"**Transverse beam profiles**"

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DITANET school
30 March - 3 April 2009
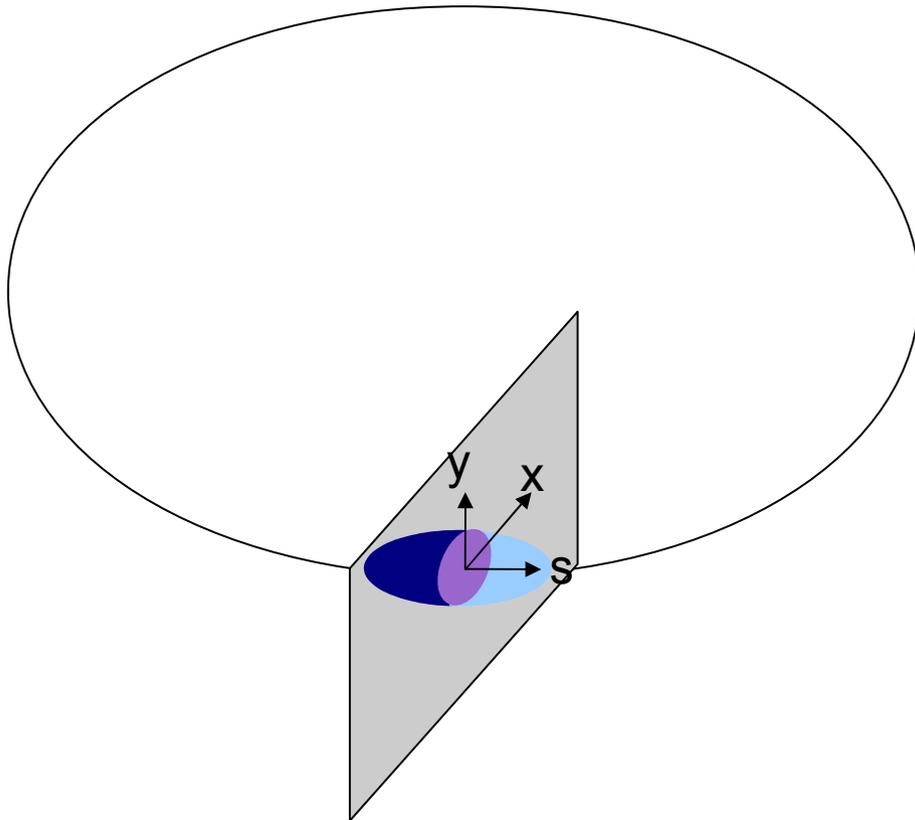
Content

- Coordinate system

- Transverse space

- Transverse phase space

- Phase space dynamics

- Interaction of particles with matter

- Radiation emission by charged particles

- Sampling of distributions in 2D space

Coordinate System

- **Longitudinal coordinate**
  - defined by the direction of motion of the beam
  - Axis indicated with *s*

- **Transverse Plane**
  - Plane orthogonal to the close orbit
  - Axes usually indicated with *x* and *y* and referred as *HORIZONTAL* and *VERTICAL*
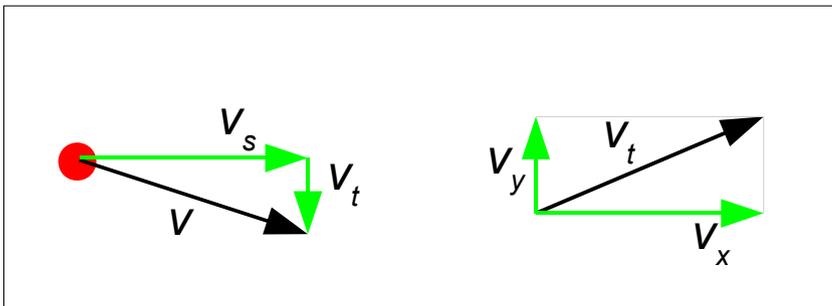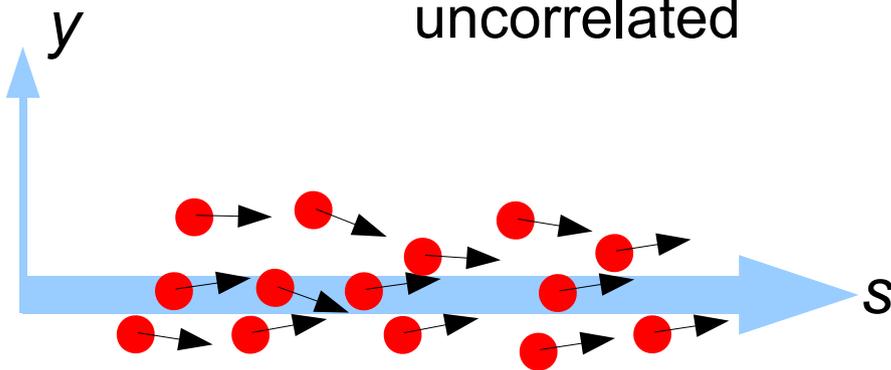
Transverse *x,y* space
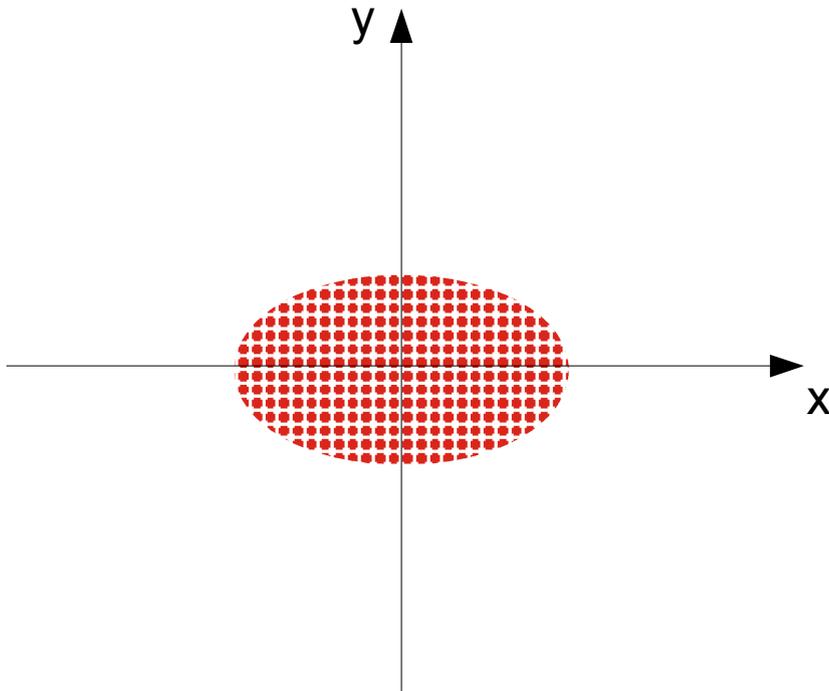
- Define a plane orthogonal to the beam trajectory at a given *s*

- Record the *x,y* coordinates of each particle crossing this plane

- Plot on a 2D chart (*x*, *y*) each particle
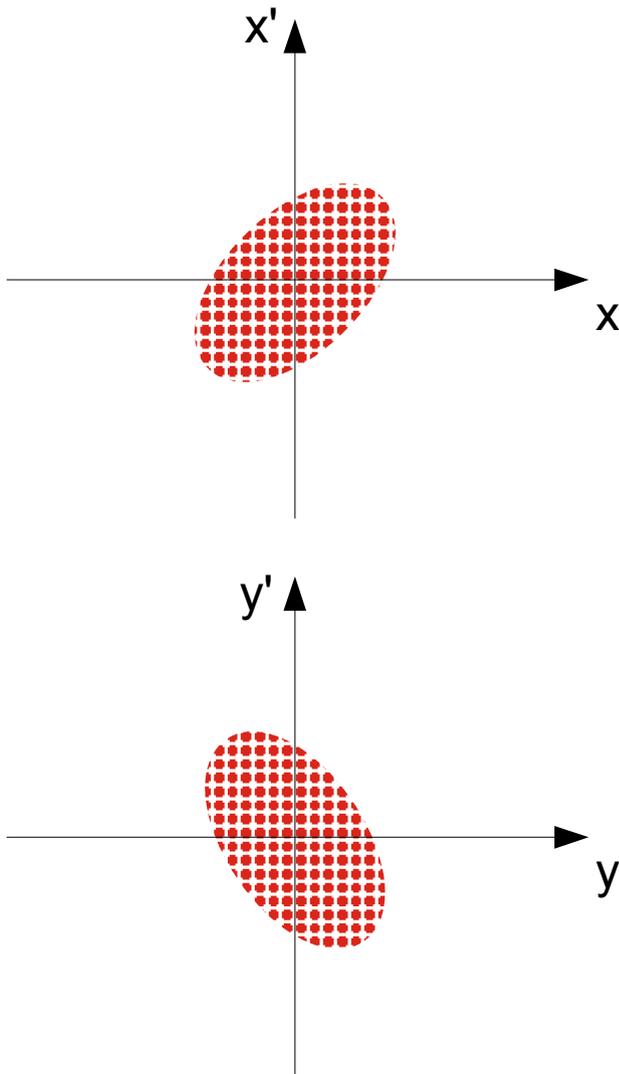
Transverse phase space
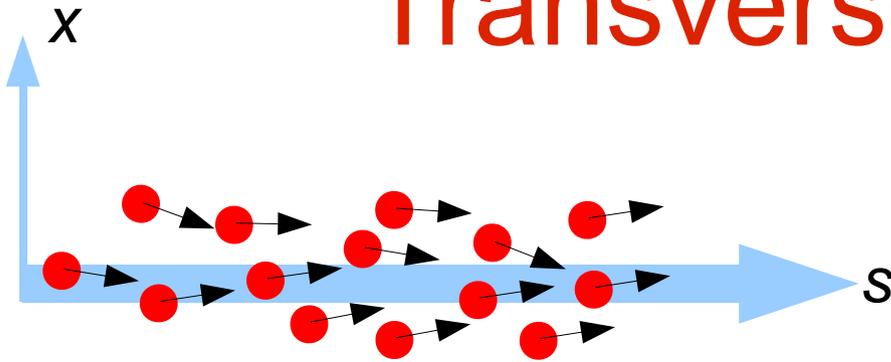
v_x, v_y and v_s are normally uncorrelated

- Beam moves along s
- Each particle moves in a different direction
- Velocity has 2 components
  - *Transverse* $v_t = v_x \hat{x} + v_y \hat{y}$
  - *Longitudinal* v_s
- Transverse components also called x' and y'
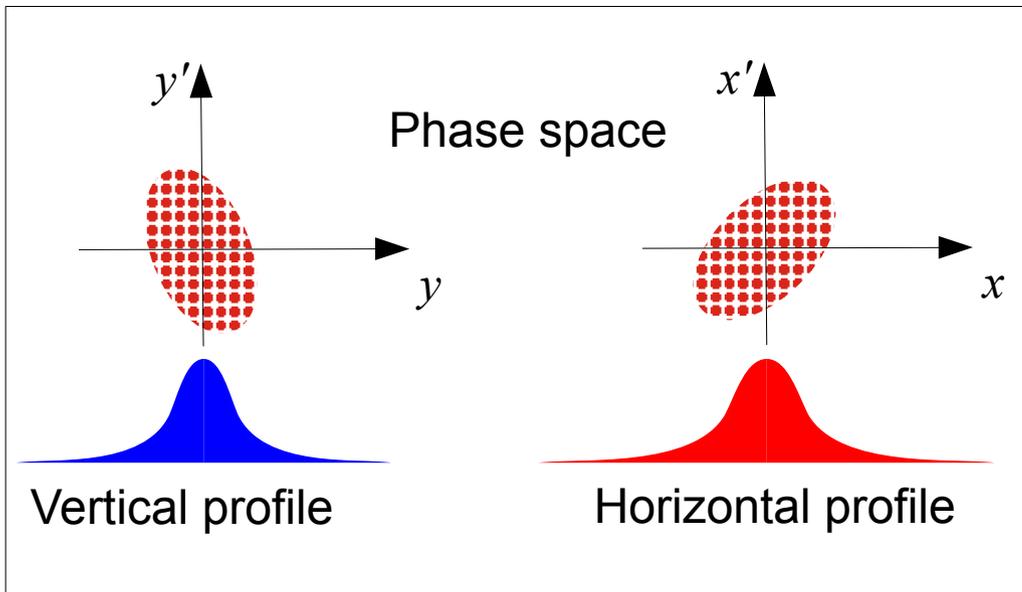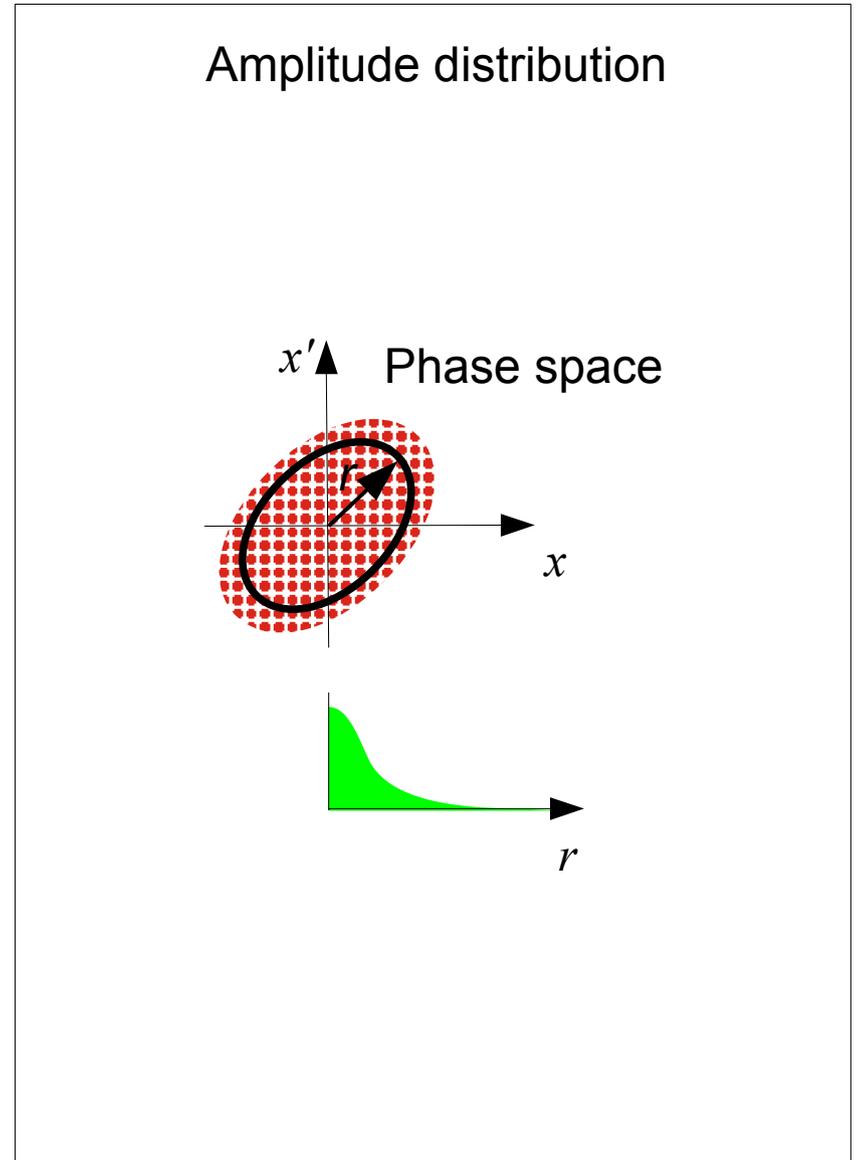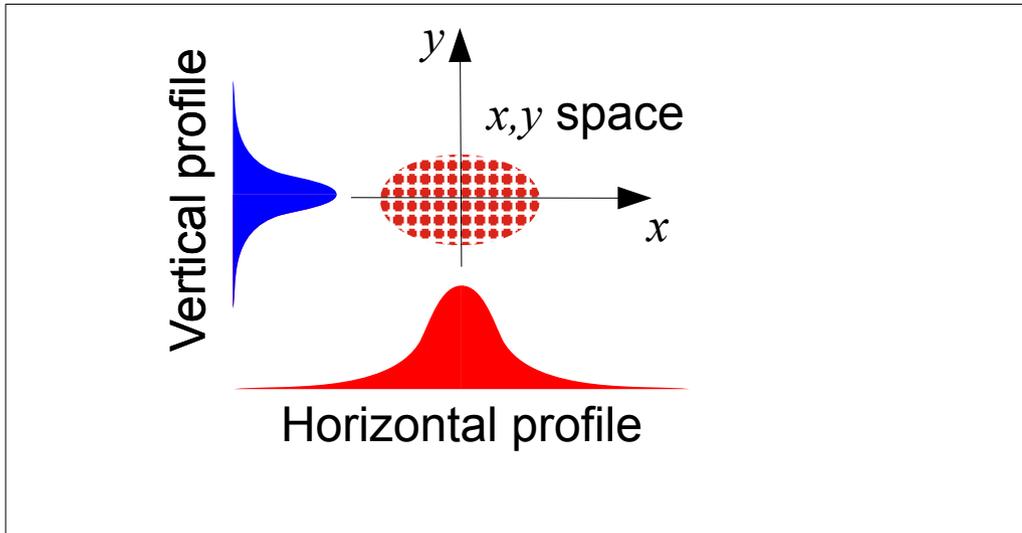
Transverse phase space (2)

- Take the same plane as before

- Note x, v_x and y, v_y for each particle crossing the plane

- Plot on a 2D chart (x, v_x OR y, v_y) of each particle

- Rename $v_x \rightarrow x'$, $v_y \rightarrow y'$

- Area of the ellipse is an invariant and is called transverse emittance $\varepsilon_x, \varepsilon_y$

Transverse spaces

- The x,y space and the phase space are different things

- Their projections along x or y are however the same thing

- Phase spaces contain the information needed for beam dynamic calculations

- x,y space is easier to sample

- Perform measurement in x,y and use optics parameters and beam dynamic theories to calculate the phase space

Beam Profiles

Vertical profile

y

x,y space

x

Horizontal profile

Phase space

y'

y

x'

x

Vertical profile

Horizontal profile

Amplitude distribution

x'

Phase space

x

r

Beam Profiles (2)

Particles distribution: $\qquad$ $i(x,y)$

Profiles:
$$\begin{cases} Prof_H(x) = \int_{-\infty}^{+\infty} i(x,y)\,dy \\[2em] Prof_V(y) = \int_{-\infty}^{+\infty} i(x,y)\,dx \end{cases}$$

