Beam Energy Measurements in Accelerators

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DITANET School

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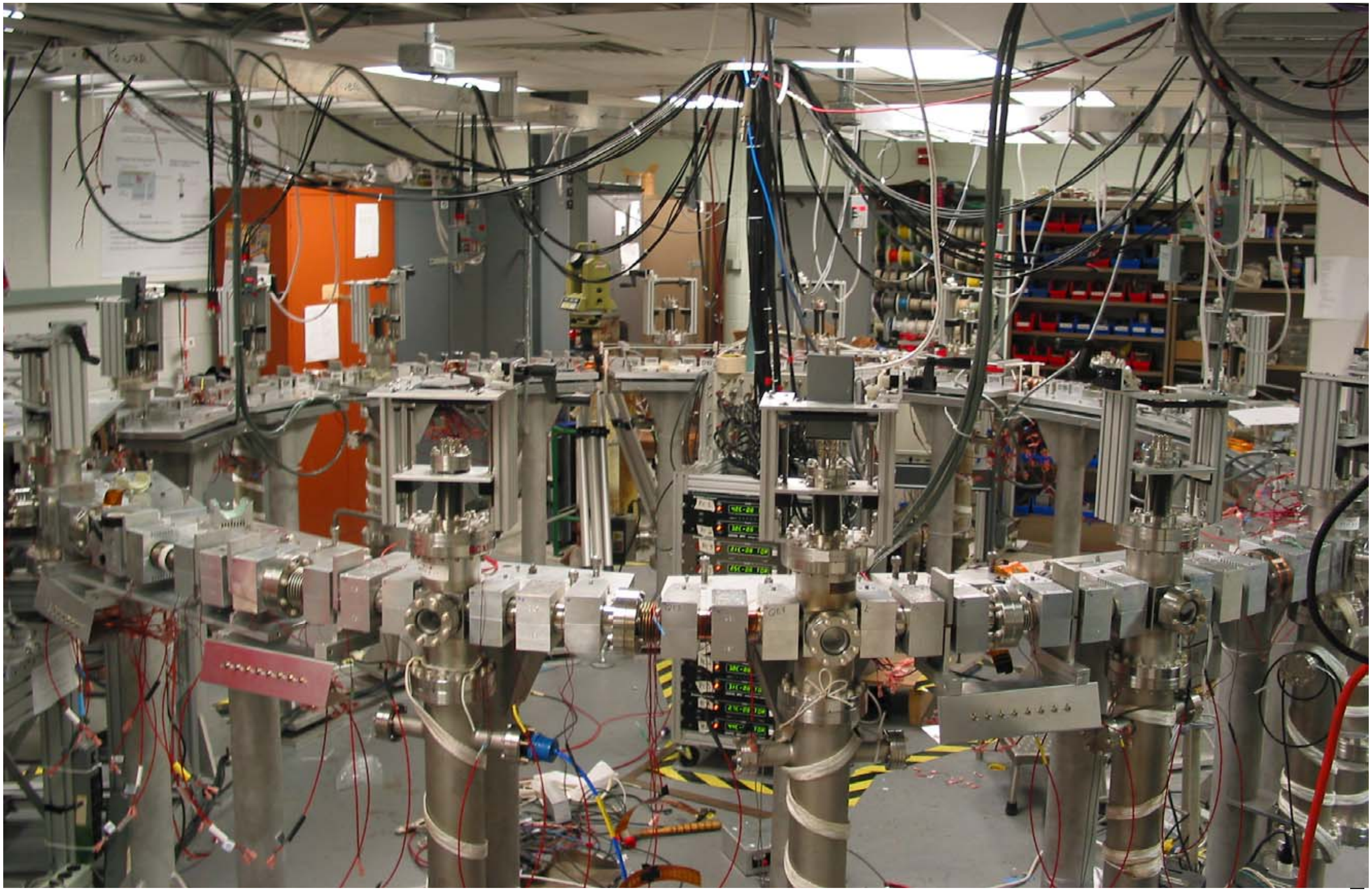
University of MD Electron Ring (UMER)



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Acronyms

AG	Alternating Gradient	ALS	Advanced Light Source (LBNL)
BEC	Bose-Einstein Condensate	ANL	Argonne National Laboratory
BPM	Beam Position Monitor	APS	Advanced Photon Source (ANL)
CRT	Cathode-Ray Tube	CEBA	Continuous Electron Beam Accel.
CTR	Coherent Transition Radiation	DAFNE	(LNF) Lab. Nazionali di Frascati
ERL	Energy Recovery Linac	DESY	Deutsches Elektronen Synchrotron
FEL	Free Electron Laser	ILC*	International Linear Collider
HEP	High Energy Physics	ISR	Intersecting Storage Rings (CERN)
HEDP	High Energy-Density Physics	KEK	“Koh-Ene-Ken”
LINAC	Linear Accelerator	LBNL	Lawrence Berkeley National Lab.
OSR	Optical Synchrotron Radiation	LEP	Large Electron-Positron STR
OTR	Optical Transition Radiation	LHC	Large Hadron Collider
RF	Radio Frequency	MGH	Massachusetts General Hospital
SR	Synchrotron Radiation	RHIC	Relativistic Heavy-Ion Collider
STR	Storage Ring	SLAC	Stanford Linear Accelerator
SNS	Spallation Neutron Source	TJNAF	Thomas Jefferson Natl. Accel. Fac.
		UMER	University of MD Electron Ring

Symbols

α	Momentum Compaction	L	Luminosity
β	Normalized velocity v/c	m	Rest mass
γ	Normalized Energy E/mc^2	ν_H	Horizontal Tune
γ_t	Transition γ	ω_{RF}	RF Angular Frequency
$B\rho$	Magnetic Rigidity	ρ	Local Bending Radius
c	Speed of Light	R	Average Orbit Radius
C	Circumference	σ	Reaction Cross Section
D	Dispersion Function	σ_b	Beam Cross-Section Radius
E_0	Rest Mass Energy	T	Period
E_d	Design Total Energy	T_0	Period of Reference Particle
η	Slip Factor		
$\varepsilon_{X,Y}$	$\text{Emittance}_{\text{Horizontal, Vertical}}$	W	Kinetic Energy
h	Harmonic Number (RF)	W_d	Design Kinetic Energy
I_b	Beam Current		

Introduction: Energy Scale, from BEC to BB

PHENOM./DEVICE	Energy	PARTICLE	Rest Mass Energy, E_0
BEC: Ultra-cold atoms (170 nK)	10^{-11} eV	Neutrino (electron), ν_e	< 2.2 eV
Liquid Helium (4 K)		Electron, e^-	511 keV
Th. Energy @ room temp.	1/40 eV	Muon, μ^-	106 MeV
CRT electron beam	20-50 keV	Proton, p	938 MeV
Electron microscope beam	100 keV	bottom quark, b	4.1- 4.4 GeV
Fermilab Tevatron	1 TeV	Vector boson, Z_0	91.2 GeV
LHC (center-of-mass energy)	14 TeV	top quark, t	169 - 174 GeV
Big Bang	?	Higgs Boson, ?	170 - 285 GeV

Introduction: Why and How

- **Why:**
  - HEP: Energy calibration for identifying particle "resonances"
  - MATERIALS SCIENCE/BIOLOGY: Crystallography
  - MEDICAL: Beam energy critical for light-ion beam therapy
  - MICROELECTRONICS, NANO-SCIENCE: Measure and control *energy spread* for high spatial-resolution electron and ion-beam lithography
  - NUCLEAR ASTROPHYSICS: Detector calibration
- **How:**
  - Beam orbit monitoring (e.g. in dispersive element)
  - Reaction with target with cross section sensitive to energy
  - Radiation monitoring and interferometry (e.g. OSR)
  - Resonant spin depolarization

Introduction: Energy in the LHC

From the LHC website:

ENERGY

How much energy are we talking about?

7 TeV = $7 \cdot 10^{12}$ eV · $1{,}6 \cdot 10^{-19}$ J/eV = $\mathbf{1{,}12 \cdot 10^{-6}}$ **J**

It doesn´t look like a lot of energy

For the **ALICE** experiment, each ion of Pb-208 reaches 1150/2 = 575 TeV.

