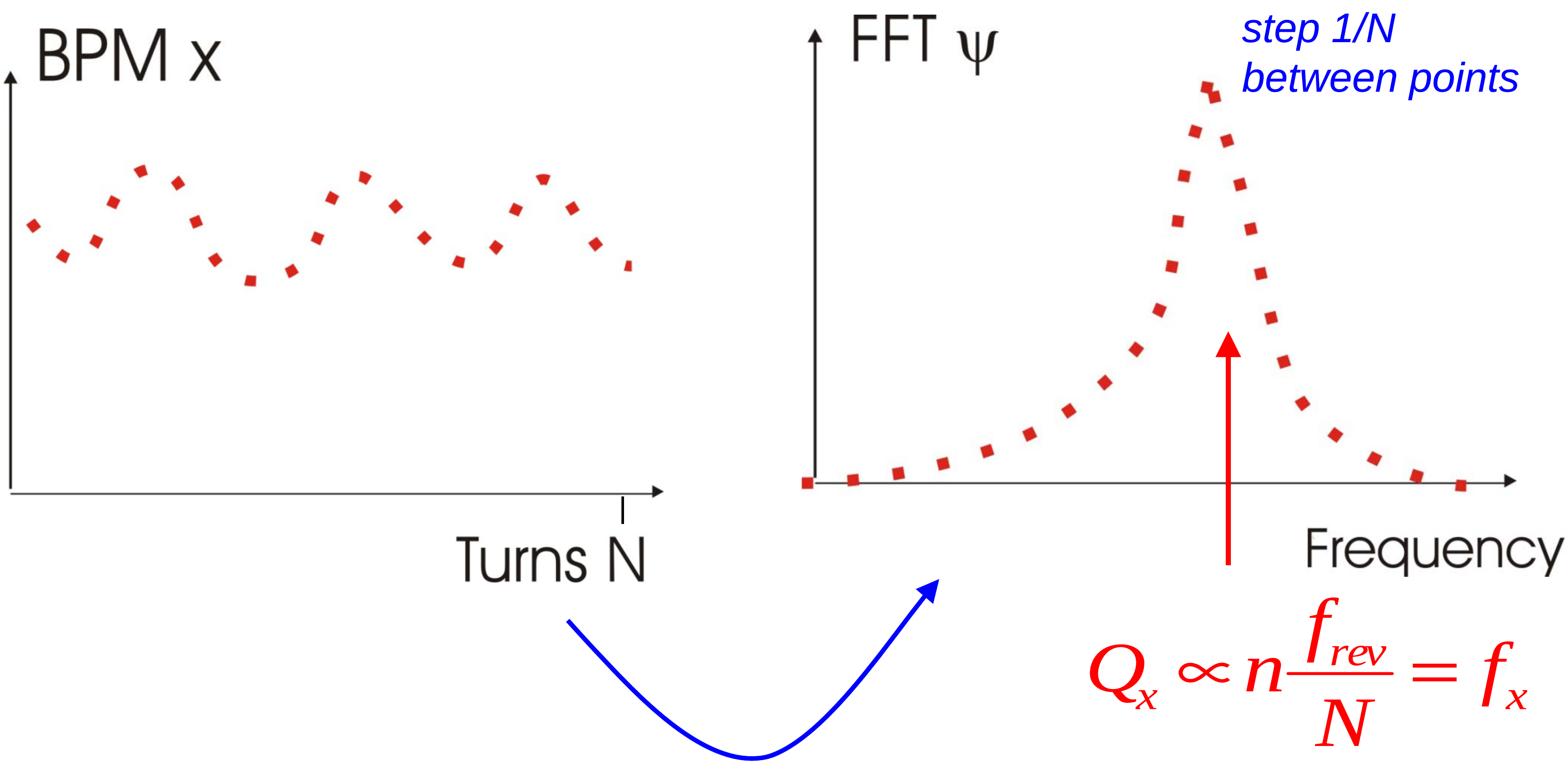
- precision tune measurements

- tune tracking
  to monitor & control fast changes
  e.g. during acceleration

FFT (Fast Fourier Transform)

(1) excite transverse beam motion + detect transverse position on a pick up over N turns

(2) compute frequency spectrum of signal; identify betatron tunes as highest peaks



$$Q_x \propto n\frac{f_{rev}}{N} = f_x$$

$$x(n) = \sum_{j=1}^{N} \psi(Q_j) e^{2\pi i n Q_j}$$

FFT signal = expansion coefficient

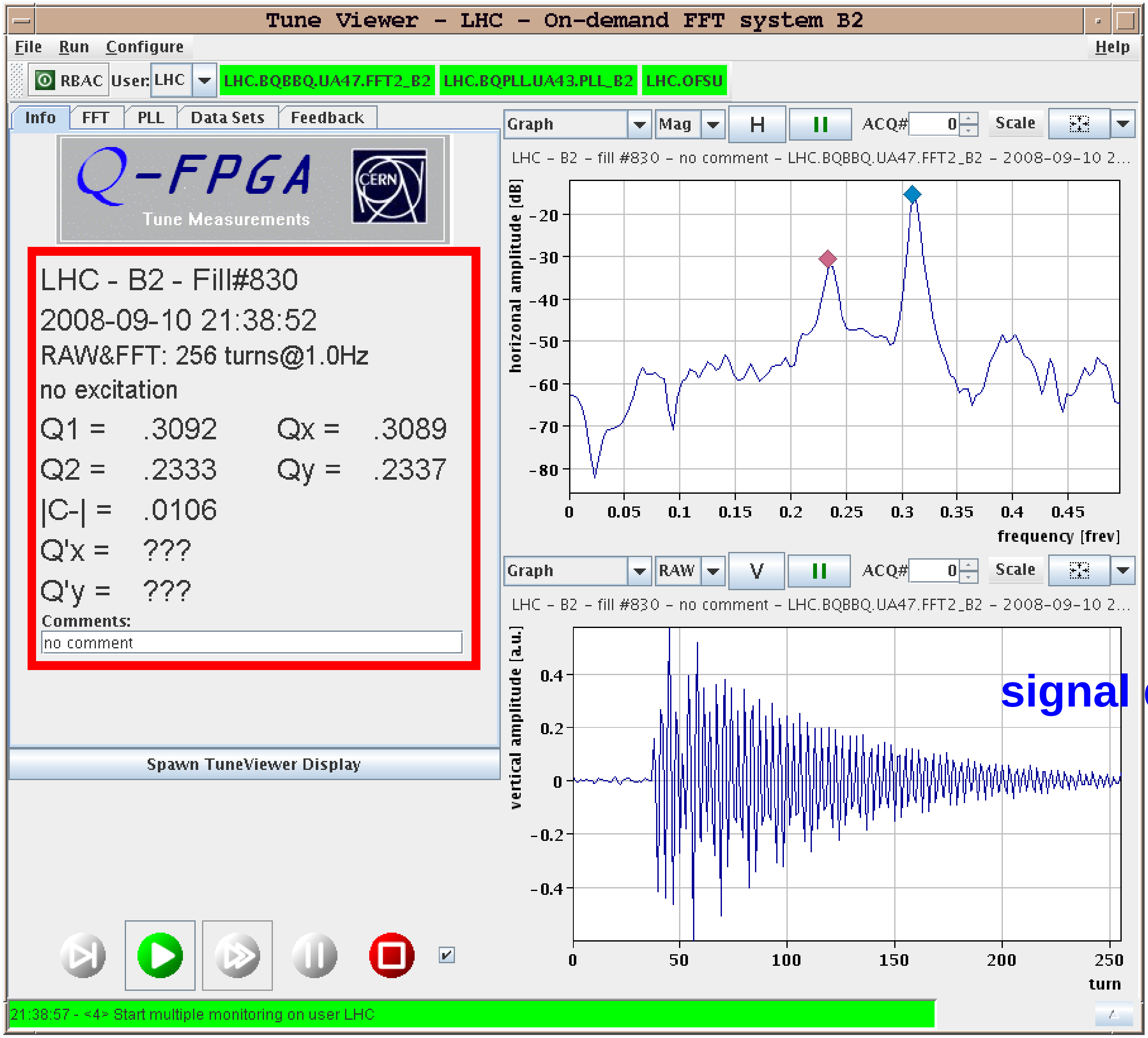
Q error of FFT: $|\delta Q| \leq \frac{1}{2N}$

due to discreteness of steps

$$|\delta Q| \leq 10^{-3} \rightarrow N \geq 500$$

checking the fractional tune

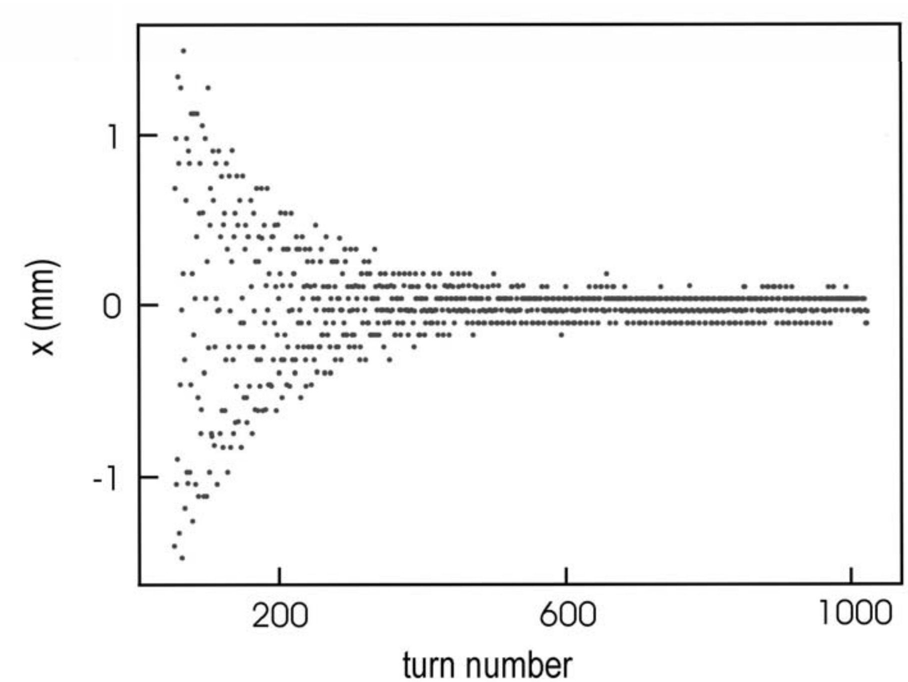
LHC beam commissioning 10 September 2008



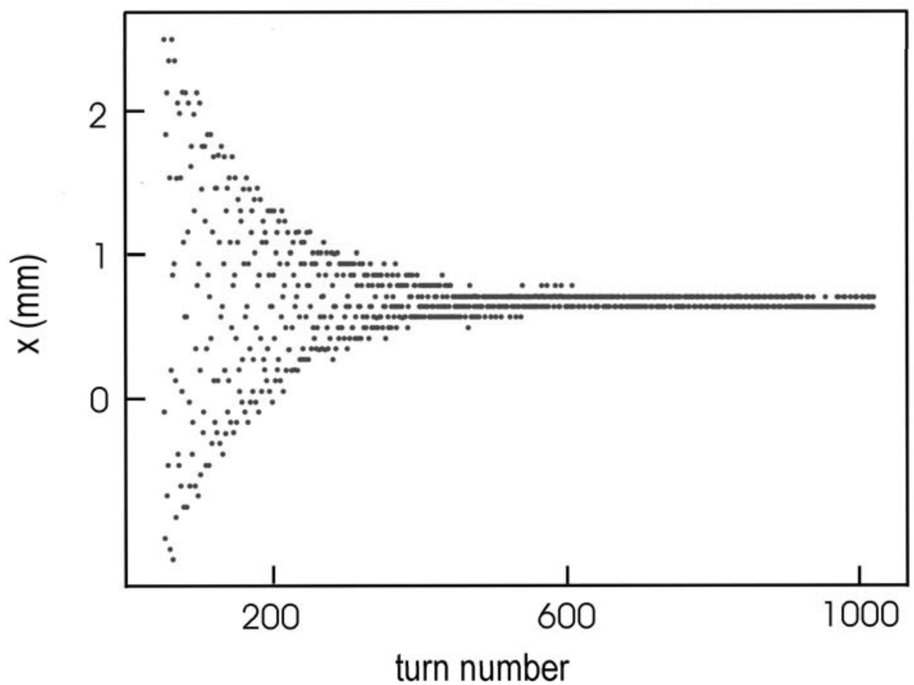
signal decay (due to de-bunching)

multi-turn orbit measurement for the motion of a single bunch in a 3-bunch train at LEP-1

signal decay (due to fast head-tail damping)

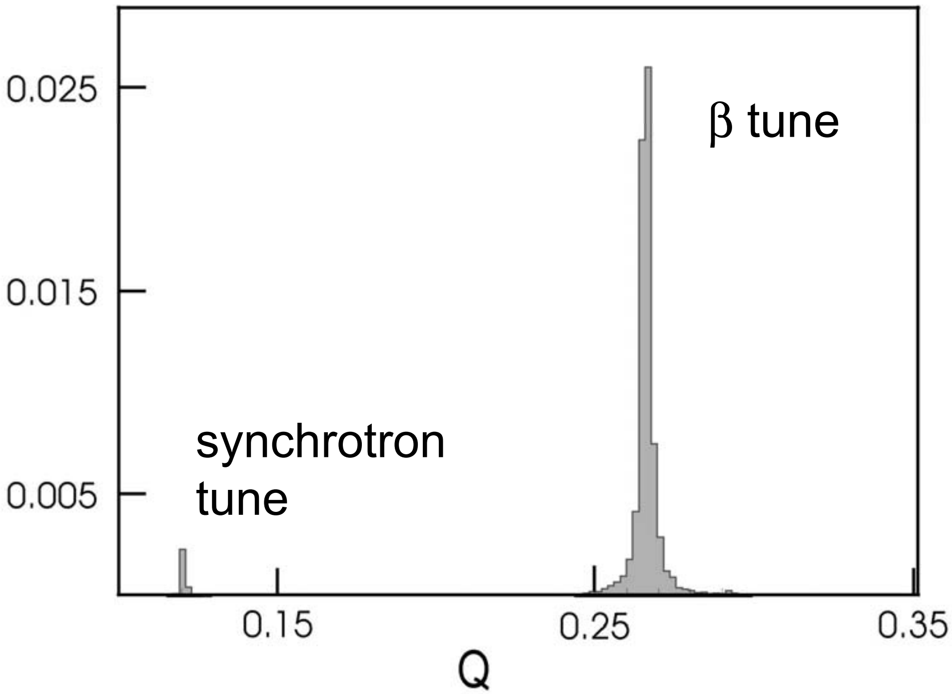


BPM in a dispersive arc region (where transverse position varies with beam momentum)

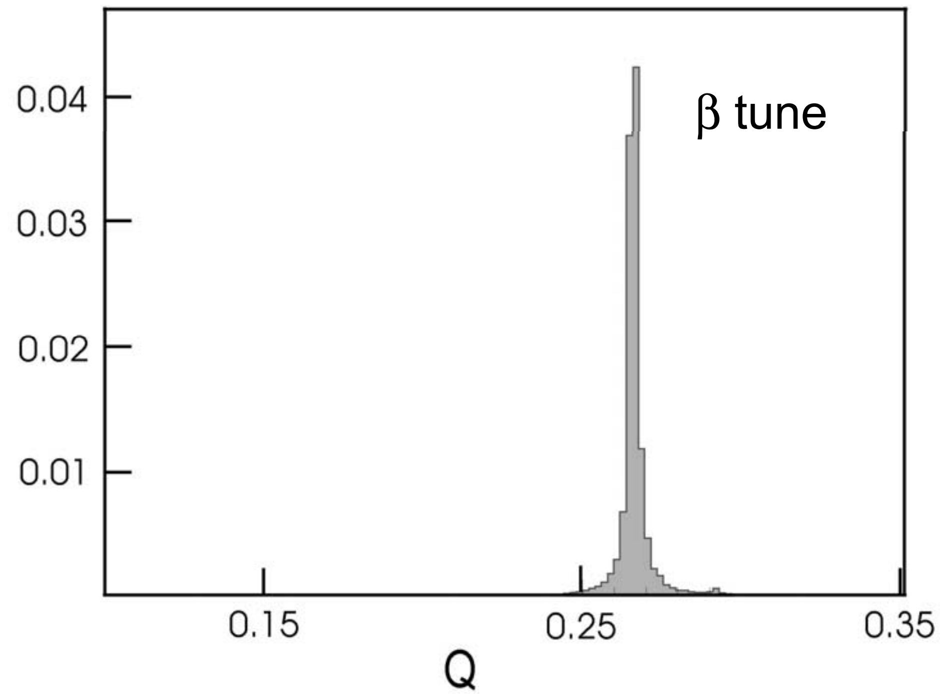


BPM in a straight section without dispersion

FFT power spectra for the two previous measurements



BPM in a dispersive arc region



BPM in a straight section without dispersion about 1000 turns are required for adequate tune measurement with FFT, *but*

• *filamentation, damping,… (see later)*

→ *spurious results?!*

further improvement in resolution, e.g. by

interpolating the shape of the Fourier spectrum around main peak *assumption:* shape = pure sinusoidal oscillation

$$\left|\psi(Q_j)\right| = \left|\frac{\sin\left(N\pi(Q_{F\,\mathrm{int}} - Q_j)\right)}{N\sin\left(\pi(Q_{F\,\mathrm{int}} - Q_j)\right)}\right|$$

$$Q_{F\mathrm{int}} = \frac{k}{N} + \frac{1}{\pi}\arctan\frac{\left|\psi(Q_{k+1})\right|\sin\left(\pi/N\right)}{\left|\psi(Q_k)\right| + \left|\psi(Q_{k+1})\right|\cos\left(\pi/N\right)}$$

uses Q at peak and highest neighbor

error: $$\left|\delta Q\right| \leq \frac{const}{N^2}$$

$N >> 1$: $$Q_{F\mathrm{int}} = \frac{k}{N} + \frac{1}{N}\arctan\frac{\left|\psi(Q_{k+1})\right|}{\left|\psi(Q_k)\right| + \left|\psi(Q_{k+1})\right|}$$

further improvement by **interpolated FFT with data windowing**

$$\psi(Q_j) = \frac{1}{N}\sum_{n=1}^{N} x(n)\chi(n)e^{-2\pi i n Q_j}$$

filter

e.g., Hanning filter of order *l*: $\chi_l(n) = A_l \sin^l\left(\frac{\pi n}{N}\right)$

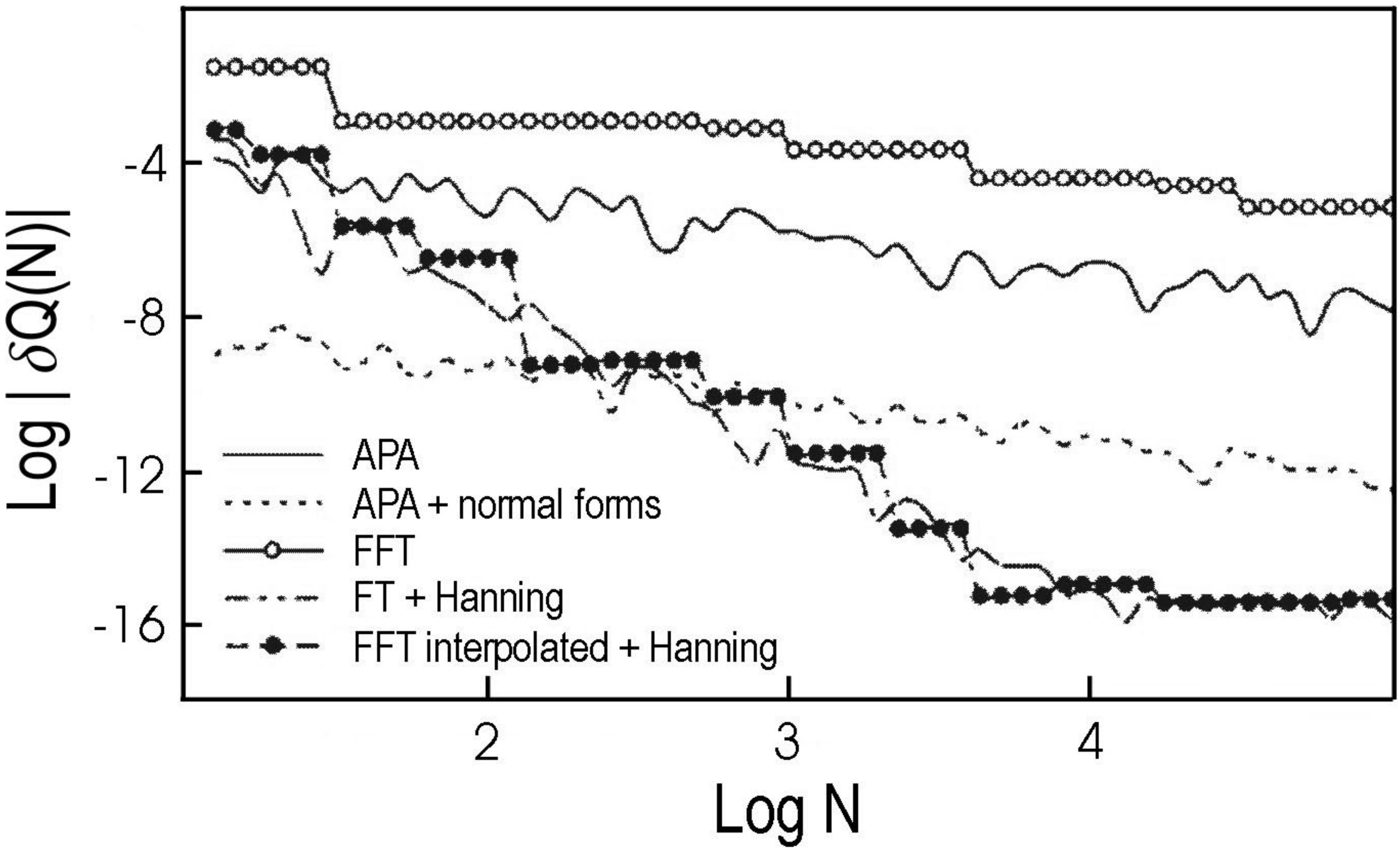
$$Q_{F\,\mathrm{int}} = \frac{k}{N} + \frac{1}{N}\left(\frac{(l+1)\left|\psi(Q_{k+1})\right|}{\left|\psi(Q_k)\right| + \left|\psi(Q_{k+1})\right|} - \frac{l}{2}\right)$$

resolution: $|\delta Q| \leq \frac{const}{N^{l+2}}$

however, with noise $|\delta Q| \leq \frac{const}{N^2}$ as for the simple interpolation

example: **refined FFTs**

(M. Giovannozzi et al.)



another approach to obtain higher accuracy than FFT

"**Lomb normalized periodogram**" *A.-S. Muller*

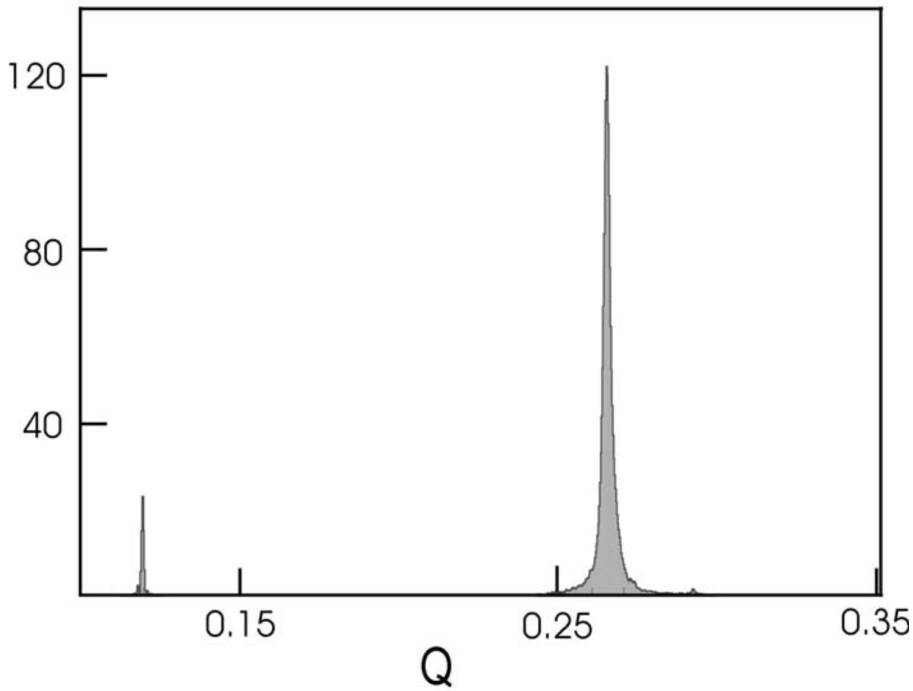
$$P_N(Q) = \frac{1}{2\sigma^2}\left\{ \frac{\left[\sum_n \cos\left(2\pi Q(n-n_0)\right)\right]^2}{\sum_j \cos^2\left(2\pi Q(n-n_0)\right)} + \frac{\left[\sum_n \sin\left(2\pi Q(n-n_0)\right)\right]^2}{\sum_j \sin^2\left(2\pi Q(n-n_0)\right)} \right\}$$

$$d_n = x_n - \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \sigma^2 = \frac{1}{N-1}\sum_{n=1}^{N} d_n^2 \qquad \tan\left(4\pi Q n_0\right) = \frac{\sum_n \sin\left(4\pi Q n\right)}{\sum_n \cos\left(4\pi Q n\right)}$$

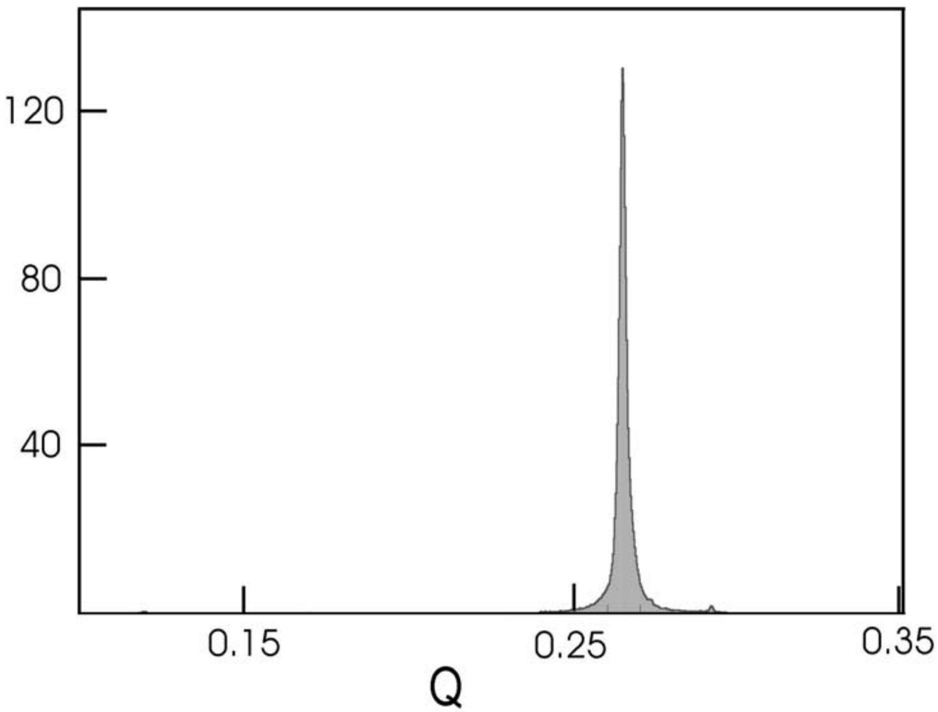
no constraint on # data point or on time interval between points (replace *Qn* by $\omega\tau_n$)

the constant n_0 is computed to eliminate the phase of the original harmonic; since the phase dependence is removed, Lomb's method is more accurate than the FFT

