Answers: Signal estimation for broad-band BPM

Peter Forck: Beam Position Monitors

1. Transfer impedance for 1 MΩ: $Z_t = \frac{1}{\beta c C} \cdot \frac{A}{\pi a} = \frac{l}{\beta c C} = 6.7\ \Omega$, with area $A = \pi a l$.
   Sum voltage for $I_{beam} = 1$ A: $\Sigma U = 2 Z_t \cdot I_{beam} = 13.3$ V.

2. Difference voltage for $I_{beam} = 1$ A:
   $x = \frac{1}{S} \cdot \frac{\Delta U}{\Sigma U} \equiv a \cdot \frac{\Delta U}{\Sigma U} \Rightarrow \Delta U(x = 0.1\text{mm}) = \frac{x}{a} \cdot \Sigma U = 10^{-3} \cdot \Sigma U = 13$ mV

3. To measure at faction of 1/1000 at least 10 bits are required.
   The dynamic range is related to the switching granulation of the amplifiers, which have typically 10 dB (corresponding to a factor of 3 in voltage) or 20 dB (corresponding to a factor of 10 in voltage) steps. Therefore, at least 4 or 5 additional bits are required to cover the beam current variation. But be aware, that the effective resolution of an ADC does not equal the least significant bit, but at least two bits worth. This leads to a requirement of a at least a 16 bit ADC, which just the state-of-the-art resolution for a sampling rate of 200 MSa/s.

4. The large dynamic range is required, because the plate voltage is digitized (be aware, that $\Delta U = 10^{-3} \cdot \Sigma U$ in this case). Frequently a hybrid is used to get the difference voltage ΔU in an analog manner. A larger amplification can now be applied to ΔU, typically by 10 to 20 dB. A digital method can the integration of the single plate voltages for each individual bunch, which improve the signal accuracy.

5. For 50 Ω termination and $Z_t(50\Omega) = Z_t(1\text{M}\Omega)/20$:
   Sum voltage for $I_{beam} = 1$ A : $U_\Sigma = 2 Z_t \cdot I_{beam} = 0.67$ V and
   difference voltage $\Delta U(x = 0.1\text{mm}) = 0.67$ mV.

6. The thermal noise voltage is given by:
   $R = 1\text{M}\Omega$: $U_{eff}(R = 1\text{M}\Omega) = \sqrt{4 k_B T R \Delta f} = 1.3$ mV
   $R = 50\Omega$: $U_{eff}(R = 50\Omega) = \sqrt{4 k_B T R \Delta f} = 9.2\ \mu$V.
   The minimum beam current for $S/N = 2$ is:
   $R = 1$ MΩ: $\Delta U = 2 \cdot U_{eff}(R = 1\text{M}\Omega) = 2.7\text{ mV} \Rightarrow I_{beam} = \frac{a}{x} \frac{1}{Z_t(1M\Omega)} \cdot \Delta U = 40$ mA.
   $R = 50\Omega$: $\Delta U = 2 \cdot U_{eff}(R = 50\Omega) = 18.4\ \mu\text{V} \Rightarrow I_{beam} = \frac{a}{x} \frac{1}{Z_t(50\Omega)} \cdot \Delta U = 5.5$ mA.

7. The scaling of Z_t for 50 Ω is given by $\Delta U \propto \omega \propto 1/\sigma_{bunch}$
   $\Rightarrow$ 50 Ω has a better S/N for bunches shorter than 720 ns.

8. The highpass cut-off frequency for the $R = 5$ kΩ case is $f_{cut} = (2\pi RC)^{-1} = 0.3$ MHz, which is below the relevant frequency case (disregarding the effect of baseline shift). Due to the transformation of the signal voltage by transformer ratio of 1:10, ΣU and ΔU are about a factor of 10 lower compared to the high impedance coupling. The thermal noise contribution scales with $U_{eff} \propto \sqrt{R}$ and is therefore a factor of $\sqrt{5\text{ k}\Omega/1\text{ M}\Omega} = 0.07$ lower. This means, that even with lower signal amplitude the S/N is slightly improved compared to the 1 MΩ case.

9. By a narrow-band processing the band-width Δf is much lower (typically 10 kHz for the narrow-band case instead of 100 MHz in the broad-band case) and therefore the thermal noise voltage via $U_{eff} \propto \sqrt{\Delta f}$ (i.e. the noise contribution is about a factor 100 lower). The digital correspondence is the numerical low-pass filtering, having even the advantage that an adaption of the filter constant can more easily be achieved in a digital manner compared to the analog counterpart.