So, the energy per nucleon is: 575/208 = 2,76 TeV

Let´s calculate the kinetic energy of an insect of 60 mg flying at 20 cm/s:

$E_k = ½\ m \cdot v^2 \Rightarrow E_k = ½\ 6 \cdot 10^{-5} \cdot 0{,}2^2 \sim$ **7 TeV**

That is, in LHC each proton will reach an energy similar to that of an annoying ... **MOSQUITO!**

But we have to keep in mind that this mosquito has 36 thousand trillion nucleons, whereas **the 7 TeV in the LHC will be concentrate in one sole proton.**

Basics: Energy-Momentum in Special Relativity

Energy-momentum 4-vector: $\left(E/c, \vec{p}\right)$, $\quad (E/c)^2 - p^2 = (mc)^2$

Energy :

$$E = \sqrt{p^2c^2 + \left(mc^2\right)^2} \rightarrow pc,$$

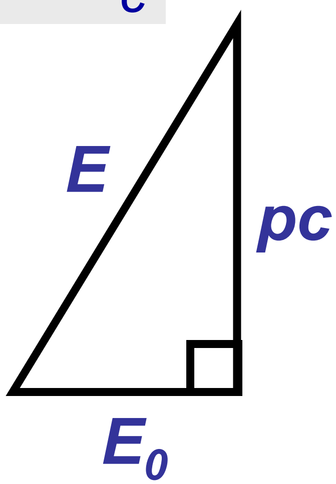
for highly relativistic particles

p^2c^2 ← γmv; $\quad (mc^2)$ ← E_0

$$\gamma = \frac{E}{mc^2}, \quad \gamma = \left(1-\beta^2\right)^{-1/2}, \quad \beta = \frac{v}{c}$$

momentum often given in units of $\frac{GeV}{c}$

Energy Triangle :



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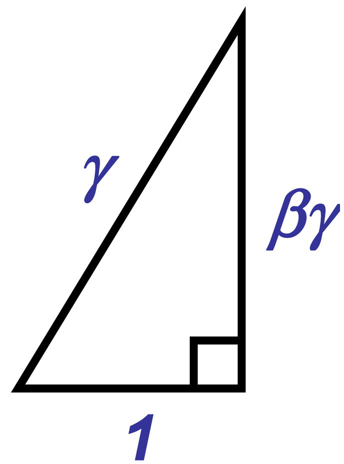


Kinetic Energy:

$$W = E - E_0 \overset{\text{NR}}{\approx} (\beta\gamma)^2 mc^2/2$$

Energy in the Tevatron and UMER

In principle, determine energy from speed of particles:

$$E = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} mc^2, \quad v = \frac{C}{T} = Cf_{RF}.$$

For Tevatron, C_0 = 2π km ± 1 mm (!), f_{RF} = 53… MHz ± 2 Hz (real).

Further,

$$\frac{\Delta E}{E_d} = \beta^2 \gamma^2 \frac{\Delta \beta}{\beta} \quad \text{(use energy□)}$$

For 1 TeV Tevatron,
γ ≈ 1000; β ≈ 1, so

$$\frac{\Delta E}{E_d} \cong 10^6 \frac{10^{-3}\,\text{m}}{2\pi \times 10^3\,\text{m}} \approx 10^{-1}!$$

For 10 keV UMER,
γ ≈ 1,02; β ≈ 0,199, so

$$\frac{\Delta E}{E_d} \cong 0.4 \frac{10^{-3}\,\text{m}}{11{,}52\,\text{m}} \approx 3 \times 10^{-5}$$

Energy in the Tevatron and UMER

But $E_d = E_0 + W_d$. In UMER, really interested in

$$\frac{\Delta W}{W_d} \cong \frac{E_0}{W_d}\frac{\Delta E}{E_d} \approx 50\times 3\times 10^{-5} = 1,5\times 10^{-3}$$

Back to Tevatron - better to use measured field of magnets (772 superconducting dipoles!) and study closed orbit to determine energy:

$$ds = \rho d\theta \rightarrow \oint \frac{ds}{\rho(s)} = 2\pi \quad \text{(closed orbit!)},$$

$$\text{with } \frac{1}{\rho(s)} = \frac{B(s)}{p/q} \quad \text{(next slide)}.$$

For 1987 run at nominal 900 GeV, get **E = 901.5 ± 0.2 GeV.** (Final error actually a bit larger b/c of dipole current regulation.)

From knowledge of E, and f_{RF} = 53104707 ± 2 Hz, get R = 1000,00610 m ± 0,00004 !

Circular Machines: Magnetic Rigidity, Tunes

$\gamma m v^2/\rho = q v B \rightarrow B\rho = p/q$: **Magnetic Rigidity**

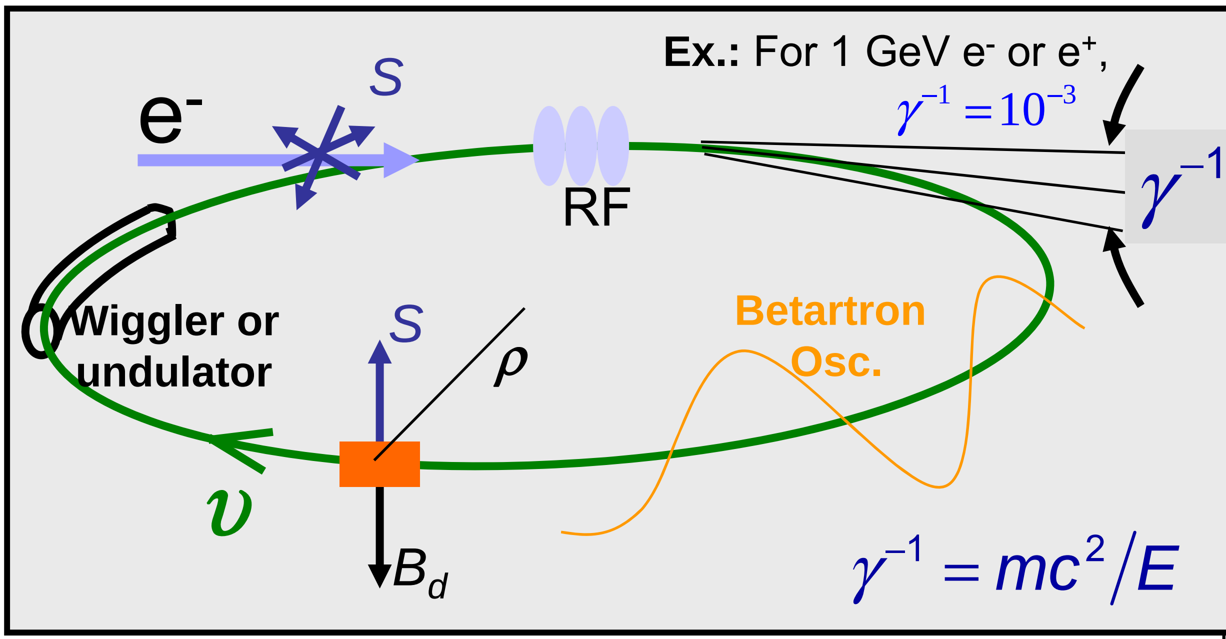
For electrons :

$pc \cong 0.3 B\rho \xrightarrow{\gamma >> 1} E \cong 0.3 B\rho$

(pc, E: GeV; B: Tesla; ρ: m)

Betatron Tune :
 # betatron oscillations per turn

Spin Tune:
 # spin rotations per turn



Circular Machines: Momentum Compaction

Revolution time: $T = C/\beta c = 2\pi R/\beta c$

Effects of "energy error":

$$\frac{\Delta T}{T_0} = \frac{\Delta C}{C_0} - \frac{\Delta\beta}{\beta_0}, \quad \Delta C = C - C_0, \ etc$$

From energy triangle:

$$\frac{\Delta\beta}{\beta_0} = \frac{1}{\gamma^2}\frac{\Delta p}{p_0} = \frac{1}{\beta^2\gamma^2}\frac{\Delta E}{E_d}$$

Momentum compaction α:

$$\frac{\Delta C}{C_0} = \alpha\frac{\Delta p}{p_0}$$

$$\therefore \quad \frac{\Delta T}{T_0} = \left(\alpha - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p_0} = \eta\frac{\Delta p}{p_0};$$