Lomb normalized periodogram for previous measurements



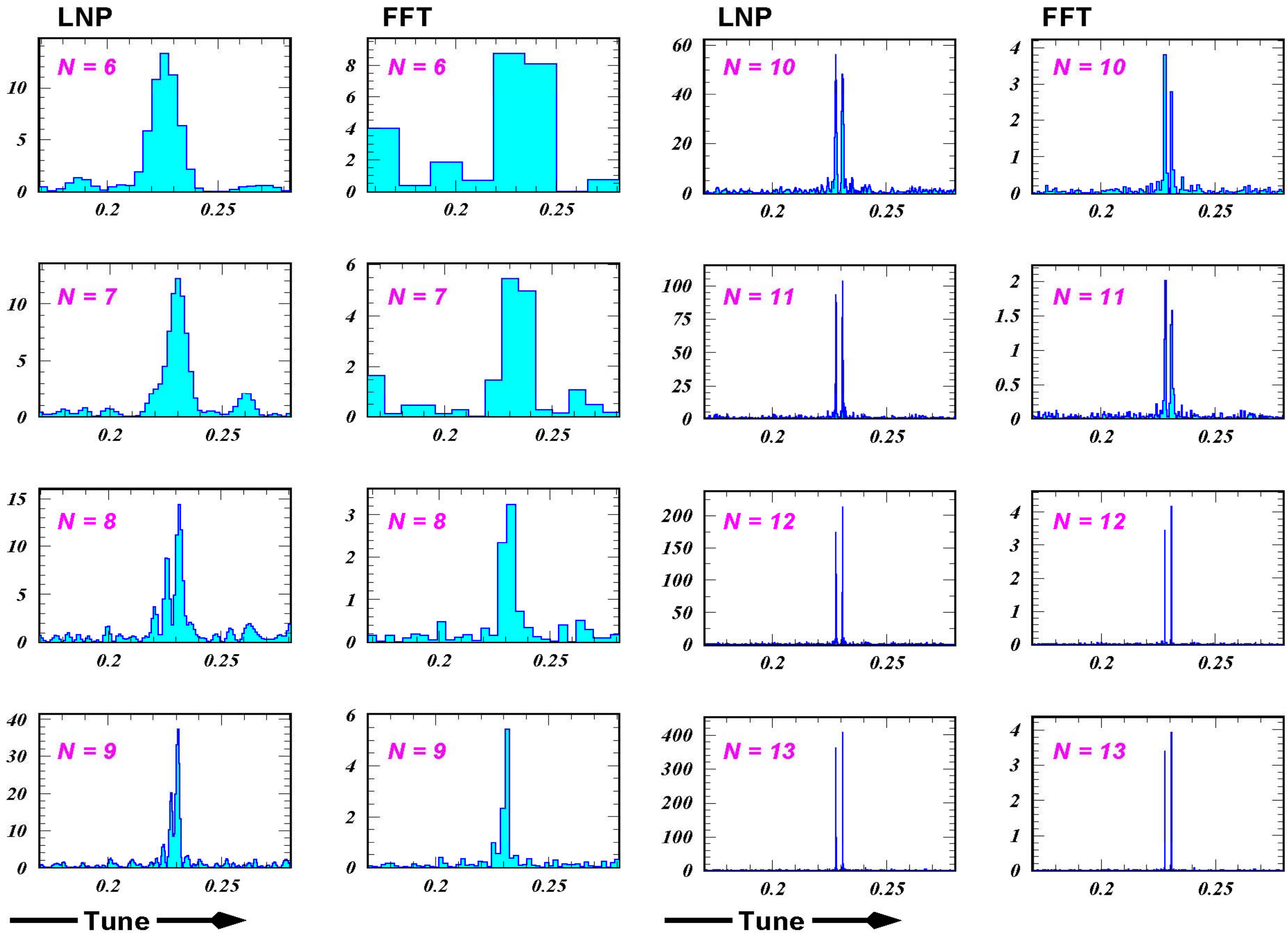
BPM in a dispersive arc region



BPM in a straight section without dispersion

(A.-S. Muller)

Lomb Normalised Periodograms versus FFT: number of turns = 2^N



comparison Lomb-FFT for CERN PS simulation with two spectral lines [A.-S.Muller, ’04]]

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
CERN — AB DEPARTMENT

CERN-AB-2004-023 BDI

Improving FFT Frequency Measurement Resolution by Parabolic and Gaussian Spectrum Interpolation

M. Gasior, J.L. Gonzalez

CERN, CH-1211,Geneva 23, Switzerland

Abstract

Discrete spectra can be used to measure frequencies of sinusoidal signal components. Such a measurement consists in digitizing a compound signal, performing windowing of the signal samples and computing their discrete magnitude spectrum, usually by means of the Fast Fourier Transform algorithm. Frequencies of individual components can be evaluated from their locations in the discrete spectrum with a resolution depending on the number of samples. However, the frequency of a sinusoidal component can be determined with improved resolution by fitting an interpolating parabola through the three largest consecutive spectrum bins corresponding to the component. The abscissa of its maximum constitutes a better frequency approximation. Such a method has been used for tune measurement systems in circular accelerators. This paper describes the efficiency of the method, depending on the windowing function applied to the signal samples. A typical interpolation gain is one order of magnitude. Better results are obtained with Gaussian interpolation, offering frequency resolution improvement by more than two orders of magnitude when used with windows having fast sidelobe decay. An improvement beyond three orders of magnitude is possible with steep Gaussian windows. These results are confirmed by laboratory measurements. Both methods assume the measured frequency to be constant during acquisition and the spectral peak corresponding to the measured component to constitute a local maximum in a given band of the input signal discrete spectrum.

Presented at BIW'04 – 3-6 May 2004 – Knoxville TE - USA

Geneva, Switzerland
May, 2004

still an active area of research…

most of the above "improvements" rely on harmonic motion

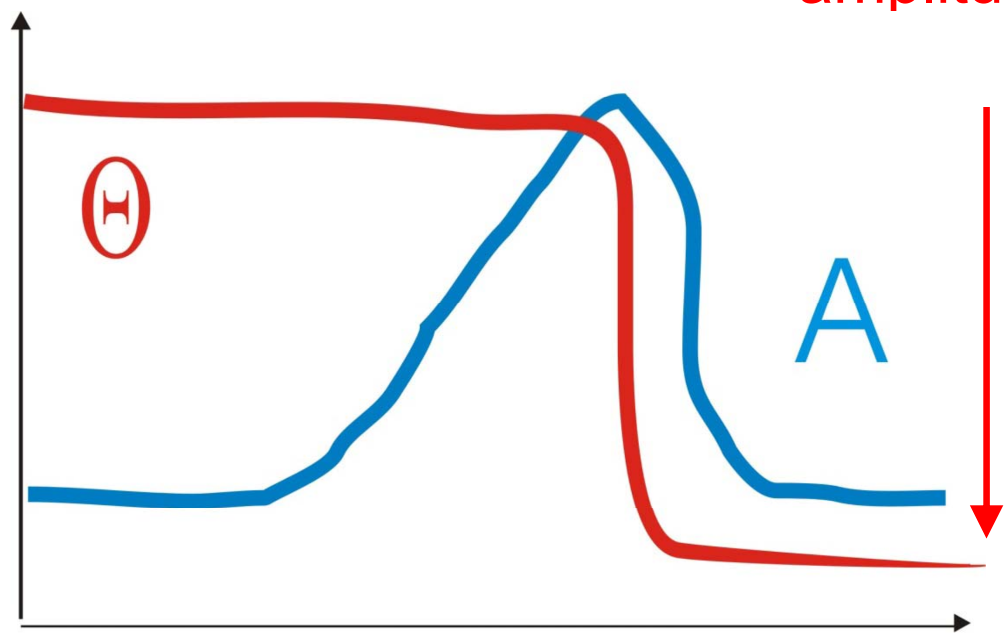
swept frequency excitation

excitation $\theta \sin \omega t$

measure response $x_b \approx A \sin(\omega t + \theta)$

amplitude phase

vary ω in steps



180° phase jump

"beam transfer function"

beam transfer function

→ transverse impedance

→ radiation damping

at betatron tune:

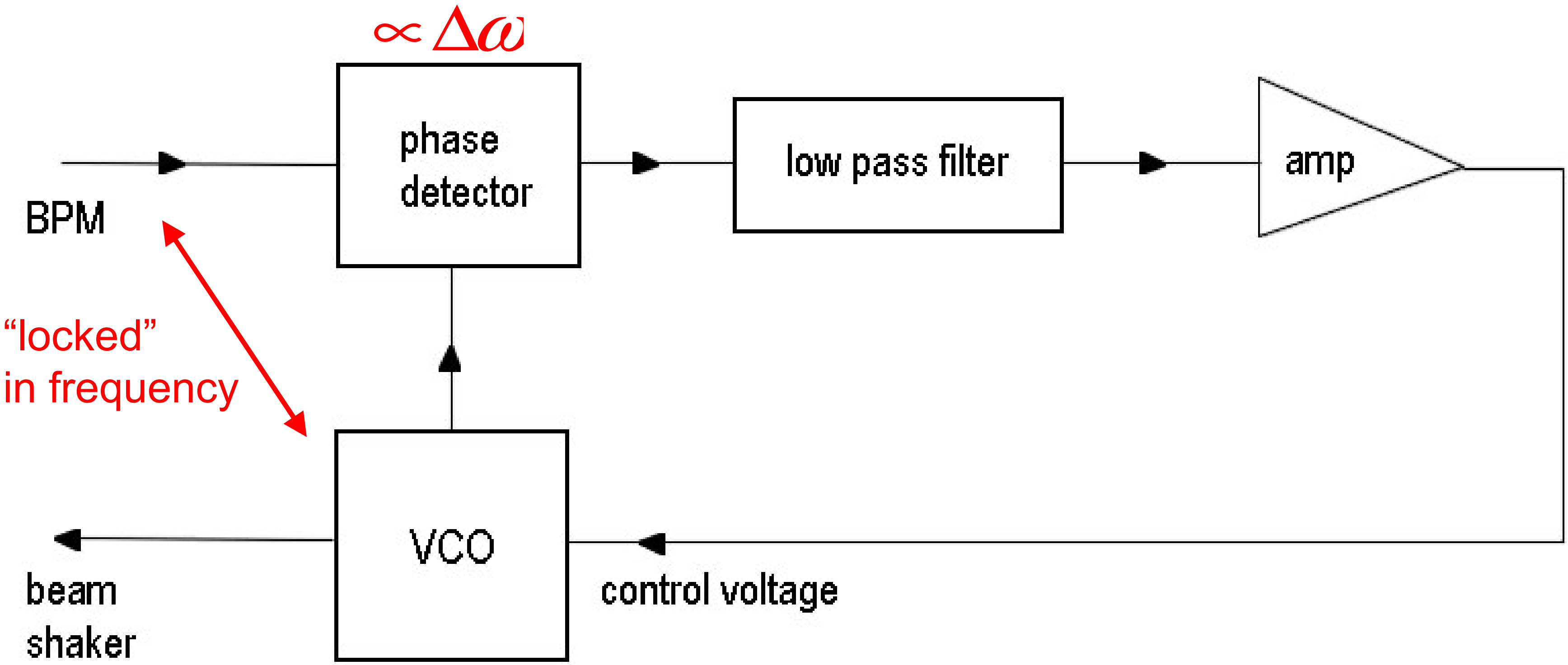
A zero slope

θ maximum slope!

phase can be monitored by phase-locked loop

if beam is excited by VCO → lock-in amplifier

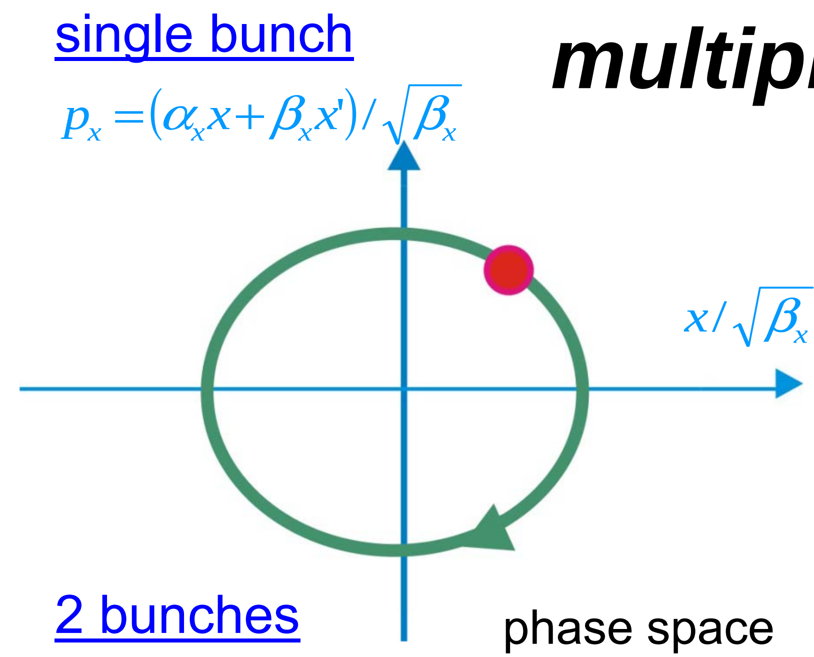
phase locked loop (for continuous tune control)

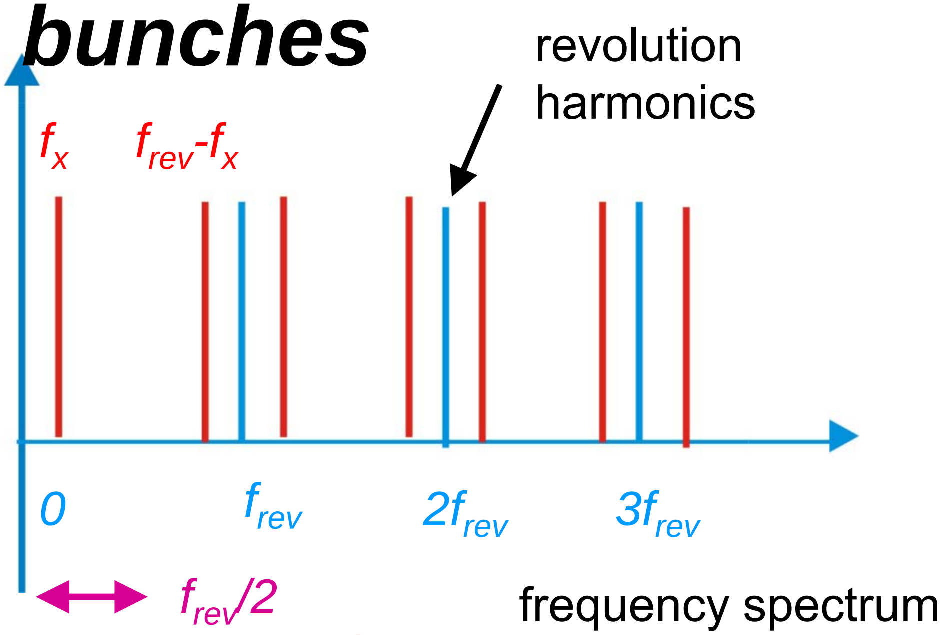


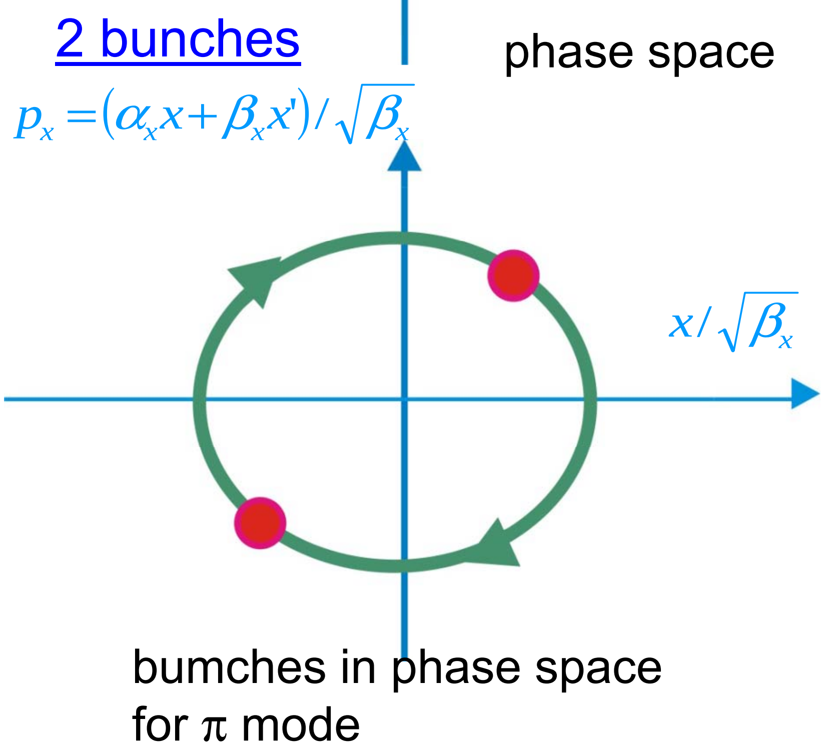
"lock-in amplifier"

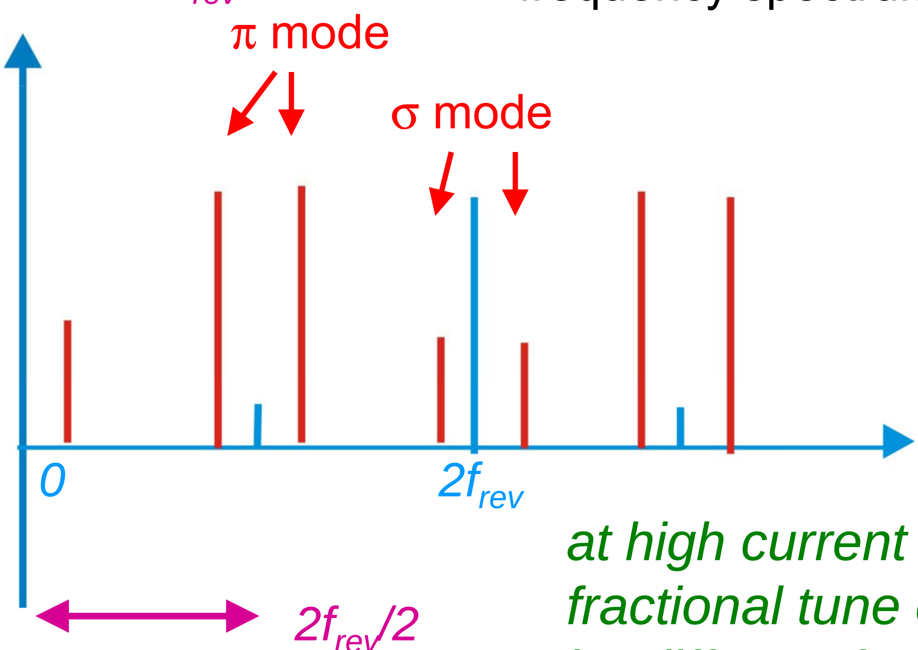
VCO frequency = betatron frequency

multiple bunches









at high current fractional tune can be different for σ and π modes!

at low current

$$x(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)e^{i\omega_x t}+\sum_{n=-\infty}^{\infty}\delta(t-nT-T/2)e^{i\omega_x t+i\phi}$$

bunch 1

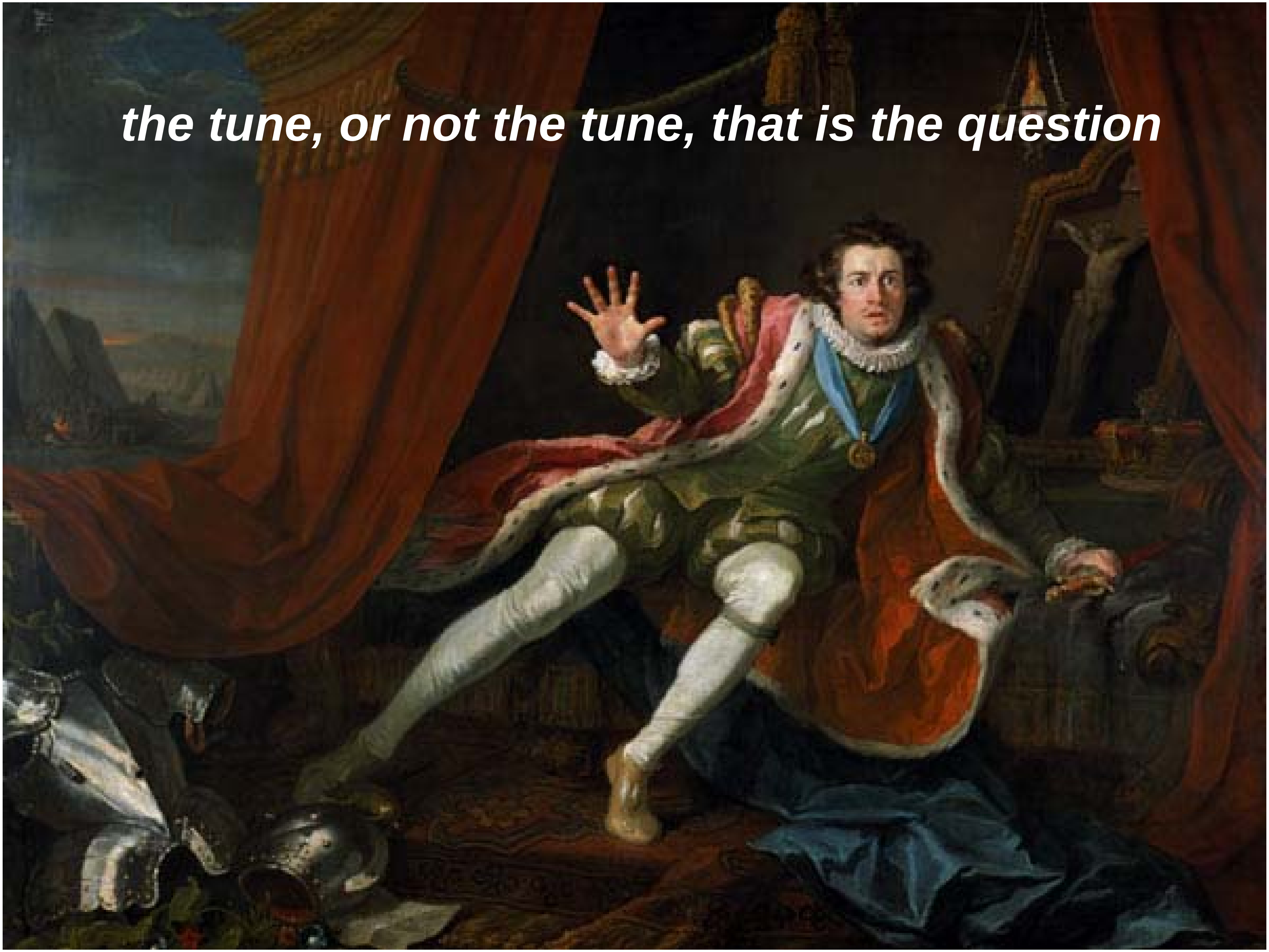
bunch 2

ϕ=0: σ mode

ϕ=p: π mode

n_b bunches ⟶ n_b multibunch modes

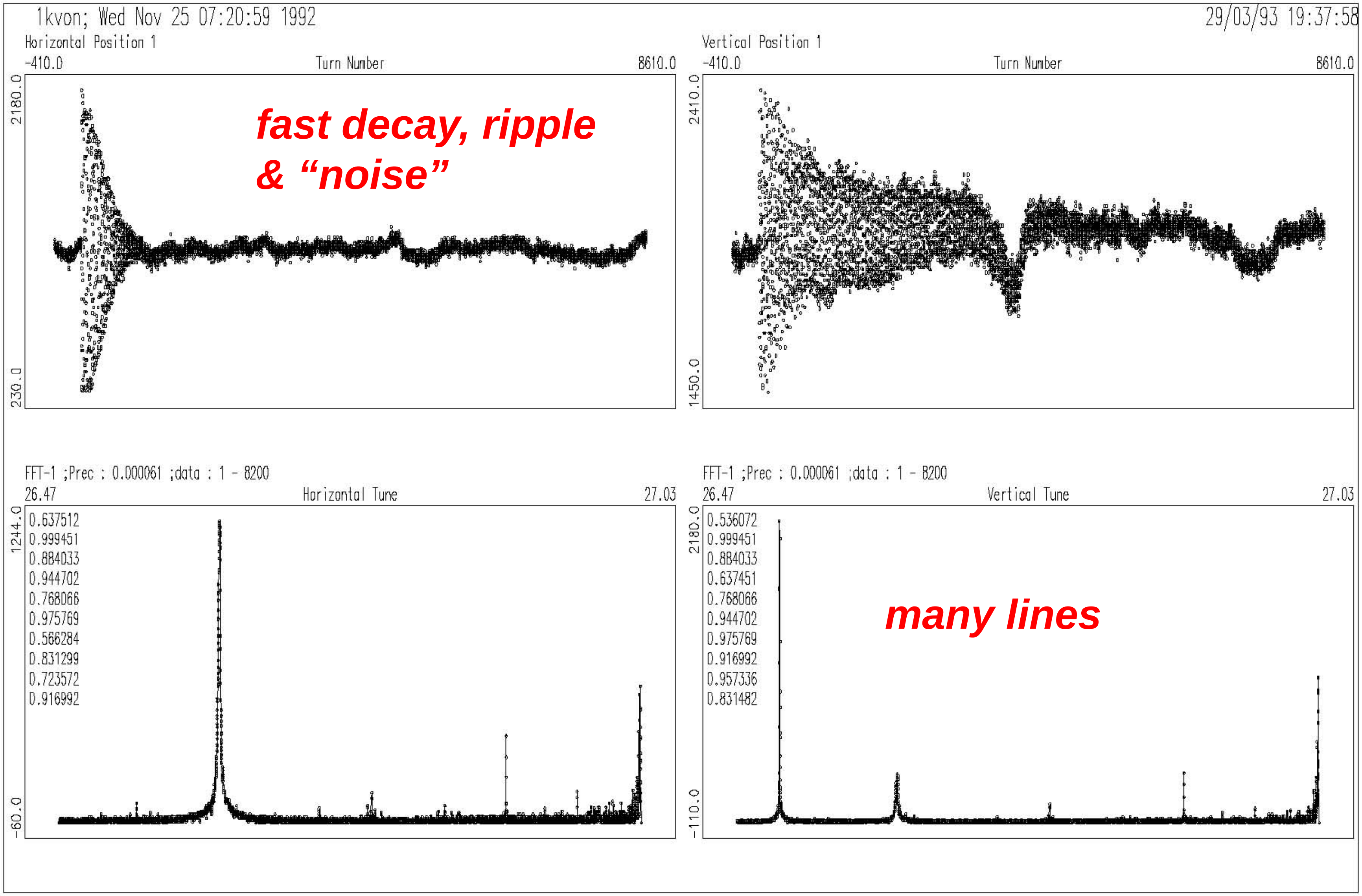
measuring spectrum from 0 to n_b *frev*/2 suffices!