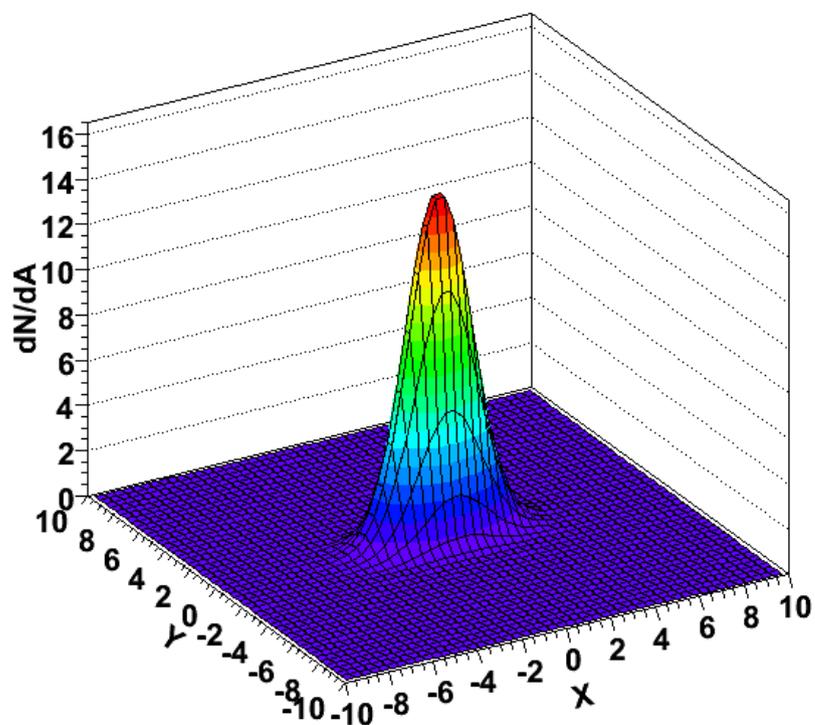Typically $i(x,y)$ is Gaussian:
N_0 = total number of particles

$$i(x,y) = \frac{N_0}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)}$$
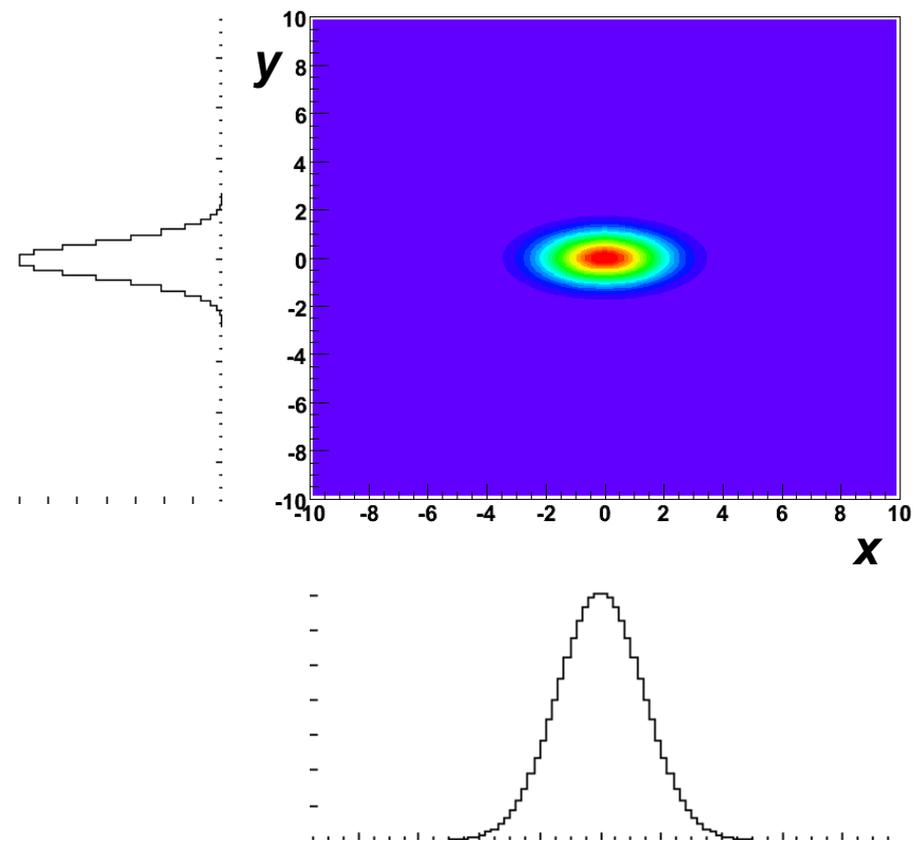
$$\Rightarrow \quad \begin{cases} Prof_H(x) = \dfrac{N_0}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}} \\[3em] Prof_V(y) = \dfrac{N_0}{\sqrt{2\pi}\,\sigma_y} e^{-\frac{y^2}{2\sigma_y^2}} \end{cases}$$
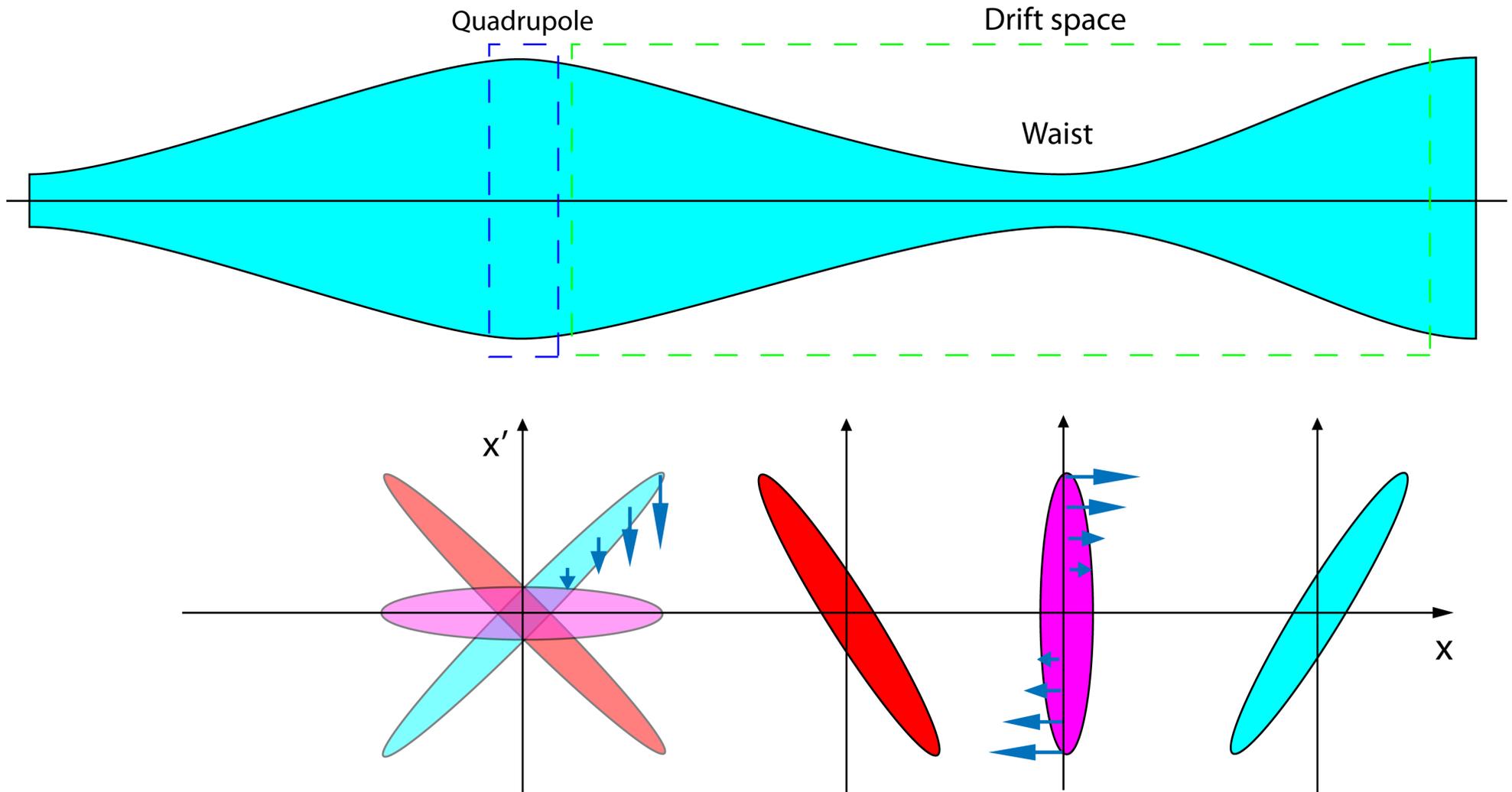
Beam Profiles (3)
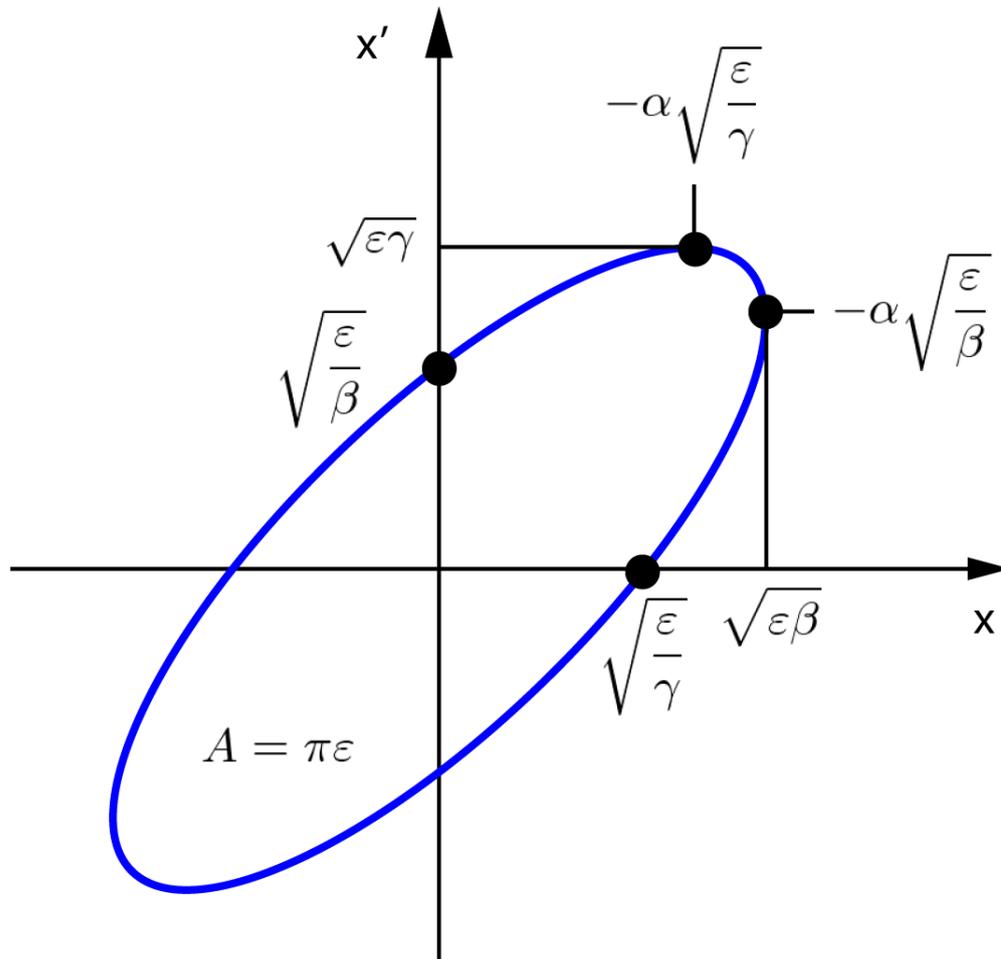
Particles Distribution

Profiles

Phase space dynamics

Courant-Snyder parameters

The phase space ellipse can be defined by 4 parameters: ε, β, α and γ, with the relation

$$\gamma = \frac{1+\alpha^2}{\beta}$$

The equation of the ellipse is

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

Particles transport

In a linear system, like a system composed of drift space and quadrupoles, the coordinates of a particle in phase space can be transported using a simple matrix notation

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = M_1 \begin{bmatrix} x_0 \\ x_0' \end{bmatrix} \qquad \begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = M_2 \begin{bmatrix} x_1 \\ x_1' \end{bmatrix} \qquad \begin{bmatrix} x_3 \\ x_3' \end{bmatrix} = M_3 \begin{bmatrix} x_2 \\ x_2' \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ x_3' \end{bmatrix} = M_3 M_2 M_1 \begin{bmatrix} x_0 \\ x_0' \end{bmatrix} = M_{0 \Rightarrow 3} \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}$$

$$M_{Drift} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \qquad M_{Quad} = \begin{bmatrix} \cos(\sqrt{k}\,L_Q) & 1/\sqrt{k}\,\sin(\sqrt{k}\,L_Q) \\ -\sqrt{k}\,\sin(\sqrt{k}\,L_Q) & \cos(\sqrt{k}\,L_Q) \end{bmatrix}$$

Twiss parameters transport

If one can transport each point of the phase space one can also transport the ellipse and thus the Courant-Snyder, a.k.a. Twiss, parameters

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} c & s \\ c' & s' \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} c^2 & 2cs & s^2 \\ -cc' & cs'+c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

From the measurement of the beam profiles one obtains $\sigma^2 = \varepsilon \beta$

$$\beta_1 = \begin{bmatrix} c_1^2 & 2c_1 s_1 & s_1^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon \beta_1 \\ \varepsilon \beta_2 \\ \varepsilon \beta_3 \end{bmatrix} = \varepsilon \begin{bmatrix} c_1^2 & 2c_1 s_1 & s_1^2 \\ c_2^2 & 2c_2 s_2 & s_2^2 \\ c_3^2 & 2c_3 s_3 & s_3^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

Twiss parameters measurement

$$\begin{bmatrix} \varepsilon\,\beta_1 \\ \varepsilon\,\beta_2 \\ \varepsilon\,\beta_3 \end{bmatrix} = \varepsilon\,M \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix} \qquad \Rightarrow \qquad M^{-1}\begin{bmatrix} \varepsilon\,\beta_1 \\ \varepsilon\,\beta_2 \\ \varepsilon\,\beta_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \varepsilon\begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

$$\begin{cases} a = & \varepsilon\,\beta_0 \\[4pt] b = & \varepsilon\,\alpha_0 \\[4pt] c = & \varepsilon\,\gamma_0 \\[4pt] \gamma_0 = & \dfrac{1+\alpha_0^2}{\beta_0} \end{cases} \qquad \Rightarrow \qquad \begin{cases} \beta_0 = & \dfrac{a}{\sqrt{ac-b^2}} \\[10pt] \alpha_0 = & \dfrac{b}{\sqrt{ac-b^2}} \\[10pt] \gamma_0 = & \dfrac{c}{\sqrt{ac-b^2}} \\[10pt] \varepsilon = & \sqrt{ac-b^2} \end{cases}$$

Sampling of distributions

- ## Intercepting methods
  - Scanning wires
  - Wire grids (Harps)
  - Radiative screens

- ## Non intercepting methods
  - Synchrotron light
  - Rest gas ionization
 - (Inverse Compton scattering / photo dissociation)

Interaction of particles with matter

- Ionization
  - Creation of electrons/ions pairs
    - Secondary electrons emission (low energy electrons)
  - Emission of photons (decay of excited states)
- Elastic and inelastic scattering
  - Dislocations
  - Production of secondary particles (high energy particles)
- Čerenkov radiation
- Bremsstrahlung
- Optical transition radiation