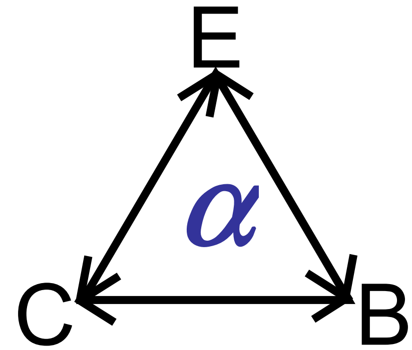
$$\eta = \alpha - \frac{1}{\gamma^2} = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2},$$

$\gamma_t mc^2$: transition energy

Can show: $\alpha \approx \left(radial\ betatron\ tune\right)^{-2}$, often very small

Circular Machines: Momentum Compaction

$$\frac{\Delta T}{T_0} = \left(\alpha - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p_0} = \frac{\eta}{\beta^2}\frac{\Delta E}{E_d}$$



E: Energy;
C: Orbit / layout;
B: Magnets

- With **bunched beams (RF),** adjusting ω_{RF}, orbit monitoring, magnet measurement/calibration help achieve design parameters. See Prob. 7.5, Minty, Zimmermann.

$$\frac{\Delta T}{T_0} = -\frac{\Delta \omega_{RF}}{\omega_{RF}}, \qquad \oint \frac{ds}{\rho(s)} = 2\pi, \text{ with } \frac{1}{\rho(s)} = \frac{B(s)}{p/q}$$

- With **coasting beams (no RF)**, determine α by measuring the *revolution frequency* as a function of momentum (e.g. UMER) or equivalent dipole field (e.g. ISR).

$$\frac{\Delta p}{p_0} = -\frac{\Delta B}{B_0}$$

Examples: Momentum Compaction

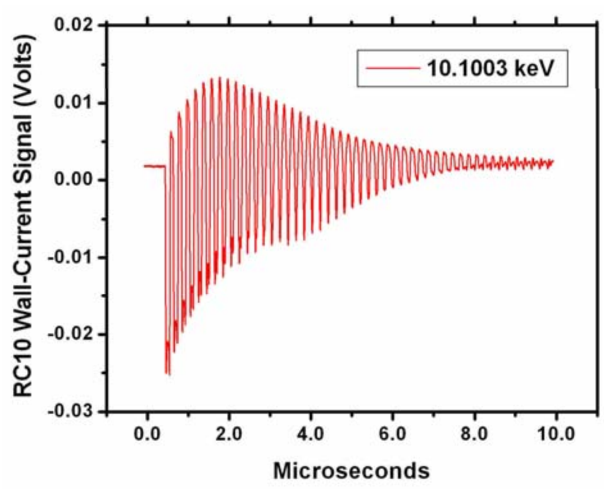
ISR (1974-1983): luminosity, Schottky scans, stochastic cooling, vacuum…

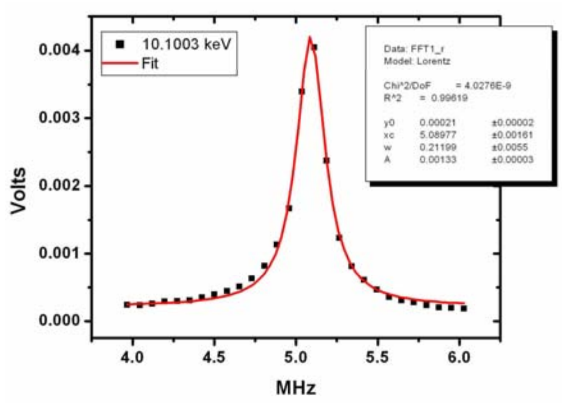
Mom. Comp. α, and slip factor η :

$$\frac{\Delta T}{T_0} = \eta \frac{\Delta p}{p_0}, \eta = \alpha - \frac{1}{\gamma^2}$$

No RF in **ISR**, so get revolution frequency data from noise (!) through Schottky signals as function of momentum.

No RF in **UMER** either, but few turns. Get revolution frequency from wall-current monitor. Increased frequency resolution by Lorentzian fit to main FFT peak.





UMER

Get $\alpha = 0.03 \pm 0.01$.
Momentum scan is $\Delta p/p_0 = 0.1$,
so $\Delta R/R_0 = 0.3\%$, or $\Delta R = 5.5$ mm
Max. meas. (BPM) $\Delta R \approx 8$ mm

Synchrotron Radiation

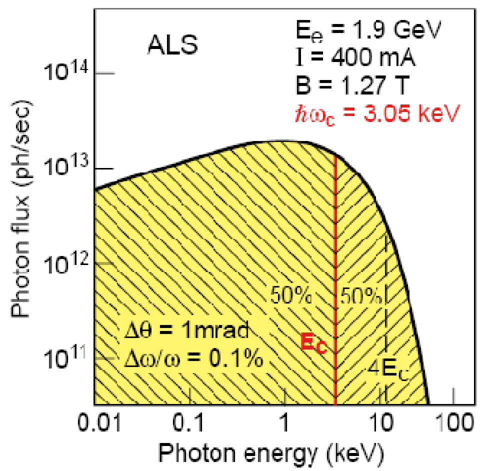
Energy lost to SR per turn :

$$\frac{\Delta E}{\text{turn}}[\text{keV}] = 88.5 \times \frac{E^4}{R} = 26.6 \times E^3 B$$

GeV (E), m (R), Tesla (B)

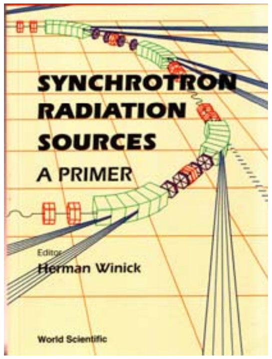
1st equality applicable to protons if multiplied by $(m_e/m_p)^4 = 0.88\text{x}10^{-13}$!

For bending magnet and wiggler sources get ***continuous spectrum***. Half the power is radiated above and half is radiated below a ***critical photon energy*** E_C:



$$E_C[\text{keV}] = 0.665 \times B \times E^2$$

Tesla (B), GeV (E)



Energy Measurement in LINACS

Bending:

$d\theta = ds/\rho(s)$, so

$$\theta_{tot} = \theta_2 - \theta_1 = \int_{s_1}^{s_2} \frac{ds}{\rho(s)},$$

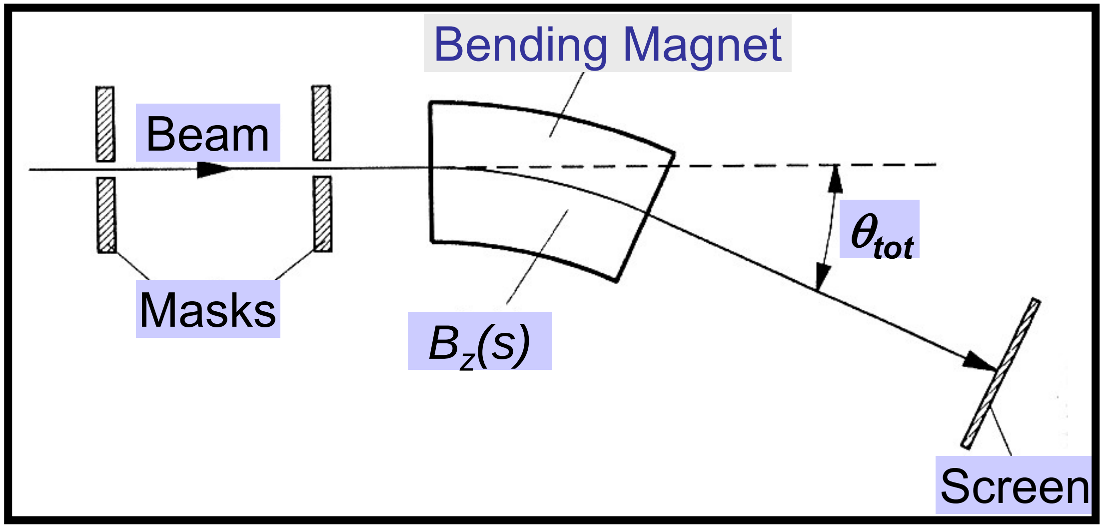
with $\frac{1}{\rho(s)} = \frac{B_Z(s)}{p/q}$.

