S-1

Specify a 1.8 m (arc length) long bending magnet that is capable to deflect a 3 GeV electron beam by an angle of $\pi/12$. This is the deflection angle of PLS-II. Assume a full gap aperture of 34 mm. What is the total excitation current in each coil? Determine the bending radius and sagitta. How many magnets do you need to define a circular accelerator?

The deflection angle is: $\psi = \frac{\pi}{12} = \frac{\ell}{\rho} = 0.29979\frac{B}{\beta E}$ or with E = 3 GeV

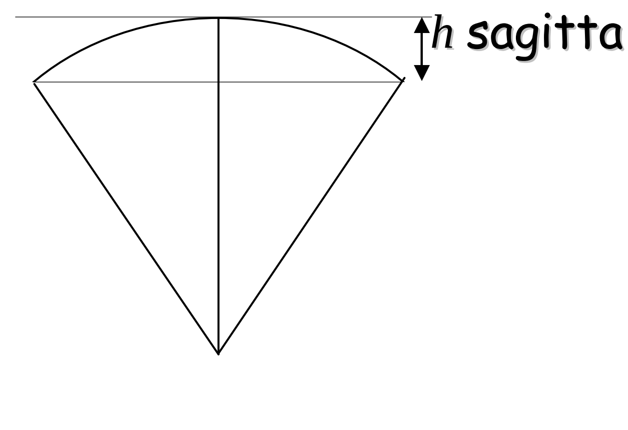
and $\beta \approx 1$ the magnetic field must be $B = \frac{\pi}{12}\frac{3}{0.29979 \cdot 1.8} = 1.4555\ \mathrm{T}$

and the bending radius $\rho = 6.8753\ \mathrm{m}$.

The coil current is $I_{\mathrm{coil}}(\mathrm{A}) = \frac{1}{\mu_0}B(\mathrm{T})G(\mathrm{m}) = \frac{1.4555 \cdot 0.017}{4\pi 10^{-7}} = 19690.0$

and the sagitta: $h = \rho(1 - \cos\psi) = 6.8753(1 - \cos\frac{\pi}{24}) = 5.8819\ \mathrm{cm}$

Finally, you need 24 magnets to make a ring.



S-2

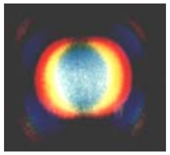
A quadrupole magnet shall focus a parallel beam to a focal point 4 m away from the center of the quadrupole. The beam energy be 3 GeV. Choose a magnet length and calculate the magnetic field gradient necessary.

The focal length of the quadrupole is $\frac{1}{f} = k\ell = 0.25\ \mathrm{m}^{-1}$.

With a magnet length of $\ell = 0.20\ \mathrm{m}$ the quadrupole strength is $k = 1.25\ \mathrm{m}^{-1}$

Magnet strength and gradient are related by $k\left(\mathrm{m}^{-2}\right) = 0.29979\frac{g\left(\mathrm{T/m}\right)}{\beta E\left(\mathrm{GeV}\right)}$

and solving for the gradient we get $g(\mathrm{T/m}) = \frac{k\left(\mathrm{m}^{-2}\right)\beta E\left(\mathrm{GeV}\right)}{0.29979} = \frac{1.25\cdot 3}{0.29979} = 12.509\ \mathrm{T/m}$



S-3

Specify a Rf-cavity (pillbox cavity) for microwaves with a wavelength of 60 cm. What is the diameter of the pill box? How long must the cavity be if an electron at the speed of light ($v \approx c$) is expected to travel through the cavity in one half Rf-period. Express this cavity length in units of the Rf-wavelength. What is the Rf-frequency?

The cavity radius is given by the first root of the Bessel's function $J_0(kr)$, which is at $kr = 2.405$ or at $r_{\mathrm{wall}} = 2.405\lambda/(2\pi)$. Numerically, the cavity wall must be at $r_{\mathrm{wall}} = \frac{2.405}{2 \cdot \pi} 0.6 = 0.22966 \text{ m}$ and the cavity diameter is $D = 0.45932 \text{ m}$.

The Rf-period is $\tau = \frac{\lambda}{c} = \frac{0.6}{2.9979 \cdot 10^8} = 2.0014 \text{ nsec}$

and the electron travels during a half period the distance of $\lambda/2$.

therefore the length of the cavity is one half wavelength long. If it were any longer, the particle would see some negative fields.

