# Signal estimation for broad-band BPM

*Peter Forck: Beam Position Monitors*

The beam position in a proton synchrotron has to be measured by a shoe-box type BPM. This BPM has a length $l = 10$ cm, a distance from the beam center to the plates is $a = 10$ cm and a total capacity of $C = 100$ pF, as used for the discussion during the lecture. The beam has a velocity of $\beta = 50$ % and a bunch length of $\sigma = 100$ ns. For the position read-out assume a linear response $\Delta U/\Sigma U = x/a$ as a function of displacement $x$.

1. Calculate the transfer impedance $Z_t$ of linear-cut plate using a 1 M$\Omega$ termination. Give the sum voltage for a peak value of the beam current of $I_{beam} = 1$ A.

2. What is the difference voltage for a $x = 0.1$ mm displacement in this case?

3. Which ADC resolution is at least required for the detection of such displacement? To allow for variation in the beam current i.e. the related signal strength one need more dynamic range. What could be a matched choice in terms of ADC bits?

4. What analog method could be applied to match the value of $\Delta U$ in a better manner to the ADC input range? What could be a digital method?

5. What are the corresponding values for $\Sigma U$ and $\Delta U$ for a 50 $\Omega$ termination assuming $Z_t(50\Omega) = Z_t(1\text{M}\Omega)/20$, corresponding to a frequency of 1 MHz as shown in the lecture?
   *Remark:* In principle, for the transfer impedance the Fourier-transformation is required (Why?). For a Gaussian function in time with a standard deviation $\sigma_t$, the Fourier-transformation is a Gaussian function (for positive frequencies centered at $f = 0$) having a standard deviation $\sigma_f = (2\pi\sigma_t)^{-1}$.

6. What is the thermal noise at the $R = 50$ $\Omega$ and $R = 1$ M$\Omega$ input-impedance amplifier for a bandwidth of $\Delta f = 100$ MHz? Use the thermal noise voltage $U_{eff} = \sqrt{4k_B T R \Delta f}$ with the temperature $T = 300$ K and Boltzmann-constant $k_B = 1.4 \cdot 10^{-23}$ J/K. What is the minimal beam current for 1 mm displacement for a signal-to-noise ratio $S/N = 2$ using only this thermal noise contribution? (For a real amplifier the noise contribution is at least a factor 2 bigger.)

7. For longer bunches the transfer impedance has a scaling $Z_t \propto \omega$ for the 50 $\Omega$ termination. At which bunch length the position sensitivity of the 1 M$\Omega$ and the 50 $\Omega$ terminations are equal? Which type is a better choice for bunches shorter than that value?

8. What quantitative behavior for $\Sigma U$ and $\Delta U$ as well as $U_{eff}$ do you expected for a 1:10 transformer coupling with an effective termination of $R = 5$ k$\Omega$.

9. How can the position sensitivity be improved and what is the main reason? Discuss an analog and a digital method.