so $E \propto \frac{1}{\theta_{tot}} \int_{s_1}^{s_2} B_Z(s)ds$

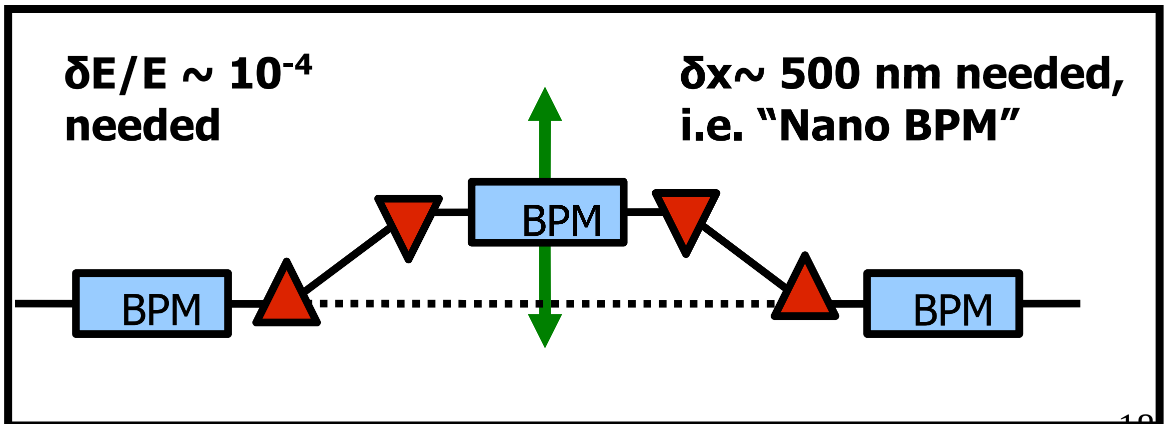
For electrons,

$$\theta_{tot} \cong \frac{3.0}{pc} \int_{s_1}^{s_2} B_Z(s)ds$$

Basic Energy Spectrometer



BPM-Based Energy Spectrometer For ILC (Maiheu, Boogert, *etal*)



Absolute Energy Measurements

DESCRIPTION / REFERENCE	SOME DETAILS	COMMENTS	machine
Two types of particles on same central orbit (e^+, p) obtained by adjusting RF REF: USPAS98 Notes	$\beta_p = \frac{\omega_{RF-CP}}{\omega_{RF-Ce^+}} \frac{h_e}{h_p},$ Error: 10^{-4}	• Circular machines only • For not very relativistic only (e.g. p = 20 GeV/c)	**LEP**
Use **SR** from bending dipoles with very different fields to get different λ_C. REF: Karabekov etal EPAC96	Determine main dipole bending angle from study of K-edge absorption Error: 10^{-5} – 10^{-4}	Linear or circular accelerators (with extraction line)	**CEBAF** **SLAC**
Elastic scattering off gaseous target (e.g. helium) REF. J.P. Burq, NIM (1980)	Measure scattering angle and recoil energy of nucleus. Use α source of known energy. Error 0.15%	• Have to extract beam • Up to 300 GeV	**SPS**
Resonant Spin Depolarization REF: Melissinos, CAS CERN95-06	Next Slide Energy error 1 MeV at Z^0 resonance (45.6 GeV)		**LEP**

Energy Measurement: Resonant Spin Depolarization

e^-, or e^+ become polarized b/c of SR (Ternov *etal*, 1962)

$$P(t) = P_\infty \left[1 - \exp\left(-\frac{t}{\tau} \right) \right], \quad \frac{1}{\tau} = \frac{1}{\tau_P} + \frac{1}{\tau_D},$$

τ_P: **Polarization time const.**

τ_D: **Depolar. time const**

$$P_\infty = 0.924 \ \ (\tau_D >> \tau_P)$$

$$\tau_P = 98 \frac{\rho^3}{E^5} \left(\frac{R}{\rho} \right)$$

E: **GeV**; R: **Ring Radius**; ρ: **Bending Radius**

For LEP, τ_p~ 340 min
at the Z^0 energy (45.6 geV)

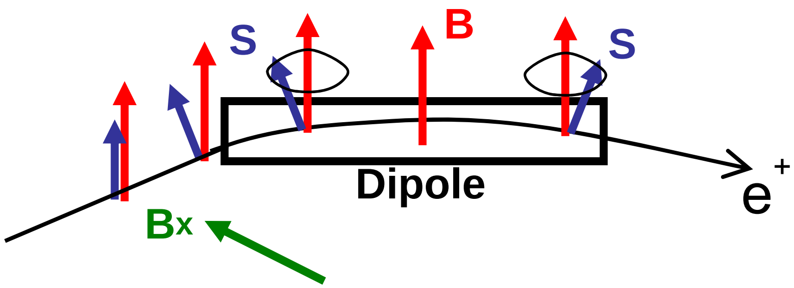
The spin tune ν_s is:

$$\nu_S = \gamma \frac{g-2}{2} = \frac{E}{0.44065\,\text{GeV}}$$

$e^\pm$ Gyromagnetic Anomaly = 0,00011596…

For LEP, ν_S~100

Energy Measurement: Resonant Spin Depolarization



$\boldsymbol{B_X}$ leads to vertical deflection and rotation of spin ***S.***

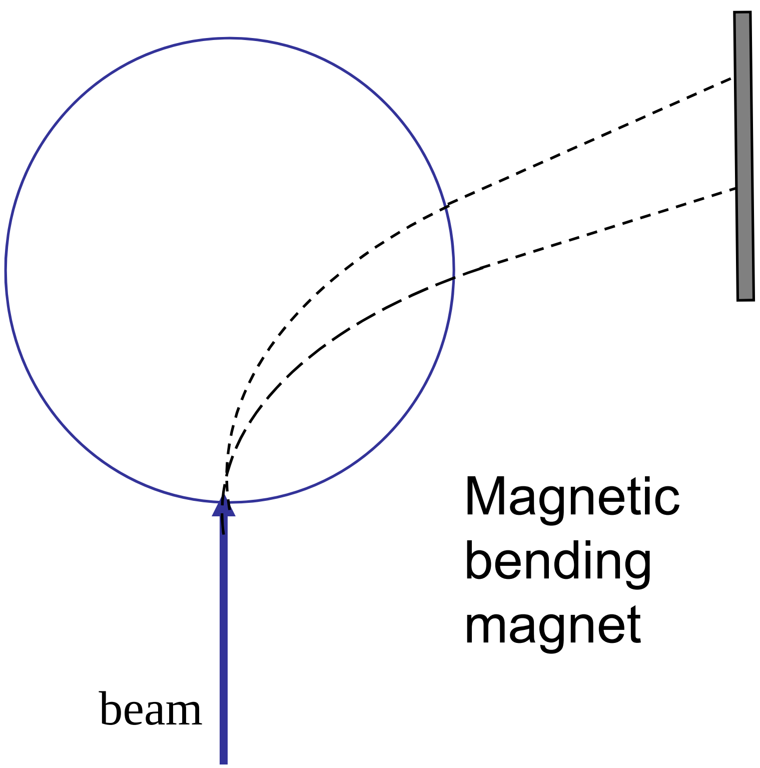
A weak depolarizing field $\boldsymbol{B_X}$ at frequency ω_D will cause resonant depolarization if

$$\nu_S \pm \frac{\omega_D}{\omega_C} = n, \text{ for integer n,}$$

where ω_C is the circulation angular frequency.

Beam polarization is measured by Compton scattering of laser photons from stored electron or positron beam. Compton cross section for scattering of polarized light is spin dependent.