The Rf-frequency is $f = \frac{1}{\tau} = \frac{1}{2.0014 \cdot 10^{-9}} = 499.65 \text{ MHz}$

Useful vector relations

$$
\begin{aligned}
\nabla(\mathbf{a}\varphi) &= \varphi\nabla\mathbf{a} + \mathbf{a}\nabla\varphi \\
\nabla \times (\mathbf{a}\varphi) &= \varphi(\nabla \times \mathbf{a}) - \mathbf{a} \times \nabla\varphi \\
\nabla(\mathbf{a} \times \mathbf{b}) &= \mathbf{b}(\nabla \times \mathbf{a}) - \mathbf{a}(\nabla \times \mathbf{b}) \\
\nabla \times (\mathbf{a} \times \mathbf{b}) &= (\mathbf{b}\nabla)\mathbf{a} - (\mathbf{a}\nabla)\mathbf{b} + \mathbf{a}(\nabla\mathbf{b}) - \mathbf{b}(\nabla\mathbf{a}) \\
\nabla(\mathbf{a}\mathbf{b}) &= (\mathbf{b}\nabla)\mathbf{a} + (\mathbf{a}\nabla)\mathbf{b} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) \\
\nabla \times (\nabla\varphi) &= 0 \\
\nabla(\nabla \times \mathbf{a}) &= 0 \\
\nabla \times (\nabla \times \mathbf{a}) &= \nabla(\nabla\mathbf{a}) - \Delta\mathbf{a}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{a}(\mathbf{b} \times \mathbf{c}) &= \mathbf{b}(\mathbf{c} \times \mathbf{a}) = \mathbf{c}(\mathbf{a} \times \mathbf{b}) \\
\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= \mathbf{b}(\mathbf{a}\mathbf{c}) - \mathbf{c}(\mathbf{a}\mathbf{b}) \\
(\mathbf{a} \times \mathbf{b})(\mathbf{c} \times \mathbf{d}) &= (\mathbf{a}\mathbf{c})(\mathbf{b}\mathbf{d}) - (\mathbf{b}\mathbf{c})(\mathbf{a}\mathbf{d}) \\
\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) &= 0 \\
(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) &= \mathbf{c}[(\mathbf{a} \times \mathbf{b})\mathbf{d}] - \mathbf{d}[(\mathbf{a} \times \mathbf{b})\mathbf{c}]
\end{aligned}
$$

Integral Relations

$$\int_V \nabla\varphi \, \mathrm{d}\mathbf{r} = \oint_S \varphi \hat{\mathbf{u}} \, \mathrm{d}\sigma$$

$$\int_V \nabla \boldsymbol{a} \, \mathrm{d}\boldsymbol{r}^3 = \oint_S \boldsymbol{a}\hat{\boldsymbol{u}} \, \mathrm{d}\boldsymbol{\sigma}$$ Gauss' Law

$$\int_S (\nabla \times \boldsymbol{a}) \hat{\boldsymbol{u}} \, \mathrm{d}\boldsymbol{\sigma} = \oint \boldsymbol{a} \, \mathrm{d}\boldsymbol{\sigma}$$ Stoke's Law

Cylindrical coordinates

$$(\rho, \varphi, z)$$

transformation:

$$(x, y, z) = (\rho\cos\varphi, \rho\sin\varphi, z)$$

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \rho^2\mathrm{d}\varphi^2 + \mathrm{d}z^2$$

$$\mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z$$

vector expressions

$$\nabla\phi = \left[\frac{\partial\phi}{\partial\rho}, \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}, \frac{\partial\phi}{\partial z}\right]$$

$$\nabla\mathbf{a} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho a_\rho) + \frac{1}{\rho}\frac{\partial a_\varphi}{\partial\varphi} + \frac{\partial a_z}{\partial z}$$

$$\nabla\times\mathbf{a} = \left[\frac{1}{\rho}\frac{\partial a_z}{\partial\varphi} - \frac{\partial a_\varphi}{\partial z}, \frac{\partial a_\rho}{\partial z} - \frac{\partial a_z}{\partial\rho}, \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho a_\varphi) - \frac{1}{\rho}\frac{\partial a_\rho}{\partial\varphi}\right]$$

$$\Delta\boldsymbol{\phi} = \frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$$

Polar coordinates

(r, φ, θ)

$$\nabla\phi = \left[\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\varphi}, \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\theta}, \right]$$

$$\nabla\mathbf{a} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 a_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(\sin\varphi\, a_\varphi) + \frac{1}{r\sin\theta}\frac{\partial a_\theta}{\partial\theta}$$

$$\nabla\times\mathbf{a} = \begin{bmatrix} \frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta\, a_\zeta)}{\partial\varphi} - \frac{\partial a_\varphi}{\partial\theta}\right), \\ \frac{1}{r\sin\theta}\left(\frac{\partial a_r}{\partial\theta} - \sin\theta\frac{\partial(r a_\theta)}{\partial r}\right), \\ \frac{1}{r}\left(\frac{\partial}{\partial r}(r a_\varphi) - \frac{\partial a_r}{\partial\varphi}\right). \end{bmatrix}$$

$$\Delta\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right)$$

transformation: $(x, y, z) = (r\cos\varphi\,\sin\theta, r\sin\varphi\,\sin\theta, r\cos\theta)$

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2 + r^2\,\mathrm{d}\theta^2$$

$$\mathrm{d}V = r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}\theta$$