Energy deposition

- Energy deposition is probably the most important aspect for all intercepting devices
  - Signals are often proportional to the deposited energy
  - Energy deposition can cause damage to the instrument
- The Bethe-Bloch formula describes energy losses in most cases
- The energy lost by the particles is not necessarily deposited in the sensor
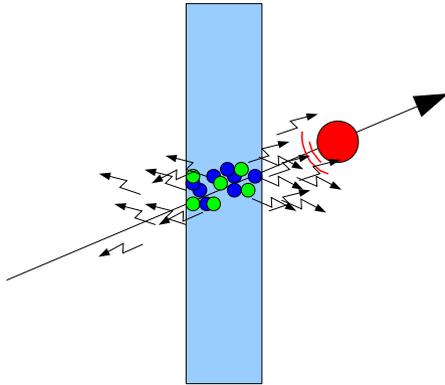
Energy deposition - dE/dx
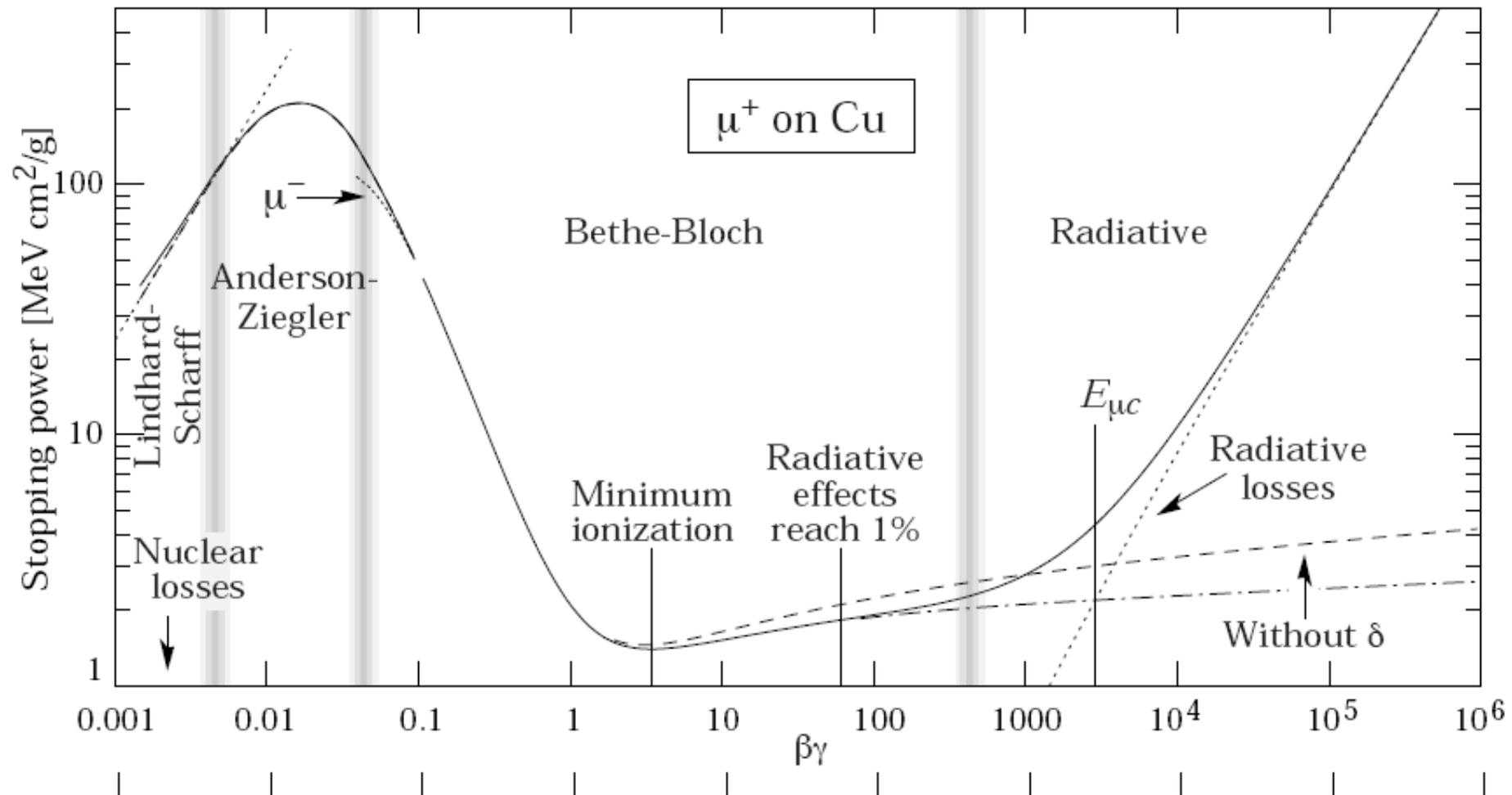
Secondary emission - SEM
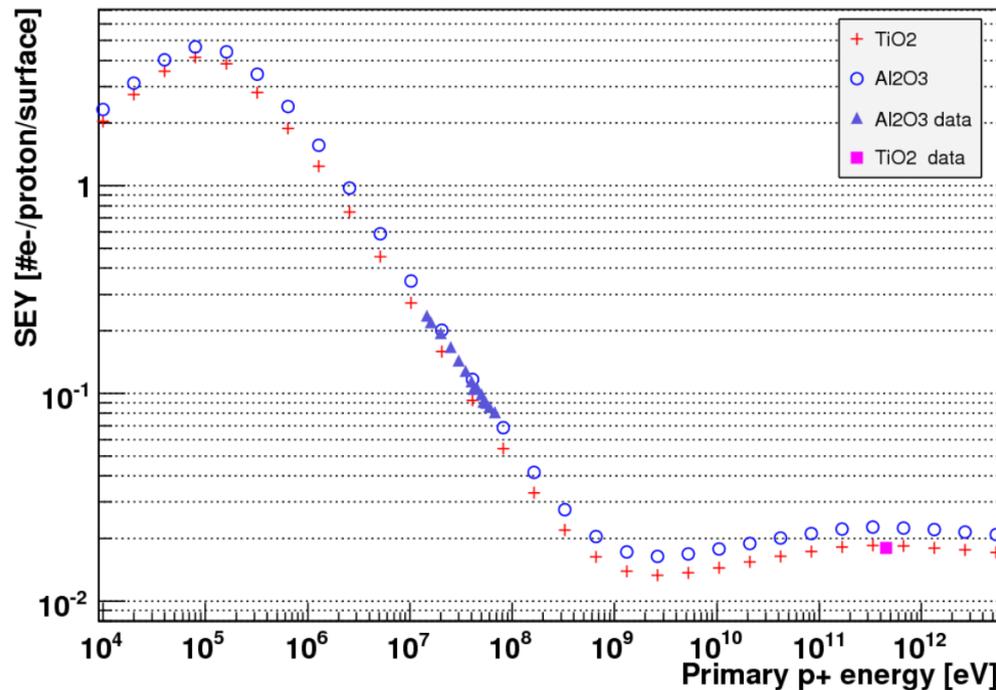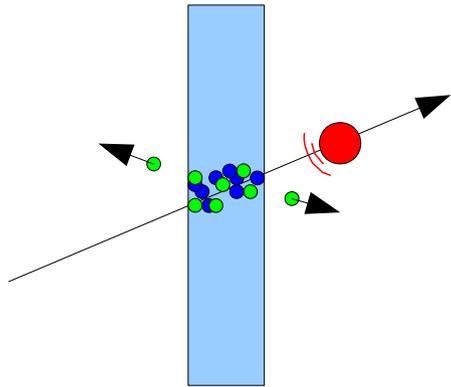
- Linked to ionization

- Surface electrons receive sufficient energy to travel to the surface and leave

- Emission yield depends on particles energy, material, surface state etc.

Scintillation

- Linked to ionization

- Photons are emitted by the de-excitation of atomic states populated by the passage of the particle

- Emission time *ns* to *hours*

Energy Conversion ((W/nm)/W)
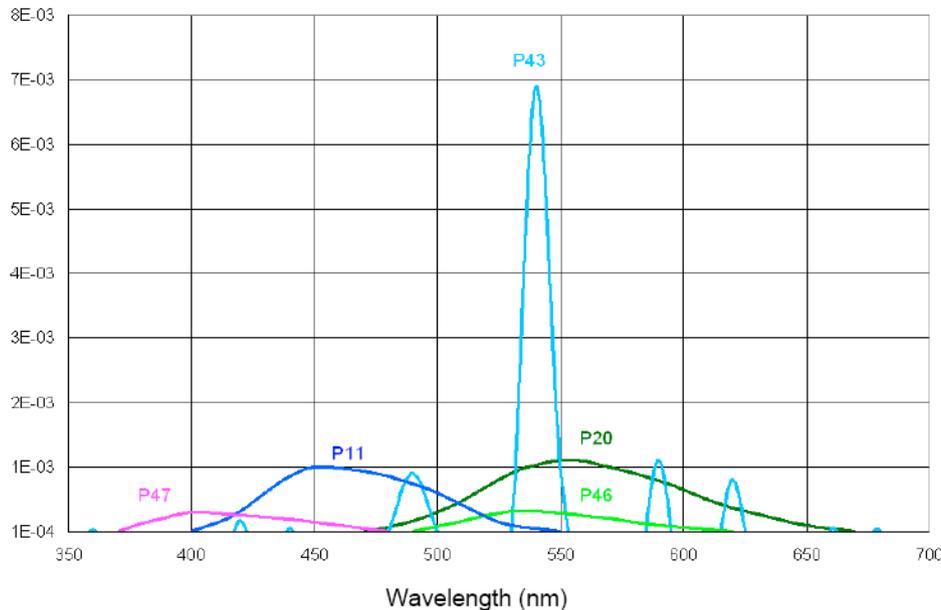
Type	Composition	Decay Time	
		Decay of Light Intensity	
		from 90 % to 10 % in	from 10 % to 1 % in
P 43	Gd_2O_2S:Tb	1 ms	1,6 ms
P 46	$Y_3Al_5O_{12}$:Ce	300 ns	90 µs
P 47	Y_2SiO_5:Ce,Tb	100 ns	2,9 µs
P 20	(Zn,Cd)S:Ag	4 ms	55 ms
P 11	ZnS:Ag	3 ms	37 ms

1MeV e⁻ on 5µm P43 yields ~ 60 ph.

Scintillation (2)

- Phosphors have very high light yields, but can only be used as thin coating on a rigid support and get damaged very quickly

- Normally used only for very low intensity beams

- Ceramics, glasses and crystals are a more popular choice in high energy accelerators

- Al_2O_3 (Alumina, Aluminium Oxide) is a very common choice (eventually doped with Cr) because it is a very robust ceramic
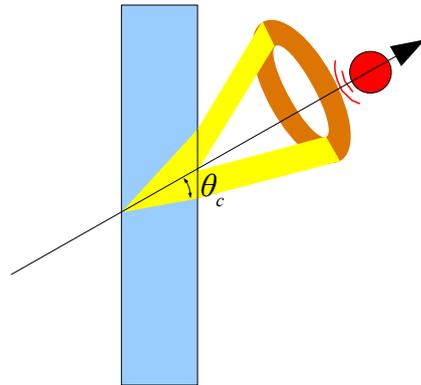
Čerenkov radiation

Index of refraction
$$n = n(\omega)$$

Velocity of particle
$$\mathrm{v} = \beta c$$

- Linked to the polarization of the atoms in the material

- Threshold effect: particle travels faster than phase velocity of light in material

- Radiation has defined angular distribution

- Not very good resolution in imaging

Čerenkov angle
$$\cos \theta_c = \frac{1}{n(\omega)\beta}$$

Velocity Threshold
$$\beta_t = \frac{1}{n(\omega)}$$

$$\frac{d^2 N}{dx\, d\omega} = \frac{\alpha z^2}{c} \sin^2 \theta_c(\omega) = \frac{\alpha z^2}{c}\left(1 - \frac{1}{\beta^2 n^2(\omega)}\right)$$

For $\theta_c = 45°$ (quartz at $\beta \approx 1$), $\lambda \in [400,500]nm$ and $\delta x = 1mm$ -> $N \sim 10$ photons

Optical Transition Radiation (OTR)

Backward emission

$\hat{n}$

2θ

Foil

Forward emission

W

θ

$$\frac{d^2 W}{d\Omega d\omega} \approx \frac{N q^2}{\pi^2 c}\left(\frac{\theta}{\gamma^{-2}+\theta^2}\right)^2$$

Maximum at $\theta = \dfrac{1}{\gamma}$

$$W \propto \begin{cases} \beta^2 & \beta \ll 1 \\ \ln(2\gamma) & \gamma \gg 1 \end{cases}$$
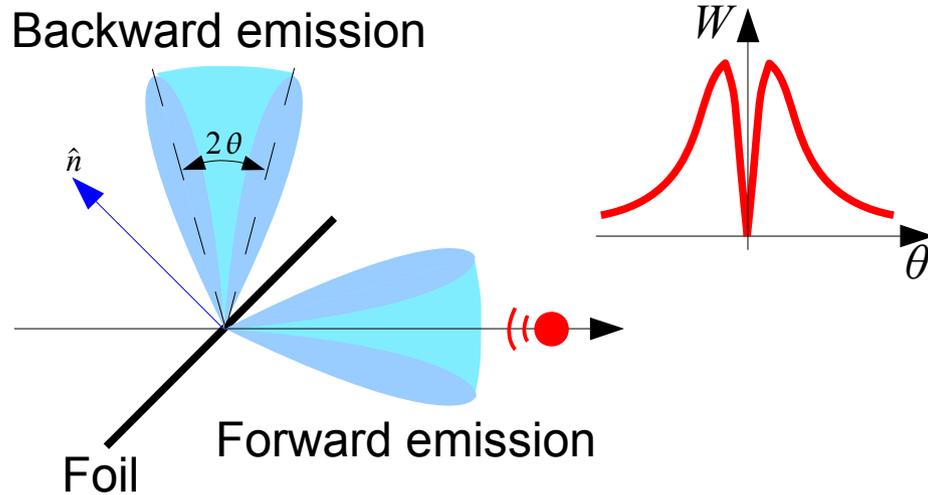
- Radiation is emitted when a charged particle crosses the boundary of different dielectric properties

- Radiation has defined angular distribution

- Radiation is radially polarized

- Thickness of radiator not important

For *50 MeV* electrons ~ *0.3 ph./el.*

For *100 keV* electrons ~ *0.001 ph./el.*

($\lambda \in$ [400, 600] nm)

Synchrotron radiation

Dipole magnet

$\theta = \dfrac{2}{\gamma}$
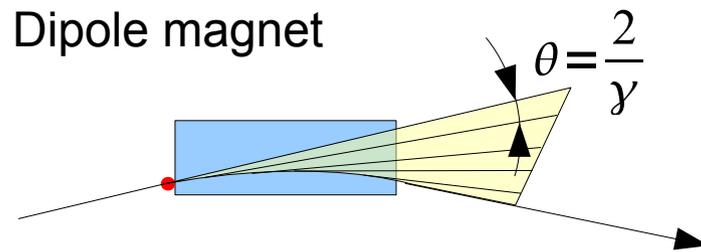
$$P = \frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{c\,e^2\gamma^4}{\rho^2}$$

Wiggler / Undulator

S   S   S

N   N   N

$dP/d\omega$

ω
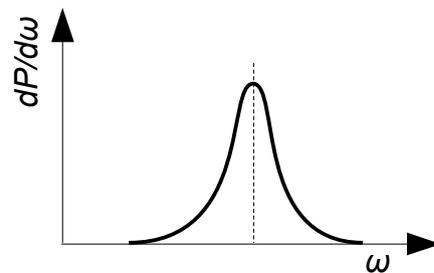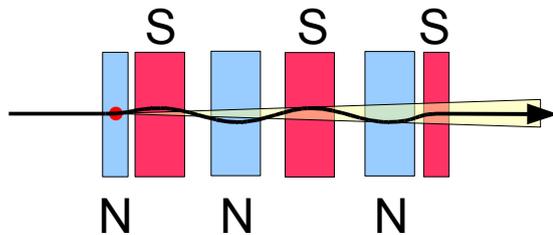
$\lambda \propto \dfrac{\lambda_u}{2\gamma^2}$ $W \propto B_0^2\gamma^2$

- Charged particles emit electromagnetic radiation when accelerated
  - Bremsstrahlung: reduction of velocity
  - Synchrotron radiation: change of direction
- Synch. rad. from dipole magnet emits in a fan
- Radiation from undulator has different properties

Synchrotron radiation (2)
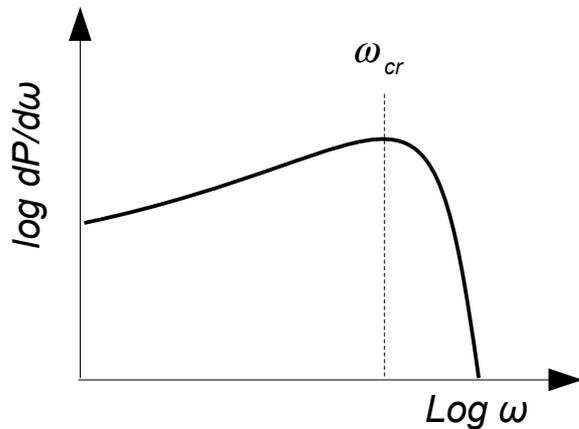
$$\omega_c = \frac{3}{2} \frac{c \gamma^3}{\rho}$$
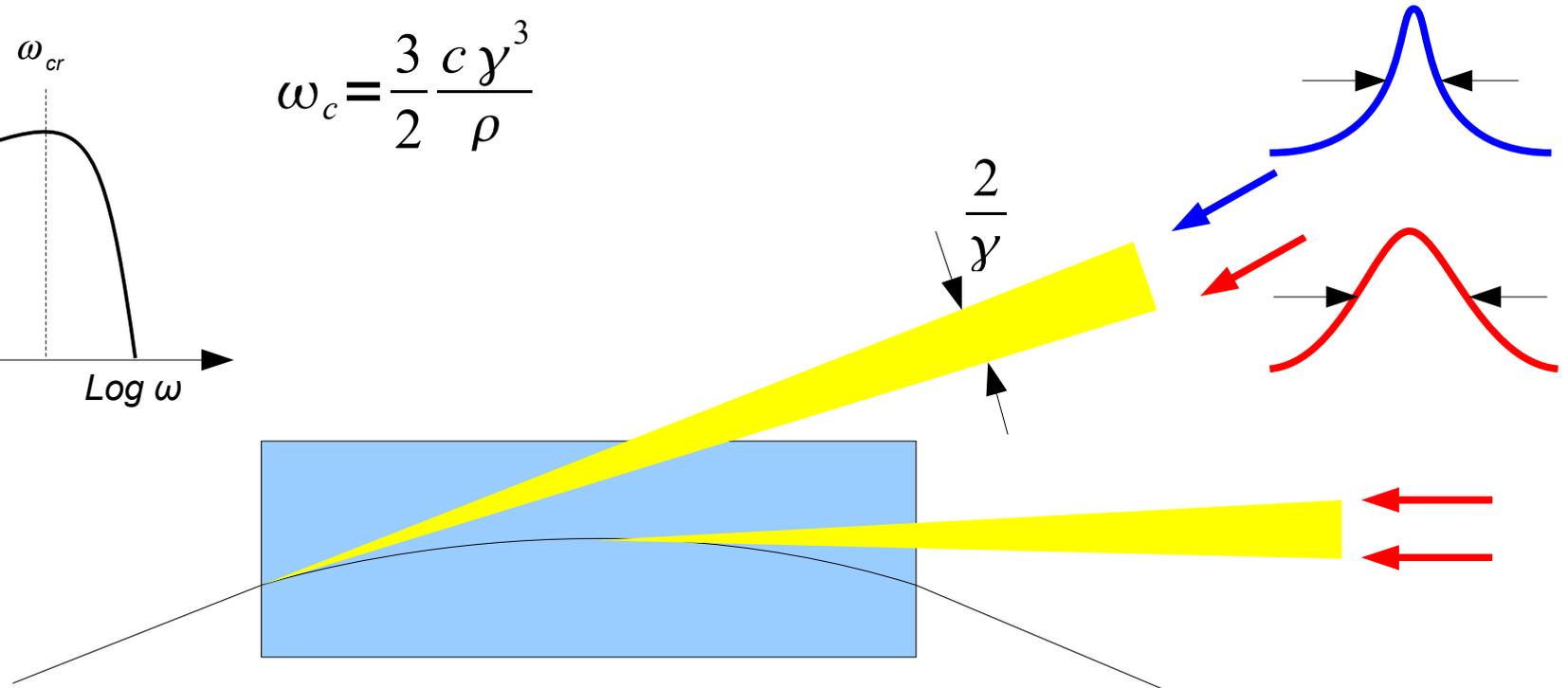
The red observers will see a pulse which duration is equal to the time it takes to the particle to be deviated by an angle 2/γ and an emission spectrum as the one depicted above.