Standard Accelerator Energy and Energy Spread Diagnostic: High resolution Magnetic Spectrometer



Energy dispersion converted to position on imaging screen

Advantage:

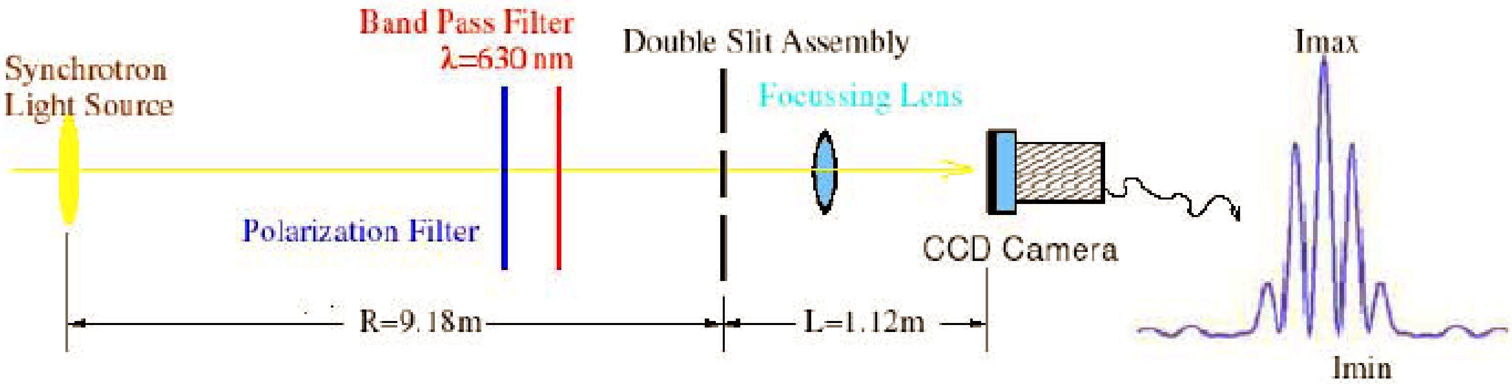
- can use any imaging method: phospor, OTR, OSR, etc.

Problems:

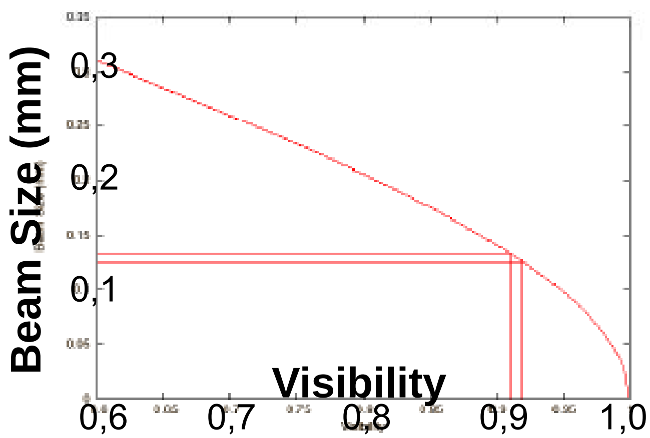
- usually large, bulky costly
- fixed position in beam line
- resolution limited by optics used to image beam

Courtesy: Ralph Fiorito

Energy Spread Measurements using
Young's Double Slit Optical Synchrotron Interferometer
(Michelson Stellar Interferometer 1921)



✔ Beam size is a function of the visibility on the interference pattern:



$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}.$$

Resolution: 10 microns for 150 micron beam (depends on experimental conditions)

Courtesy: Ralph Fiorito

Modern optical method: uses spatial coherence of beam radiation

1. Spatial coherence and profile of the object (e.g.beam)

According to van Citterut-Zernike theorem, the complex degree of spatial coherence $\gamma(\upsilon_x, \upsilon_y)$ is given by the Fourier Transform of the profile $f(x,y)$ of an object (beam)

$$\gamma(\upsilon_x, \upsilon_y) = \int f(x,y) \exp\{-i2\pi(\upsilon_x \cdot x + \upsilon_y \cdot y)\} dy dx$$

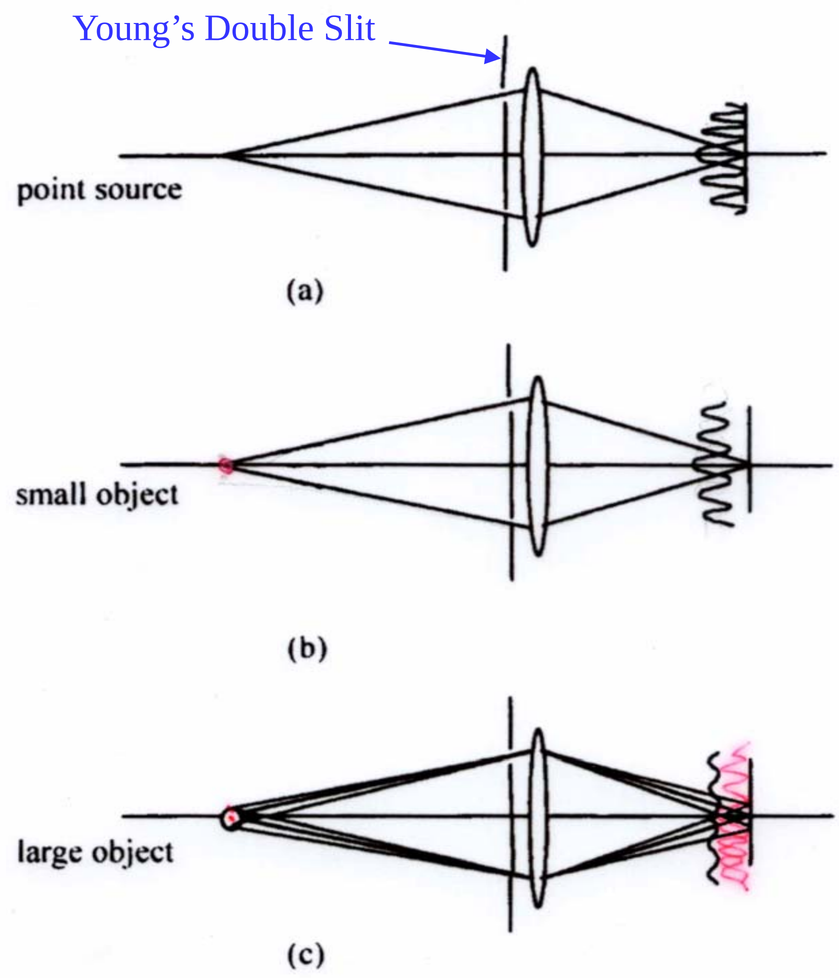
where υ_x, υ_y are spatial frequency

$$\upsilon_x = \frac{2\cdot\pi}{\lambda\cdot R_0} D_x, \quad \upsilon_y = \frac{2\cdot\pi}{\lambda\cdot R_0} D_y$$

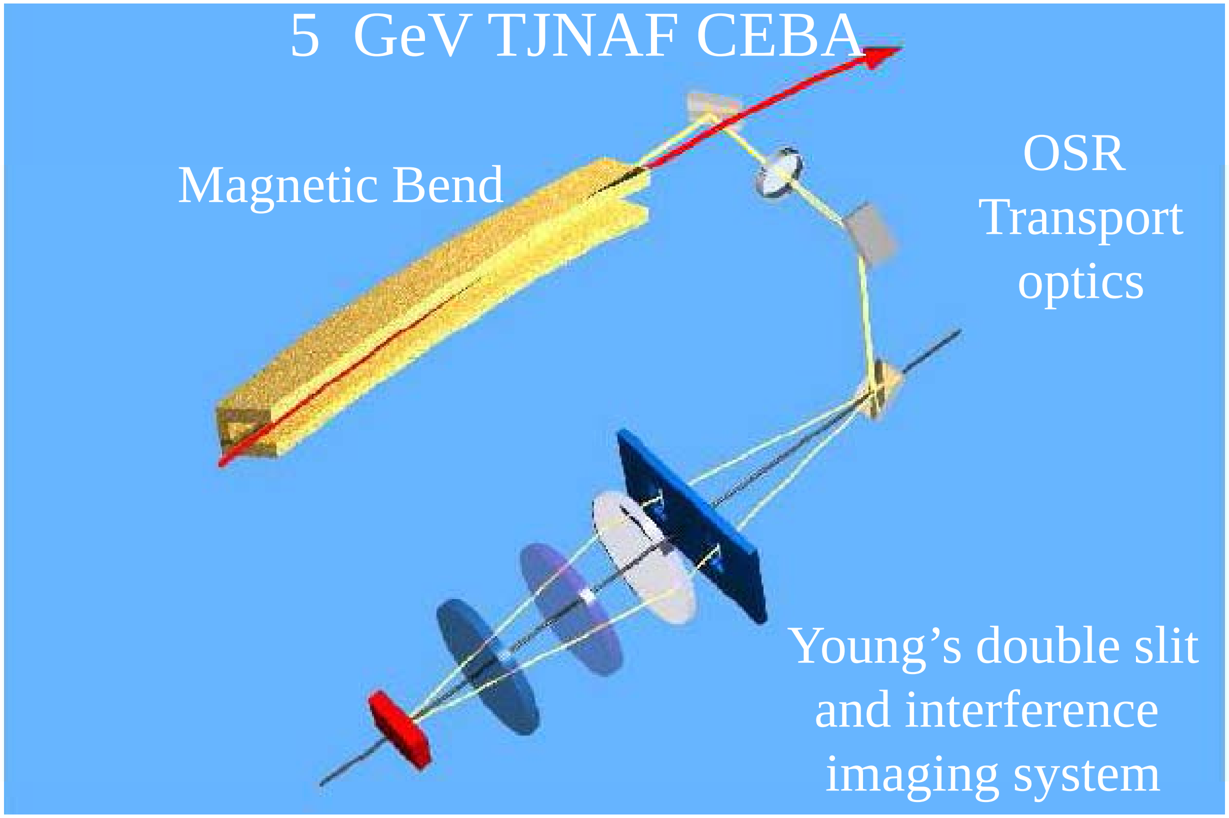
γ (spatial coherence function)
is related to the **visibility** of a Young's Double slit interference pattern
(used by Michelson in 1921 to measure the size of a star)