the tune, or not the tune, that is the question

modifications of tune signal

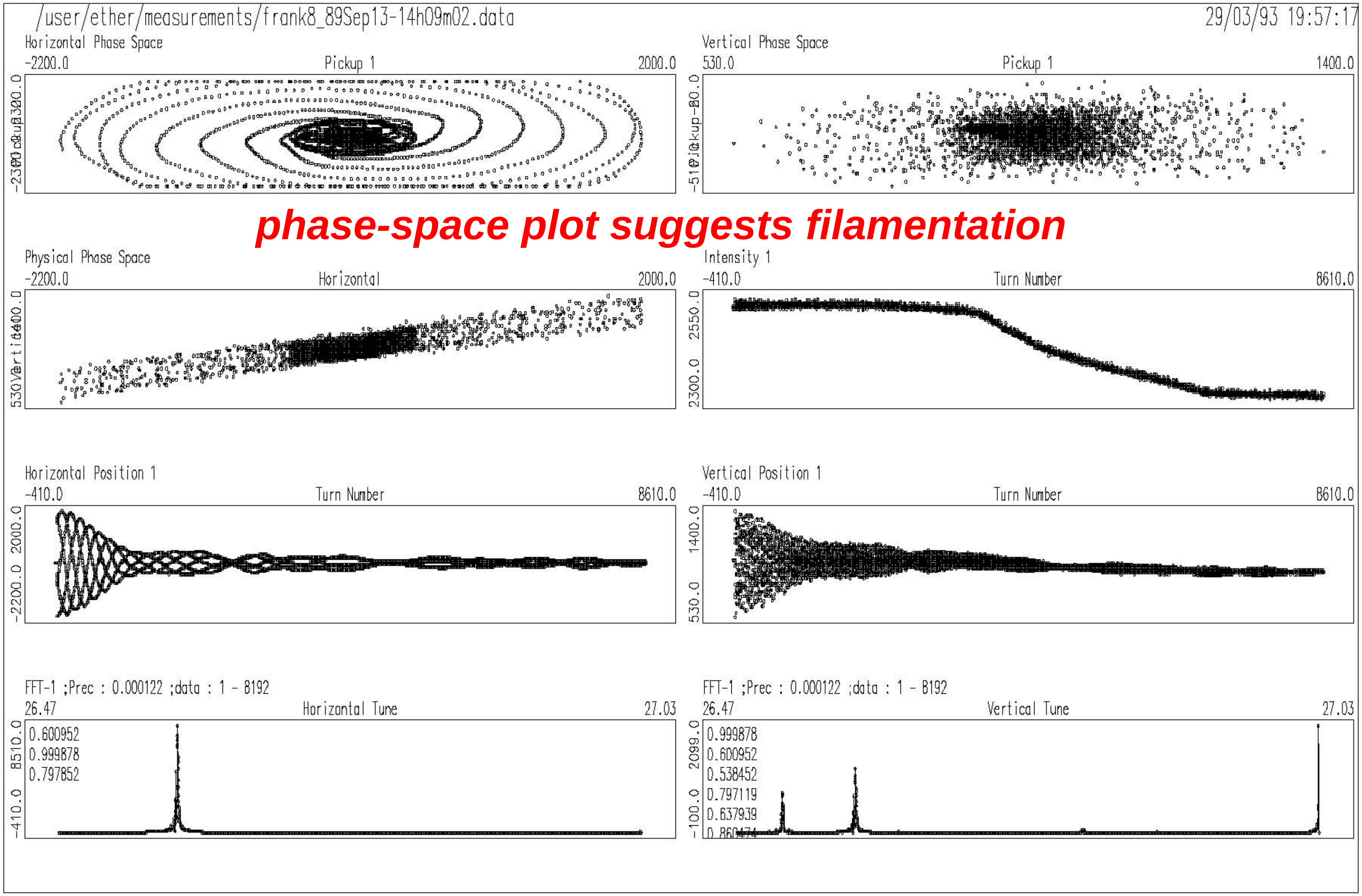
- damping
- filamentation
- chromaticity
- linear coupling

tune measurements for proton beams in the CERN-SPS



W. Fischer and F. Schmidt, CERN SL/Note 93-64 (AP)

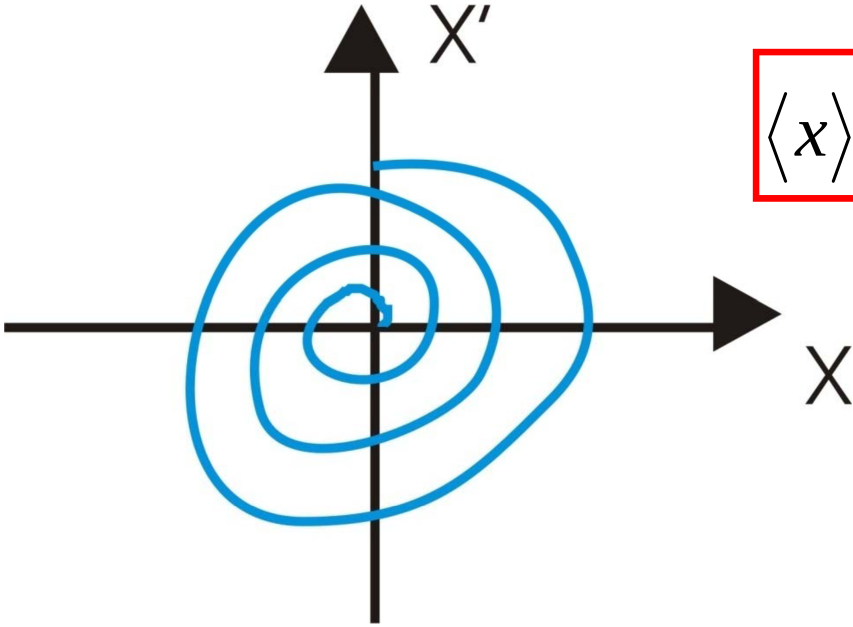
tune measurements for proton beams in the CERN-SPS



W. Fischer and F. Schmidt, CERN SL/Note 93-64 (AP)

coherent oscillations, damping & filamentation

response to a kick: ***coherent damping***



$$\langle x \rangle = x_0 e^{-t/\tau} \cos(\omega(J)t + \pi/2)$$

exp. decay

$$J = \frac{x^2 + (\beta x' + \alpha x)^2}{2\beta}$$

amplitude-dependent oscillation frequency

$$\frac{1}{\tau} = \frac{1}{\tau_{SR}} + \frac{1}{\tau_{HT}} + \ldots$$

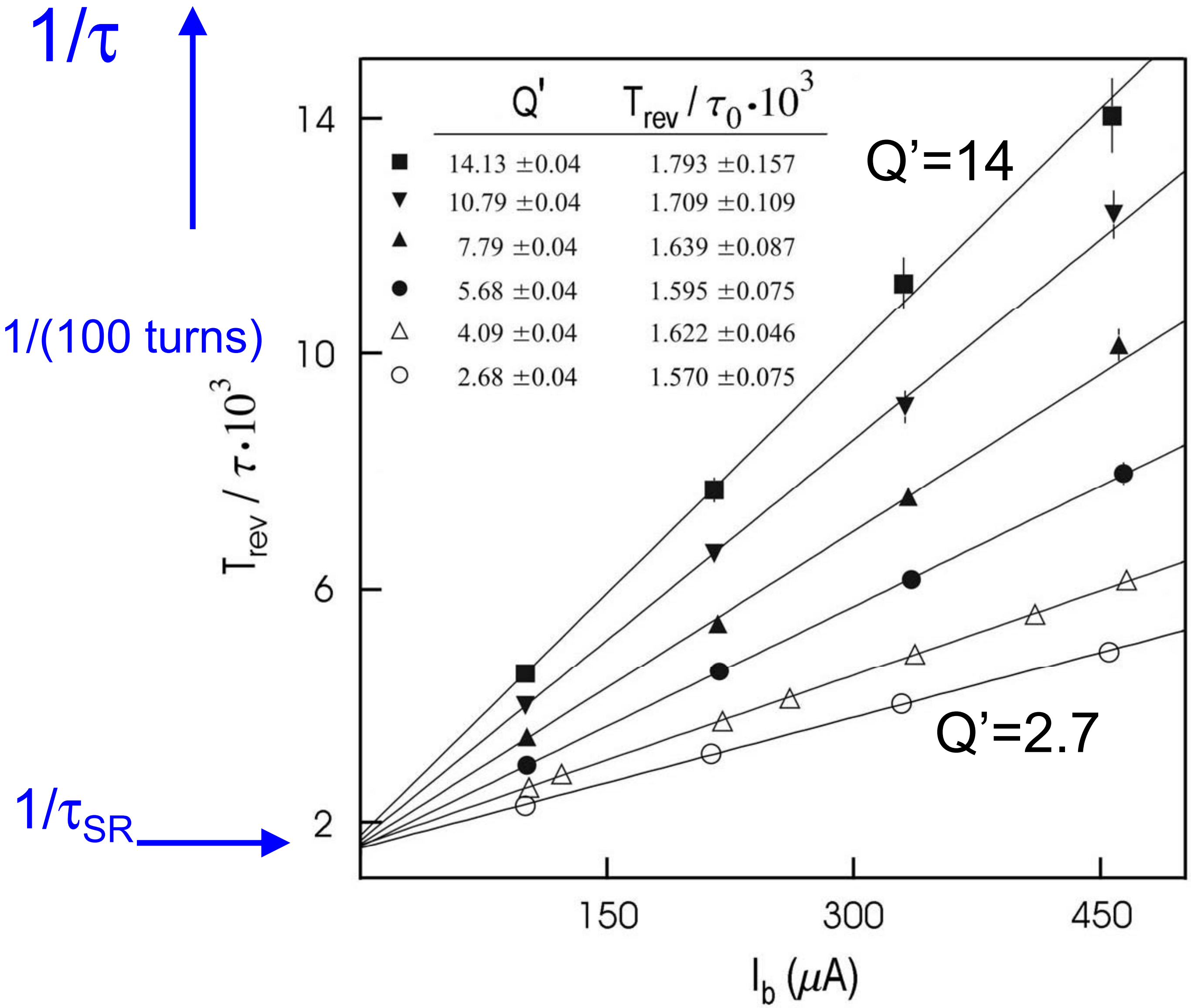
synchrotron-radiation damping

head-tail damping

$$\frac{1}{\tau_{HT}} \propto N_b \xi (\mathrm{Re}\, Z_{eff}(\sigma_z))$$

bunch population

chromaticity



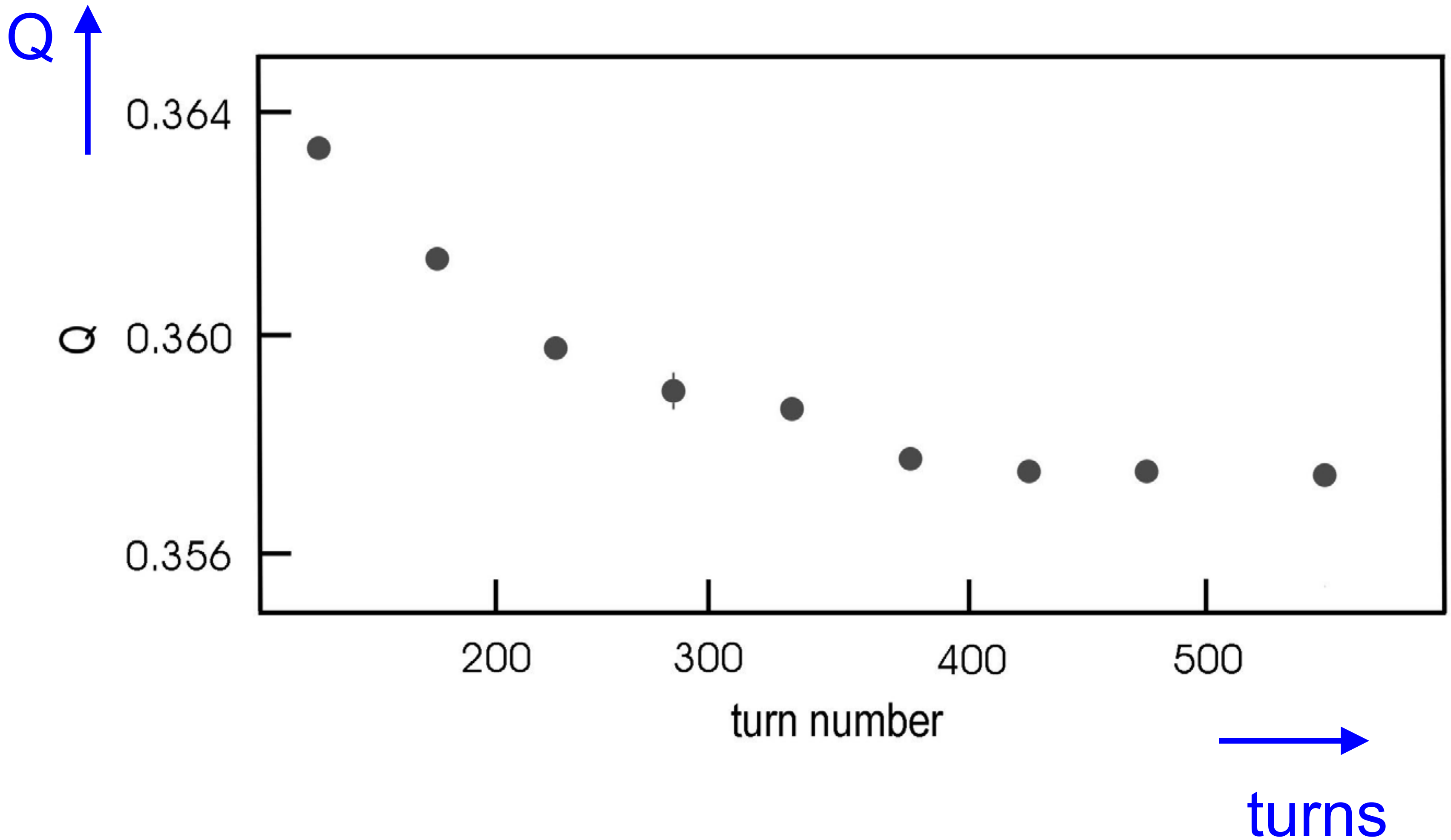
LEP 45.63 GeV,
damping rate
$1/\tau$ vs. I_{bunch}
for different
chromaticities
[A.-S. Muller]

$$\frac{1}{\tau_{SR}} = \frac{1}{2}\frac{U_0 f_{rev}}{E_0} J_x$$

horizontal
damping
partition number

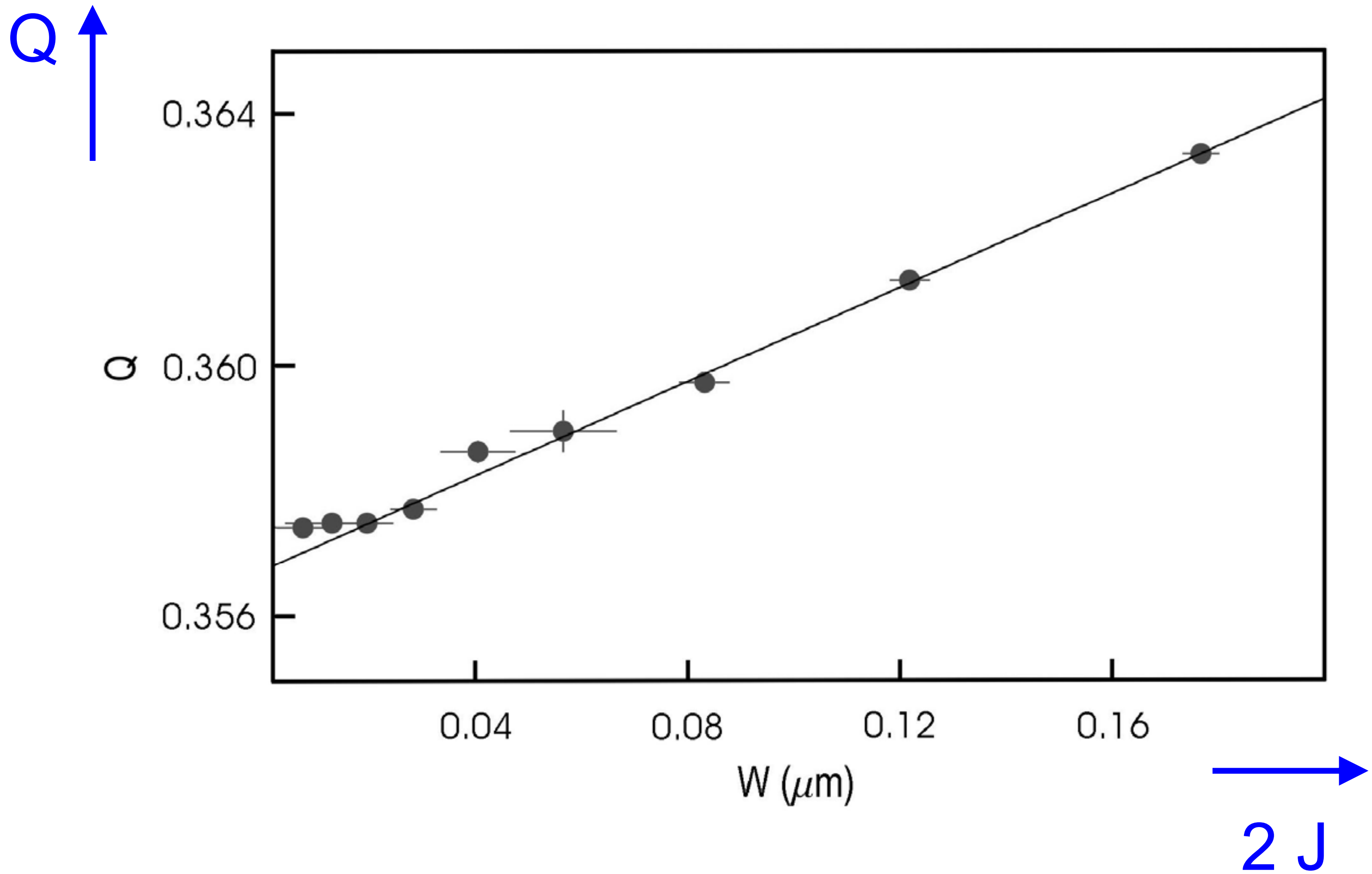
I_b

LEP: tune change during damping



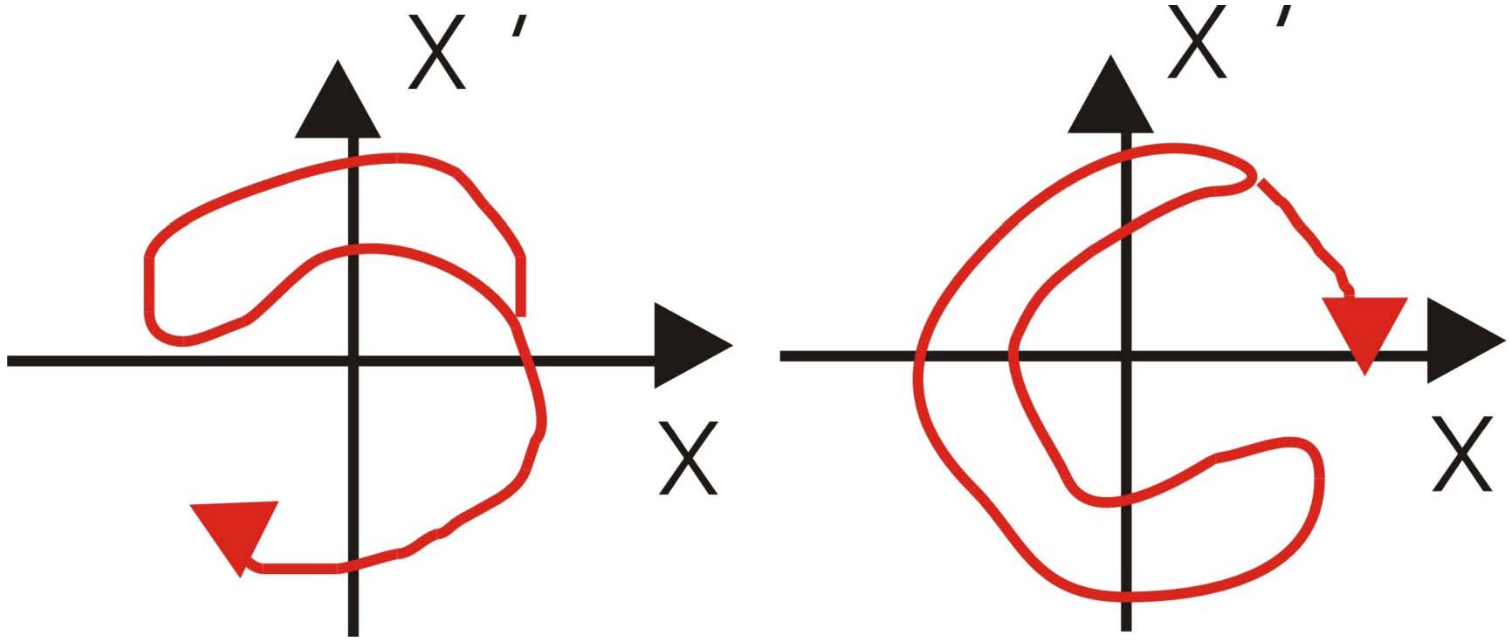
[A.-S. Muller]

LEP: detuning with amplitude from single kick



[A.-S. Muller]

another response to a kick: ***filamentation***



$$\langle x\rangle = \frac{Z\sigma_x}{1+\alpha t^2}\, e^{-\frac{Z^2}{2}\frac{\alpha t^2}{1+\alpha t^2}} \cos\left(\omega t + \Delta\phi(t)\right)$$

R. Meller et al.,
SSC-N -360, 1987

$t \ll 1/\sqrt{\alpha}$: $\quad e^{-c_1 t^2}$

$t \gg 1/\sqrt{\alpha}$: $\quad \sim \frac{1}{1+\alpha t^2}$

both different from $e^{-t/\tau}$

Z: kick in σ

$\alpha = (2\mu\omega_0)^2$

$Q = Q_0 - \mu a^2$

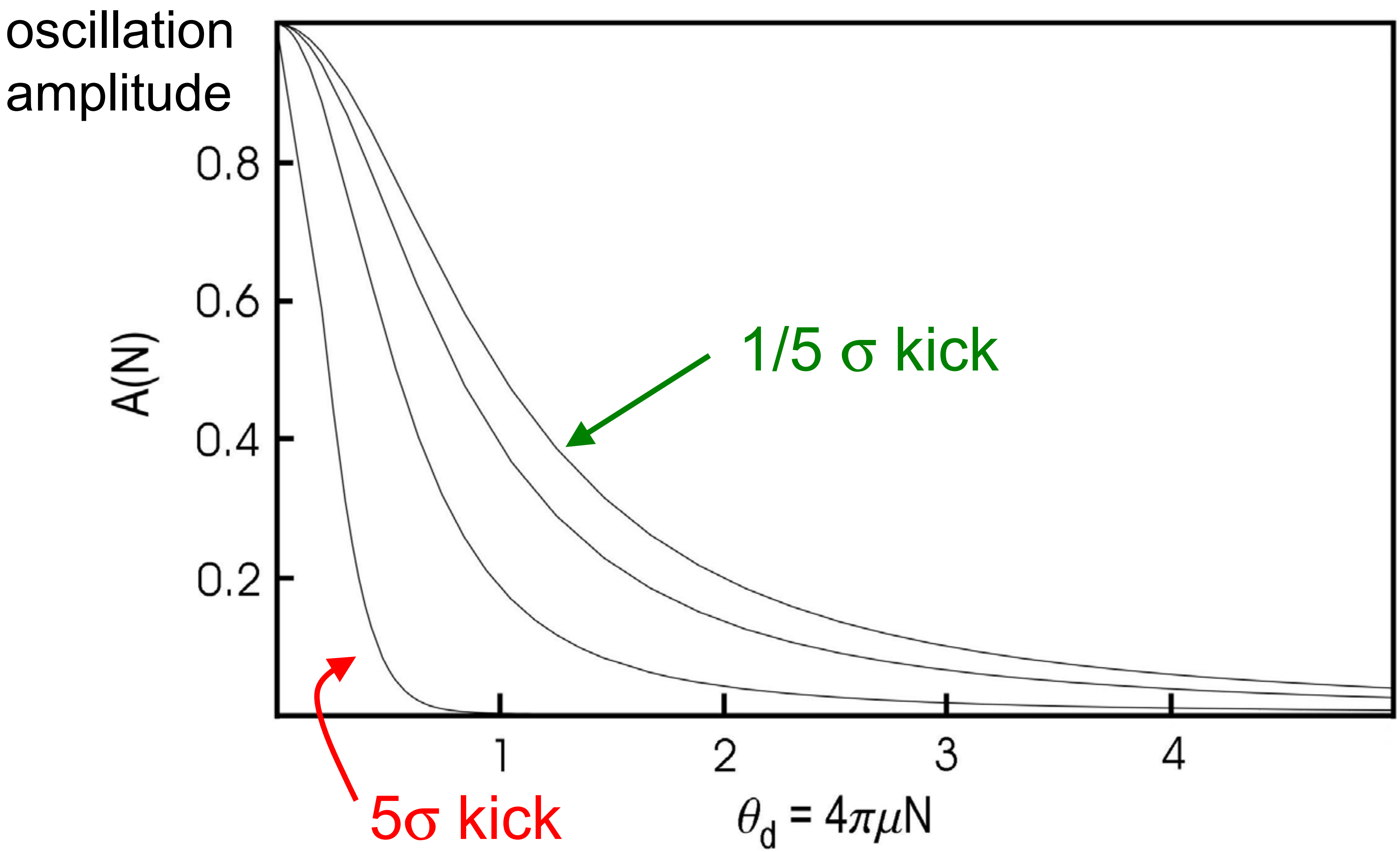
amplitude in σ

ex.:

$\mu = 10^{-4}$: $\sqrt{\alpha} t = 1$

after ≈ 1000 turns

amplitude decoherence factor vs. turn number



chromaticity

normalized $\xi = \frac{\Delta Q / Q}{\Delta p / p}$ unnormalized $Q' = \frac{\Delta Q}{\Delta p / p}$

relation $\xi = Q' / Q$

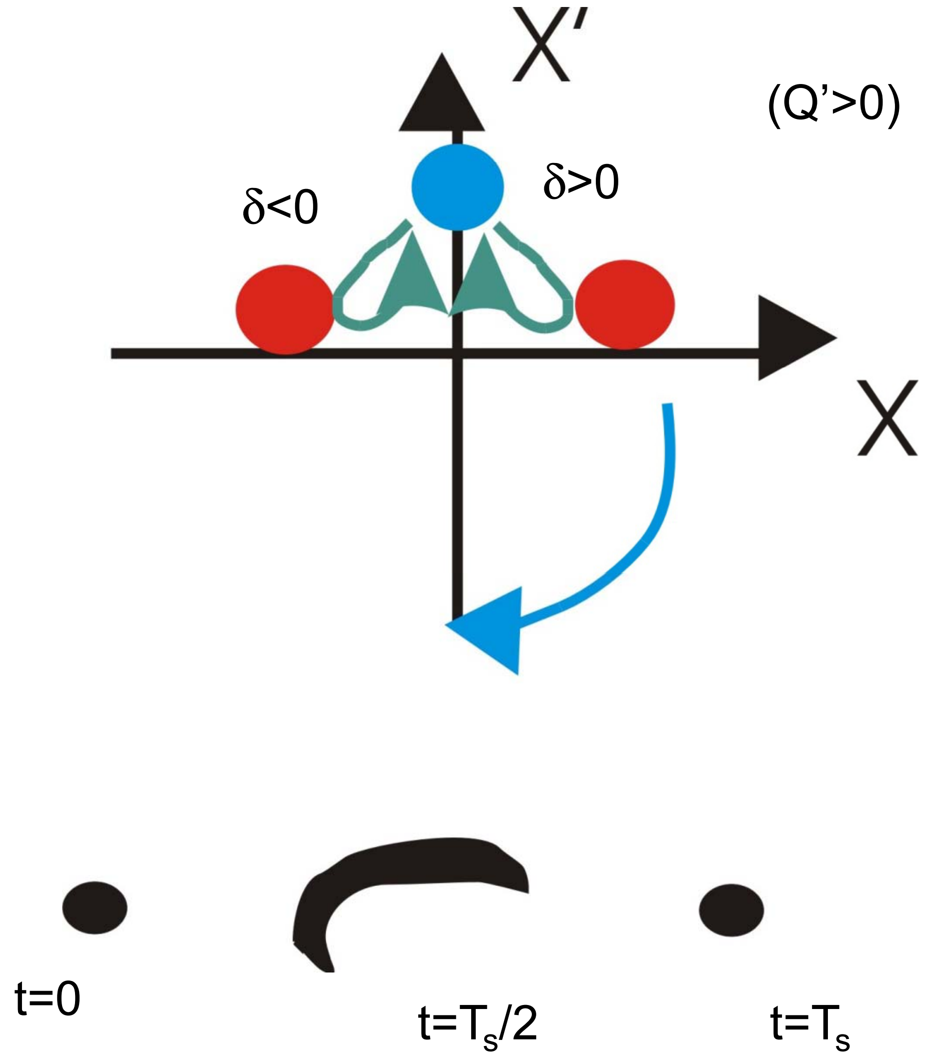
chromaticity describes the change of focusing and tune with particle energy

usually 2 or more families of sextupoles are used to compensate and control the chromatcity

small chromaticity is desired to minimize tune spread and amount of synchrobetatron coupling (maximize dynamic aperture)