The blue observer being at the edge of the emission cone will see a shorter pulse. As a consequence the spectrum of the emission will be broadened and extend to higher frequencies (shorter wavelengths)

(Inverse) Compton scattering

$$h\nu'$$

$$\nu' = \gamma^2 \nu$$

$$h\nu$$
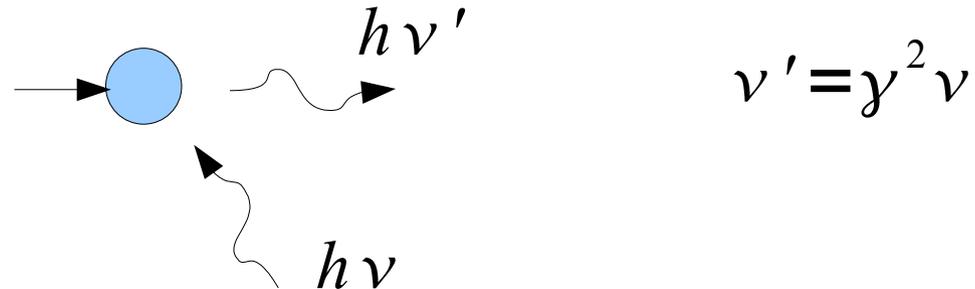
- A low energy photon (few eV) interacts with a high energy charged lepton (e-, e+)

- The photon gets boosted and gains energy to the expense of the particles

- Cross section is small, but usable for leptons, it is however too small for hadrons (protons)

Photo dissociation (H⁻ beams)
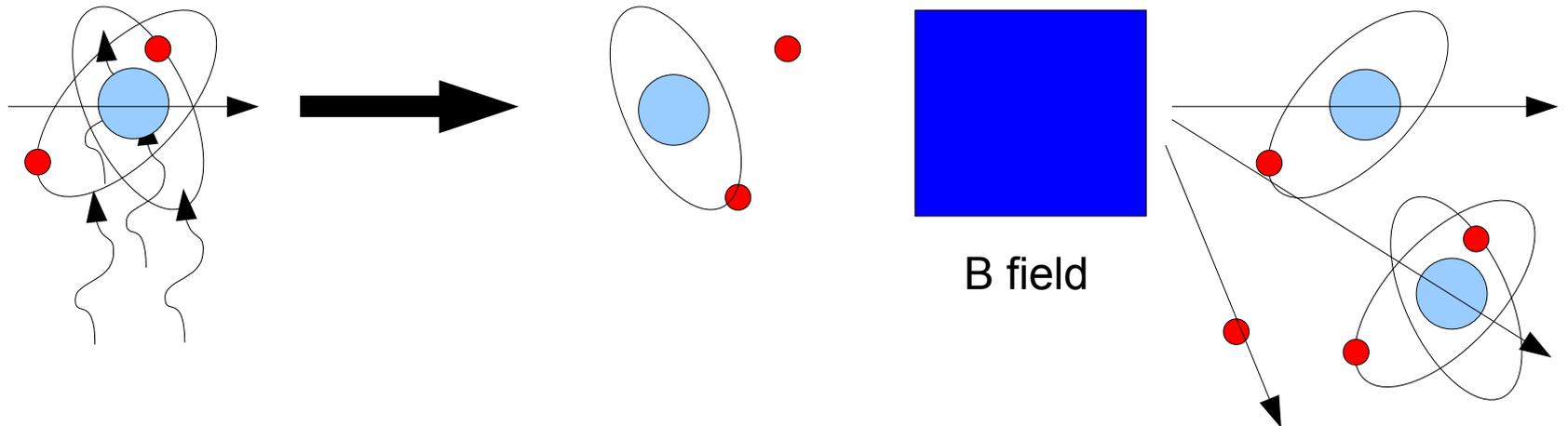
B field

- Photons from a laser are used to separate one electron from the H⁻ ion. This can be facilitated by external electric or magnetic fields

- Some of the ions will lose the extra electron and become neutral H_0

- The different particles can be separated by a bending magnet

Sampling particle distributions

- ## One dimension sampling
  - Wire scanners
  - Wire grids
  - Rest gas ionization monitors
  - Laser Wire Scanner
- ## Two dimension sampling
  - Screens and radiators
  - Synchrotron radiation

Wire scanner

- Scans a thin wire or a needle across the beam

- Detects secondary emission current or high energy secondary particles (scintillator + PMT)
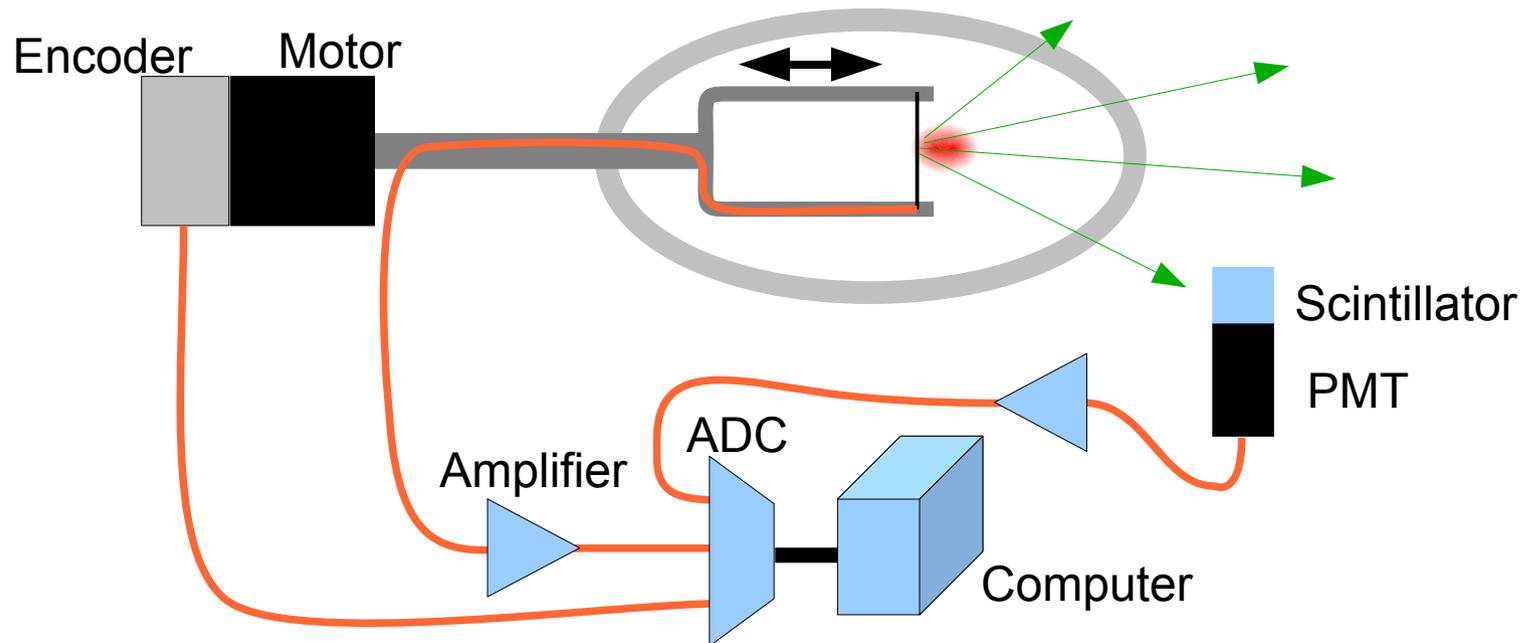
Wire scanner (2)
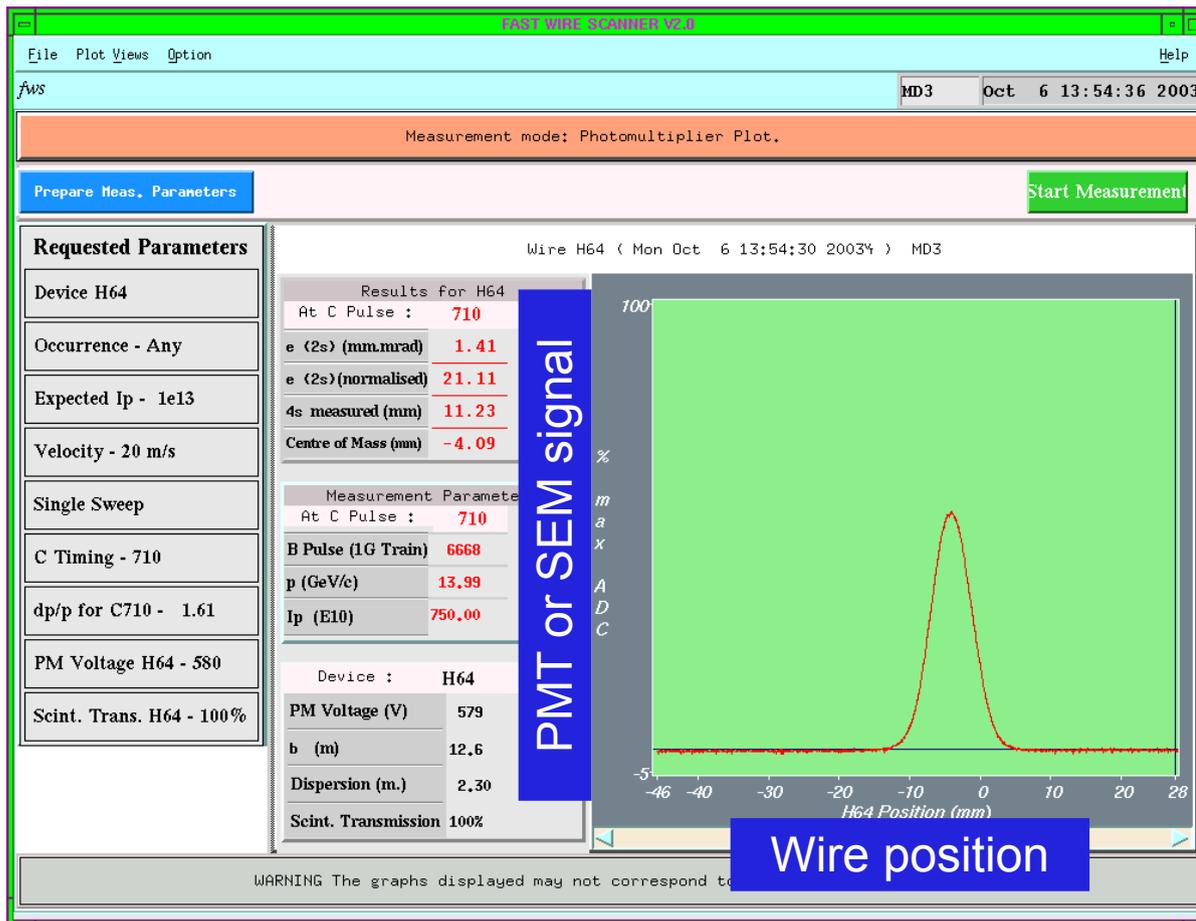
- The position of the wire is read by resolver or an encoder and sampled simultaneously with the signal

- On complex, fast mechanism the error on the position can be the largest contribution, need calibration

Wire scanner (3)

- Secondary emission
  - Good for low energy beams (no high energy secondary)
  - Small signal
  - If the wire becomes too hot it can start to emit thermionic electrons spoiling the measurement
- High energy secondary
  - No problem with wire heating (well...)
  - Strong signal
  - Detection may be non homogeneous leading to distorted profiles
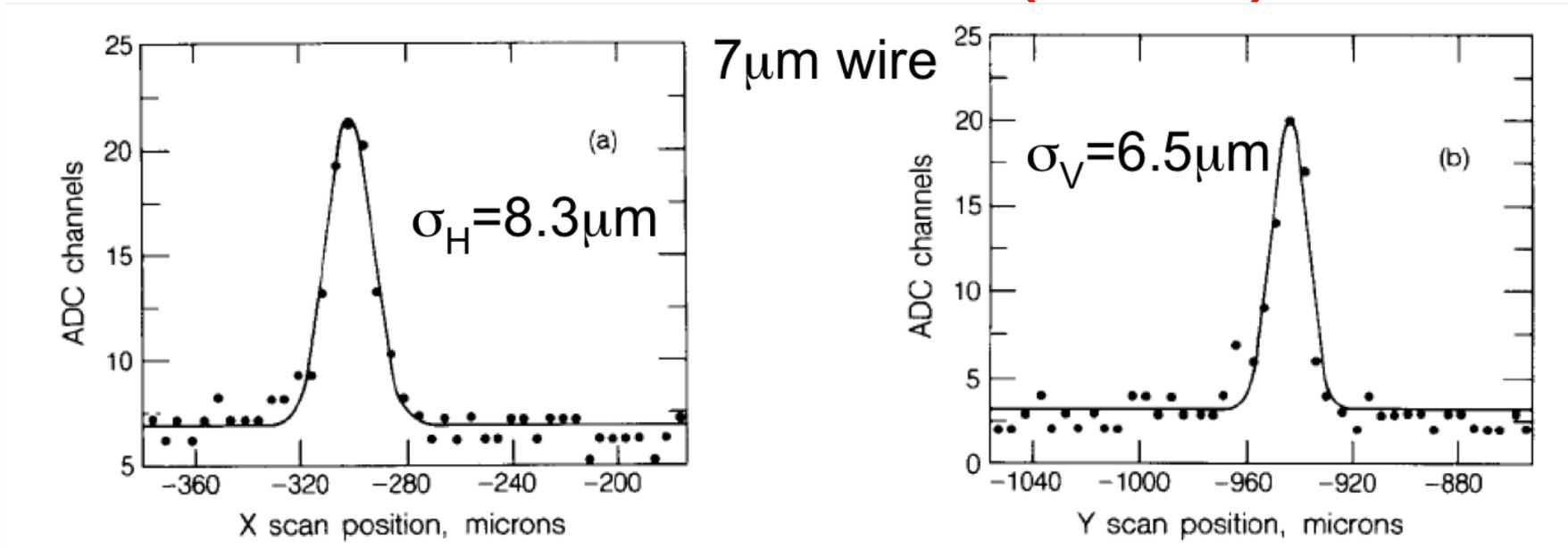
Wire scanner (4)

CERN "flying wires"
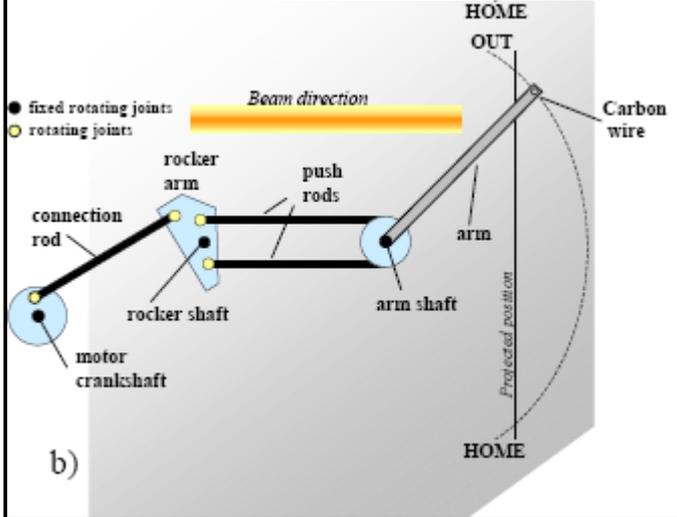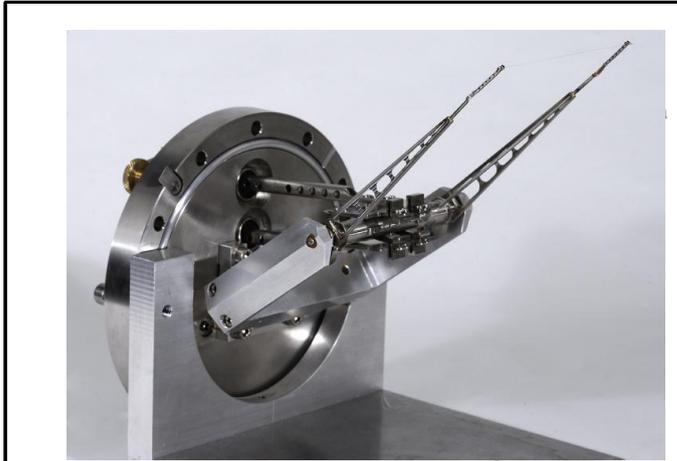
SLAC SLC high resolution 3 axis scanners
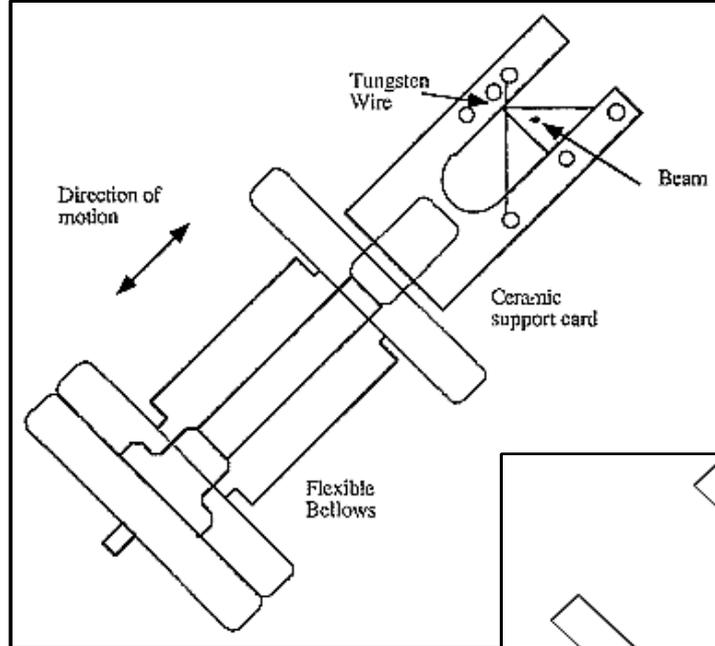
KEK ATF high resolution 3 axis scanners

Wire Scanner (SLC)