Courtesy: Ralph Fiorito

Simple understanding of van Cittert-Zernike theorem



Energy spread measurement using spatial coherence of optical synchrotron radiation



In dispersive (magnetic) region, beam size is directly related to energy spread

$$\sigma_E / E = \sigma_{beam} / R$$

e.g. $\sigma_E / E \sim 3 \times 10^{-5}$
when $\sigma_{beam} \sim$ 80 microns, R = 4 m

Boersch Effect

Electrons from thermionic sources follow Maxwell-Boltzmann distribution. At temperature $\boldsymbol{T_S}$,

$$\text{FWHM} = \Delta E_S[eV] = 2.45 \times k_B T_S$$

Ex.: Tungsten filament at T_S = 2800 K $\rightarrow$ ΔE_S = 0.6 eV
Thermionic cathode at T_S = 1000 K $\rightarrow$ ΔE_S = 0.2 eV
[Note: FWHM is 2.35xRMS for gaussian]

Initial energy spread measured in UMER, electron microscopes and other low-energy devices is $\Delta E_0 > \Delta E_S$.

Difference $\Delta E_0 - \Delta E_S$ is referred to as the *Boersch Effect (1954).*

Boersch Effect increases with electron current. If ΔE_0 is too large, energy spread leads to *chromatic aberration*…

The Boersch Effect H. Boersch, Z. Phys. 139, 115 (1954).

Reiser 6.4.1

Transverse temperature *unaffected* by acceleration

Therefore, acceleration results in temperature anisotropy

In equilibrium (due to collisions) beam wants to be equipartitioned

$$T_{eq} = T_{\perp f} = T_{\parallel f}$$

Assume longitudinal temperature very small, due to acceleration $\rightarrow$ neglect

Since 2 transverse degrees of freedom, conservation of energy gives:

$$3T_{eq} = 2T_{\perp o} \quad \Rightarrow \quad T_{eq} = \frac{2}{3}T_{\perp o}$$

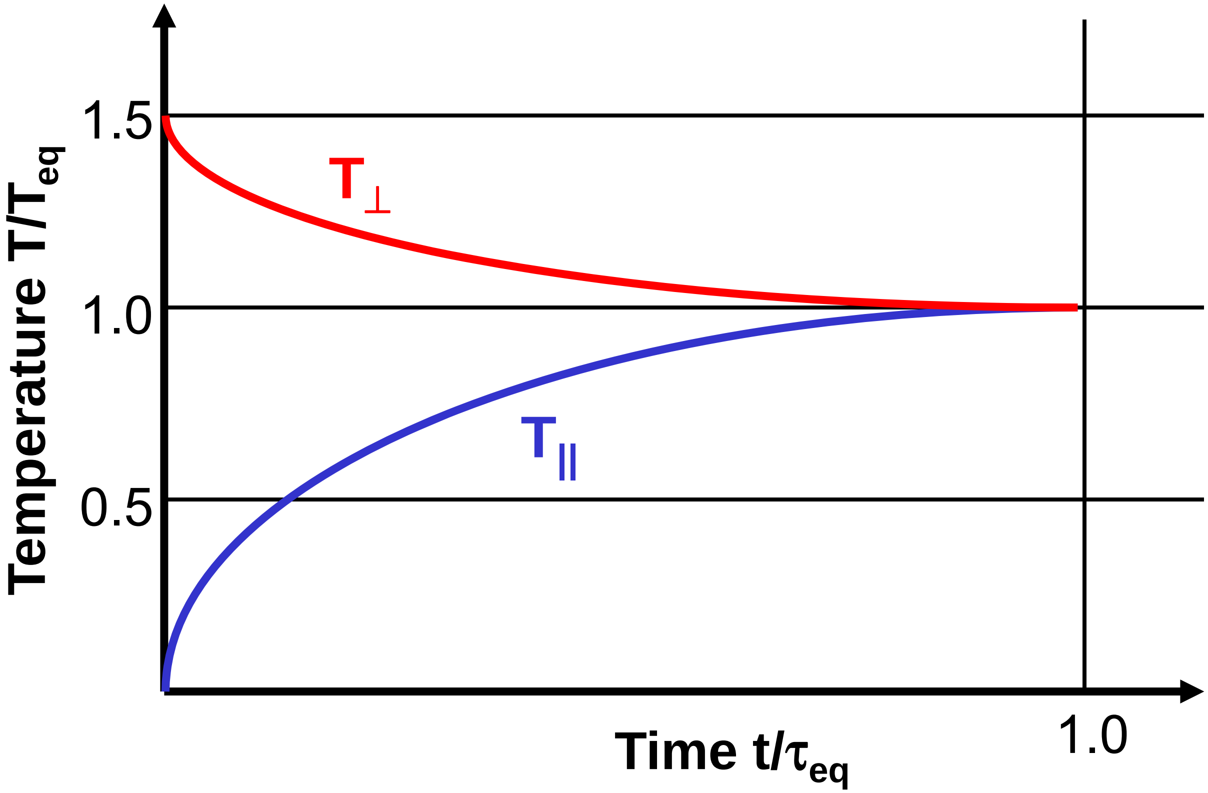
Approximate behavior:

$$T_{\perp}(t) = \frac{2}{3}T_{\perp o}\left(1 + 0.5e^{-3t/\tau_{eff}}\right)$$

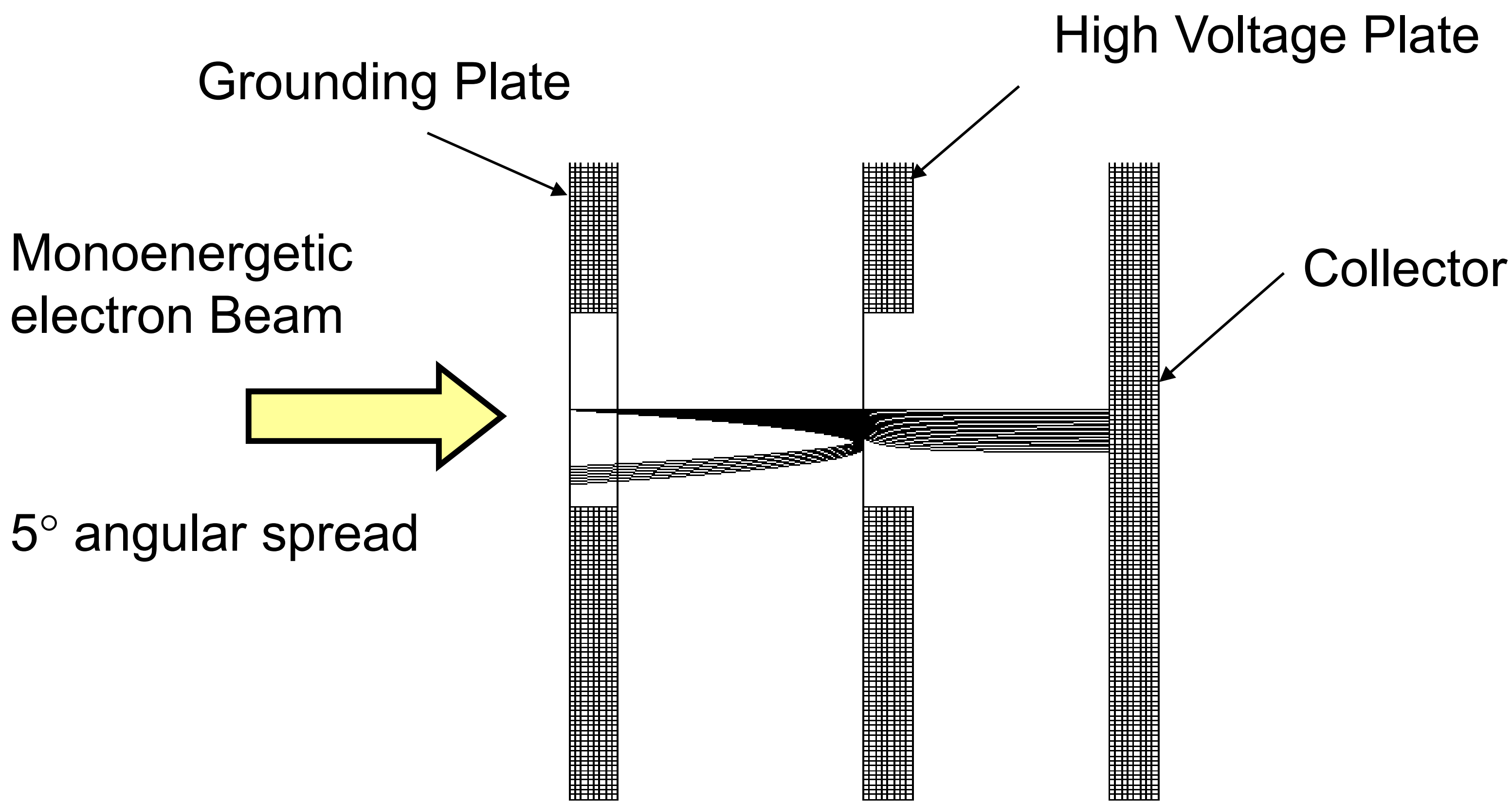
$$T_{\parallel}(t) = \frac{2}{3}T_{\perp o}\left(1 - e^{-3t/\tau_{eff}}\right)$$

Boersch Effect

See Fig 6.22 (M. Reiser)



Parallel-Plate Energy Analyzer Problem



Transverse expansion of beam causes apparently larger energy spread.

Energy Resolution Not Good (20 eV / 10 keV)

Credits: Yun Zou, SimIon simulation

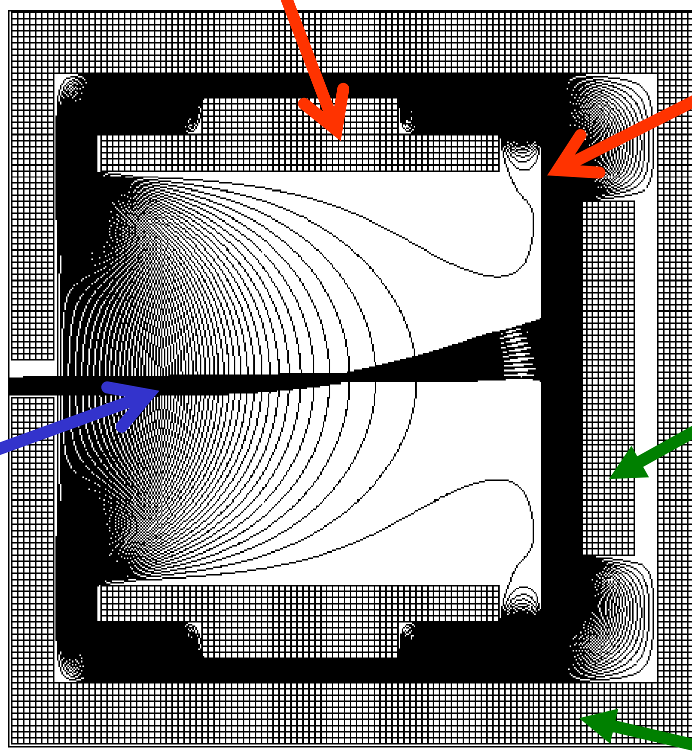
High resolution retarding-field energy analyzer

Collimating Cylinder
-10.13kV

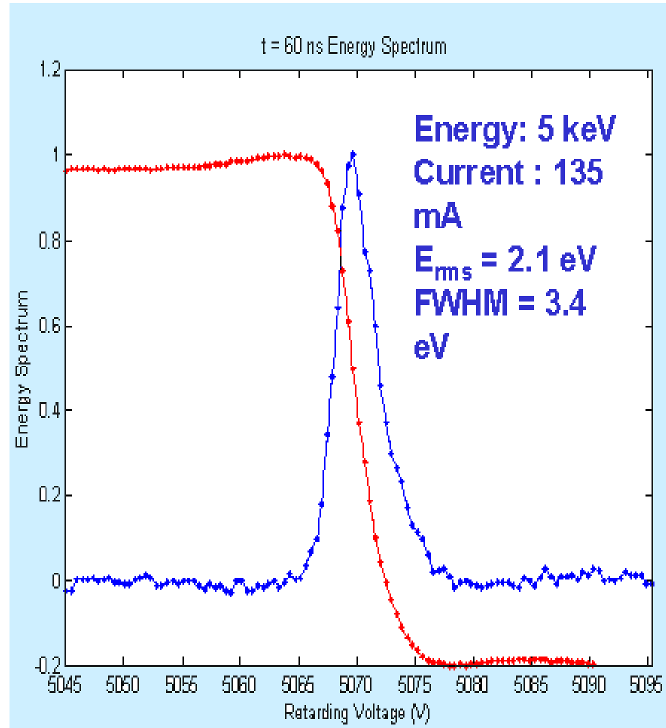
Retarding Mesh
-9999.5 V

Collector

10 keV Beam



Grounded Housing



3rd Generation: Res. < 1 eV

Y. Zou and Y . Cui

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Power, Luminosity, Emittance, Brightness

Power : $P = EI_b$

Luminosity : $L \propto \frac{I_{b1} I_{b2}}{\sigma_{b1}\sigma_{b2}}, \dot{N} = \sigma L$

σ_{b1}, σ_{b2}: beam radii,
σ: reaction cross section

Transverse Emittance: $\varepsilon_X, \varepsilon_Y \propto$

Beam radius × beam divergence.
(Units are *metre,* also *mm-mrad)*

Longitudinal Emittance : $\varepsilon_Z \propto \Delta E \times \Delta t$

Energy spread × Bunch's duration
(Units are *eV•s*)

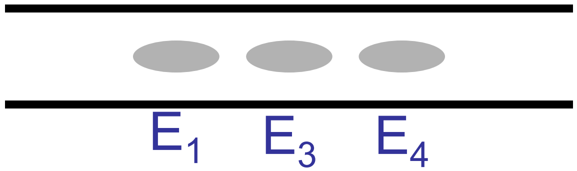
Brightness : $B = \frac{I_b}{\pi \varepsilon_X \varepsilon_Y} \propto$

Current density at source

Accelerator Examples

MACHINE	Circumf. (m)	*E or W* (GeV)	γ, $B\rho$ (Tm)	I_b (A)	ν_H	γ_t	α, η
LHC p-p^- collider	26658,883	7000			64,31		
TEVATRON p-p^- collider	$2\pi \times 10^3$	980					
RHIC Au^{79+} collider	3833,845		81,113782		28,19		
ISR p-p collider	943	31		40 DC			
MGH p cyclotron		0.235					
UMER e^-STR	11,52	10^{-5}	1,02 0,0003389	0,1	6.6		