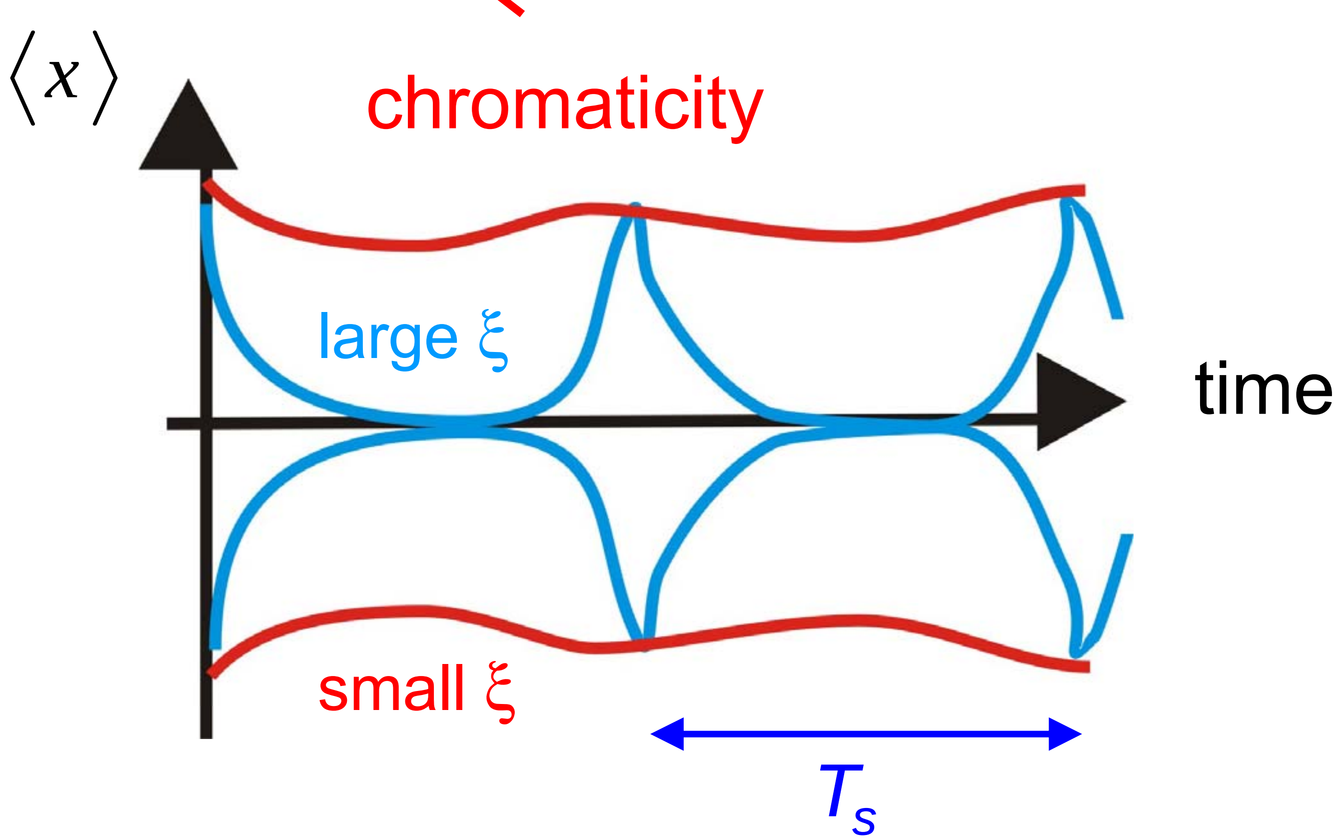
but large positive chromaticity is often employed to damp instabilities (ESRF, Tevatron, SPS,…)

response to a kick: decoherence due to chromaticity



$$\langle x \rangle = Z\sigma_x e^{-\frac{2\sigma_\delta^2 Q'^2}{Q_s^2}\sin^2(\omega_s t/2)} \cos(\omega t + \Delta\phi(t))$$

R. Meller et al.,
SSC-N -360, 1987



FFT over small number of turns → *widening of tune peak*

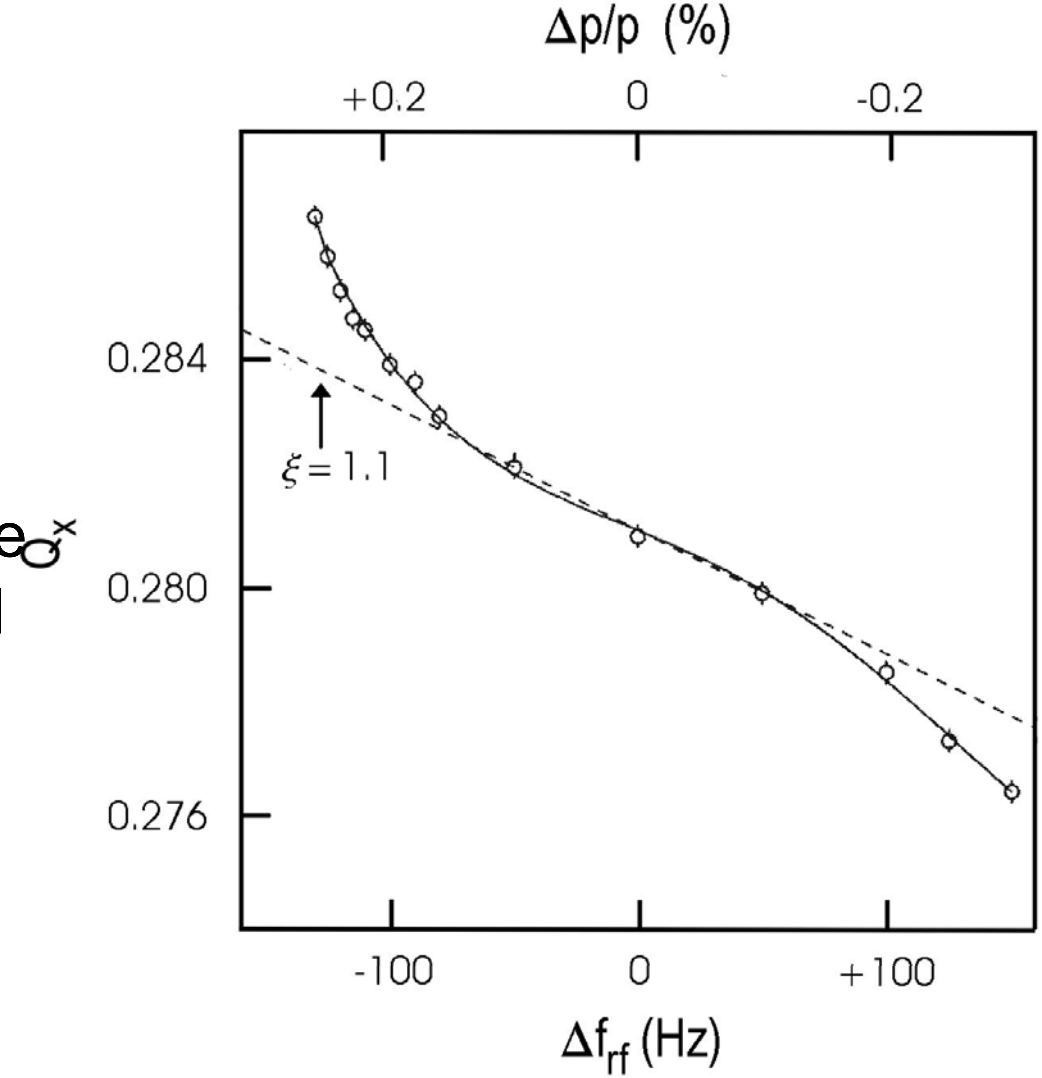
FFT over several synchrotron periods

→ *synchotron sidebands around betatron tune* another method to determine total chromaticity

tune shift as a function of rf frequency

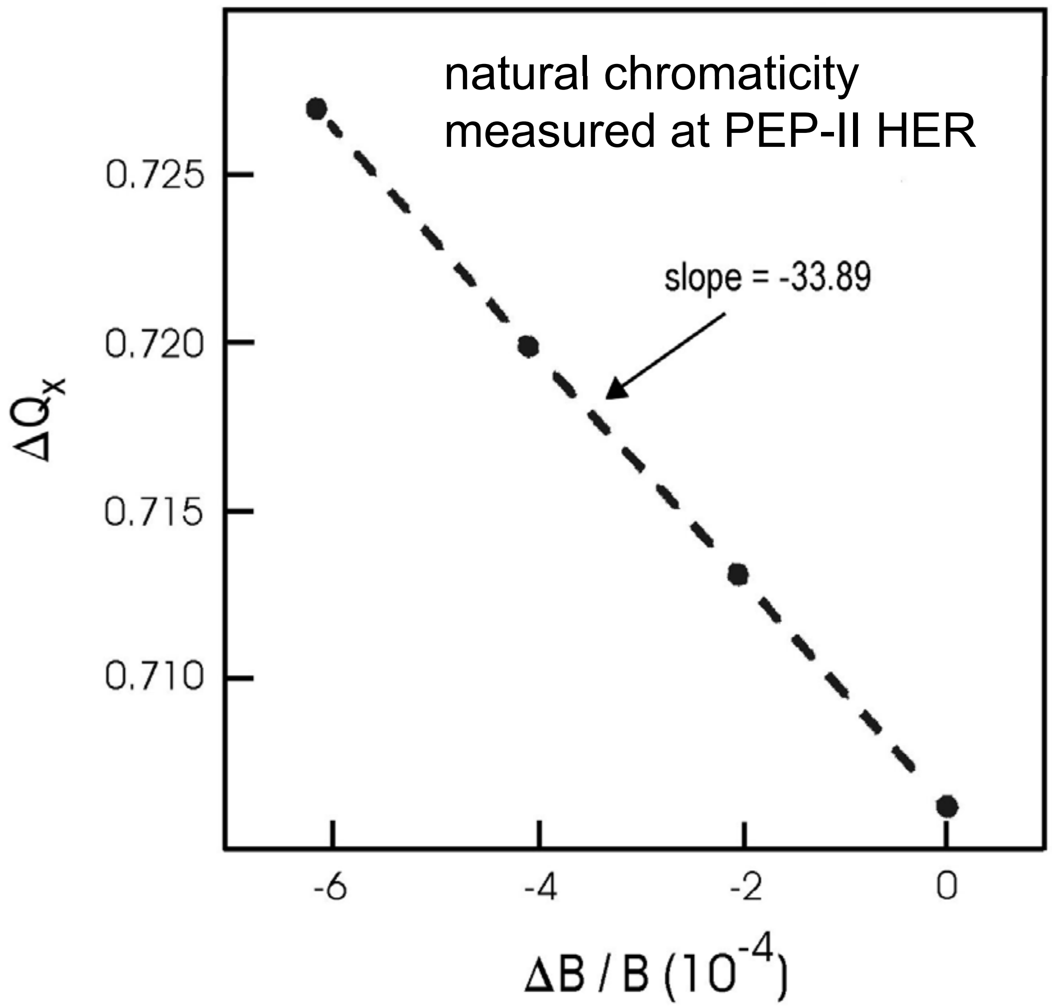
$$Q'_{x,y} = \frac{\Delta Q_{x,y}}{\Delta p / p}$$

$$= -\left(\alpha_C - \frac{1}{\gamma^2}\right) \frac{\Delta Q_{x,y}}{\Delta f_{rf} / f_{rf}}$$



horizontal tune vs change in rf frequency measured at LEP; the dashed line shows the linear chromaticity as determined by measurements at +/- 50 kHz

measuring the *natural* chromaticity (*Q'* w/o sextupoles)
from tune shift vs. dipole field



$$\frac{\Delta p}{p} = \frac{\Delta B}{B}$$ electron ring

for e-, the orbit is unchanged (determined by rf!)

$$Q'^{nat}_{x,y} \approx \frac{\Delta Q_{x,y}}{\Delta B / B}$$

for p, simultaneous change in rf frequency required to keep the same orbit:

$$\frac{\Delta \omega_{rf}}{\omega_{rf}} = \frac{1}{\gamma^2}\frac{\Delta B}{B}$$

linear coupling: model of 2 coupled oscillators

$$\frac{d^2x}{d\theta^2} + Q_0^2 x = -\kappa y$$

κ: coupling strength

$$\frac{d^2y}{d\theta^2} + Q_0^2 y = -\kappa x$$

normal-mode coordinates: $u = \frac{x+y}{2}, \quad \mathrm{v} = \frac{\mathrm{x-y}}{2}$

decoupled equations

$$\frac{d^2u}{d\theta^2} + \left(Q_0^2 + \kappa\right)u = 0$$

$$\frac{d^2v}{d\theta^2} + \left(Q_0^2 - \kappa\right)v = 0$$

$$Q_u^2 = Q_0^2 + \kappa$$

$$Q_v^2 = Q_0^2 - \kappa$$

new eigen-frequencies

$$Q_u^2 - Q_v^2 = 2\kappa$$

frequency split:
measure of strength of coupling

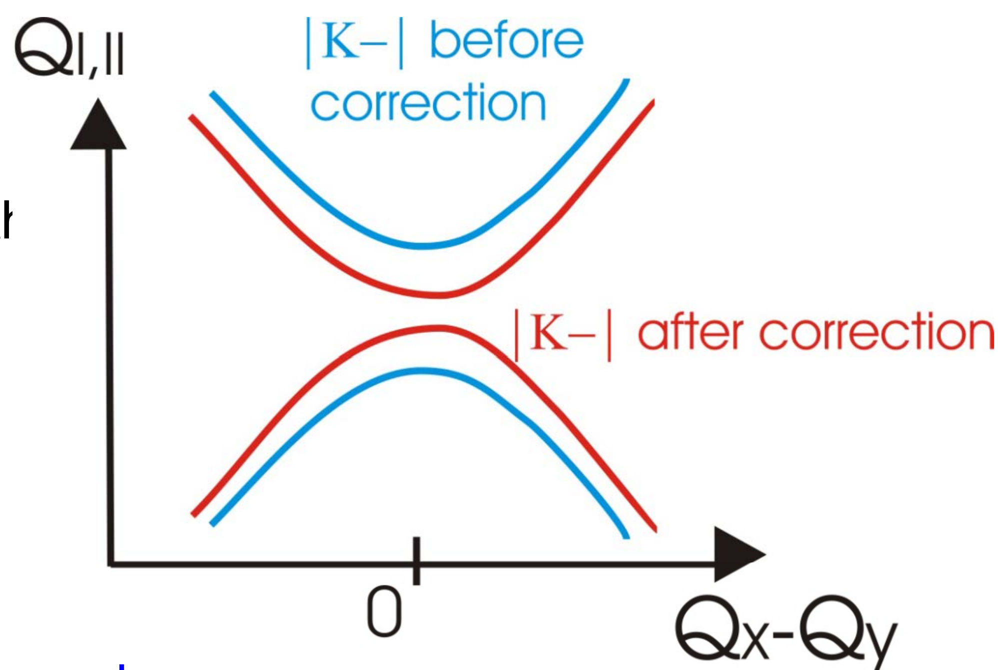
closest tune approach

near the difference resonance $Q_x - Q_y + q_{\_} \approx 0$

the tunes of the two eigenmodes, in the vertical plane, are

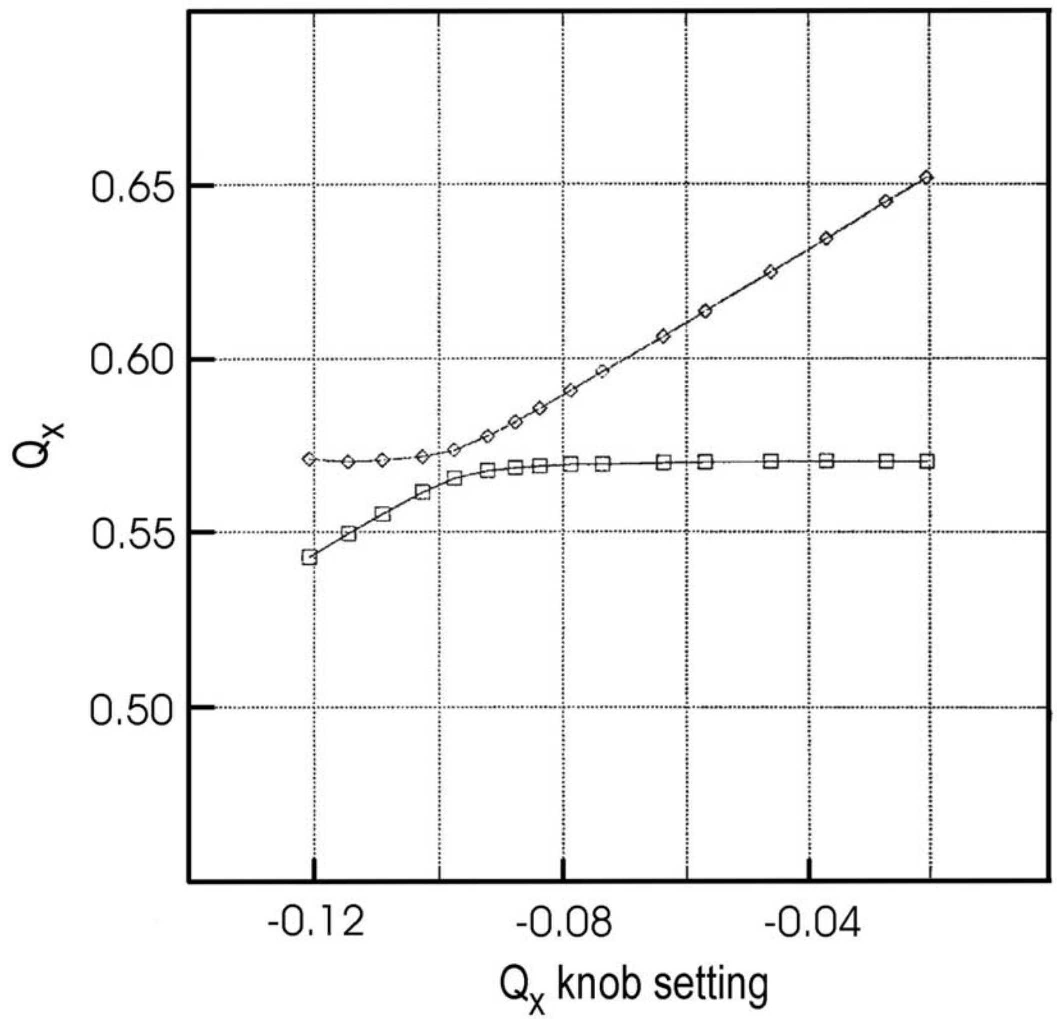
$$Q_{I,II} = \frac{1}{2}\left(Q_x + Q_y + q_{-} \pm \sqrt{\left(Q_x - Q_y + q_{\_}\right)^2 + \left|\kappa_{-}\right|^2} \right)$$

uncoupled tunes



tunes can approach each otl
only up to distance |κ_|

correction strategy;
use two skew quadrupoles
(ideally with $\Delta(\phi_x - \phi_y) \sim \pi/2$) to
minimize |κ_|, namely
the distance of closest tune approach

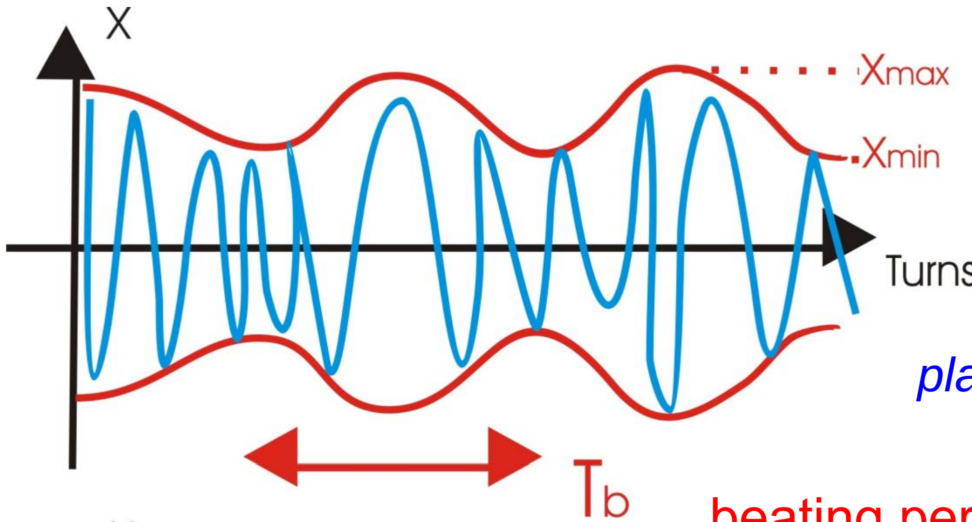


closest tune approach in the PEP-II HER before final correction; shown are the measured fractional tunes as a function of the horizontal tune knob; the minimum tune distance is equal to the driving term $|\kappa_{-}|$ of the difference resonance another way to measure $|\kappa_-|$: kick response over many turns

define

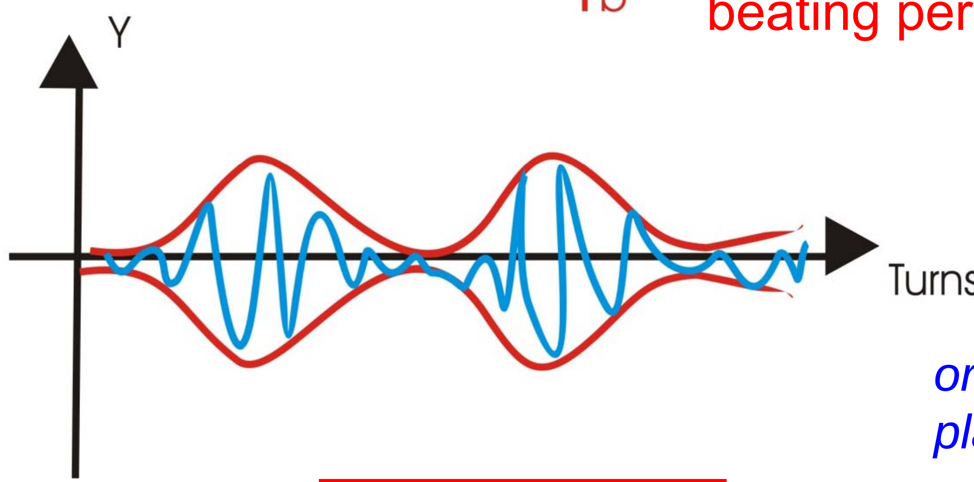
$$S = \frac{x_{\min}^2}{x_{\max}^2}$$

envelopes of horizontal and vertical oscillations exhibit beating



plane of kick

beating period



orthogonal plane

one can show that

$$\left|\kappa_-\right| = \frac{\sqrt{1-S}}{f_{rev} T_b} \quad !$$

example ATF

coupling transfer function

excite beam in x ⟶ detect coherent y motion

used for continuous monitoring of coupling at the CERN ISR in the 1970s;
is considered for LHC coupling control