7 μm wire

$\sigma_H = 8.3 \mu m$

$\sigma_V = 6.5 \mu m$
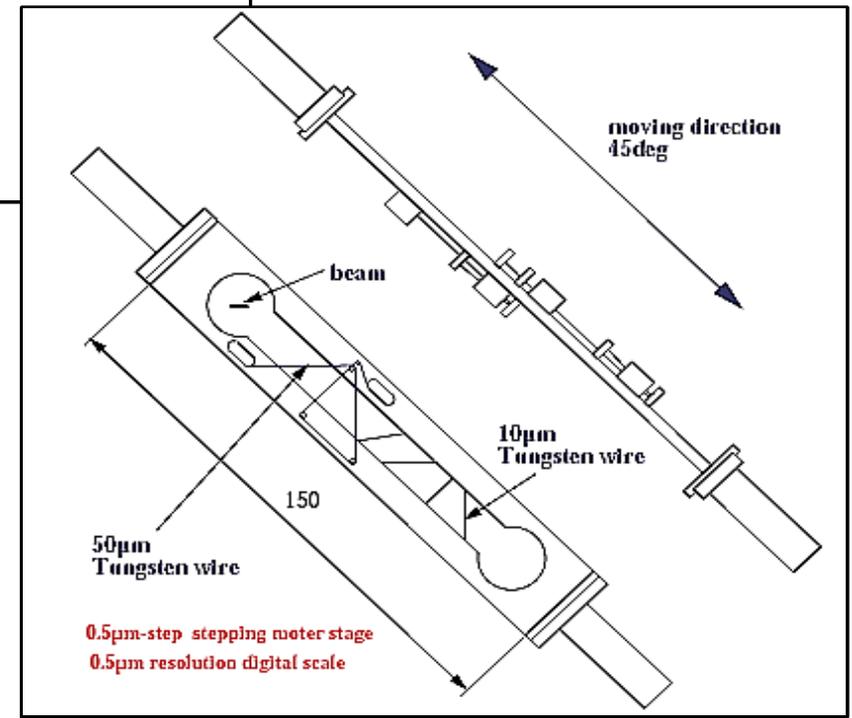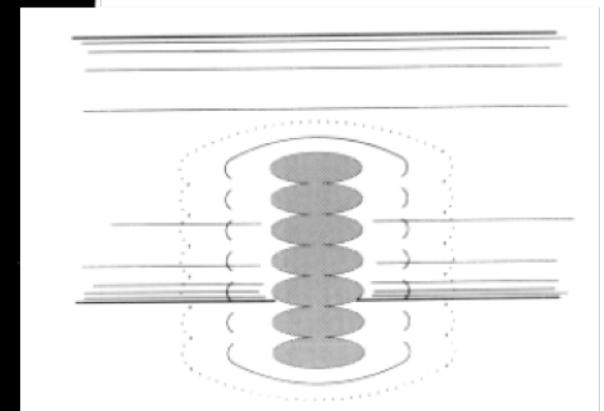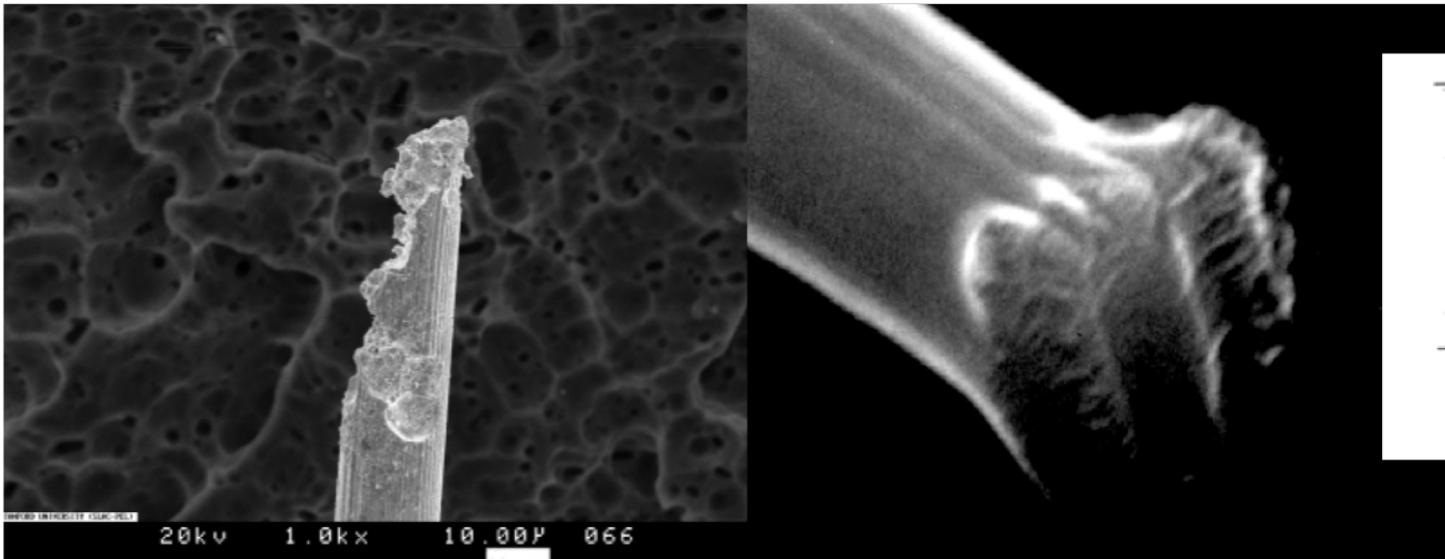
Wire scanner (5)

- Fast scanners

  - Present limit is around 20 m/s

  - Usually rotational mechanism

  - Acquire profile snapshots during acceleration without need of plateaus

  - Reduce wire heating (short scan time)

- Slow scanners

  - High wire position accuracy

  - Possibly thinner wires (low accelerations)

  - More reliable mechanisms

  - Long(er) measurement time

  - Tighter intensity limits

SEM Grids (Harps)

- The SEM current from each wire or strip is acquired independently

- Complex (=expensive) cabling/electronics

- Wire spacing down to a few hundreds microns

Wires

Head amplifier   Integrator   ADC   Computer
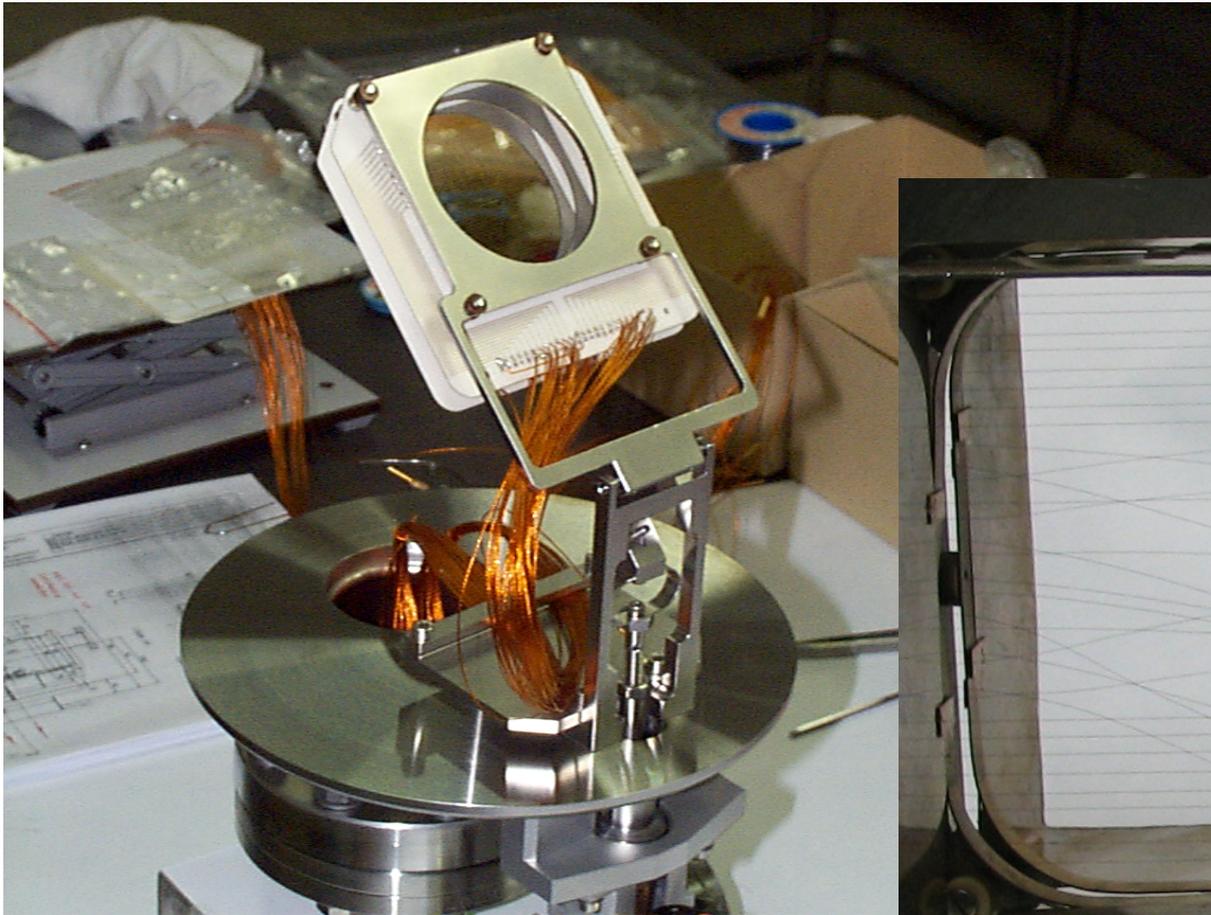
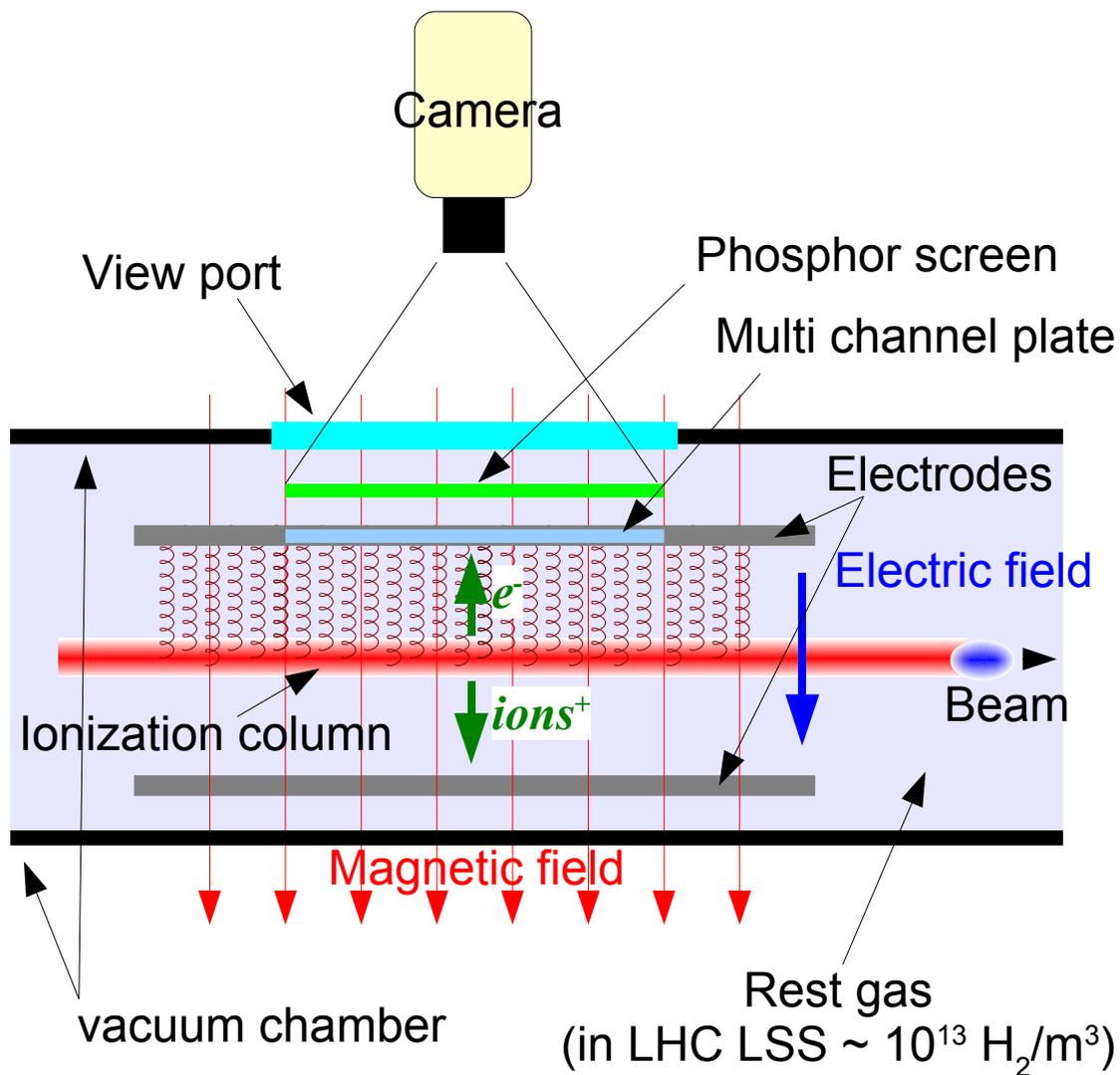Ceramic support

Can be hundreds of meters
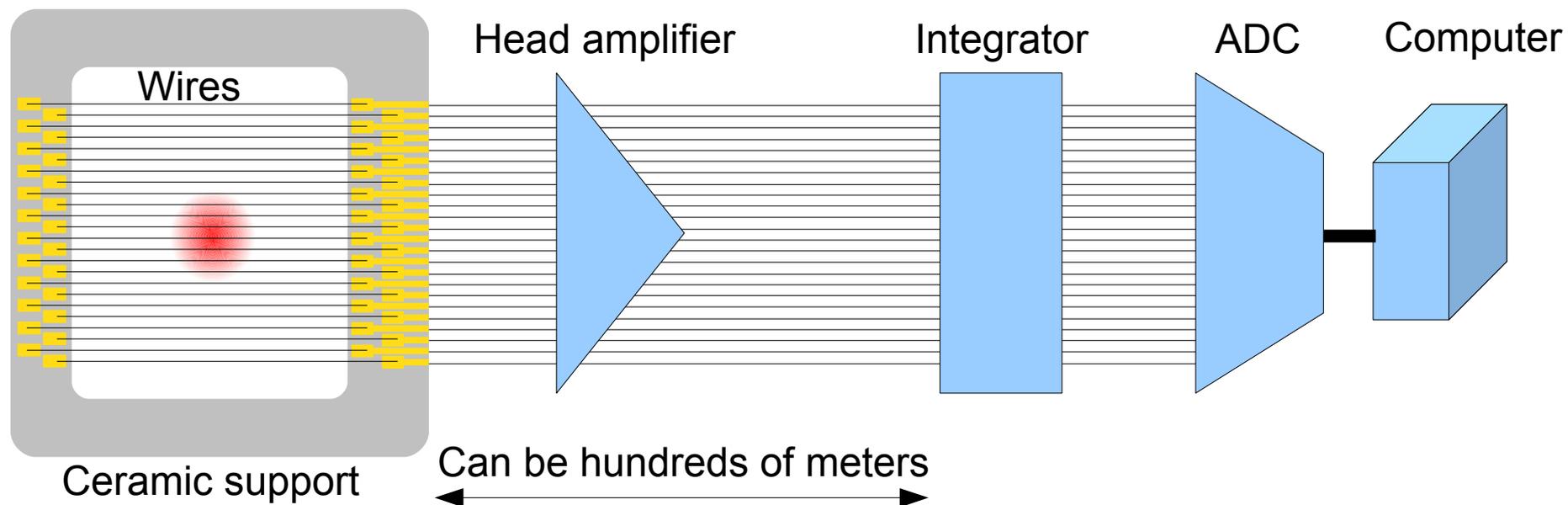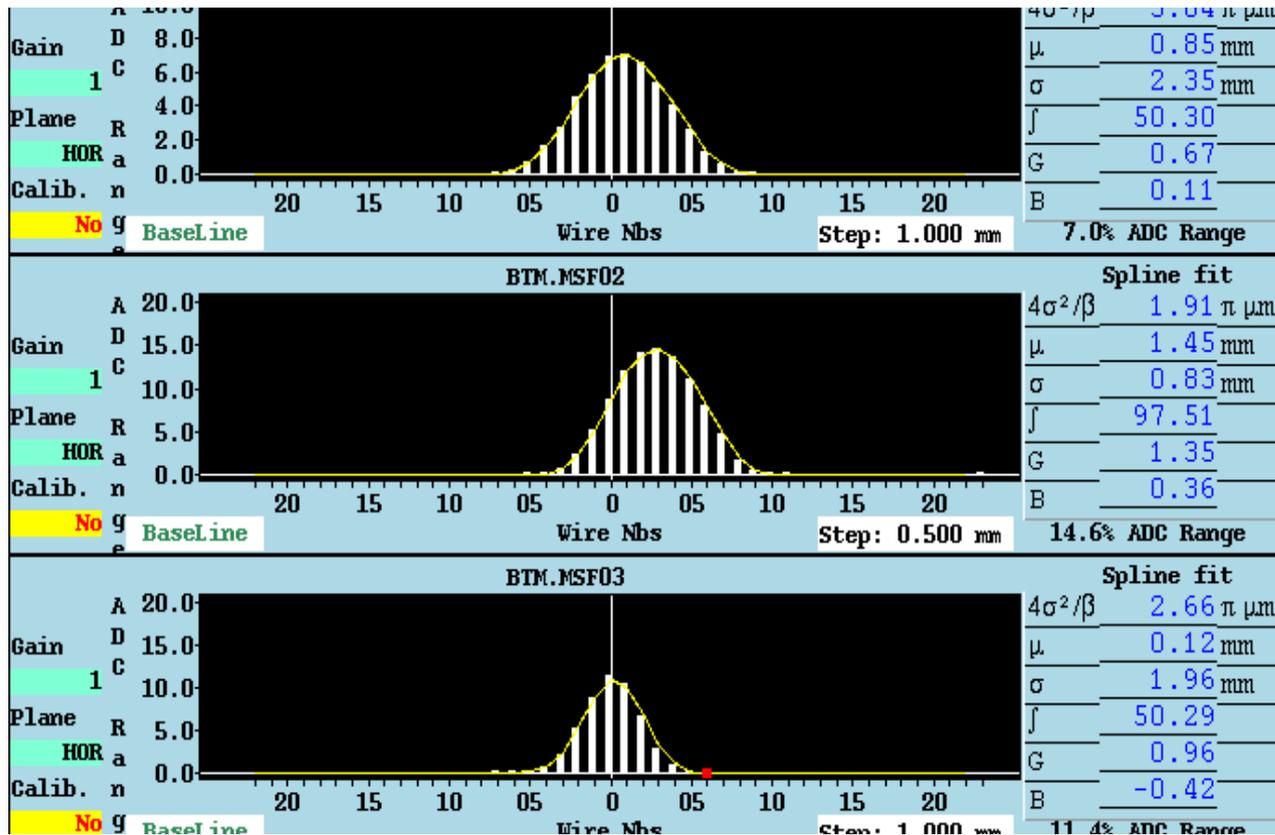
SEM Grids (2)

- Advantage of Grids is single shot measurement

- Time resolved measurement is possible (up to ~100 MHz)

- Damage to a single wire can make device unusable

# SEM Grids (3)

Ionization profile monitor

Camera

View port

Phosphor screen

Multi channel plate

Electrodes

Electric field

e^-

Ionization column

$ions^+$

Beam

Magnetic field

vacuum chamber
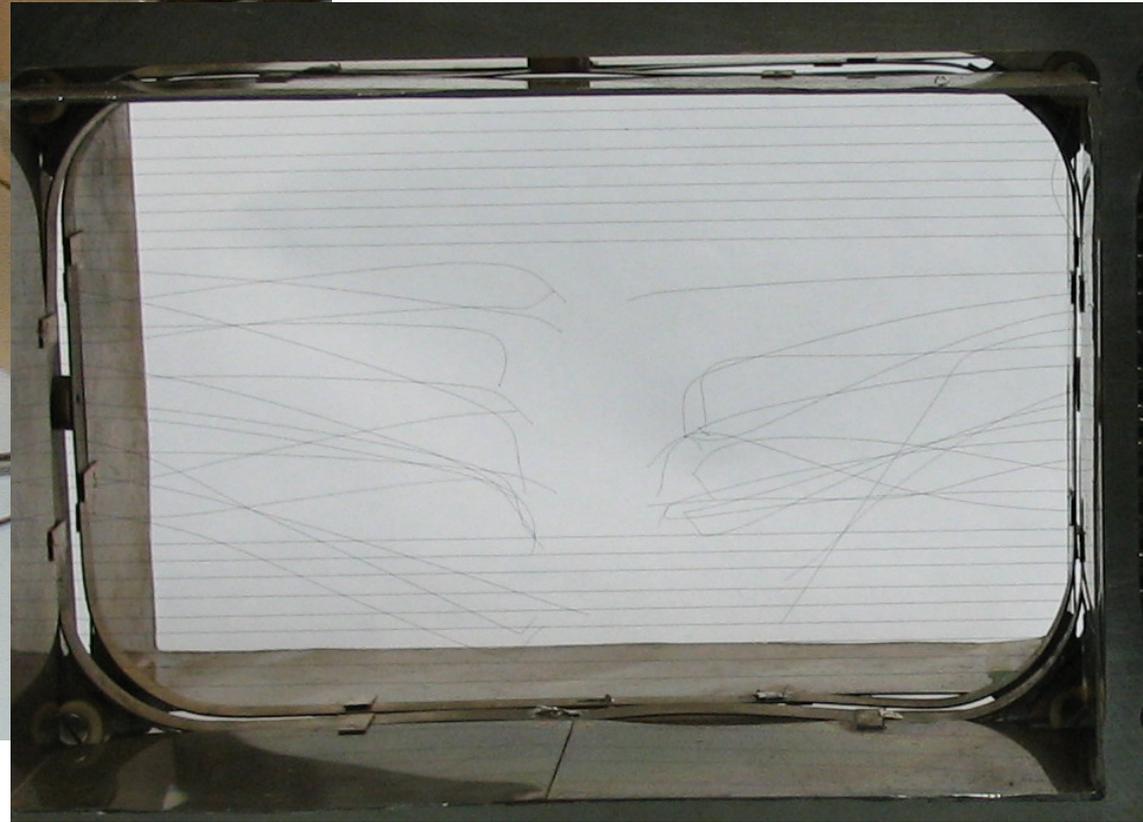
Rest gas
(in LHC LSS ~ 10^{13} H_2/m^3)