Coasting vs. Bunched Beams



Coasting Beams (ISR, ISABELLE)	**Bunched Beams** (e^+e machines and modern hadron colliders: Tevatron, SPS, LHC)
Ex.: 31 GeV, 40 A p-p at ISR	Ex.: 1-100 ps bunches separated by few ns
No RF, phase displacement accel.?	RF used for bunching/acceleration
High *Average* Luminosity	High *Peak* Luminosity
Better for Detector	Dead Times in Detector
Large aperture b/c of large current	Smaller aperture magnets
Beam-beam effects important	Reduced beam-beam effects
Efficient stochastic cooling	Stochastic cooling harder

Backup Slides

Intersecting Storage Rings (ISR)

"The Leap in the Hadron Collider Area"

pp collider up to 31.4 GeV /beam
2π R = 942 m, injection from CPS
Combined-function lattice, large $\Delta p/p$
8 Intersection points (5 used for exp.)
Constr.: 66-70, Operated:71-83

L= 4 x 10^{30} (des.) to 1.4 x 10^{32} $cm^{-2}s^{-1}$,
dc proton current: up to 40 A (57A)
Notable features:

- Ultra-high vacuum and ion clearing
- Low-impedance vacuum envelope
- High-stability of power supplies (10^{-7} ripple tolerance on dipoles)
- Superconducting low-β insertion (L increased by 6.5)

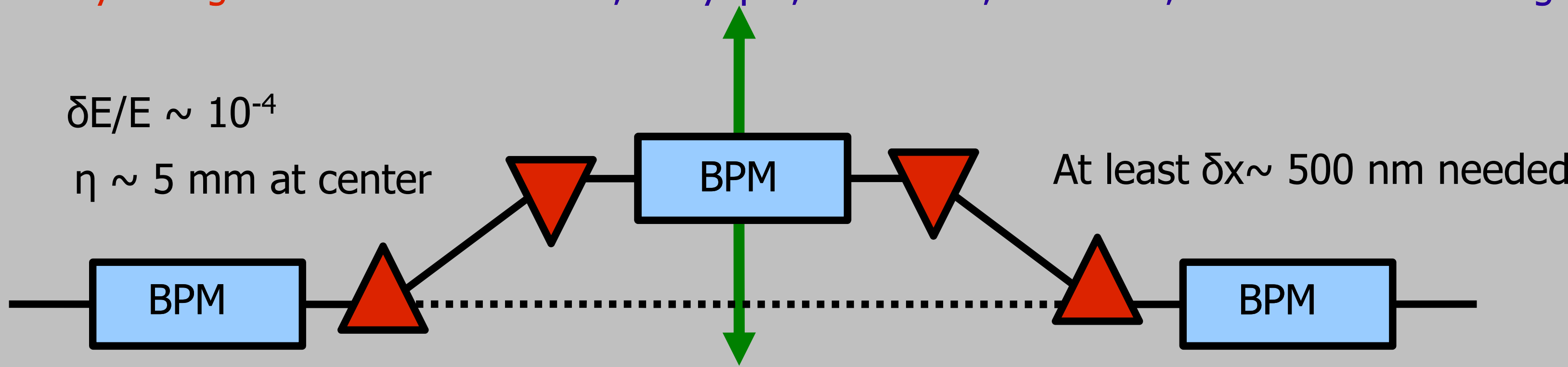
but experiments not fully exploiting it.

View of intersection point 5 in 1974

BPM based beam energy measurement

WP 4.2 mission statement :
Study & design magnetic chicane for beam energy measurement using BPMs for a future linear collider

Royal Holloway University London: S. Boogert
Cambridge : M. Slater, M. Thomson and D. Ward
University College London: F. Gournaris, A. Lyapin, B. Maiheu, S. Malton, D. Miller and M. Wing



NanoBPM@ATF : test resolution, try different analysis methods, BPM stability tests, multi bunch operation, advanced electronics techniques, inclination of beam in BPMs.

-> spectrometer aspects of BPMs can be tested

ESA@SLAC : test stability and operational issues with a full implementation of 4 magnet chicane and 3 BPM stations

-> test of chicane prototype

Beam Cooling Due to Acceleration for a beam with a lot of particles

Reiser 5.4.6

Longitudinal temperature related to *velocity spread* (though it has units of energy) for a beam :

$$k_B T_{\parallel}/m = \langle v^2 \rangle - \langle v \rangle^2$$

Assume no longitudinal focusing or compression, long-bunch beam

Initial: $\langle v_i \rangle = 0$ $\qquad v_o = \sqrt{2qV_o/m}$

Final: $\langle v_f^2 \rangle = \langle v_i^2 + v_o^2 \rangle = \langle v_i^2 \rangle + v_o^2$

$$\langle v_f \rangle = \left\langle \sqrt{v_i^2 + v_o^2} \right\rangle \approx v_o \left(1 + \frac{1}{2}\frac{\langle v_i^2 \rangle}{v_o^2} - \frac{1}{8}\frac{\langle v_i^4 \rangle}{v_o^4} + \cdots \right)$$

$$k_B T_{\parallel f} = \frac{m}{4 v_o^2}\left(\langle v_i^4 \rangle - \langle v_i^2 \rangle^2 \right)$$

$$\left(k_B T_{\parallel i}/m\right)^2$$

How do we calculate the 4th moment?

$$k_B T_{\square f} = \left(\left\langle v_i^4 \right\rangle - \left(k_B T_{\square i} \right)^2 \right) \Big/ 2qV_o$$

Assume a half-Gaussian velocity distribution function:

$$f_i\left(v_i\right) = f_o \exp\left(-\frac{m v_i^2}{2 k_B T_{\square f i}} \right), \qquad v_i > 0$$

$$\left\langle v_i^4 \right\rangle = \int_0^\infty v_i^4 f_i\left(v_i\right) dv_i$$

$$\boxed{k_B T_{\square f} = \frac{\left(k_B T_{\square i}\right)^2}{2qV_o} = \frac{\left(\Delta E\right)^2}{2qV_o}}$$

Same as before (p. 4)

Example

$$k_B T_{\parallel f} = \frac{\left(k_B T_{\parallel i}\right)^2}{2qV_o} = \frac{\left(\Delta E\right)^2}{2qV_o}$$

UMER:

at cathode: $k_B T_c = 0.1$ eV; $V_o = 10$ keV

$k_B T_f = 5 \times 10^{-7}$ eV

Cooling is very severe!

Note: Where beam is bunched or longitudinal focusing is present, cooling is less severe.

Timescale for Relaxation

Time scale determined by collisions:

$$\tau_{eff} = 1.34\tau_o = 0.42\tau_{eq}$$

For electrons:

$$\tau_{eff} = 4.44 \times 10^{20} \frac{\left(k_B T_{eff} / mc^2\right)^{3/2}}{n \ln \Lambda}$$

$$\ln \Lambda = \ln \left[5.66 \times 10^{21} \frac{\left(k_B T_{eff} / mc^2\right)^{3/2}}{\sqrt{n}} \right]$$

For UMER:

$s_{eff} = v\tau_{eff} \sim 1000$ m ~ 100 turns

However: Boersch observed that even in a short distance (~ 1m), the energy spread can grow considerably!

UMER example again

$v_{\perp o} / v = x'_{rms} = \varepsilon/2a = 50\text{ E-6} / (2*0.01) = 2.5$ mrad

$T_{\perp o} = m\beta^2c^2(x'_{rms})^2 = 0.13$ eV

$T_{||o} = 5\text{ E-7}$ eV (see p. 7)

After 1 m = 0.001 τ_{eff}

$$T_{||}(t) = \frac{2}{3} T_{\perp o}\left(1 - e^{-3t/\tau_{eff}}\right) = 0.00026$$

But using the relation on page 6:

$$\Delta E = \sqrt{2qV_o k_B T_{||}}$$

$$= \sqrt{2(10\text{keV})(5E-7)} = 0.1 \qquad \text{before relaxation}$$

$$= \sqrt{2(10\text{keV})(0.00026)} = 2.28 \qquad \text{after}$$

This is a factor 23 growth in energy spread over just 1 m !!