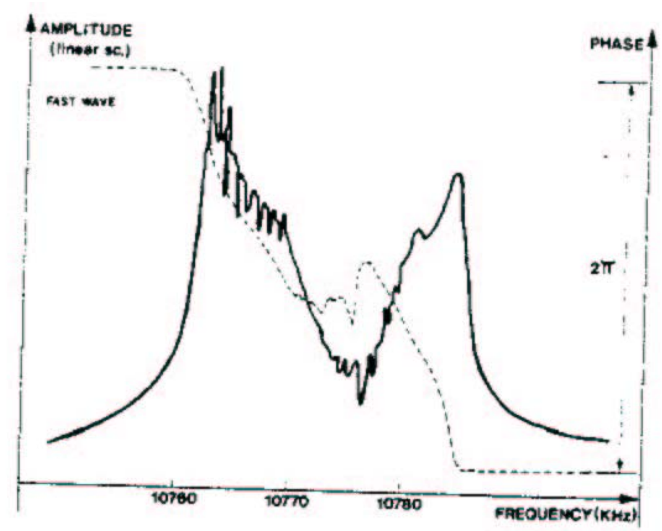
$$A(\omega) = \sqrt{c_r^2 + c_i^2}$$

$$\phi(\omega) = \mathrm{arctg}\left(\frac{c_i}{c_r}\right)$$

← amplitude and phase of vertical response; complex value of κ_-

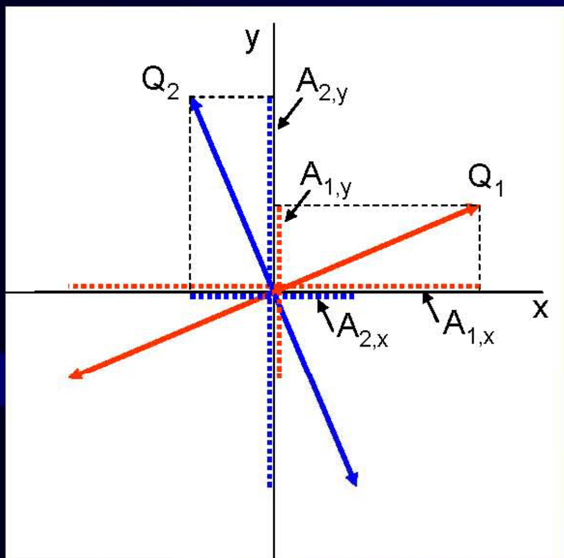
J.-P. Koutchouk, 1980

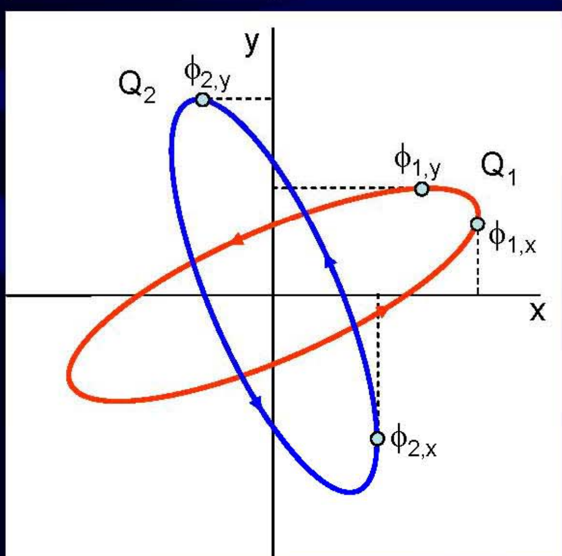
ISR coupling transfer function

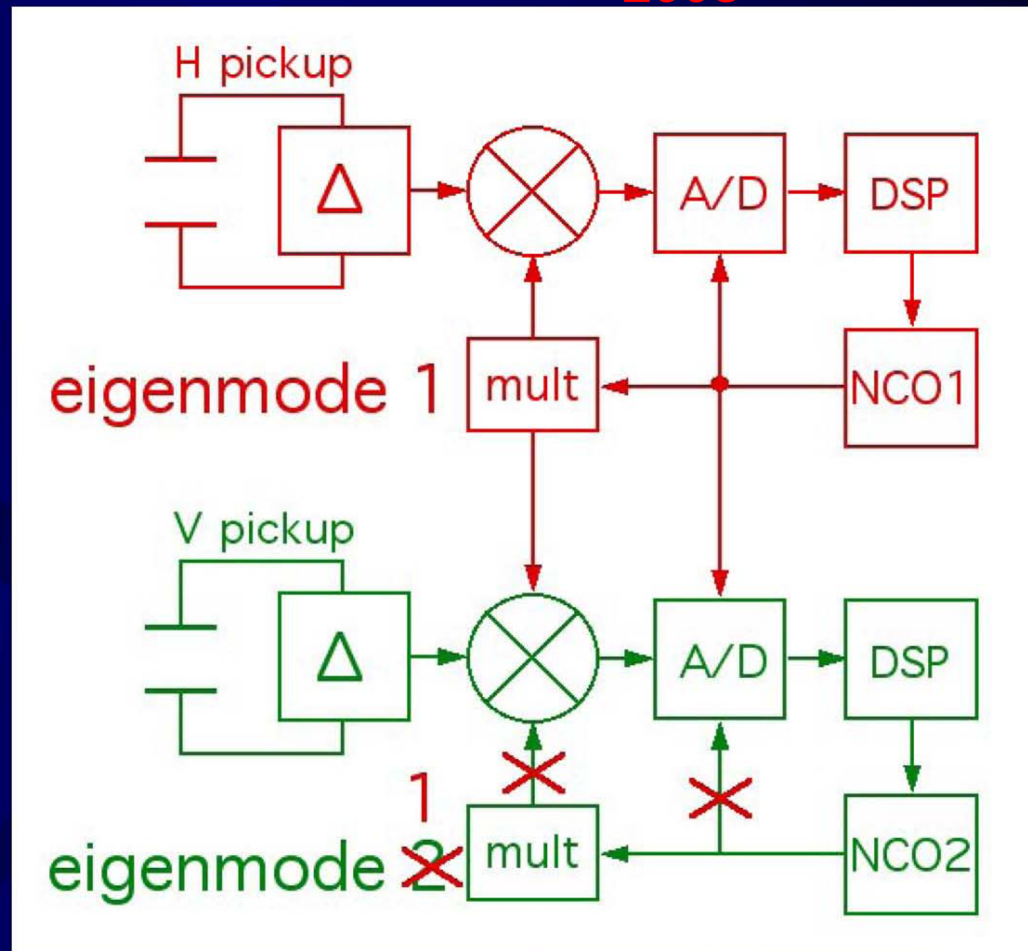


Measurement of Coupling using a PLL Tune Tracker

RHIC-Tevatron-LHC 2005

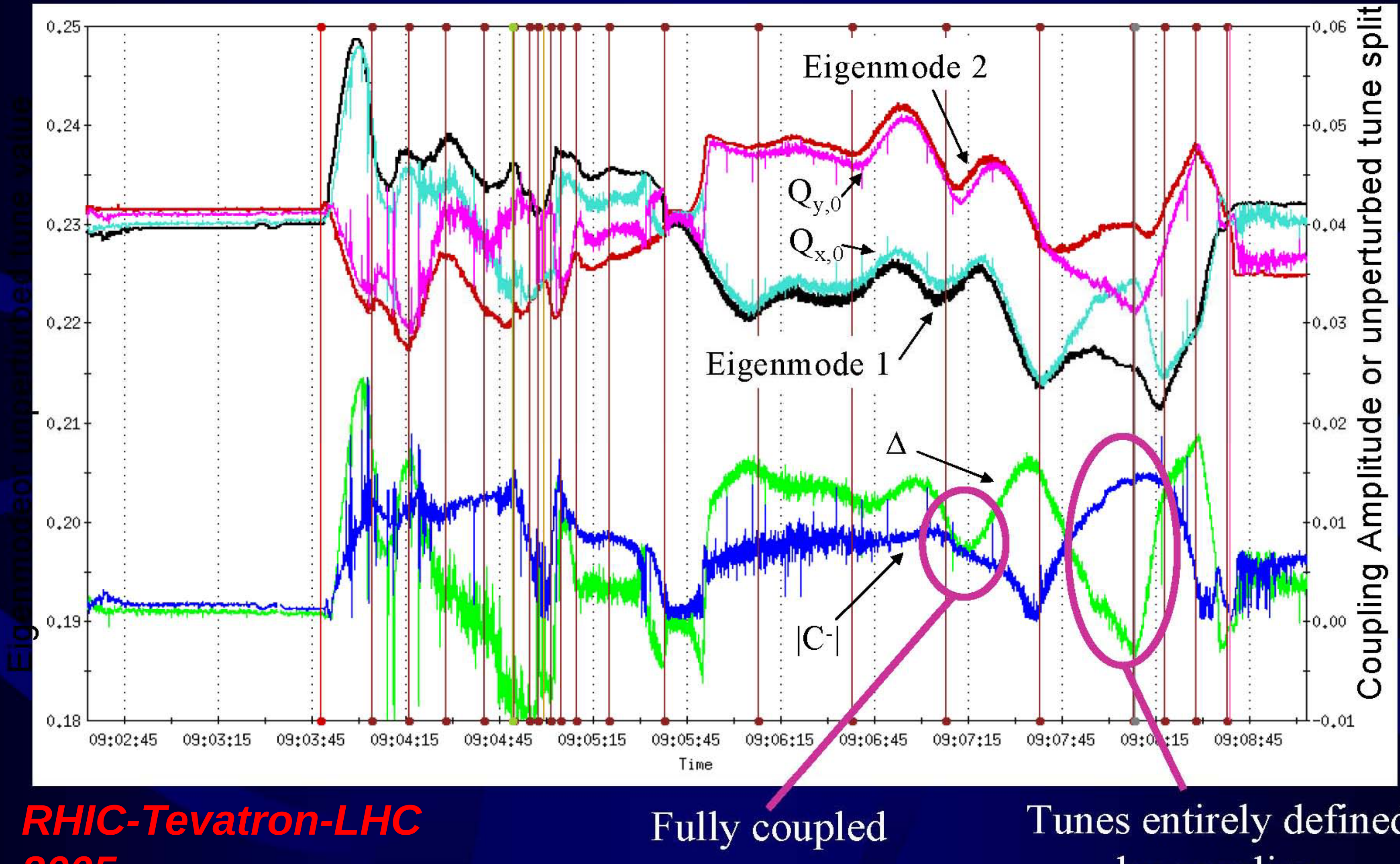






Tracking the vertical mode in the horizontal plane & vice-versa allows the coupling parameters to be calculated

Measurement of Coupling using a PLL Tune Tracker (RHIC Example)



RHIC-Tevatron-LHC 2005

many other complications and challenges,

for example:

- space charge
- ionized gas molecules
- electron cloud
- beam-beam interaction
- radiation damping
- .. etc etc …

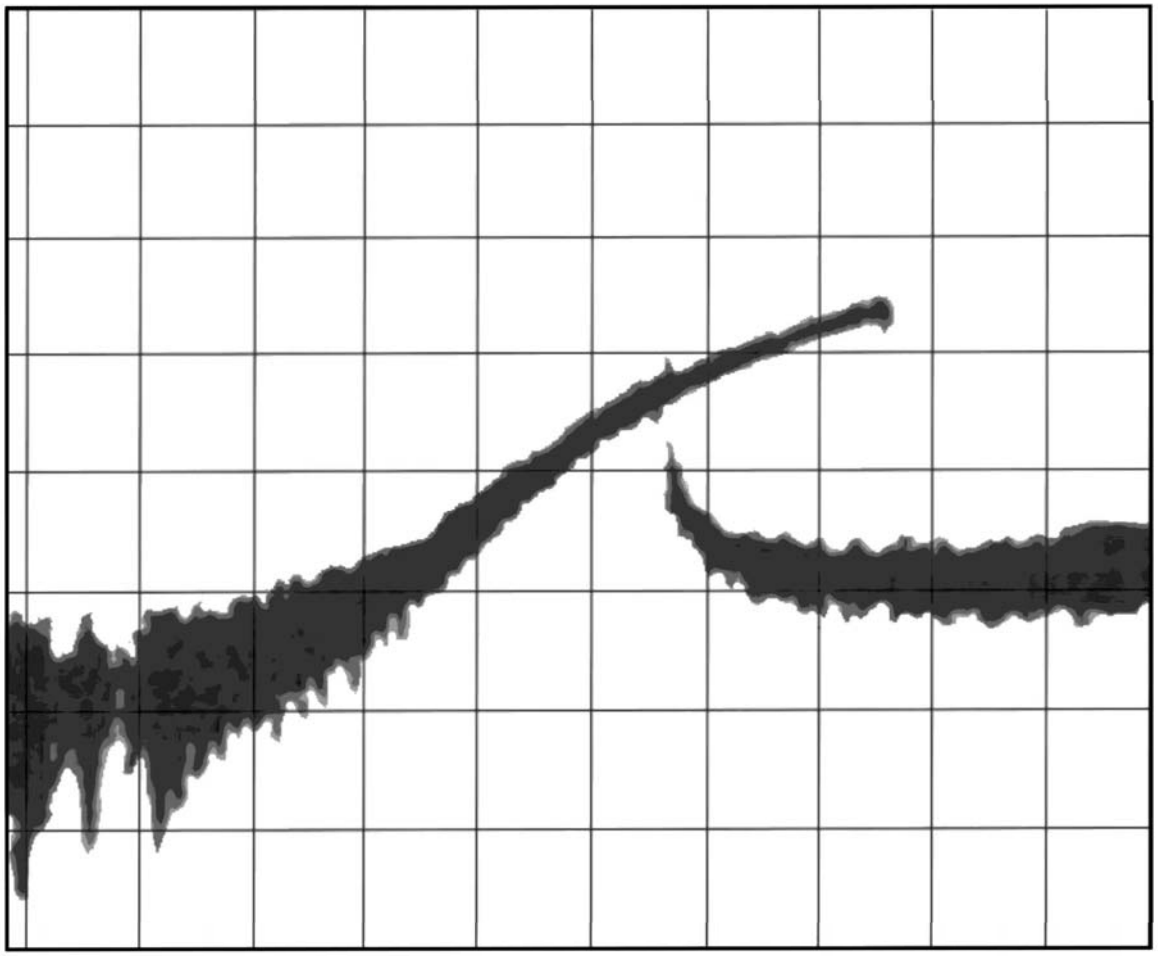
⟶ all these phenomena affect beam response to excitation

Courtesy B. Goddard

some “complications”:

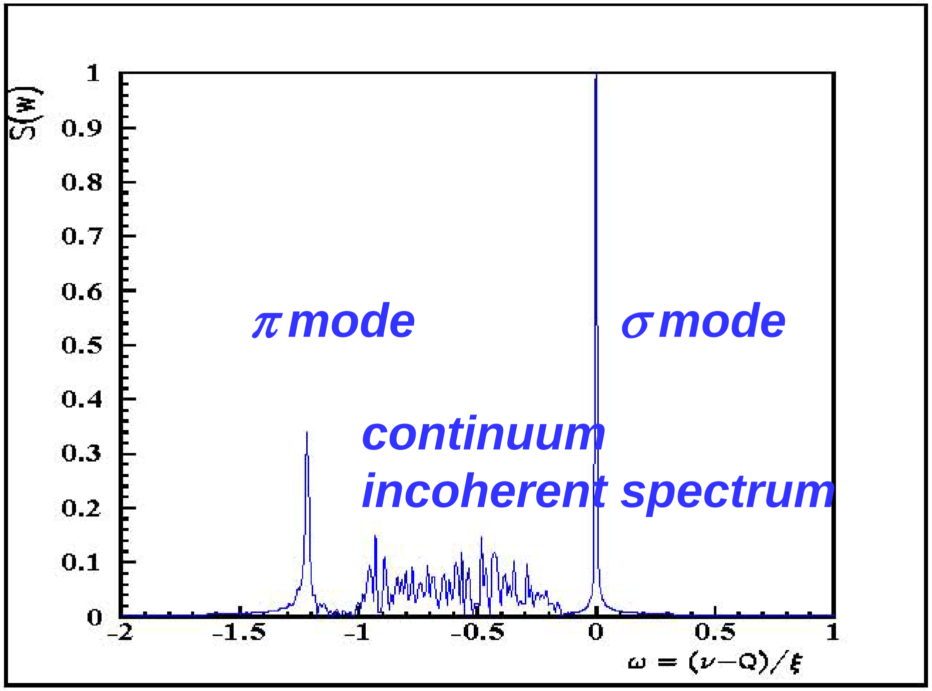
colliding beam tune spectra
space charge tune spread
measuring the incoherent tune

transverse tune measurement (swept-frequency excitation) with 2 colliding bunches at TRISTAN. Vertical axis: 10 dB/div., horizontal axis: 1 kHz/div [K. Hirata, T. Ieiri]

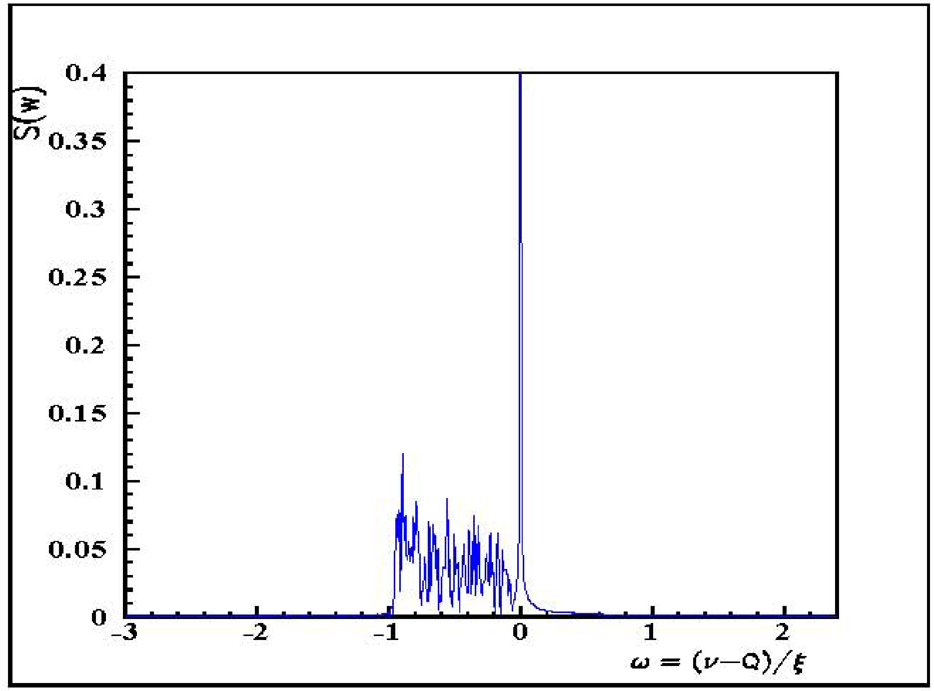


Hysteresis:
2 fixed points
due to nonlinear
beam-beam force

simulated tune spectrum for two colliding beams



equal intensity



intensity ratio 0.55

π mode not "Landau damped"

π mode "Landau damped"

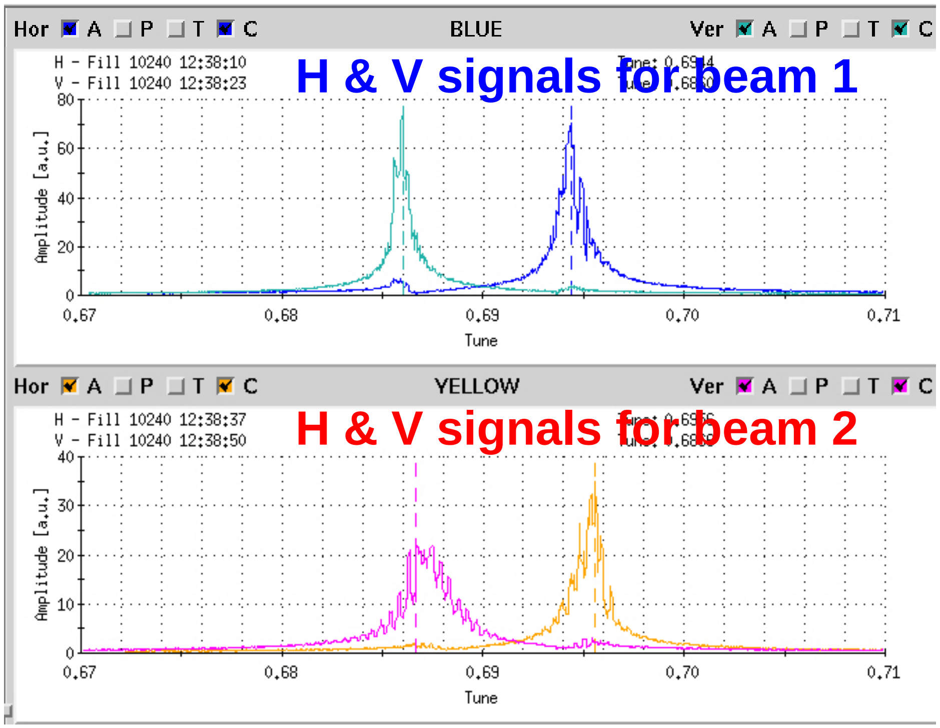
M.P. Zorzano & F.Z., PRST-AB 3, 044401 (2000)
W. Herr, M.P. Zorzano, F. Jones, PRST-AB 4, 054402 (2002)

if the coherent tune lies outside the continuum "Landau damping" is lost and the beam can be unstable; prediction for the LHC

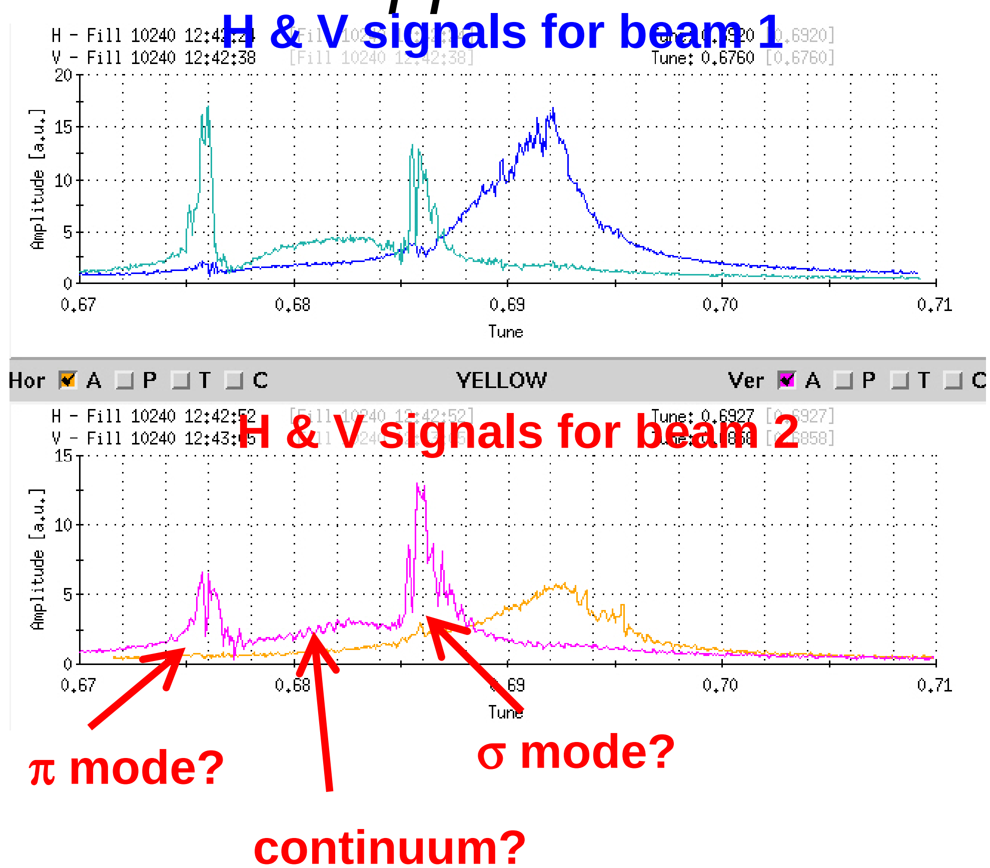
evidence for coherent beam-beam modes@RHIC

RHIC BTF amplitude response at 250 GeV

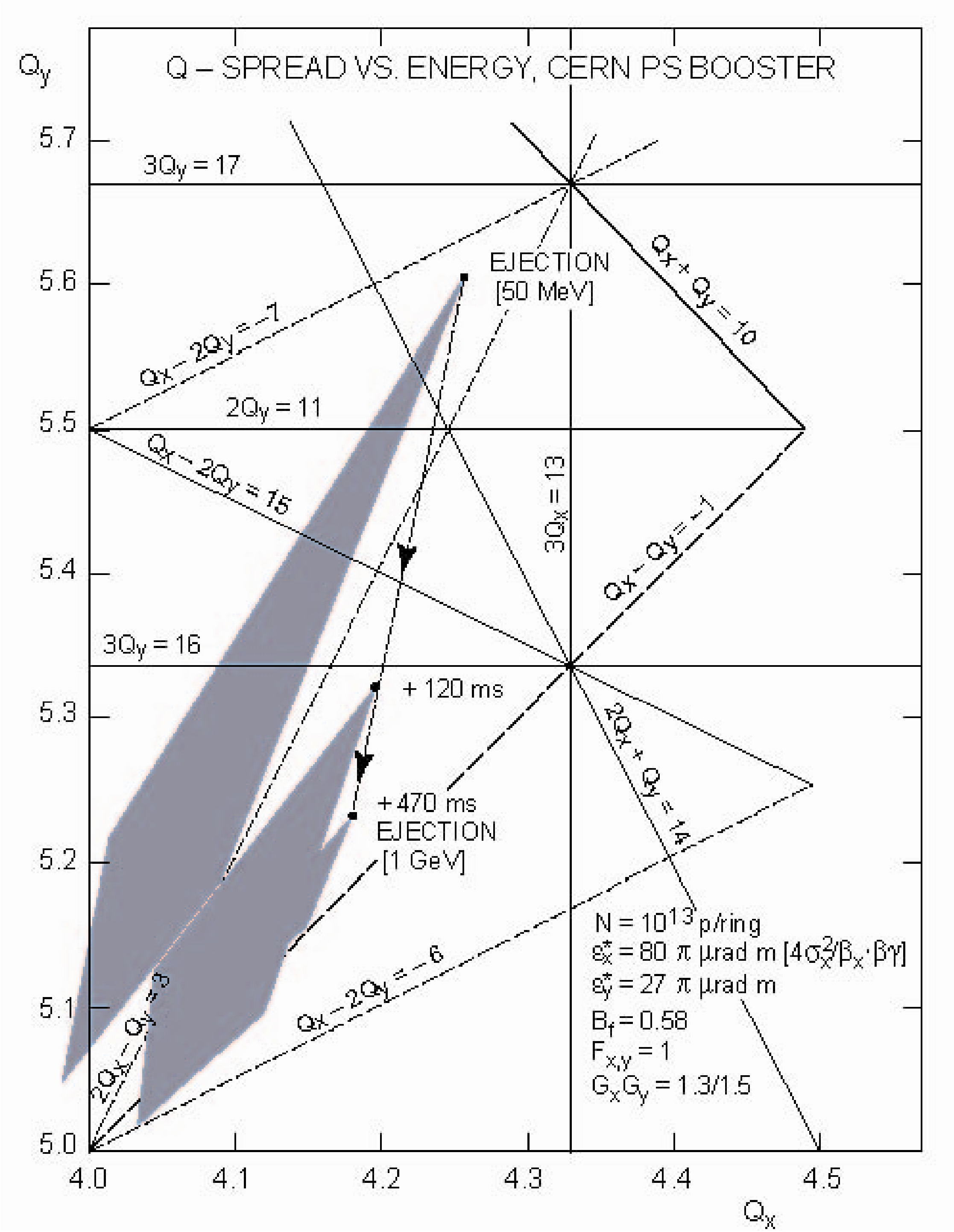
without collisions



with *pp* collisions



26. February 2009, Michiko Minty

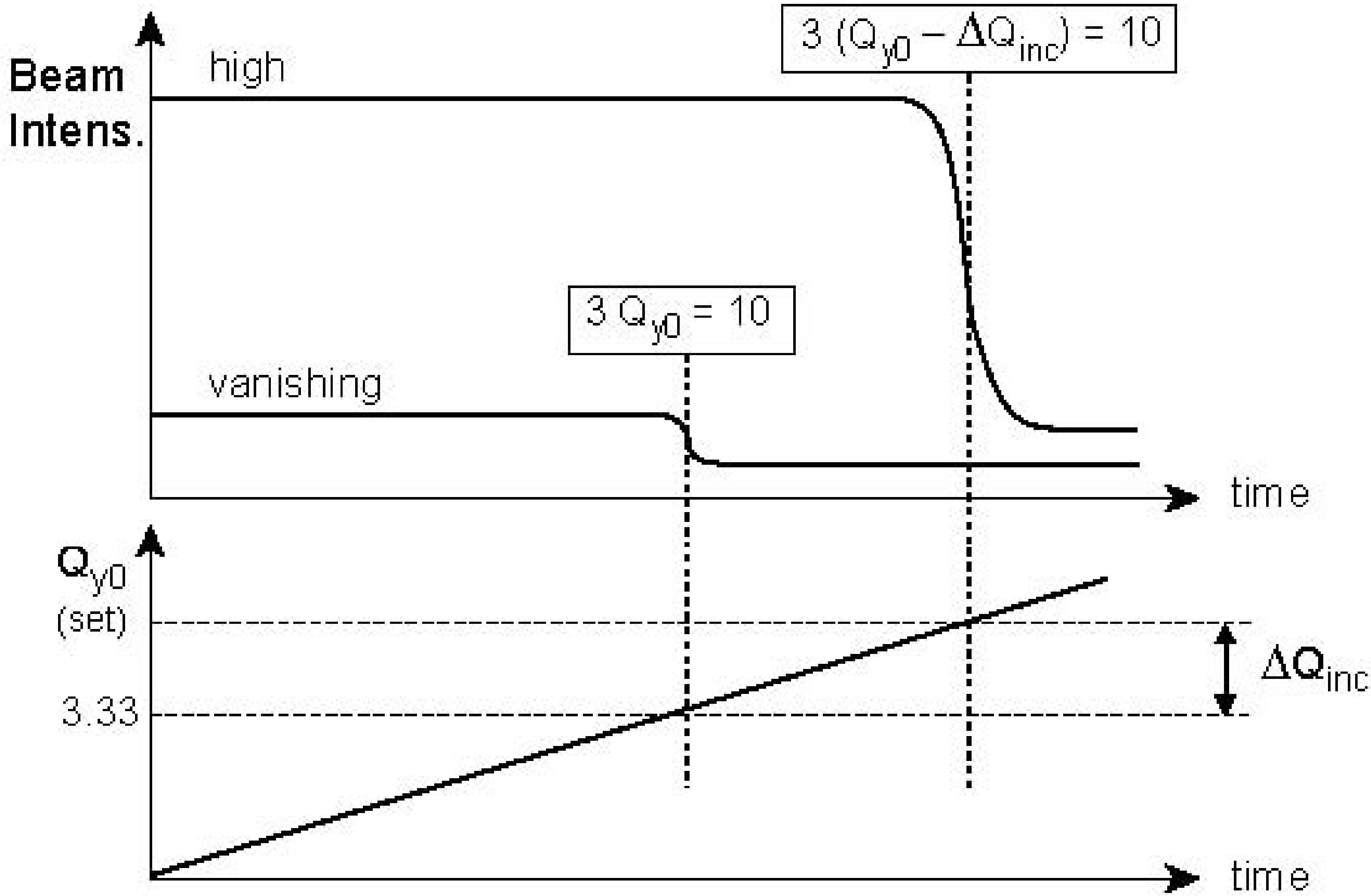


space charge

example for space-charge limited synchrotron: betatron tune diagram and areas covered by direct tune spread at injection, intermediate energy, and extraction, for the CERN Proton Synchrotron Booster. During acceleration, acceleration gets weaker and the "necktie" area shrinks, enabling the external machine tunes to move the "necktie" to a region clear of betatron resonances (up to 4th order)

K.H. Schindl

how to measure the incoherent tune shift/spread?