- Beam creates ionization column in rest gas

- Electric field drifts electrons toward detector

- Magnetic field guides the electron

- MCP+phosphor+CCD detects electrons

- If E is reversed ions can be detected instead of electrons (less need for B field)

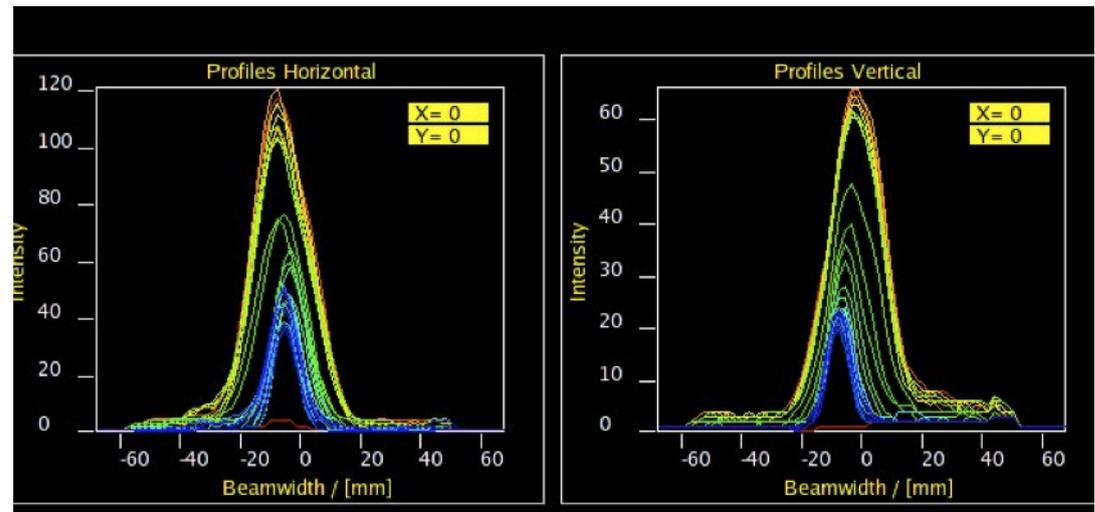# Ionization profile monitor (2)
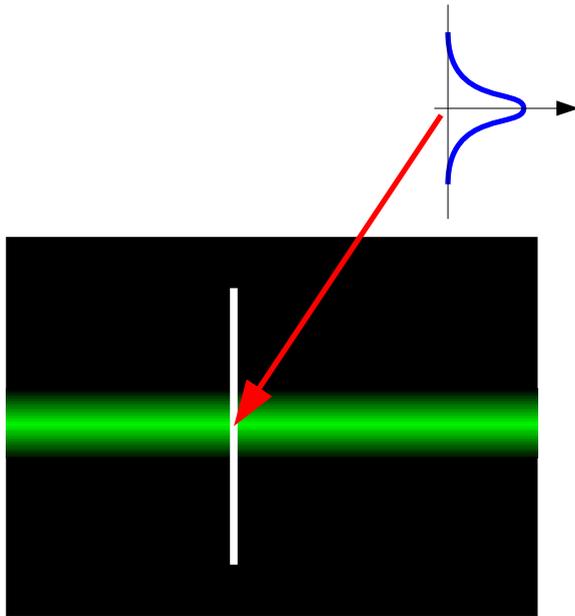
- Image shows a stripe

- Intensity profile of stripe proportional to beam profile

- Detector measures only one plane

- Transverse drift of electrons introduces broadening (need intense B field) and creates "tails"

IPM (GSI)
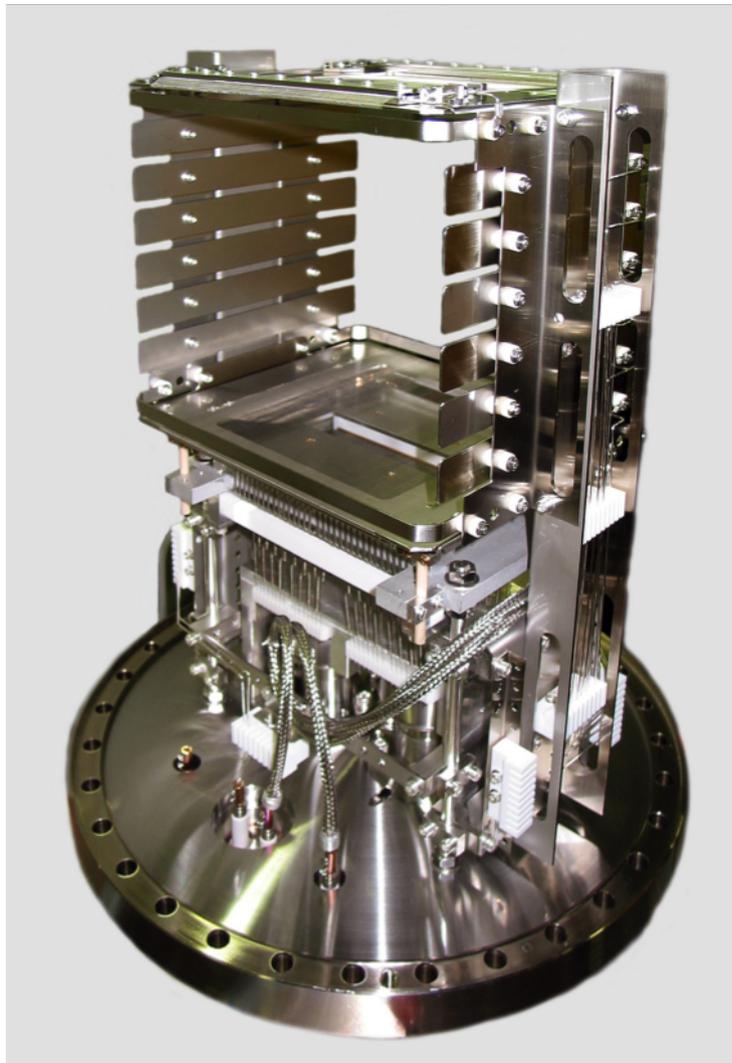
- IPMs allow the continuous monitoring of the transverse plane

- Needs a "minimum" of rest gas

Laser Wire Scanner

- Collide a high power, focused, pulsed laser with an electron beam

- X-ray or γ-ray are produced by Inverse Compton Scattering

- Detect the x-ray / γ-ray or the degraded electrons downstream

- Can also be used on H$^-$ beams exploiting the photo neutralization detecting either the neutral H atoms or the freed electrons
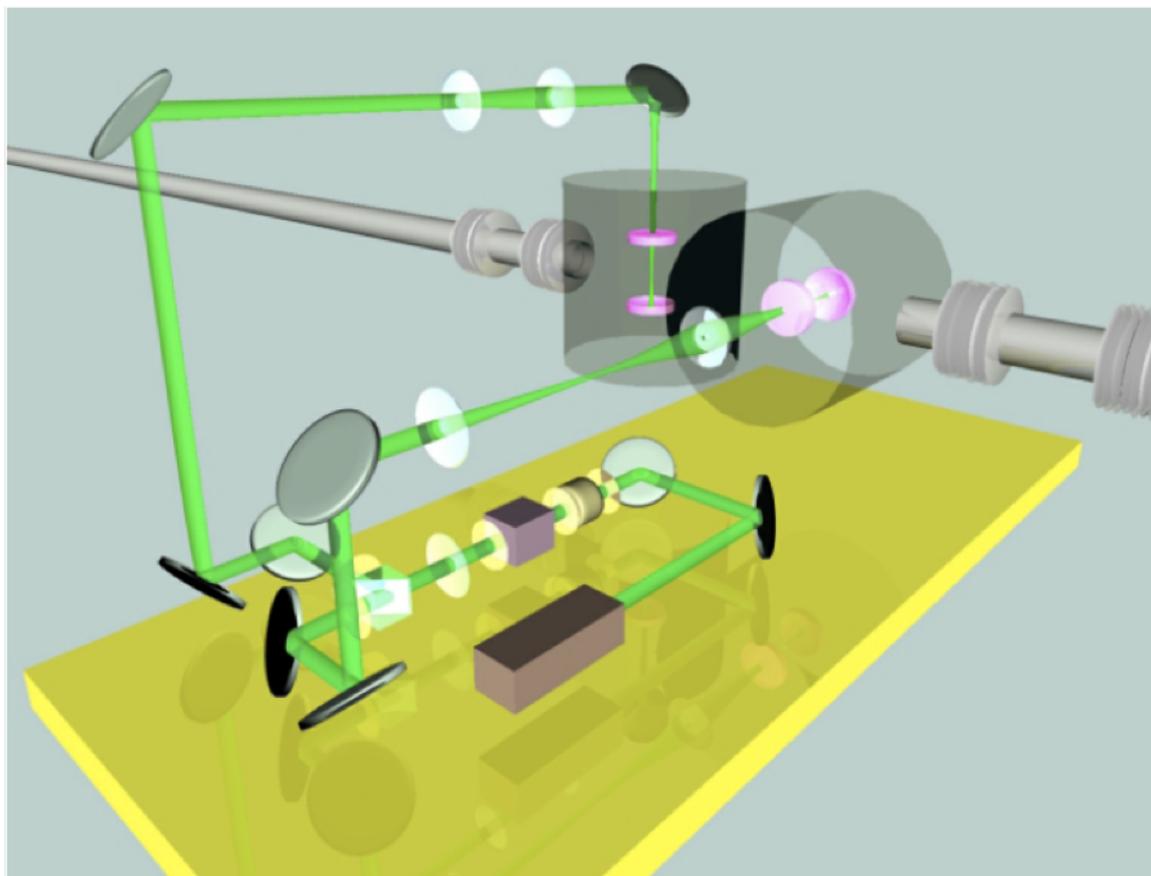
$$h\nu_{sc} \propto \gamma^2 h\nu_{laser}$$
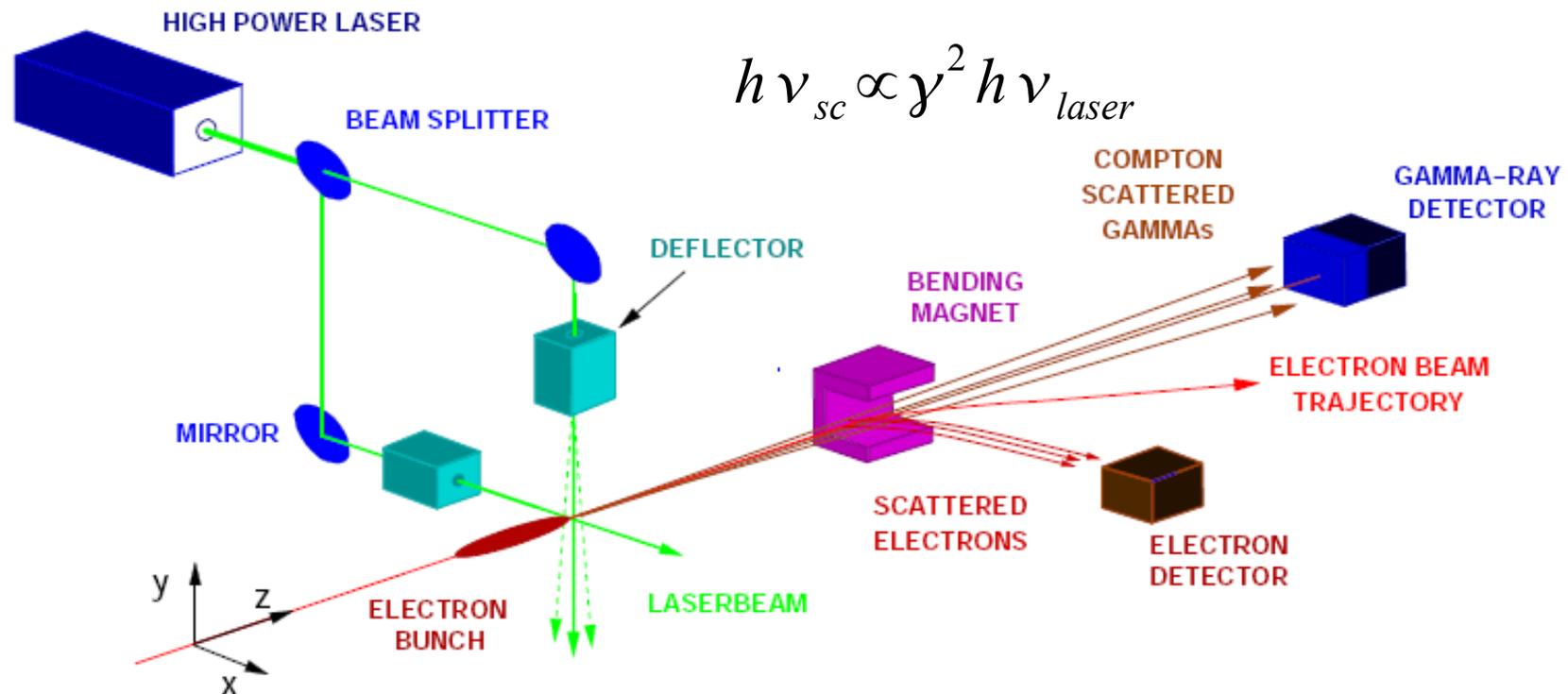
Laser Wire Scanner (2)
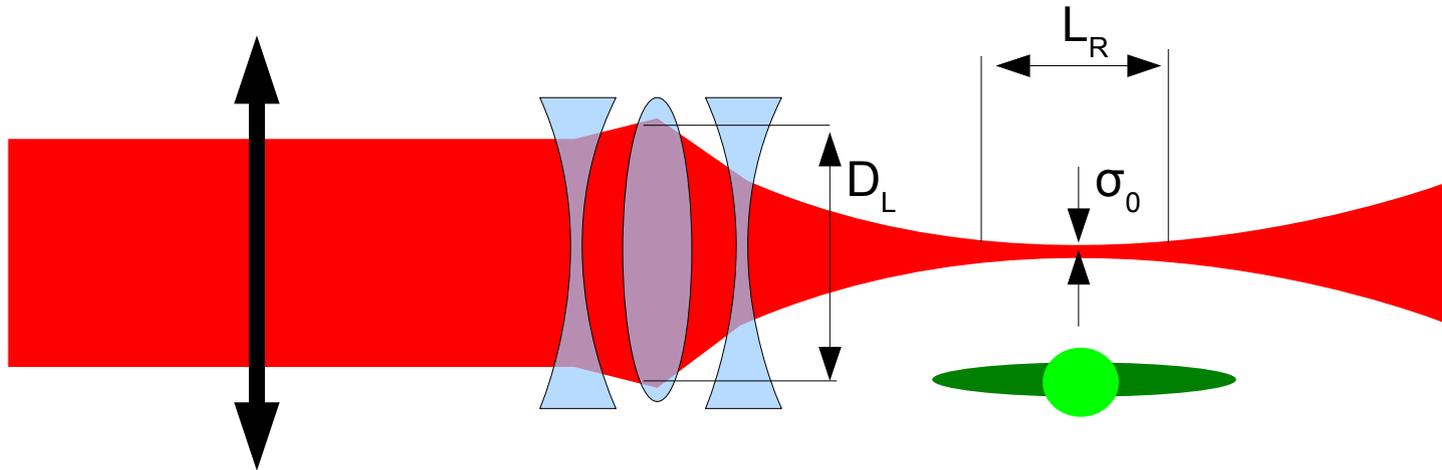
There is a physical limit on the smallest laser spot size and on the distance over which it can remain focused

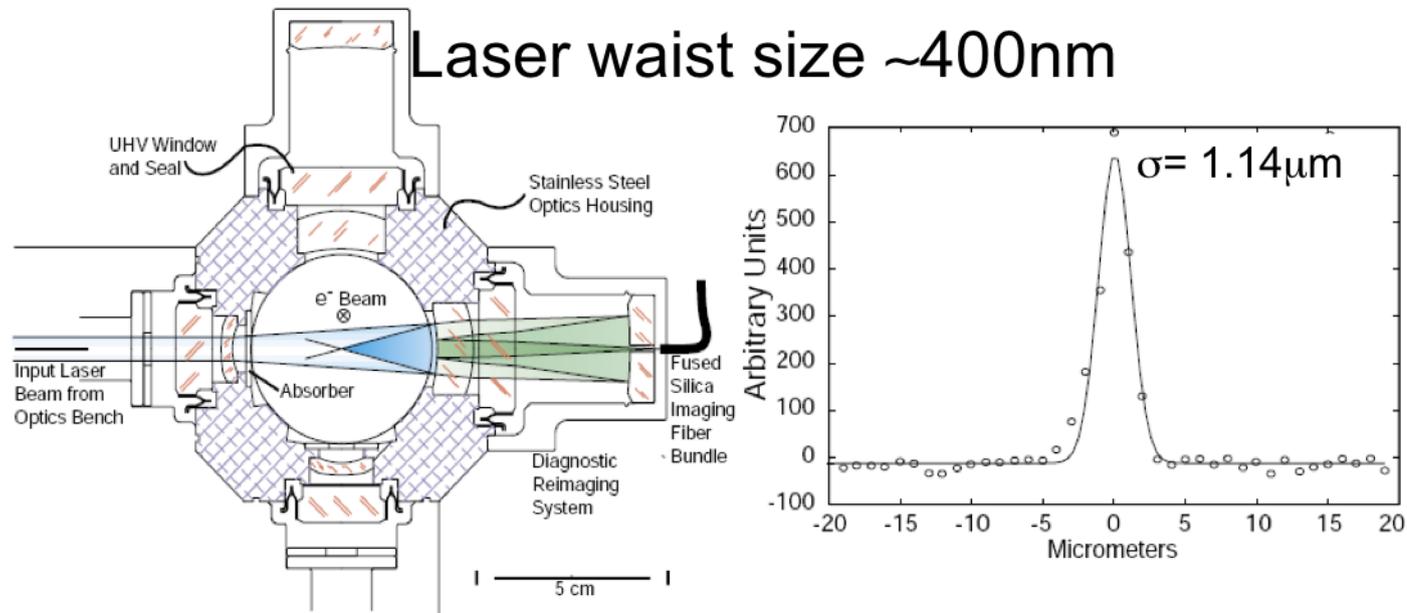$$\sigma_0 = \frac{\lambda f}{D_L} = \lambda f / \#$$

$$L_R = \frac{2\pi\sigma_0^2}{\lambda}$$

Laser Wire Scanner (3)