K.H. Schindl

Schottky monitor

directly measures "*incoherent*" tune
(oscillation frequency of individual particles)

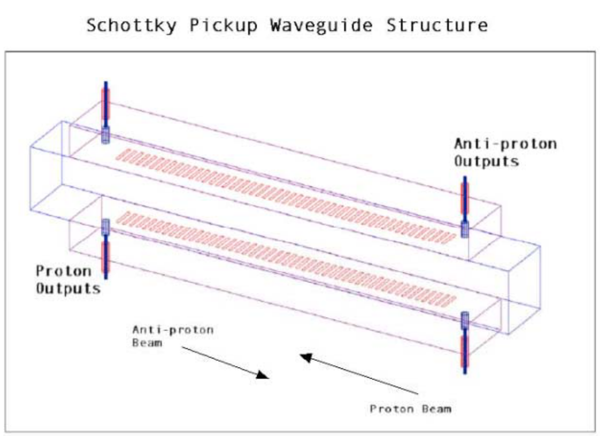
w/o centroid motion

incoherent signal $\propto \sqrt{\varepsilon N \Delta f}$

emittance — #particles — frequency bandwidth

"Schottky monitors" ⟶ stochastic cooling

FNAL slotted waveguide 1.7-GHz Schottky pickup design



fabricated FNAL Schottky pickup design

R. Pasquinelli et al

applications

tune measurements are useful

to determine:

- beta functions β, coupling strength $|\kappa\_|$
- chromaticity ξ, Q'
- transverse impedance $Z_\perp$
- nonlinear fields a_n, b_n

to improve:

- dynamic aperture A
- emittance ε
- lifetime τ
- instability thresholds I_{thr}

example applications

- chromaticity
- betatron coupling
- damping & decoherence
- tune shift with amplitude
- high-order resonances
- tune scans
- β function measurement
- measurement of nonlinear field errors

some applications of tune measurements

1. tune shift with amplitude

$$Q = Q_0 + \frac{\partial Q}{\partial I} I + \frac{1}{2}\frac{\partial^2 Q}{\partial I^2} I^2 + \ldots$$

due to nonlinear fields (octupoles, 12-poles, sextupoles,…)

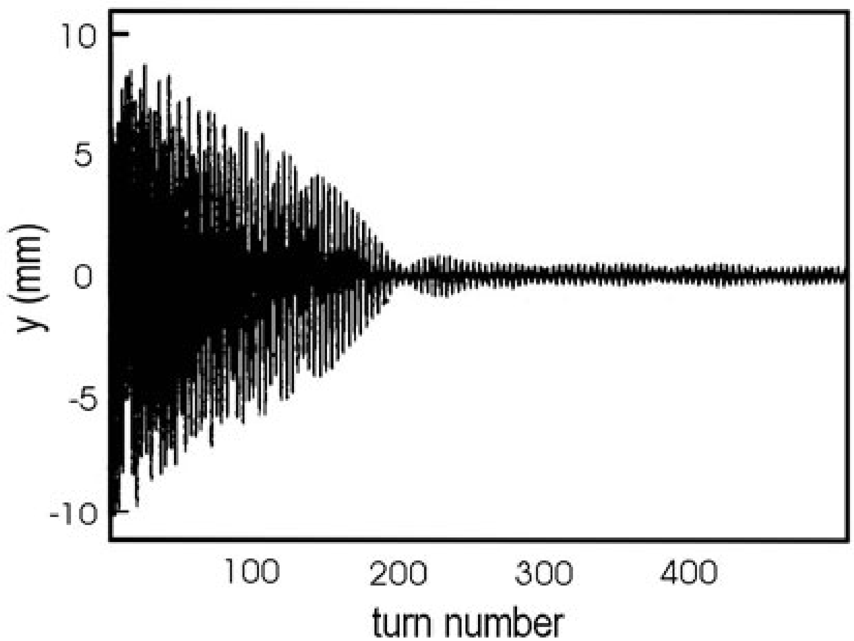
action-angle-coordinates *I*, ψ

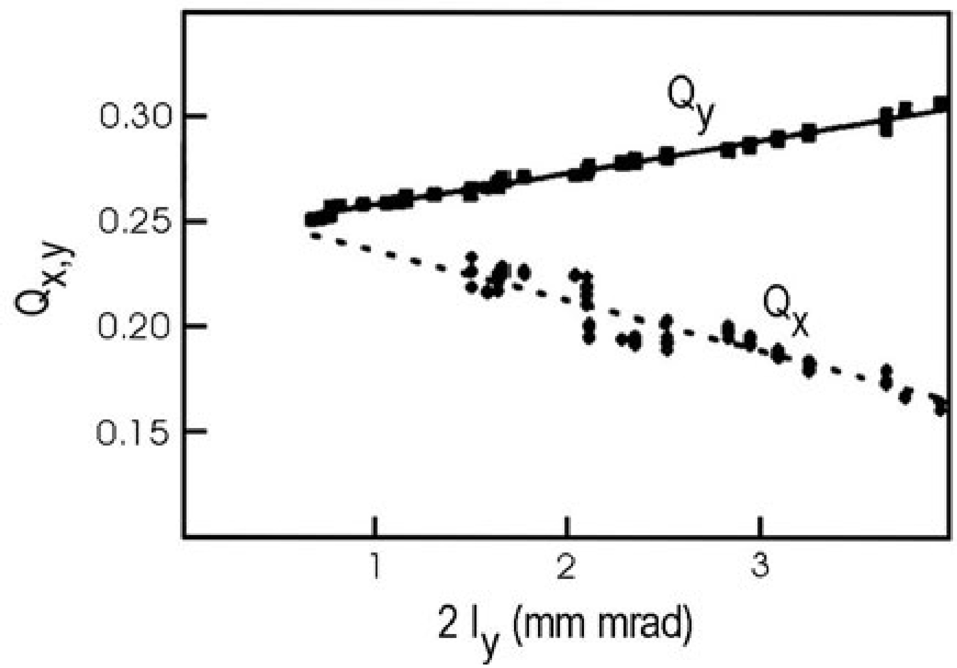
$$x = \sqrt{2I\beta}\cos\psi$$

$$x' = -\sqrt{\frac{2I}{\beta}}\sin\psi - \alpha\sqrt{\frac{2I}{\beta}}\cos\psi$$

e.g., use FFT with data windowing

→ accurate tune evaluation over 32 turns can measure <u>entire curve *Q(I)* after single injection</u> by calculating *Q&I* for each time interval (making use of radiation damping)





Measurement of tune shift with amplitude in LEP at 20 GeV using a high-precision FFT tune analysis

2. higher-order resonances

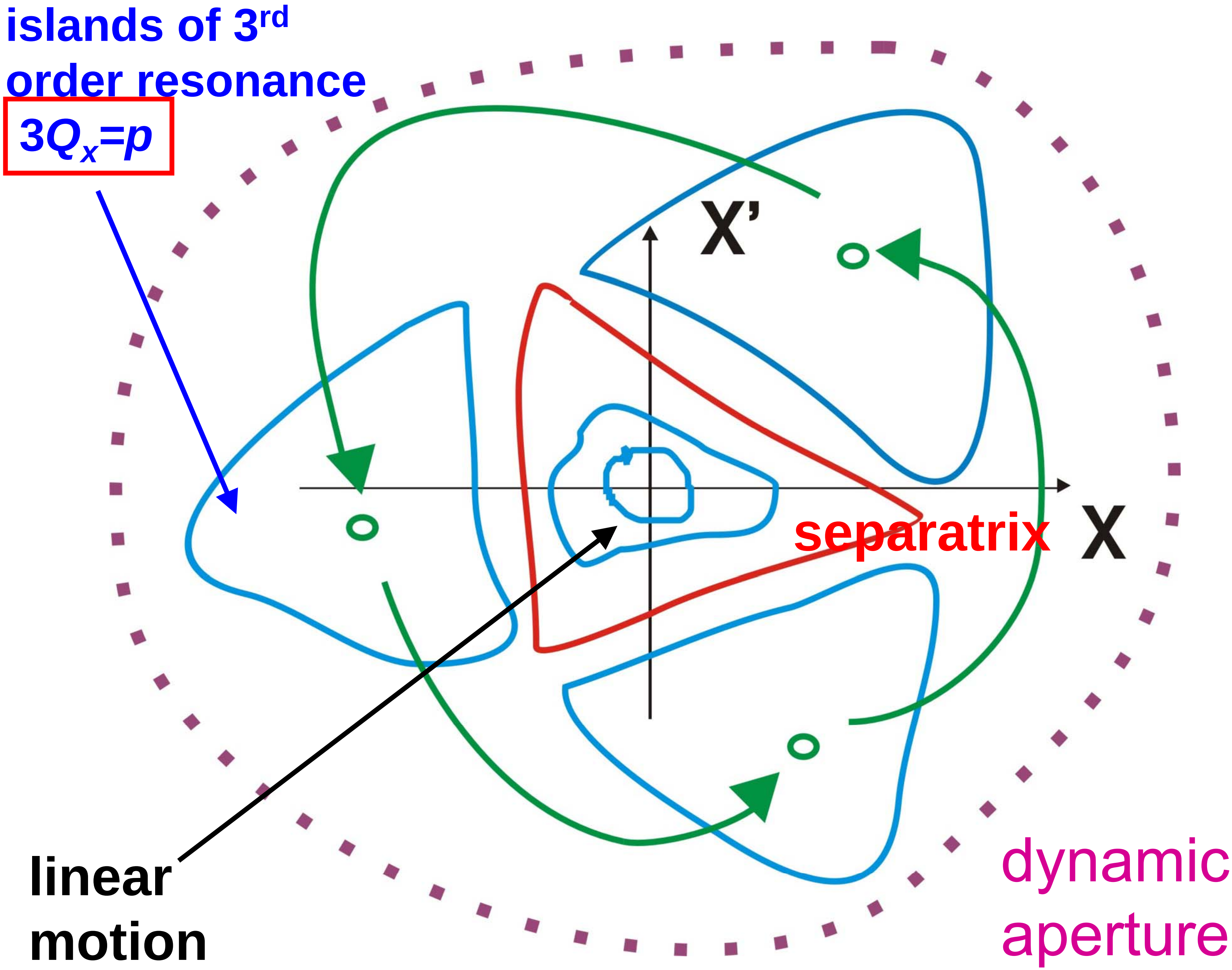
$$kQ_x + lQ_y = p$$

k,*l*,*p* integer

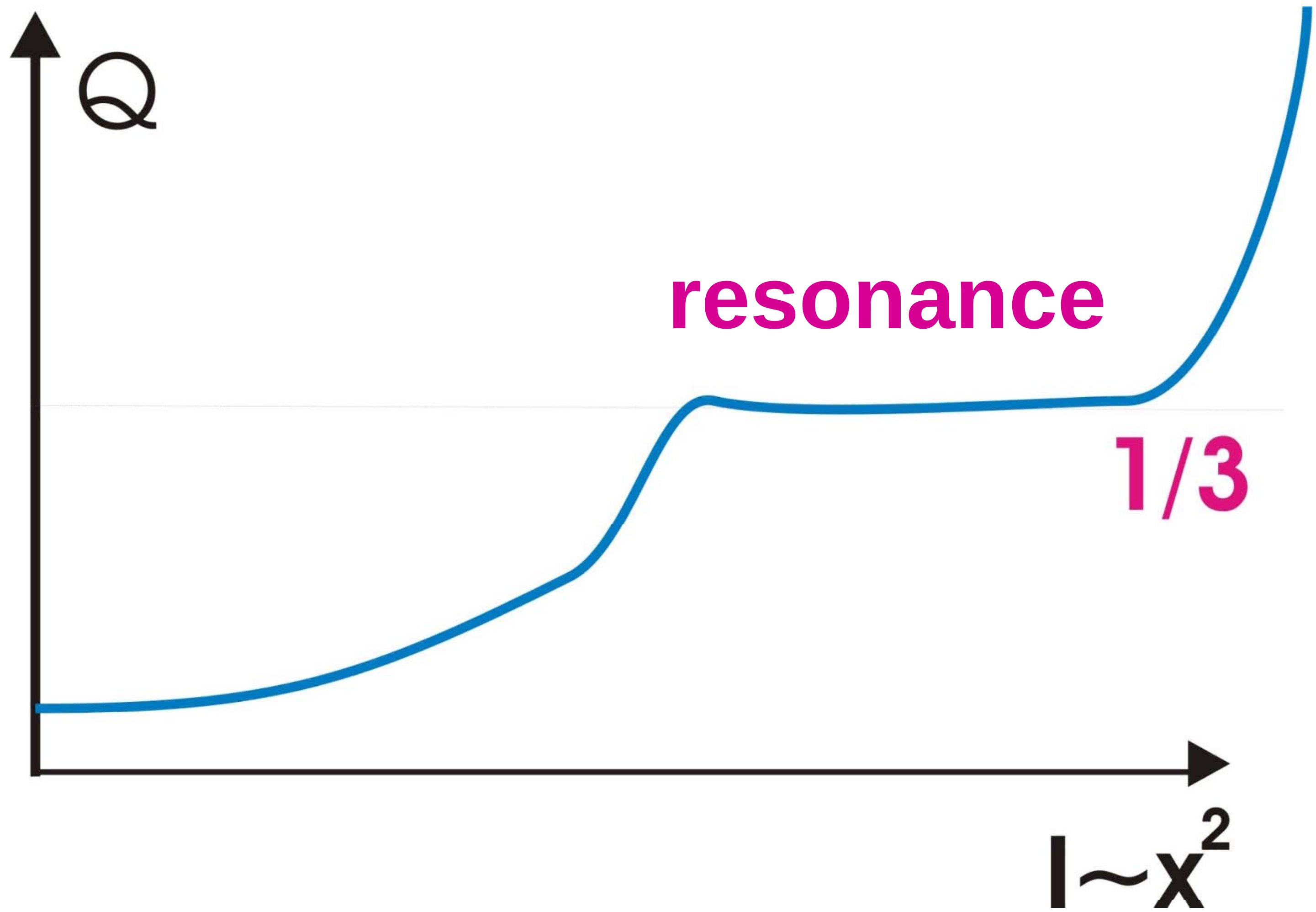
excited by nonlinear fields

→ distortion in phase space

→ chaos, dynamic aperture, particles loss …

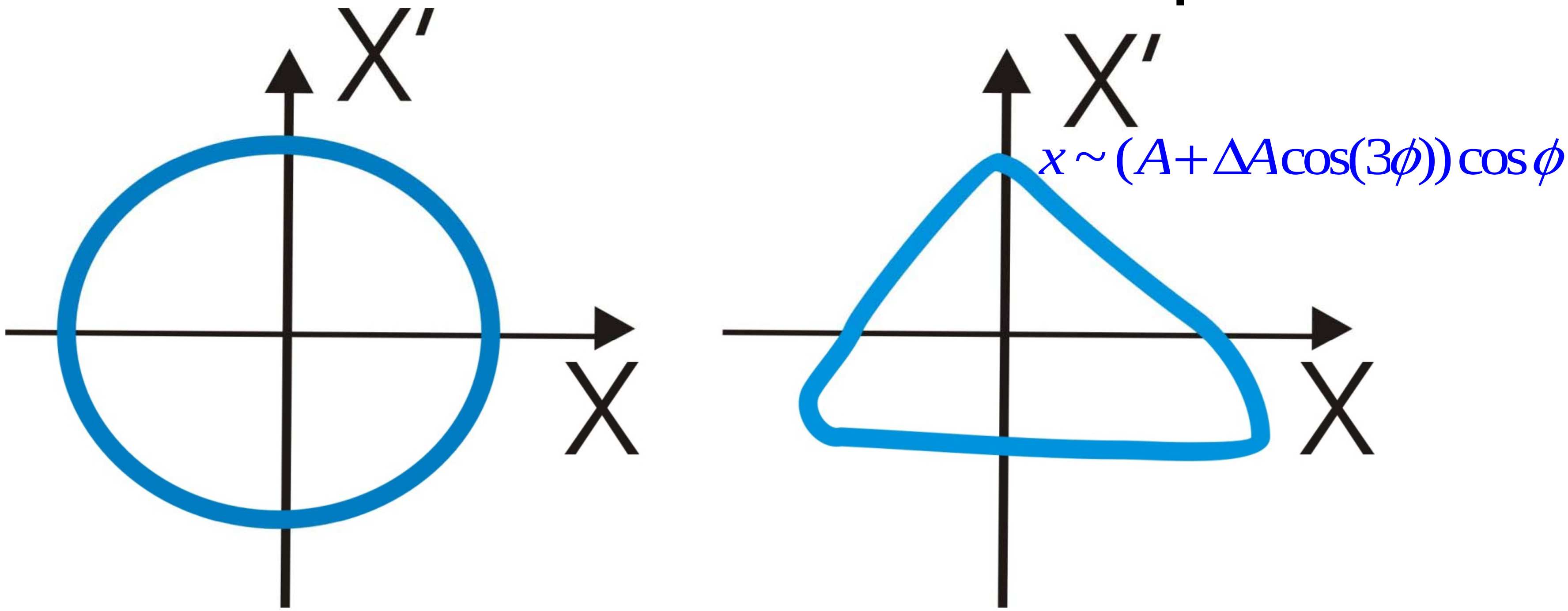

islands of 3rd order resonance
$3Q_x=p$
X'
X
separatrix
linear motion
dynamic aperture

tune vs. amplitude



phase-space distortion

→ additional lines in Fourier spectrum



line at Q_x

lines at Q_x, at $2Q_x$, at $4\ Q_x$

no line at $3\ Q_x$!

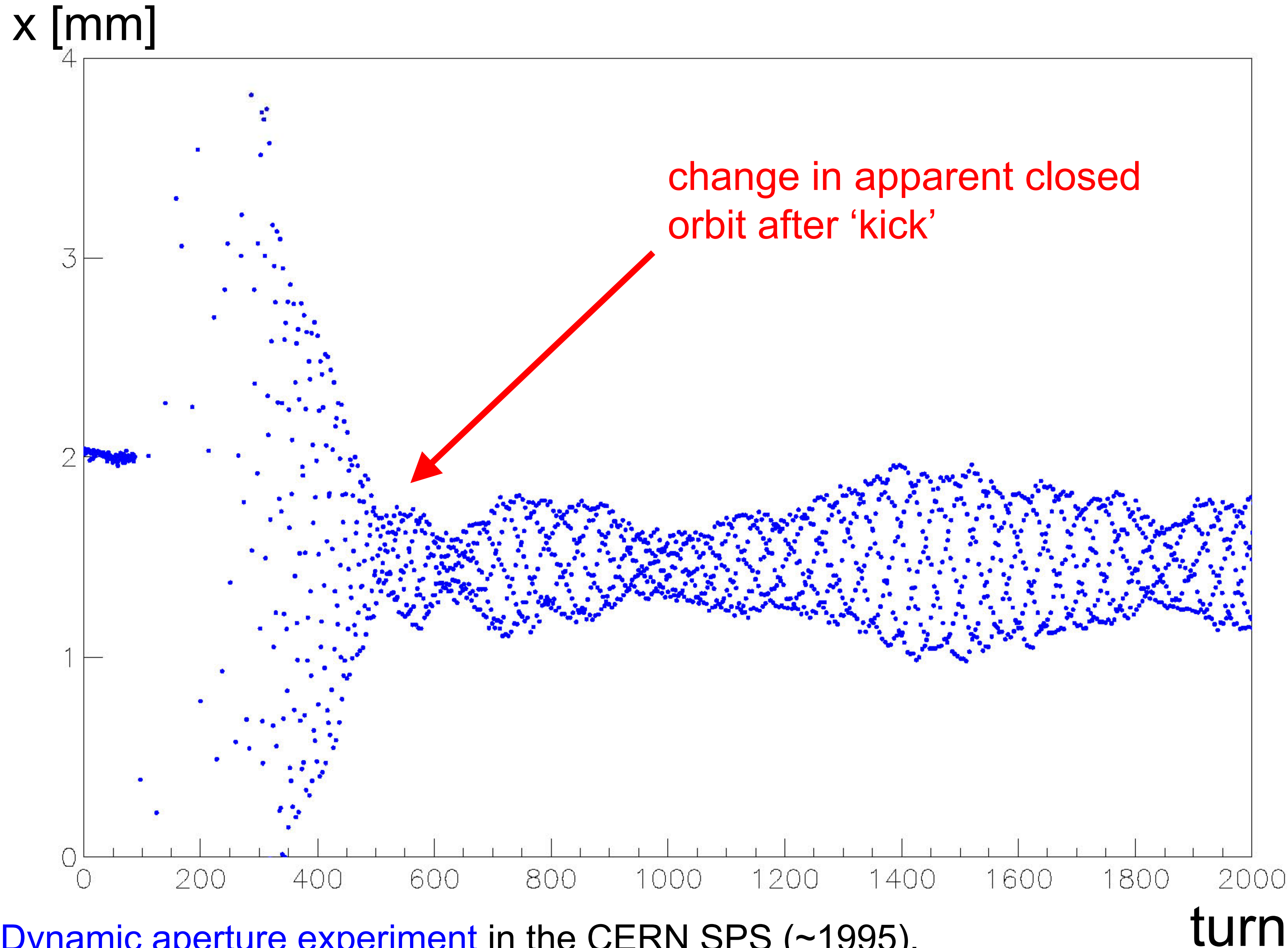
resonance

$$kQ_x + lQ_y = p$$

$(k \pm 1)Q_x + lQ_y$ in x signal

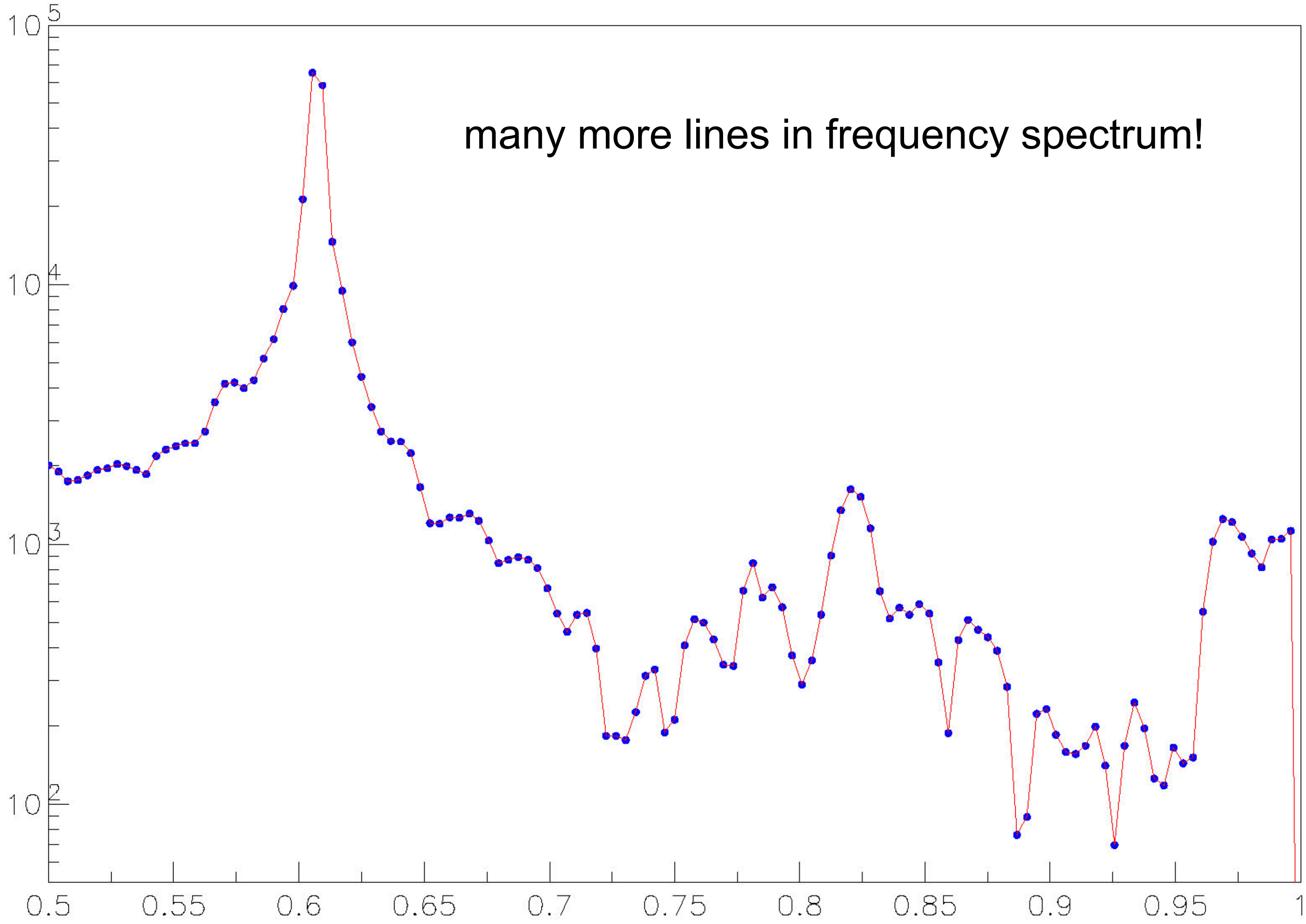
$kQ_x + (l \pm 1)Q_y$ in y signal

reconstruct nonlinearities from FFT → SUSSIX



Dynamic aperture experiment in the CERN SPS (~1995). Beam was kicked at about turn 100. Change in offset related to (1,0) resonance or (0,0) line [Courtesy F. Schmidt, 2000].