- At ATF original solution
- Instead of using a powerful laser an optical cavity surrounds the electrons beam
- The whole table is scanned

Laser Wire Scanner (SLC)

Laser waist size ~400nm

Laser Wire Scanner (4)

- High resolution LWS require

  - High power, high quality lasers (mJ, ps, $M^2\sim1$)
  - Complex focusing systems
  - Precise scanning systems (as an alternative the beam can be moved around)

- The resolution of the laser wire scanners is limited by the minimum waist size (of the order of the wavelength)

- A strongly focused laser beam will have a short waist length (Rayleigh length) and is not adapted for small beams with large aspect ratios

- Other limiting factors are laser stability, vibrations, x-ray detection (if low energy x-rays)
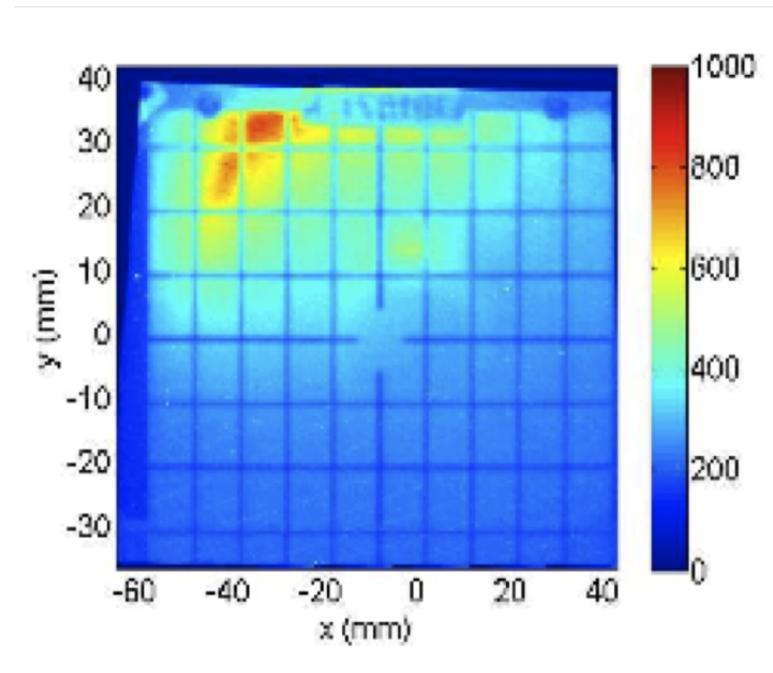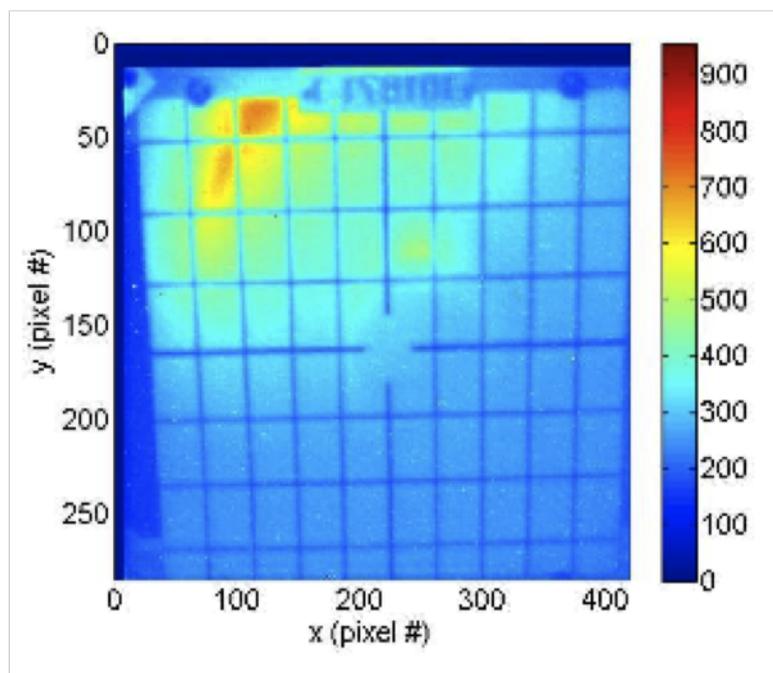
Scintillating screens
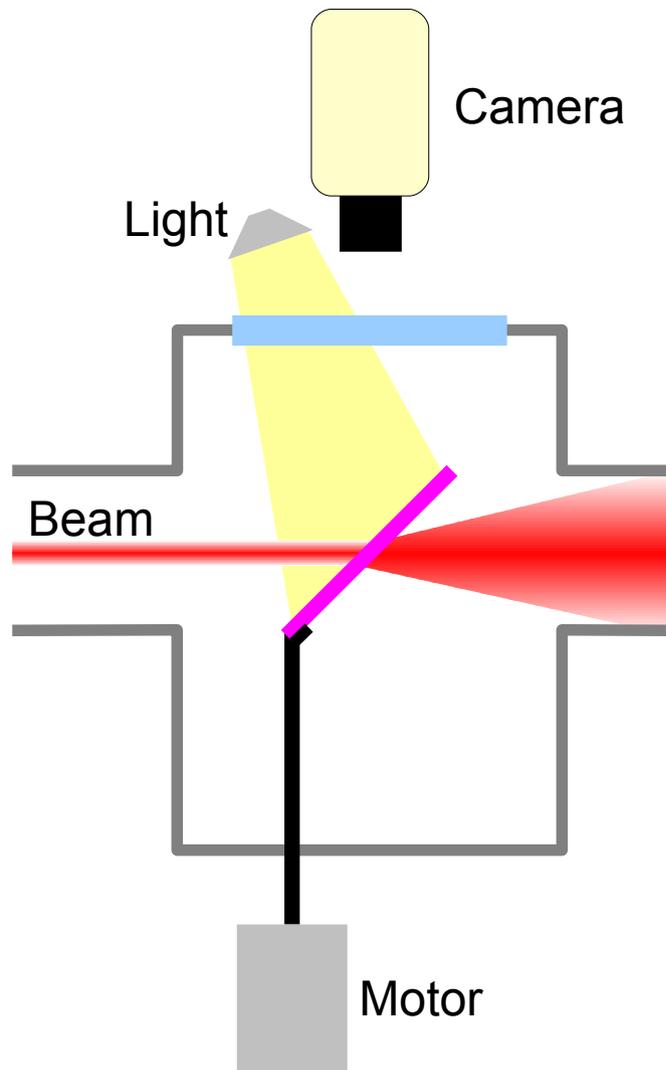
- Particles passing trough the screen excite atoms and molecules

- The screen emits photons that can be observed with a TV camera (CCD)

- Multiple scattering inside screen increases beam divergence

- Typical screens are Al_2O_3 1mm thick. Robust and good for beam observation, but not for precise profile measurements.
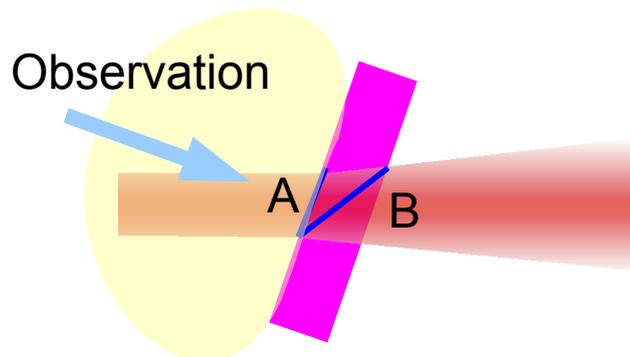
Scintillating screens (2)

- Optical setup may introduce deformations (ex 45° screens)

- Need to perform off line corrections and calibrations

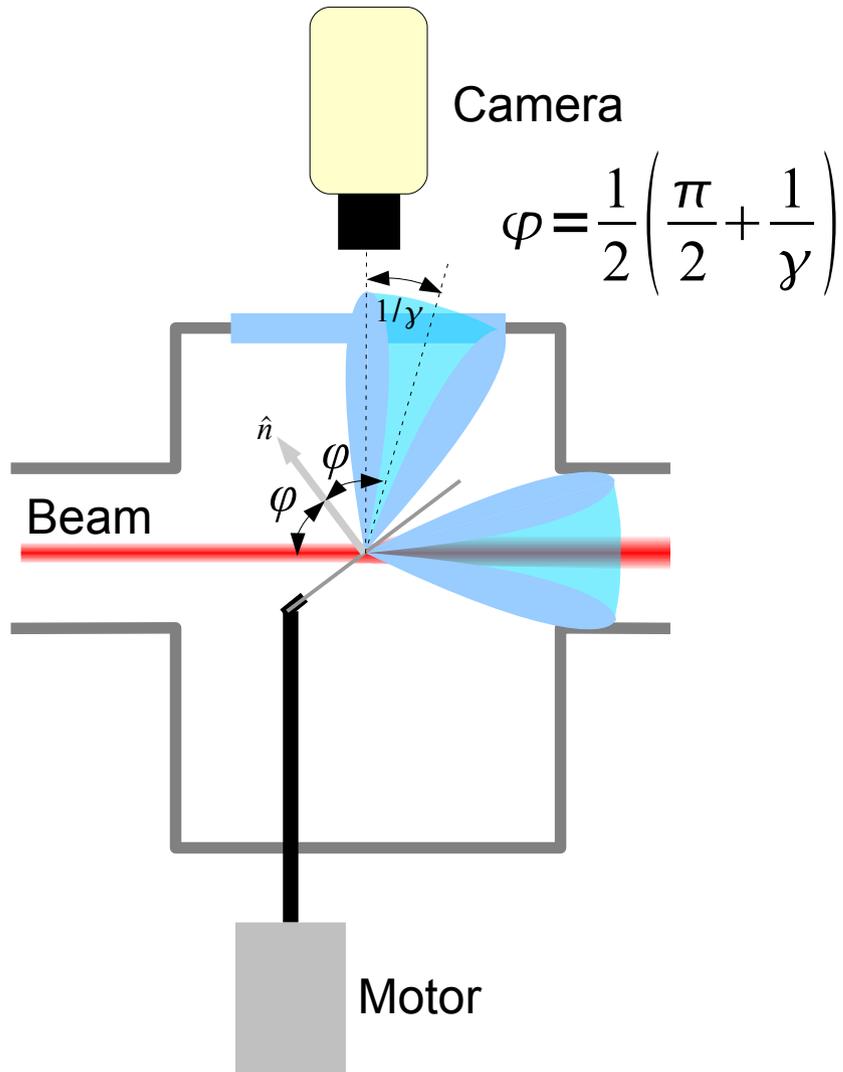# Scintillating screens (2)
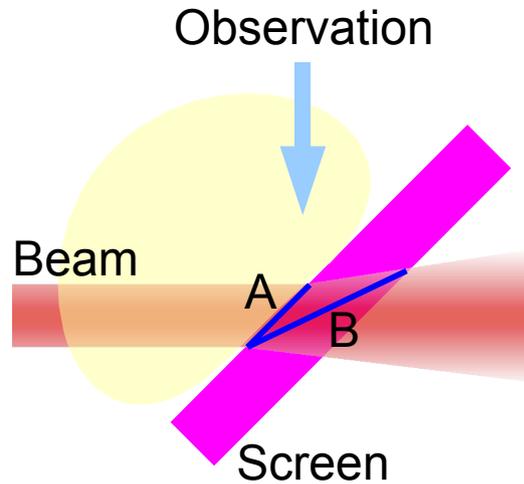
A is what we would like to observe
B is what we really obtain

- Photons created inside the screen can escape

- The image observed is distorted

- Thickness of the screen should be small (compared to beam size)

- Observation at 90° is easy to use, but very bad for quality, also for field depth and aberrations

OTR radiators

$$\varphi = \frac{1}{2}\left(\frac{\pi}{2} + \frac{1}{\gamma}\right)$$

- Use backward emission

- Reflecting properties of radiator are important (metal foil or metal coating)

- Use thin foil (few µm) or "wafers", typically Al coated Si ~300 µm

- Angle of radiator depends on beam momentum

- For dense beams use carbon foils or SiC wafers

Optics primer
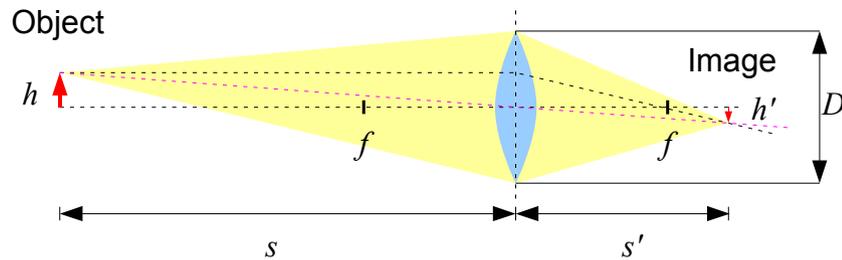
$$m = \frac{h'}{h} = \frac{s'}{s} \qquad\qquad f = \frac{s\,m}{(m+1)}$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \qquad\qquad s' = \frac{s\,f}{s-f}$$

Typical case: *s= 300 mm, m=0.2 ⇒ f= 50 mm, s'= 60 mm*

For OTR if $1/\gamma < D/(2\,s)$ the whole lobe can be collected and the observation angle is equal to the specular reflection. If $1/\gamma > D/(2\,s)$ only part of the lobe can be collected and the specular reflection differs from the observation angle
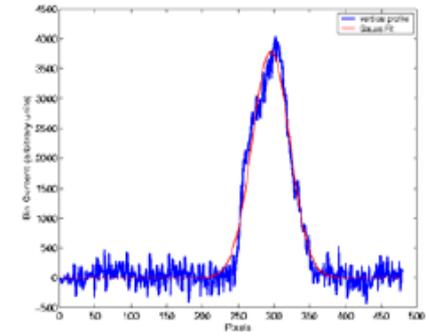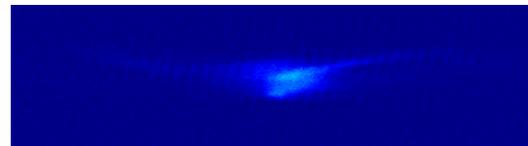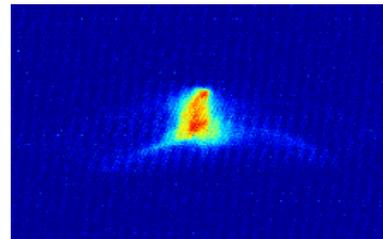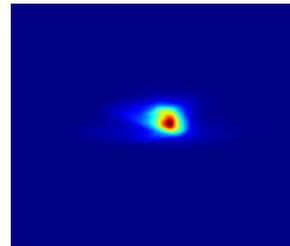
A "good" CCTV *50 mm* lens has *f/# ~ 1.4*

$$D = \frac{f}{f/\#} = \frac{50}{1.4} = 36\,mm \qquad\qquad \frac{D}{2\,s} = \frac{36}{600} = 0.06 \qquad\qquad \gamma_{min} = \frac{1}{0.06} = 17$$

For γ larger than γ_{min} the whole lobe can be observed
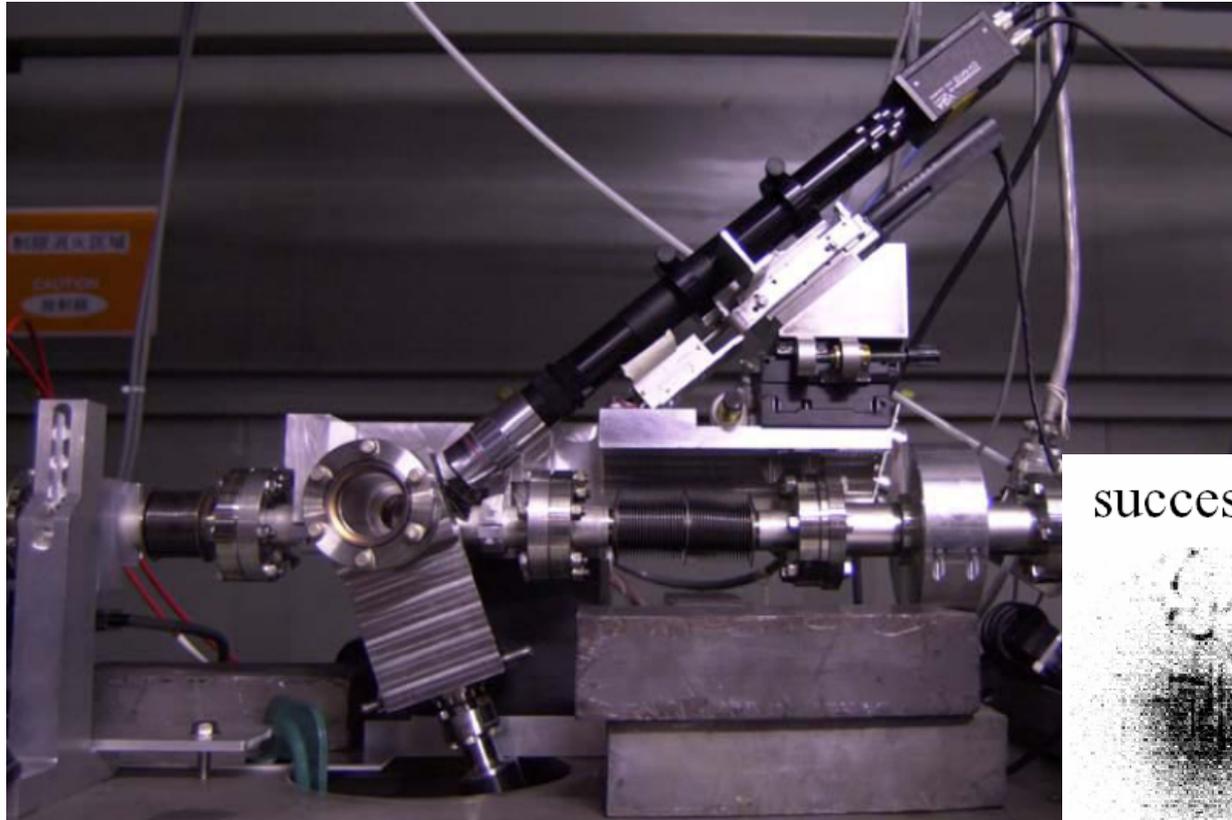
OTR example (DESY)
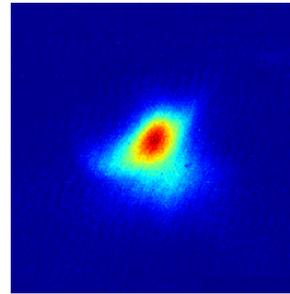
Often beams are far from Gaussian especially in linacs