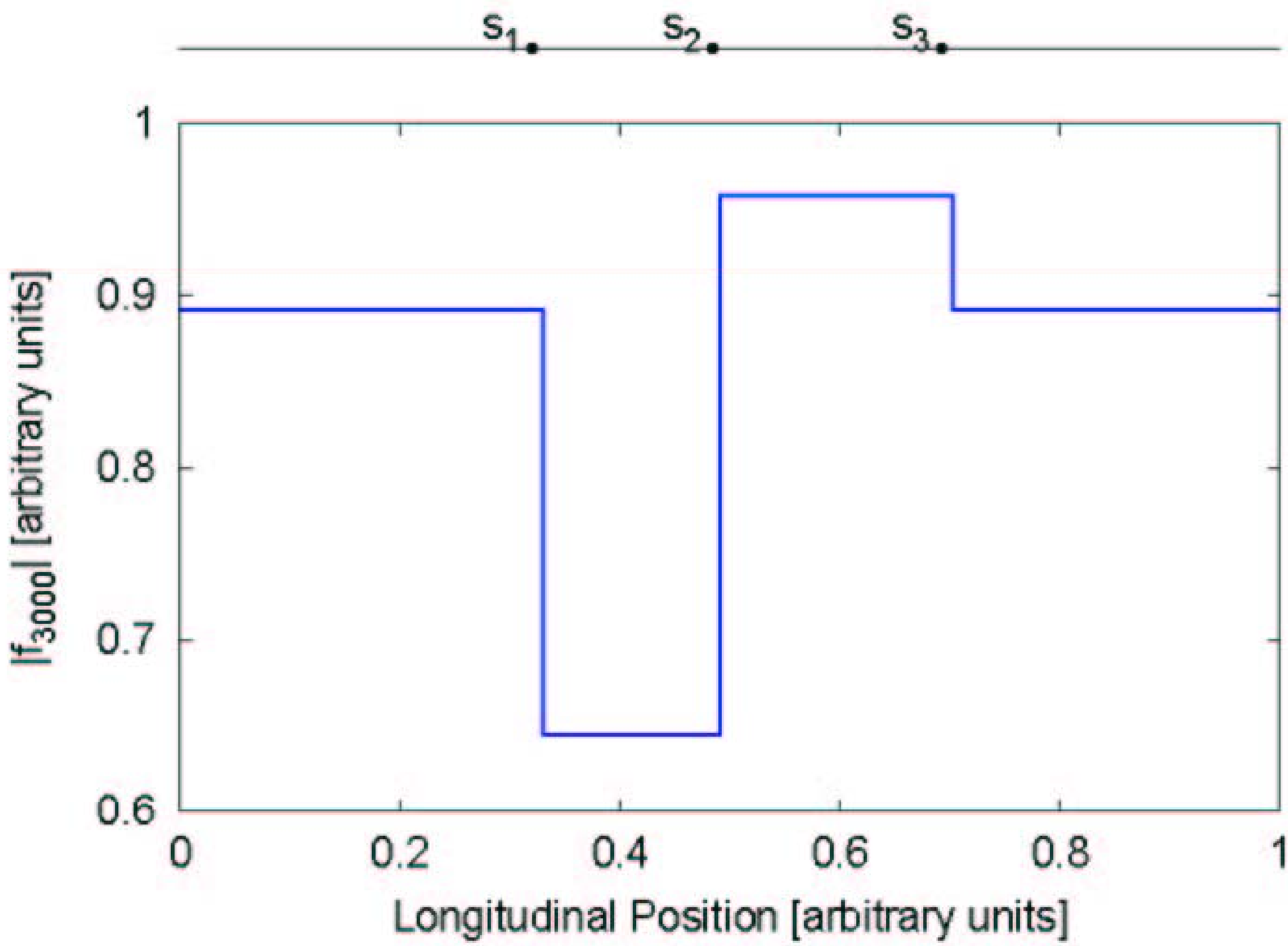
FFT amplitude



fractional tune

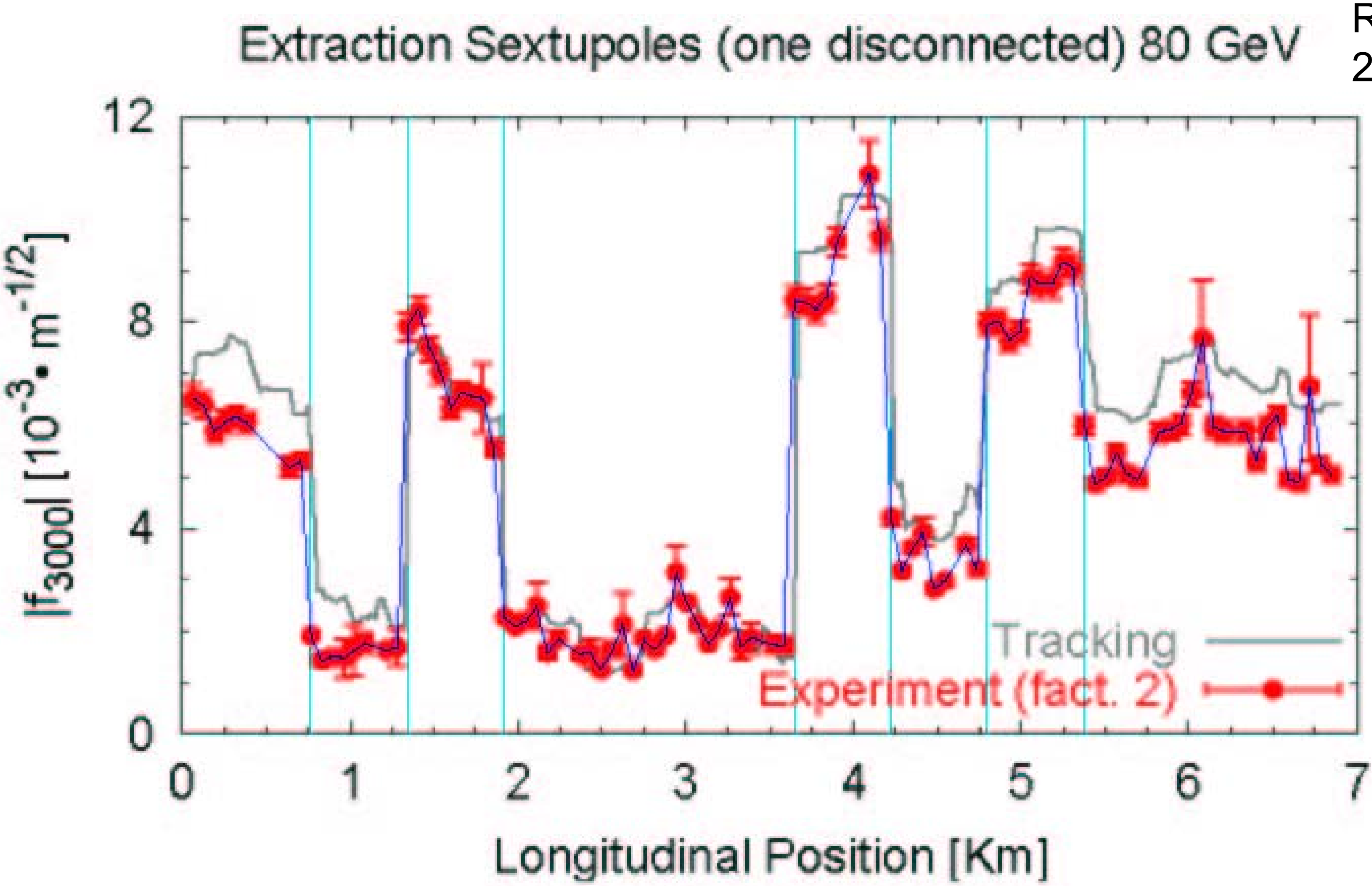
FFT of beam position, evidencing nonlinear resonance lines
[Courtesy F. Schmidt, 2000]

Ph.D. thesis
R. Tomas,
2003



amplitude of (-2,0) line vs. longitudinal position for a hypothetical ring with three sextupoles – **the strength of the nonlinear line depends on *s***

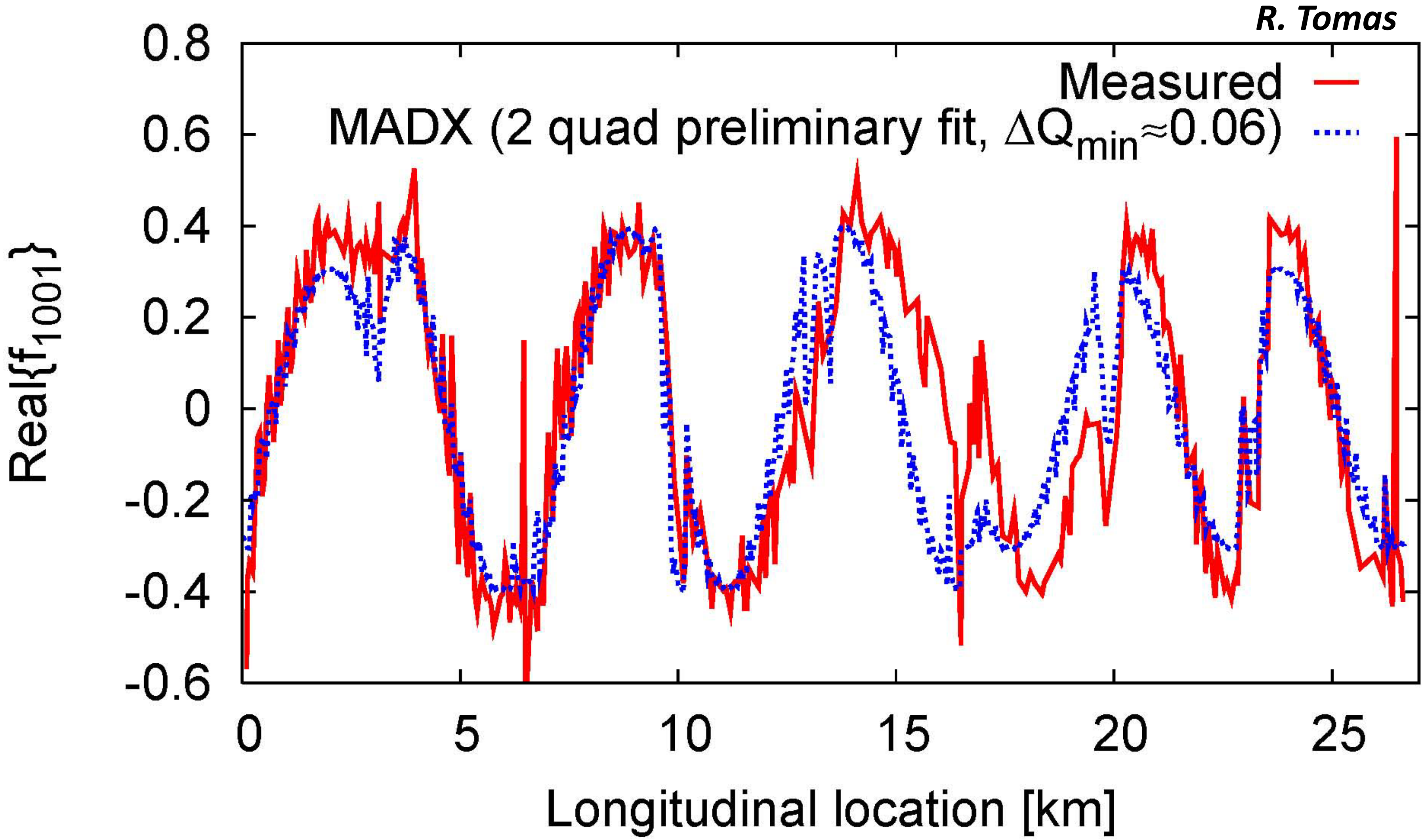
Ph.D. thesis
R. Tomas,
2003



measurement compared with simulation for the SPS;
locations of 7 strong sextupoles are indicated

example - checking the coupling strength

LHC beam commissioning 11 September 2008



beating confirms x-y tune split by 5 integers!

3. tune scans

beam lifetime ↔ *dynamic aperture*

↔ *beam-beam interaction*
sensitive to tunes!

measure $\tau(Q_x, Q_y)$

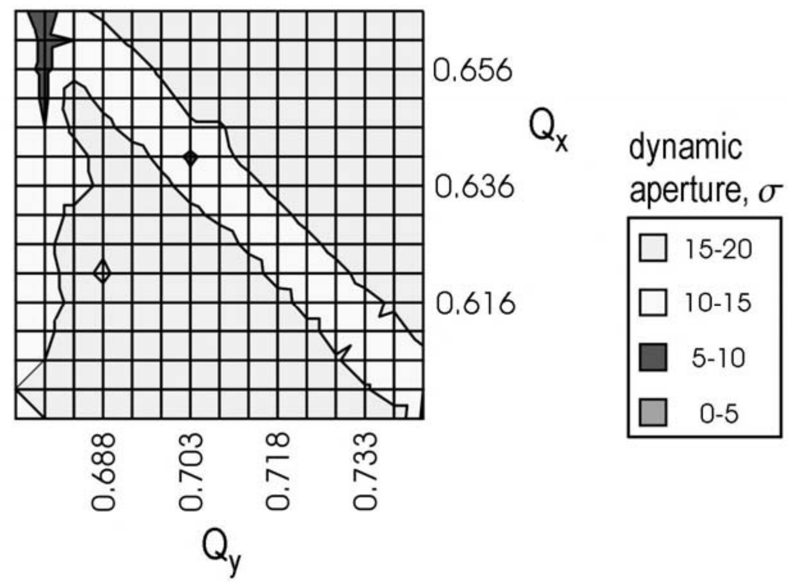
↓

tune diagram

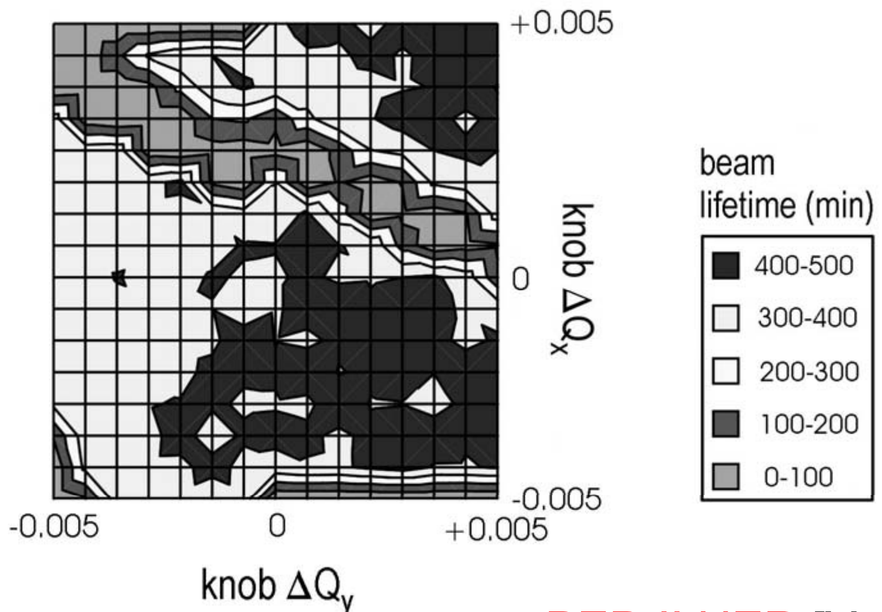
higher-order resonances appear as
stripes with reduced lifetime

- identify (&compensate) harmful resonances
- find optimum working point
- compare with simulations

dynamic aperture simulation; tune scan around 24.709 (x), 23.634 (y)



measured beam lifetime around same working point



different slope attributed to calibration error of tune knobs

PEP-II HER [Y. Cai, 1998]

4. measuring the β function with "K modulation"

vary quadrupole strength ΔK, detect tune change ΔQ, obtain β at quadrupole

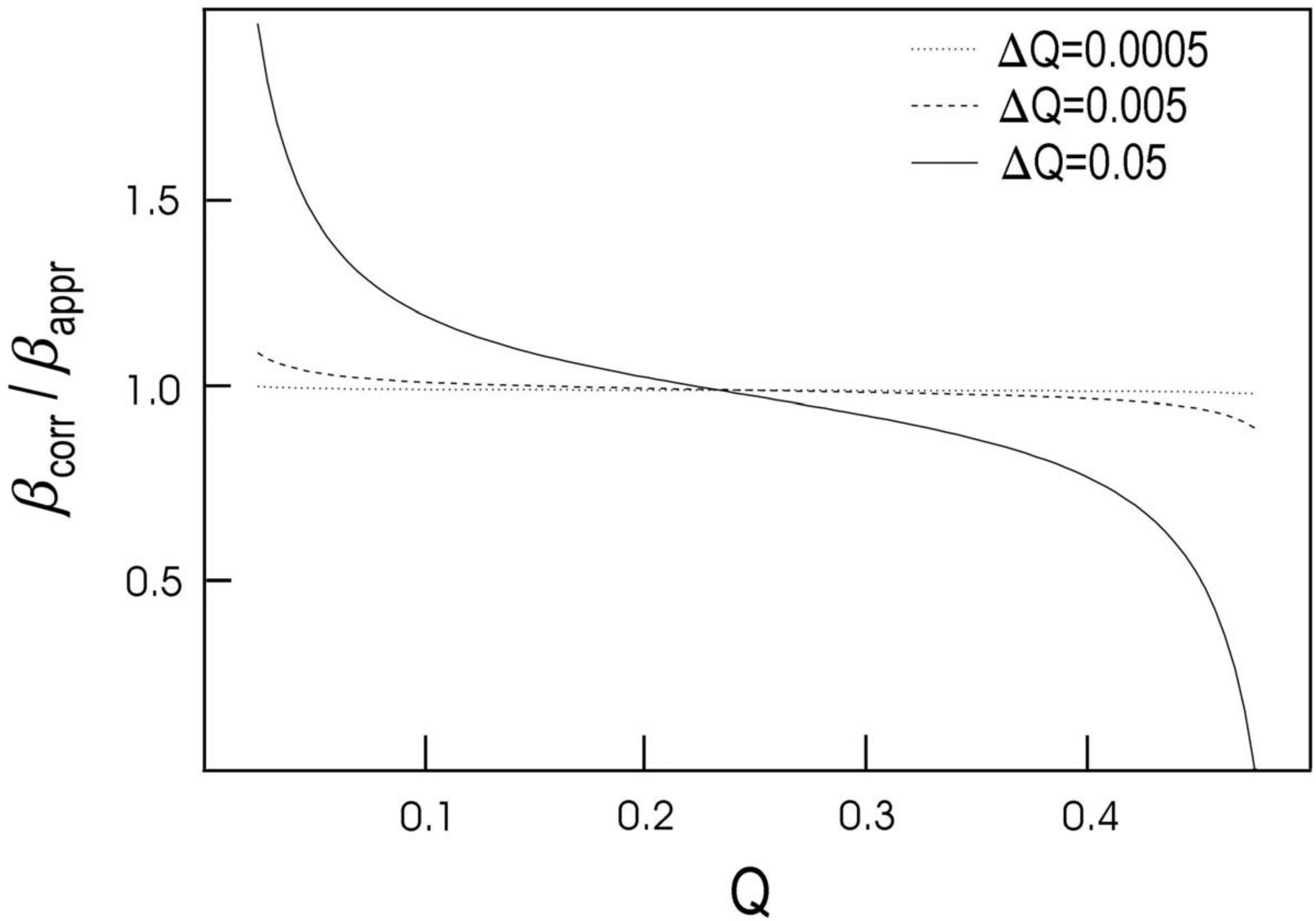
$$\beta = \frac{2}{\Delta K}\left[\cot(2\pi Q_0)\{1-\cos(2\pi \Delta Q)\}+\sin(2\pi \Delta Q)\right]$$

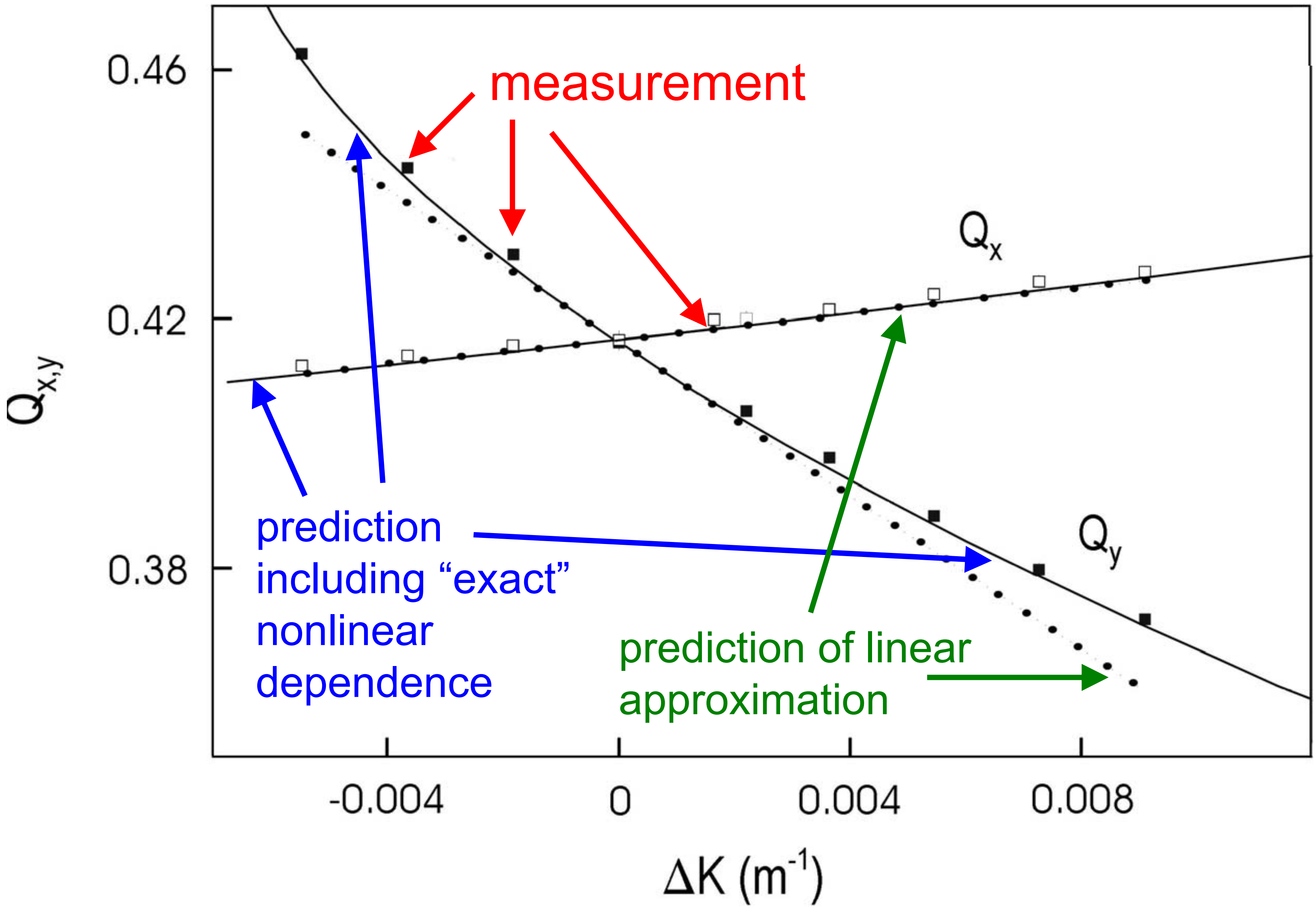
if $\cot(2\pi Q_0) \leq 1$ (tune not near integer or half integer)

and $\Delta Q \ll 1$

$$\beta \approx \frac{4\pi \Delta Q}{\Delta K}$$

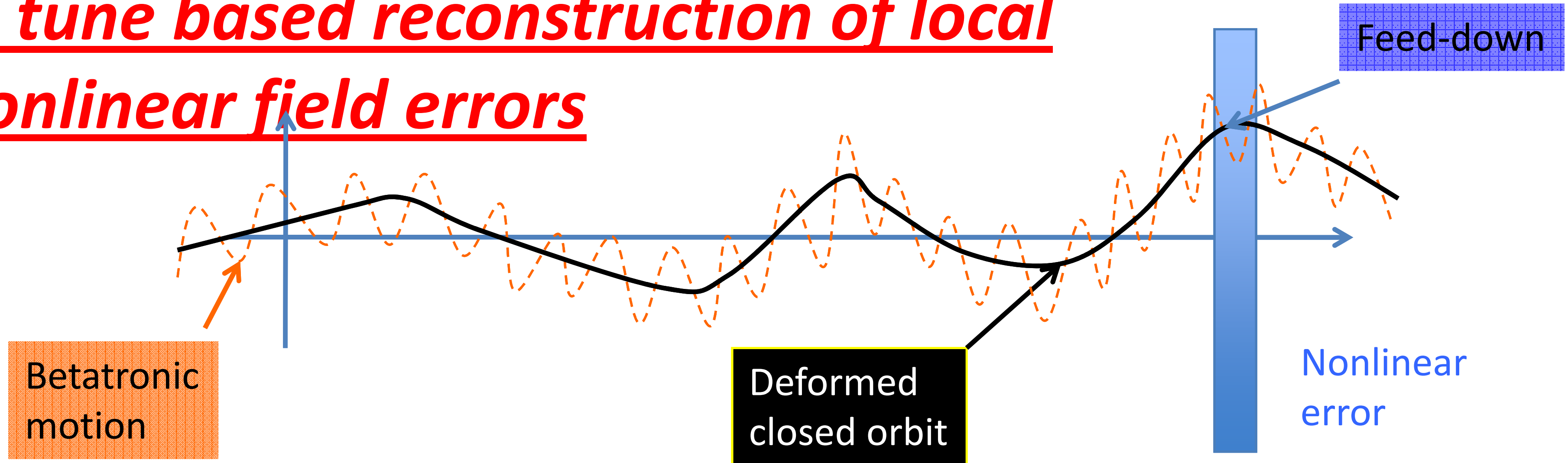
quality of approximation





Optics test in Fermilab Recycler Ring, March 2000.
Betatron tunes vs. strength of quadrupole QT601.

5. tune based reconstruction of local nonlinear field errors



General tune response for sextupolar and octupolar errors when deforming CO with steerers

$$\Delta Q_x = {}_xQ + \sum_{t=1}^{N_t} ({}_xQ_t^x \theta_{xt} + {}_xQ_t^y \theta_{yt}) + \sum_{t,i=1}^{N_t} ({}_xQ_{ti}^{xx} \theta_{xt}\theta_{xi} + {}_xQ_{ti}^{yy} \theta_{yt}\theta_{yi} + {}_xQ_{ti}^{xy} \theta_{xt}\theta_{yi}),$$

The element of the nonlinear tune response matrix are

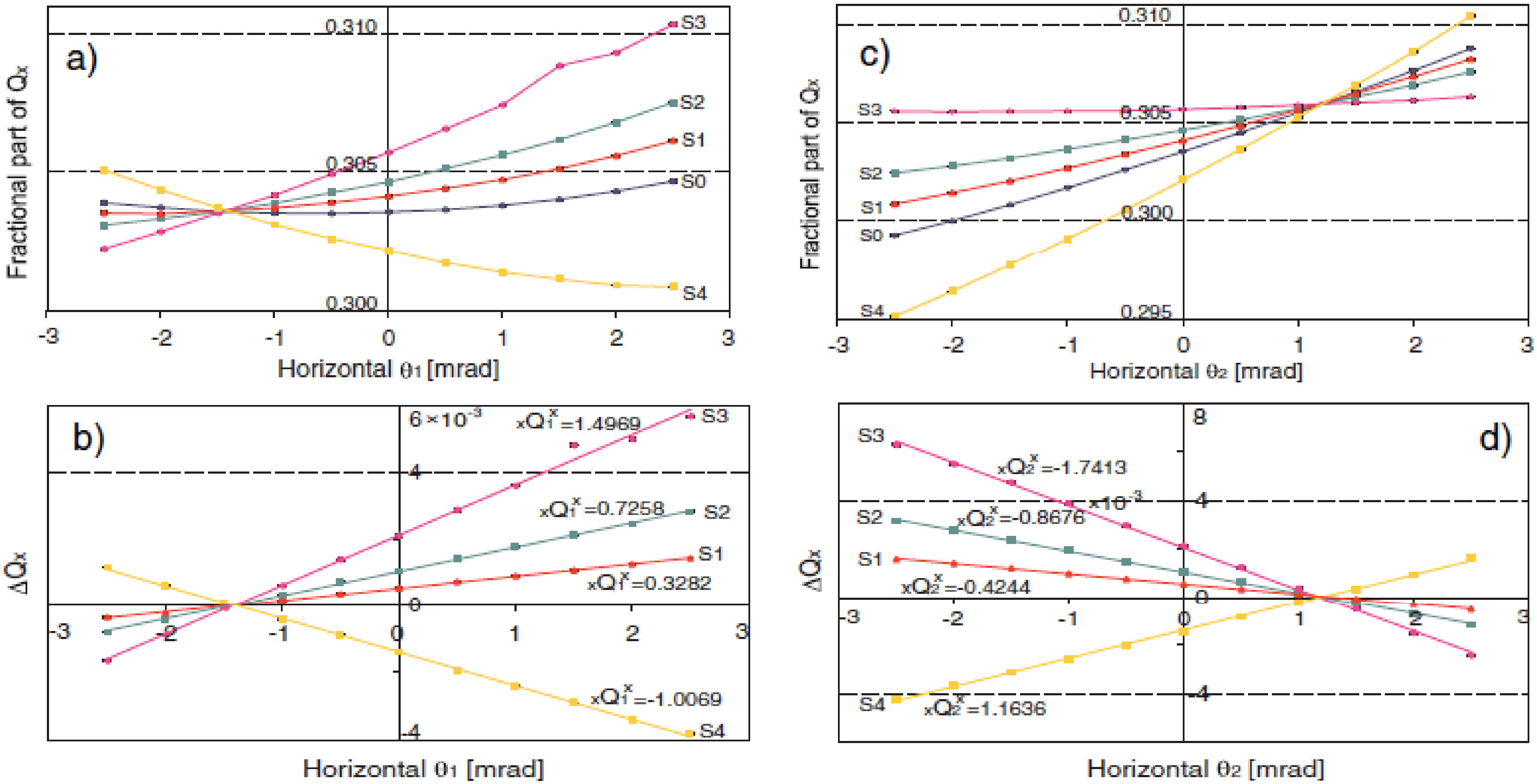
$$\begin{aligned}
{}_xQ &= \sum_{l=1}^{N_l} K_{1l}\tilde{\mathcal{K}}_l^x + \sum_{l,q=1}^{N_l} K_{1l}K_{1q}\tilde{\mathcal{K}}_{lq}^x, \\
{}_xQ_t^x &= \sum_{l=1}^{N_l} K_{2l}\tilde{\mathcal{K}}_l^x M_{lt}^x, \\
{}_xQ_t^y &= -\sum_{l=1}^{N_l} J_{2l}\tilde{\mathcal{K}}_l^x M_{lt}^y, \\
{}_xQ_{ti}^{xx} &= \frac{1}{2}\sum_{l=1}^{N_l} K_{3l}\tilde{\mathcal{K}}_l^x M_{lt}^x M_{li}^x \\
&\quad + \sum_{l,q=1}^{N_l} (J_{2l}J_{2q}\tilde{\mathcal{J}}_{lq}^x + K_{2l}K_{2q}\tilde{\mathcal{K}}_{lq}^x) M_{lt}^x M_{qi}^x, \\
{}_xQ_{ti}^{yy} &= -\frac{1}{2}\sum_{l=1}^{N_l} K_{3l}\tilde{\mathcal{K}}_l^x M_{lt}^y M_{li}^y \\
&\quad + \sum_{l,q=1}^{N_l} (K_{2l}K_{2q}\tilde{\mathcal{J}}_{lq}^x + J_{2l}J_{2q}\tilde{\mathcal{K}}_{lq}^x) M_{lt}^y M_{qi}^y, \\
{}_xQ_{ti}^{xy} &= -\sum_{l=1}^{N_l} J_{3l}\tilde{\mathcal{K}}_l^x M_{lt}^x M_{li}^y + \sum_{l,q=1}^{N_l} [J_{2l}K_{2q}(\tilde{\mathcal{J}}_{lq}^x + \tilde{\mathcal{J}}_{ql}^x) \\
&\quad - J_{2q}K_{2l}(\tilde{\mathcal{K}}_{lq}^x + \tilde{\mathcal{K}}_{ql}^x)] M_{lt}^x M_{qi}^y.
\end{aligned} \tag{20}$$

G.Franchetti,A.Parfenova,I.Hofmann PRTAB 11, 094001 (2008)

G. Franchetti

Tune measurements in SIS

G. Franchetti



From the ``slopes'' $_xQ^x_i$ the probing sextupolar errors can be reconstructed

A. Parfenova, PhD Thesis 2008, Frankfurt University
A.Parenova, G.Franchetti,I.Hofmann
Proc. EPAC08, THPC066, p. 3137

TABLE III. Additional strength applied in the normal sextupoles and retrieved values via NTRM.

Setting	l	ΔK_2 $[m^{-2}]\times 10^{-2}$	Simulation $[m^{-2}]\times 10^{-2}$	Experiment $[m^{-2}]\times 10^{-2}$	Relative error %
S1	1	−2	−1.999	−1.797	−10.5
	2	1	1.001	1.018	−1.8
S2	1	−4	−3.998	−4.133	3.3
	2	2	2.002	1.546	−22.7
S3	1	−8	−7.995	−7.609	−4.9
	2	4	4.007	3.902	−2.5
S4	1	5	5.008	4.971	−0.6
	2	−3	−2.997	−2.739	−8.7

synchrotron tune

LEP model
for synchrotron
tune

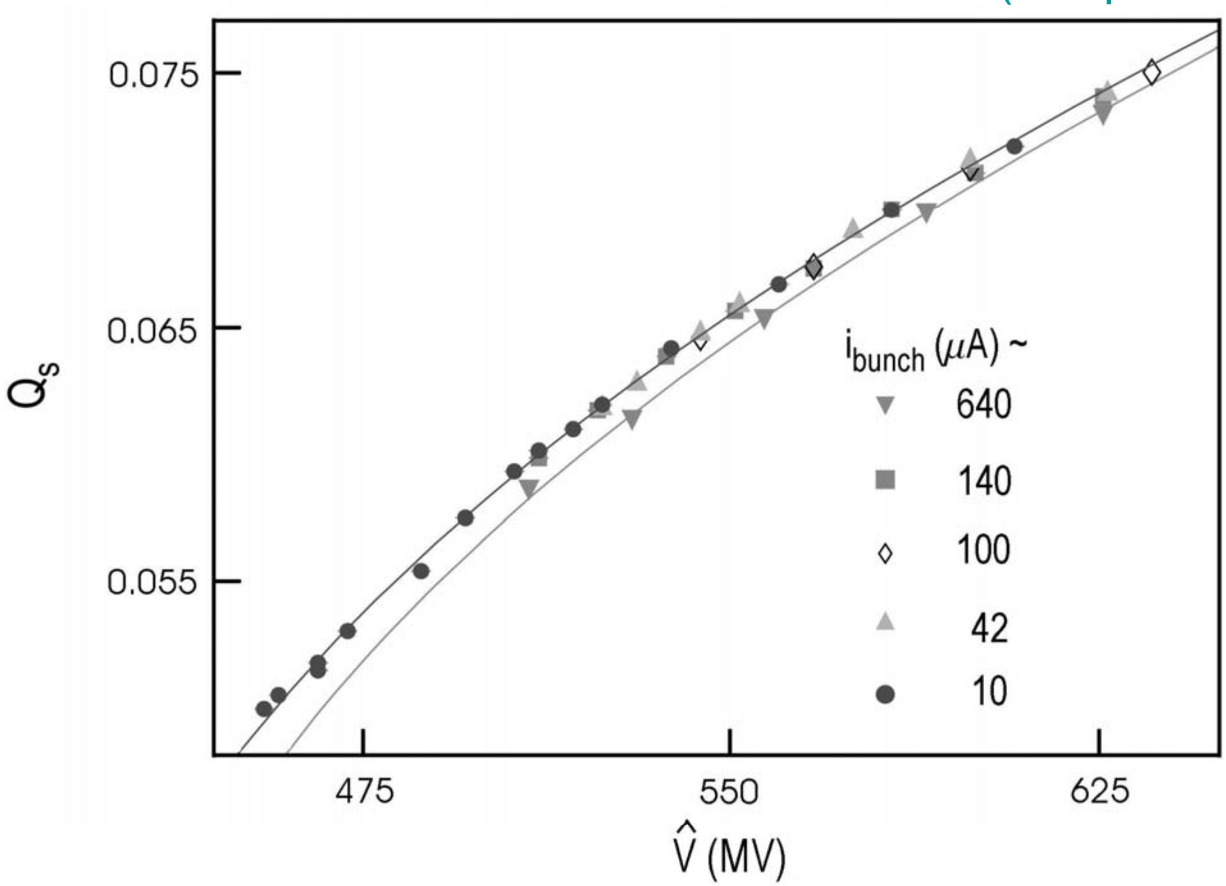
$$Q_s^2 = \frac{\left(\alpha_C - 1/\gamma^2\right)h}{2\pi}\left(\frac{g^2 e^2 \hat{V}^2}{E^2} + \frac{Mg^4\hat{V}^4}{E^2} - \frac{1}{E^2}U^2\right)^{1/2}$$

determined with 10^{-3} precision

voltage calibration

from localization of rf cavities (computed)

energy loss due to SR and impedance

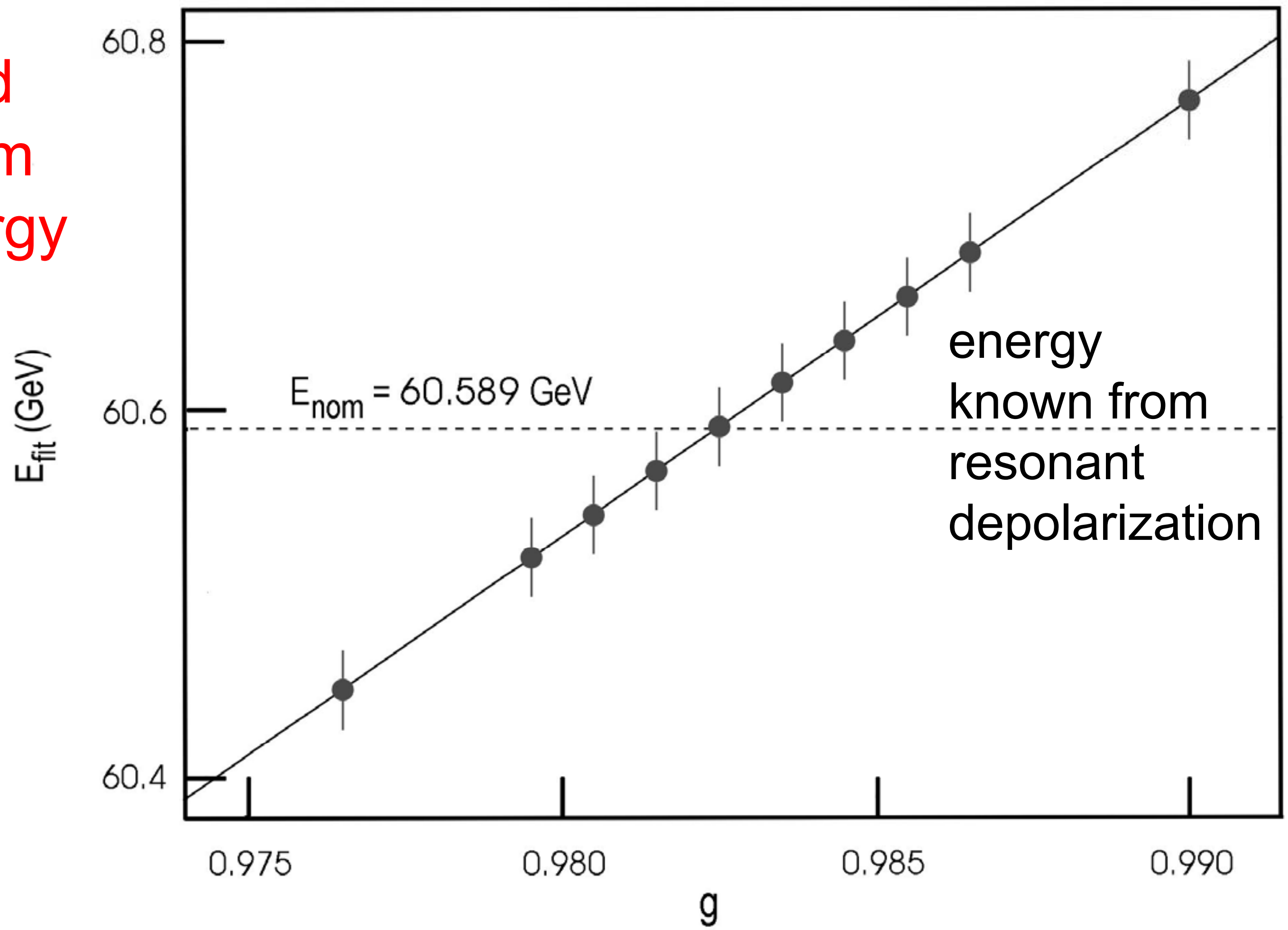


synchrotron tune as a function of total rf voltage in LEP at 60.6 GeV; the two curves are fits to the 640 µA and 10 µA data; tfe difference due to current-dependent parasitic modes is clearly visible

(A.-S. Muller)

if the energy is known at one point, i.e., on a spin resonance, the rf voltage can be calibrated from the Q_s vs V_c curve

fitted beam energy



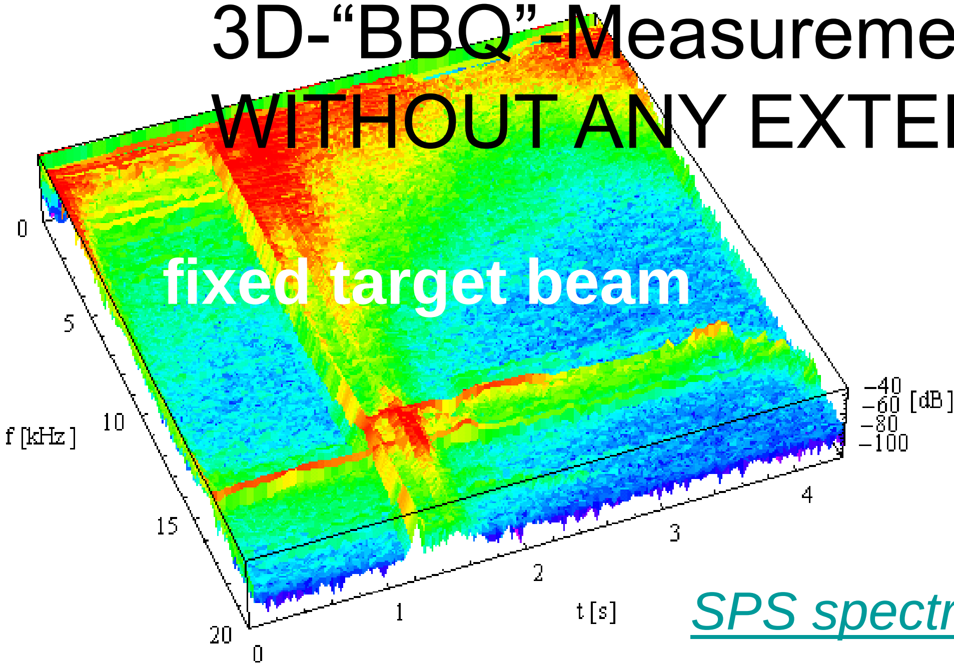
voltage calibration factor *g*

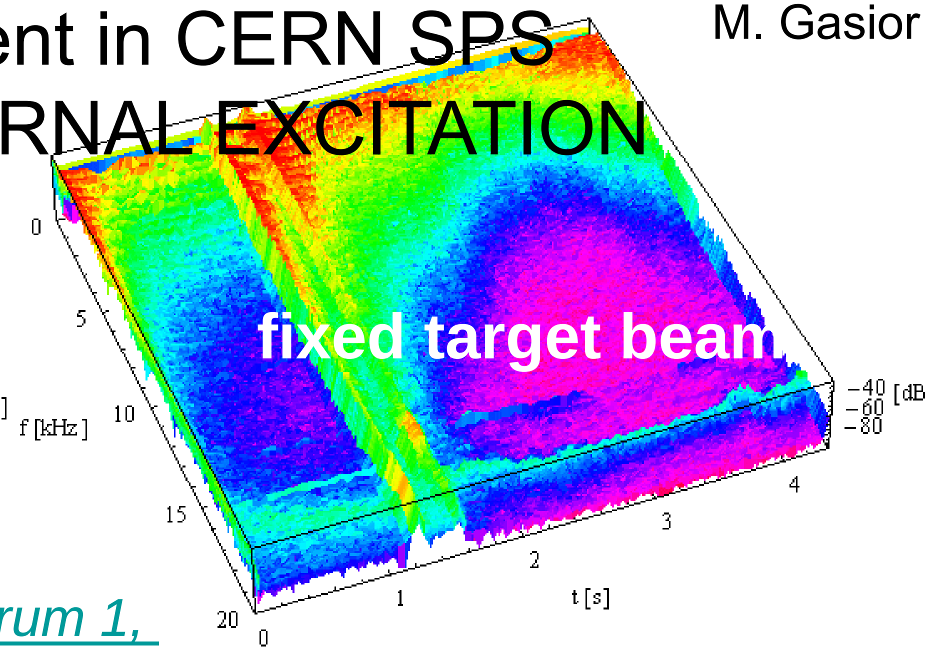
(A.-S. Muller)

display of complex tune signals

3D-“BBQ”-Measurement in CERN SPS WITHOUT ANY EXTERNAL EXCITATION

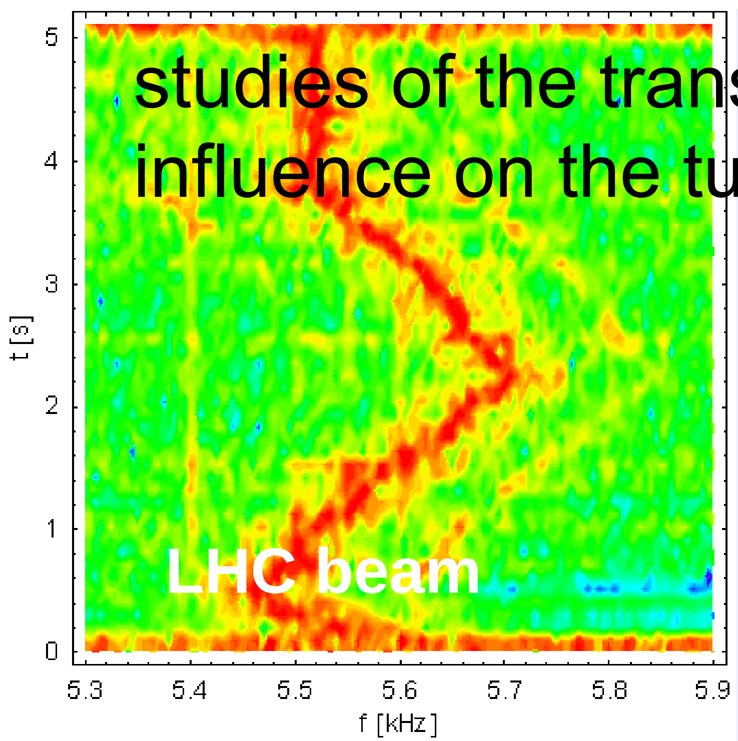
M. Gasior

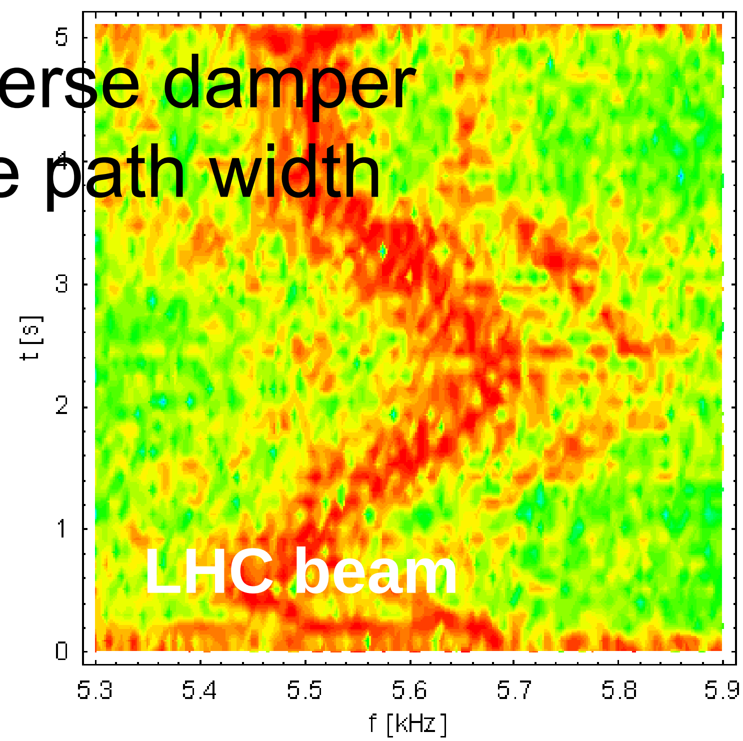




SPS spectrum 1,

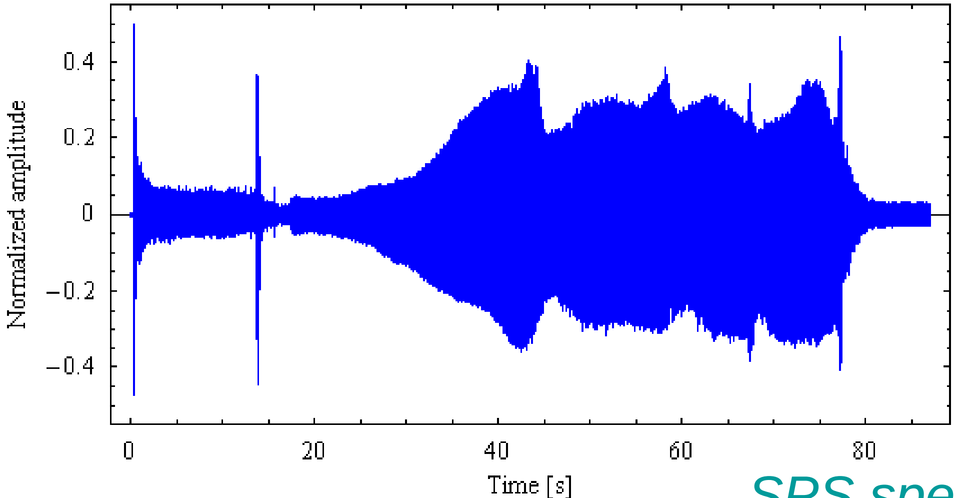
studies of the transverse damper influence on the tune path width

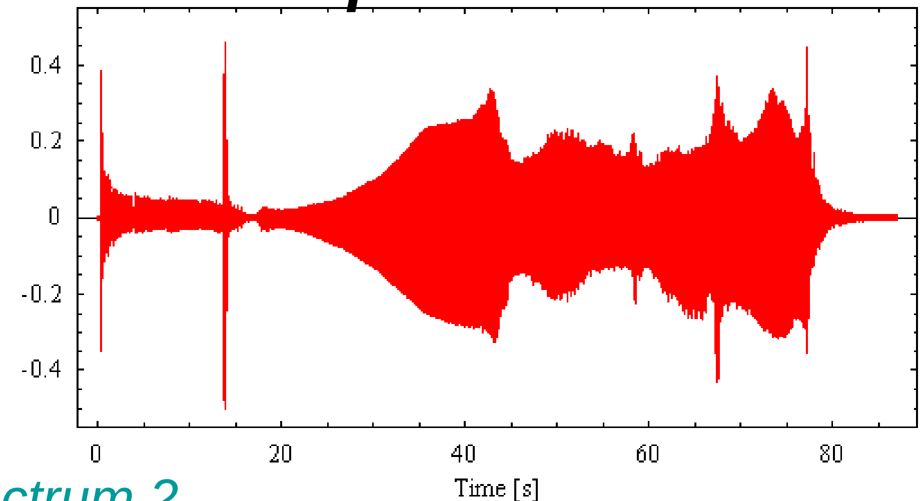




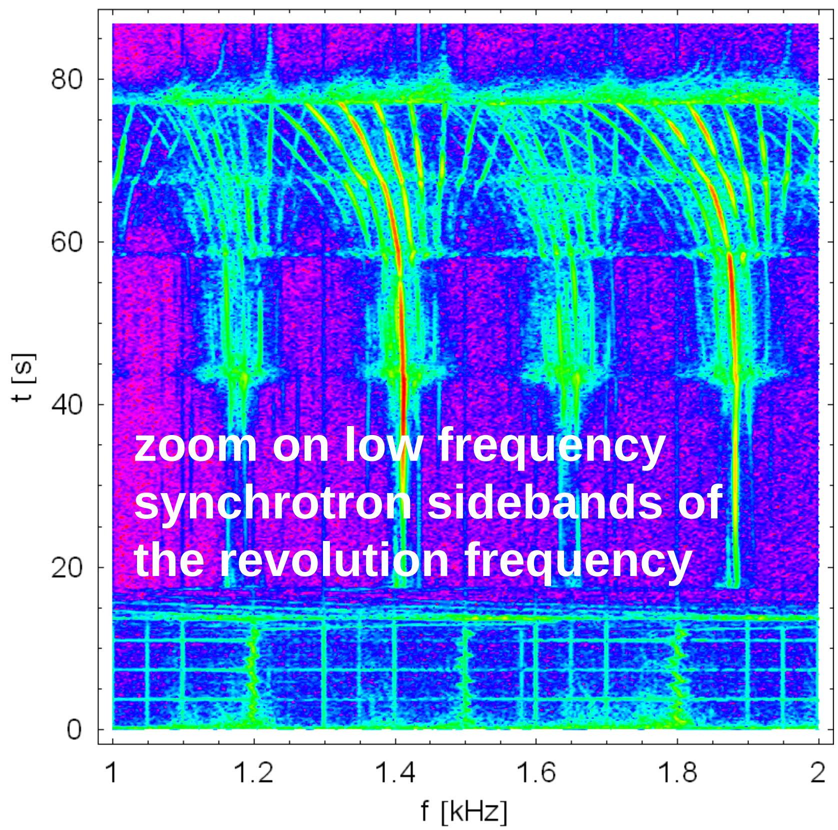
SPS continuous tune measurement example

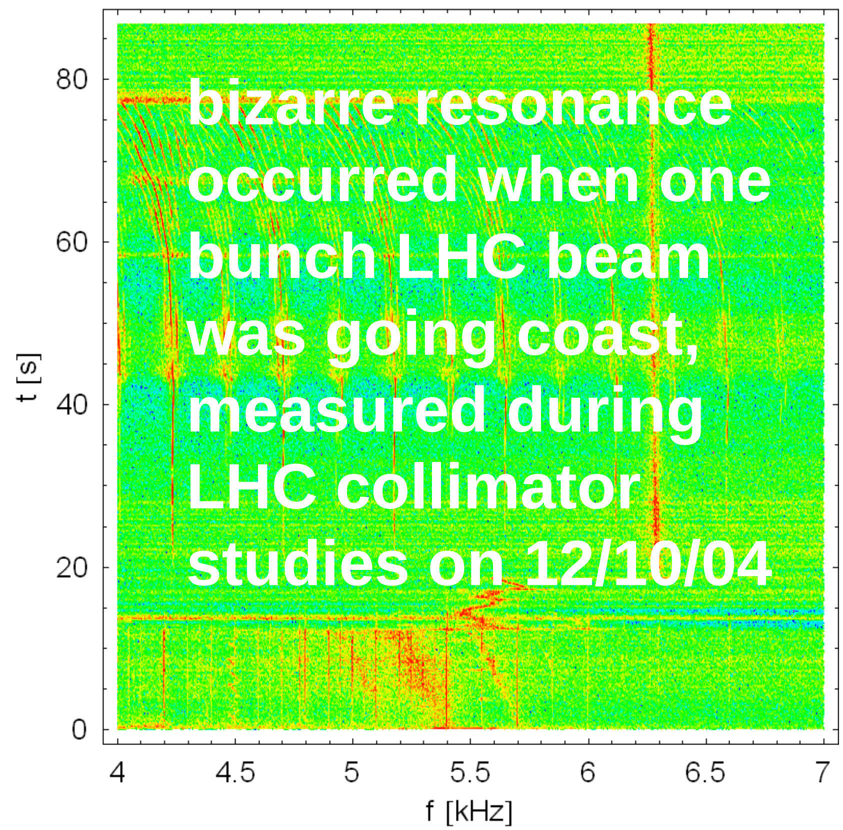
M. Gasior





SPS spectrum 2





continuous BBQ tune diagnostics at the CERN SPS

summary

introduction

tune, coherent & incoherent tune, detectors

integer betatron tune

fractional betatron tune

precision measurement, tune tracking, multiple bunches

modifications of tune signal

damping, filamentation, chromaticity, linear coupling

some “complications”

colliding beams, space charge, measuring incoherent tune

applications

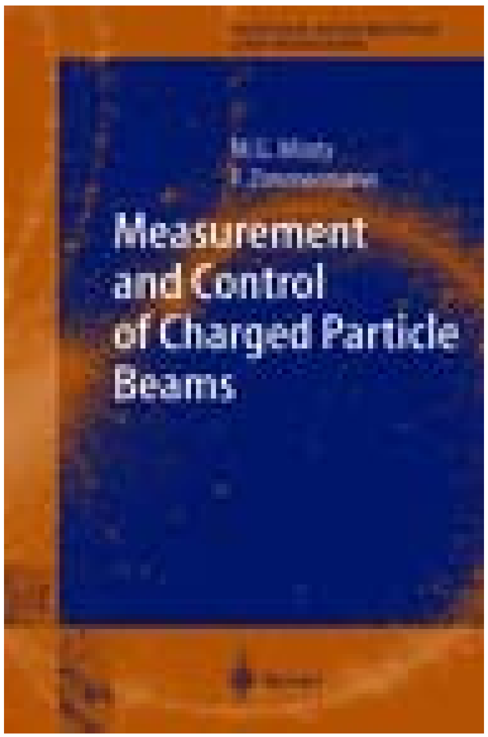
tune shift with amplitude,high-order resonances,tune scans, β function measurement, nonlinear field errors

synchrotron tune

display of complex tune signals

further literature

CARE-HHH-ABI workshop on Schottky, Tune and Chromaticity Diagnostics (with Real-Time Feedback), Chamonix, France, 11-13 December 2007, Proceedings CARE-Conf-08-003-HHH (editor Kay Wittenburg)



more examples and other types of measurements may be found in this book

Measurement and Control of Charged Particle Beams
M.G. Minty, F. Zimmermann,
Springer Verlag, Berlin, N.Y., Tokyo, 2003.