Camera must be protected from radiation requiring a complex optical lines

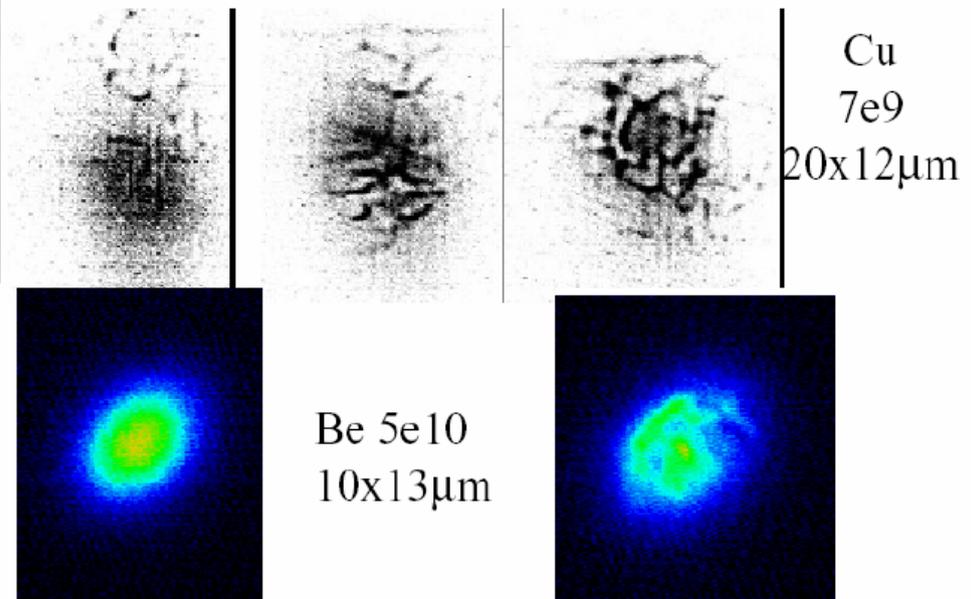Filters are needed to avoid saturating the camera

OTR example (KEK)

Small beams can be very dense posing a serious problem with radiator damage
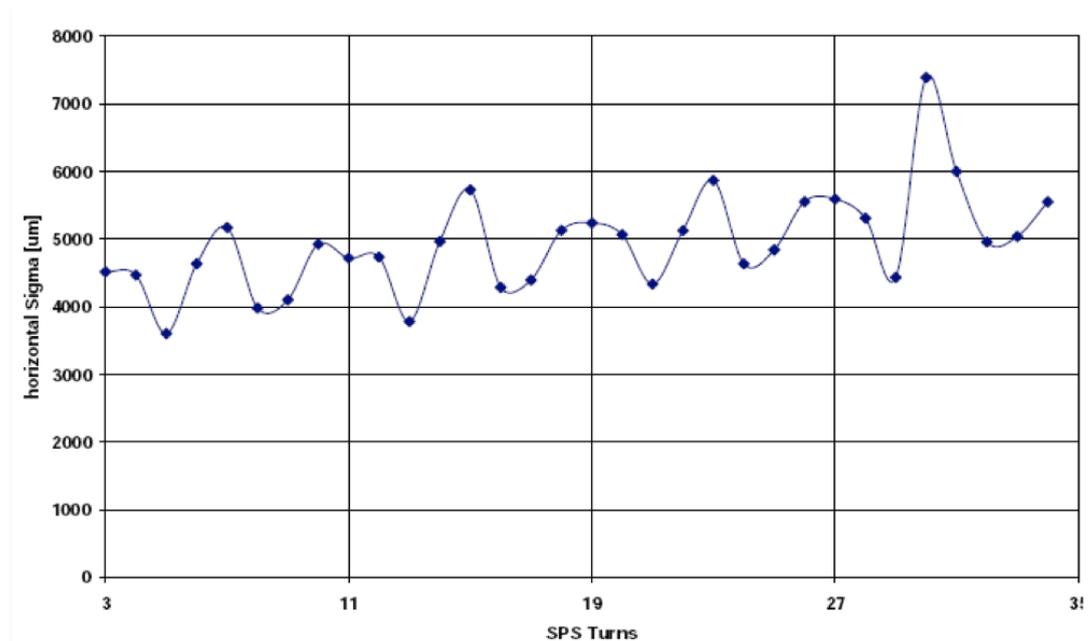
Choosing the right material is essential

successive images illustrating damage:
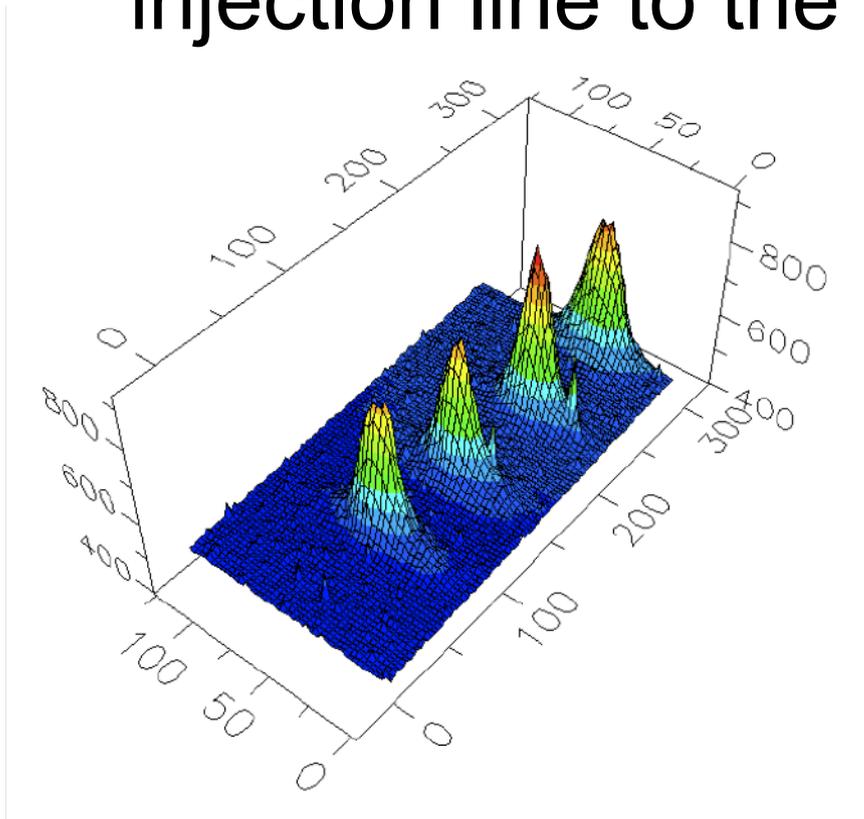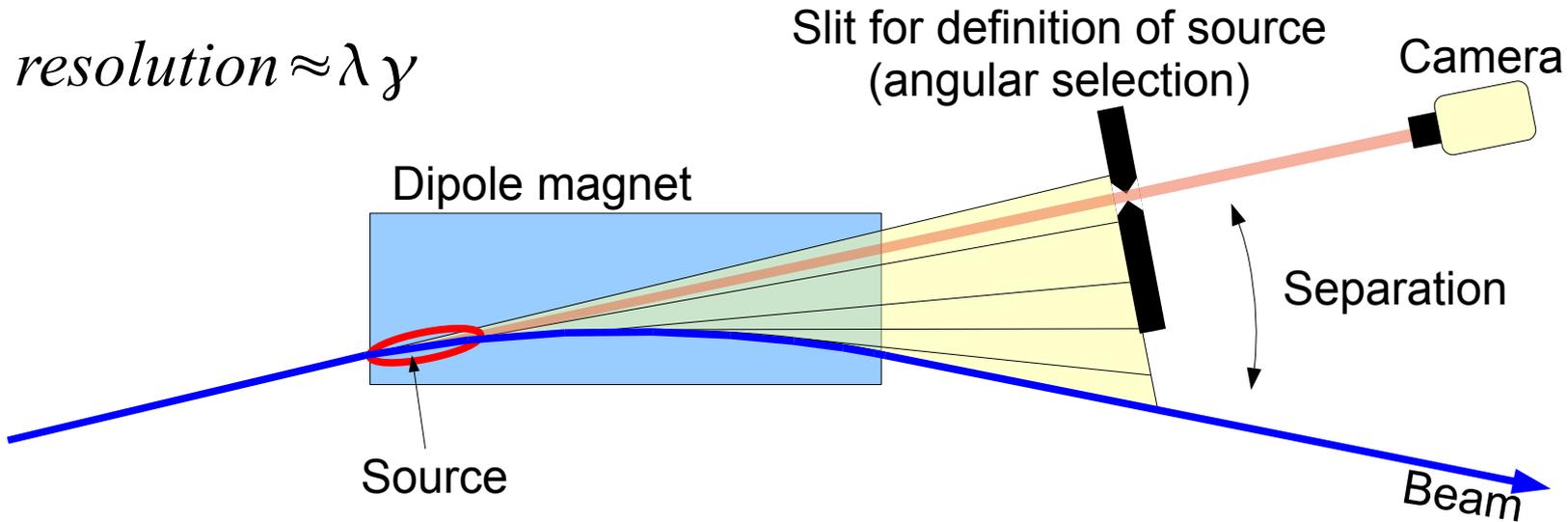
Cu
7e9
20x12μm

Be 5e10
10x13μm

$$\Delta T(x,y) = \frac{dE}{dx} \cdot \frac{i(x,y)}{c_p}$$

OTR example (SPS)

- Consecutive images at each turn are acquired
- This can be used to tune the matching of the injection line to the ring

Synchrotron radiation

$$resolution \approx \lambda\,\gamma$$

Slit for definition of source (angular selection)

Camera

Dipole magnet

Separation

Source

Beam

- Radiation inside magnet is constant

- Radiation at the entrance and exit edge has higher frequency components (shorter pulse) "edge radiation"

- Magnet also useful for separating photons from particles

- Source normally near entrance or even entrance edge

- Resolution limited by diffraction

Light sensors

- 1D sensors (Can be fast up to hundreds of MHz)
  - Photo diode array
  - Linear CCD
  - Segmented photomultiplier
- 2D sensors (usually slow ~50Hz, possible up to 100 kHz)
  - image CCD
  - image CMOS
  - (Segmented photomultiplier)

Light sensors (2)

- 1D arrays (photo diodes or photo multipliers)
  - Parallel readout of each channel allows high speed, but limits resolution
- 1D CCD
  - serial readout, good resolution, but reduced speed
- 2D CCD or CMOS
  - serial readout, very good resolution, but reduced speed.
  - Special sensors with local memory and partial parallel readout allow higher acquisition speed.

Image Intensifiers
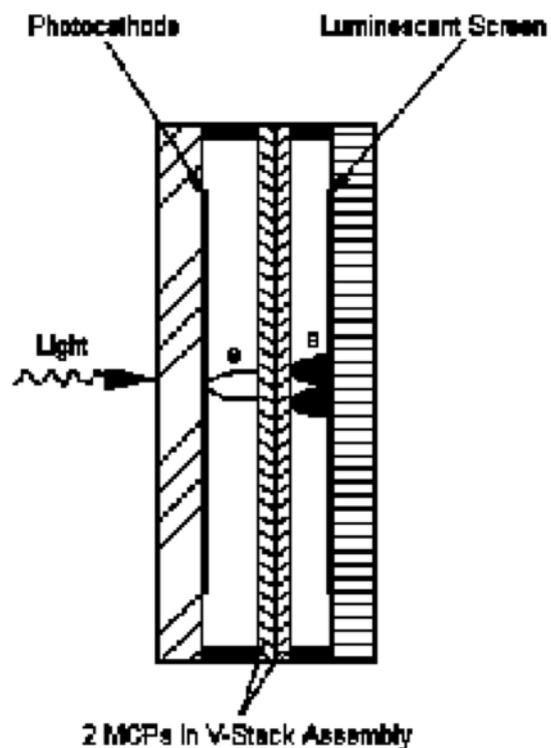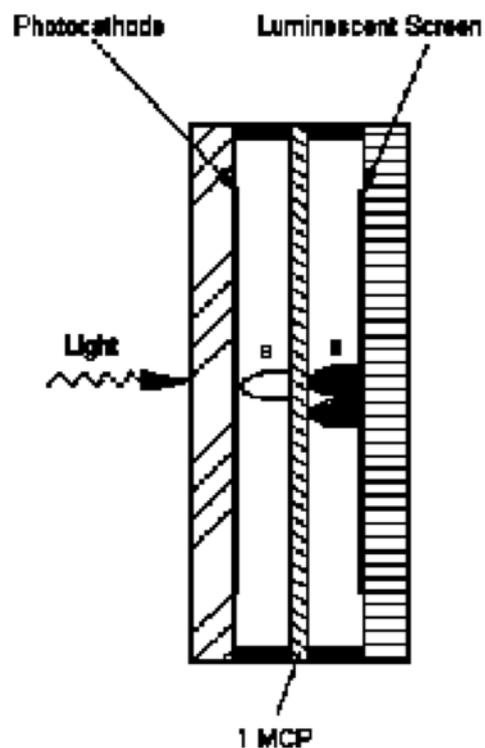
- Image intensifiers converts photons into electrons, multiply them, and then convert them back into photons, maintaining spatial information

- It is also possible to gate them and use them as shutters

Light sensors (3)

- Photomultipliers are radiation resistant (glass and metal)

- CCD and CMOS are silicon based and thus not very tolerant to radiation

- Tube cameras (ex. VIDICON) are radiation hard, but have worse resolution and sensitivity

- Special fast cameras contain loads of memory and electronics and are very sensitive to radiation (and expensive)

- Sensitivity of image sensors can be increased using image intensifiers, but usually at the expense of resolution

1D vs. 2D
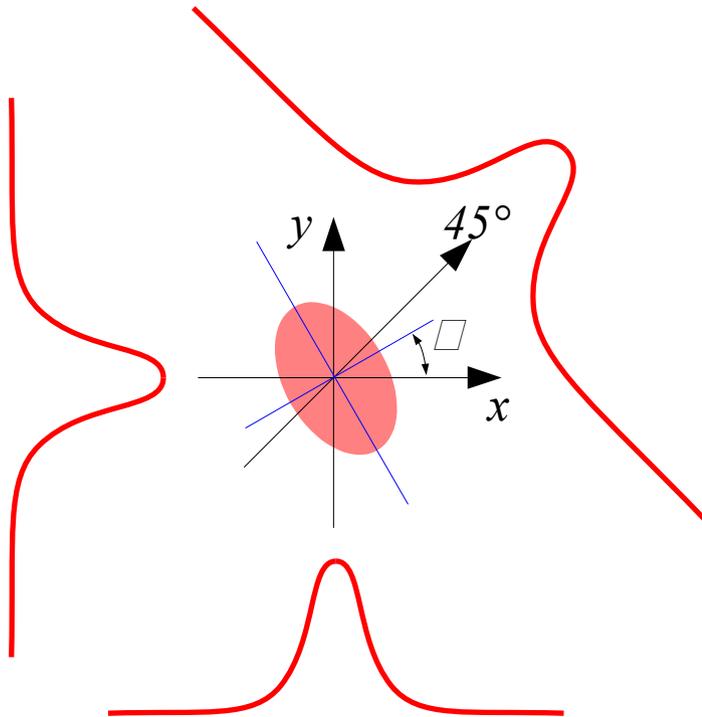
- The 2D image contains the whole spatial information

- With only 2 1D profiles (X and Y) it is impossible to see coupling (rotated ellipse in x,y) or other effects

- Need at least the profile along a third direction (45°)

- Assuming bi-Gaussian beam with tilt: σ_I, σ_{II}, θ.

- 3 D.O.F. need 3 samples

Measurement accuracy
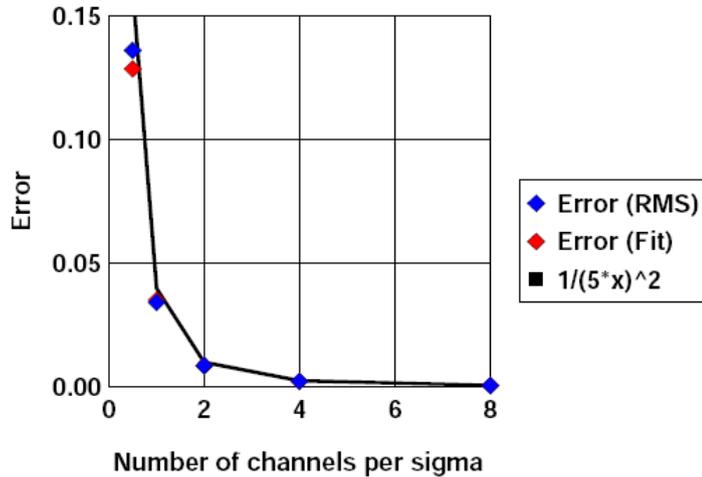
- Accuracy of measurement depends on
  - Detector size
    - Min $\pm 3\sigma$
  - Number of points
    - Min 2 points per σ
  - Accuracy of each point
    - Both position and signal
  - Noise level
- Use fit wherever possible
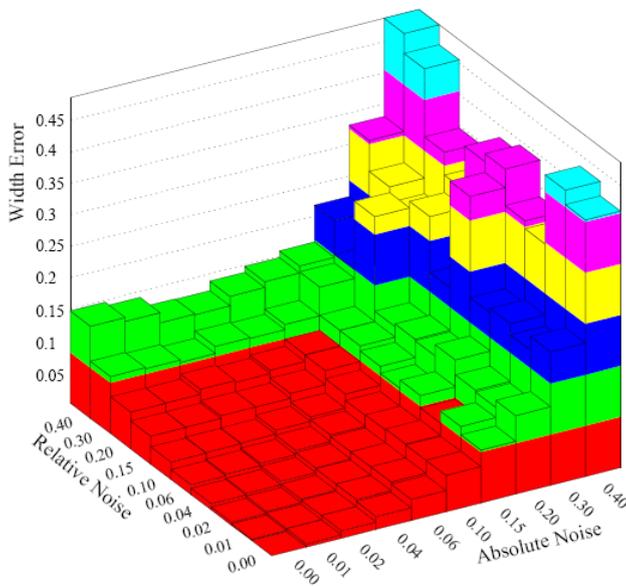
Thank you very much for your attention

Questions?