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1st DITANET School,
at Royal Holloway Univ. London
2nd April 2009

- Introduction
- Mathematical treatment
- Proton emittance
- H^- machines
- ILC emittance measurement
- Laser-wire – practical considerations
- Summary

Luminosity - Emittance

$$L = f \frac{n_1 n_2}{4\pi\sigma_x\sigma_y}$$

Luminosity is dominated By the spot-sizes σ_x, σ_y

$n_{1,2}$ is the number of particles in bunch 1,2

f is the frequency of bunch collisions

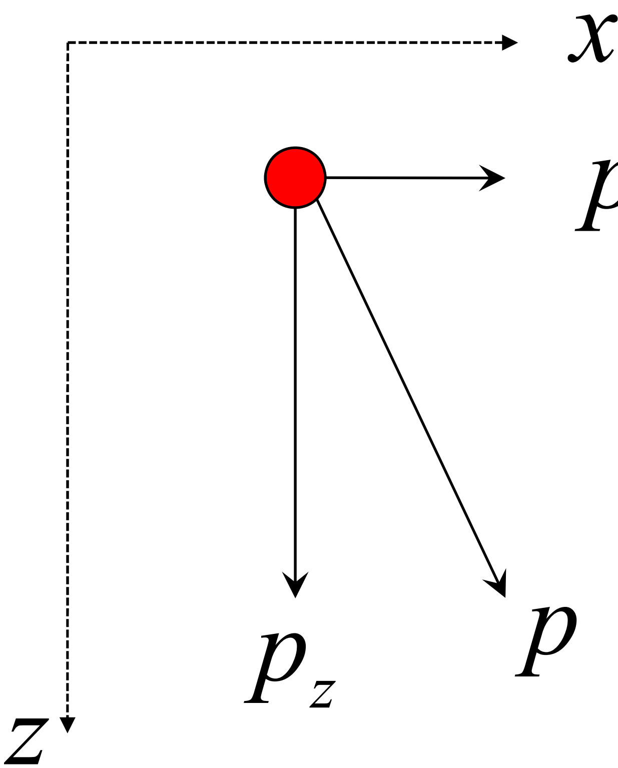
β is the "beta function"; determined by the optical elements

$$\varepsilon = \frac{\pi\sigma^2}{\beta}$$

ε is the "emittance"; and is a constant along the beam-line

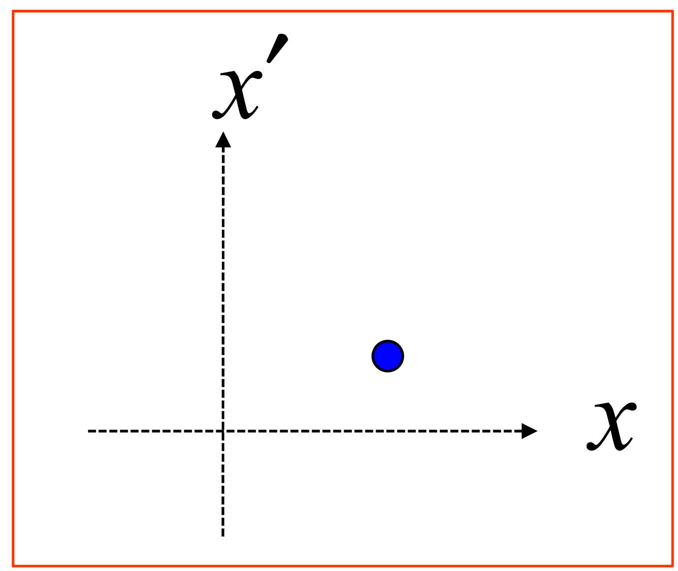
Conjugate Variables

View from the top:

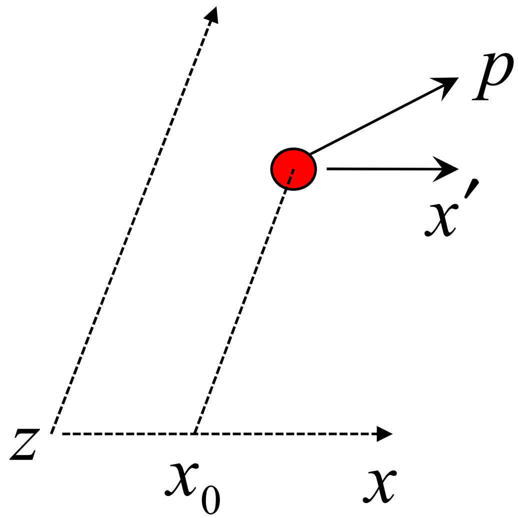


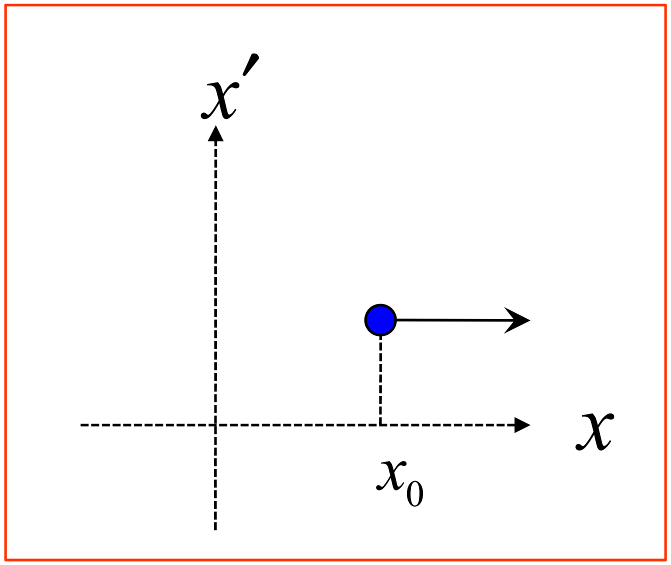
$$p_x = p_z \frac{dx}{dz} \equiv p_z x'$$

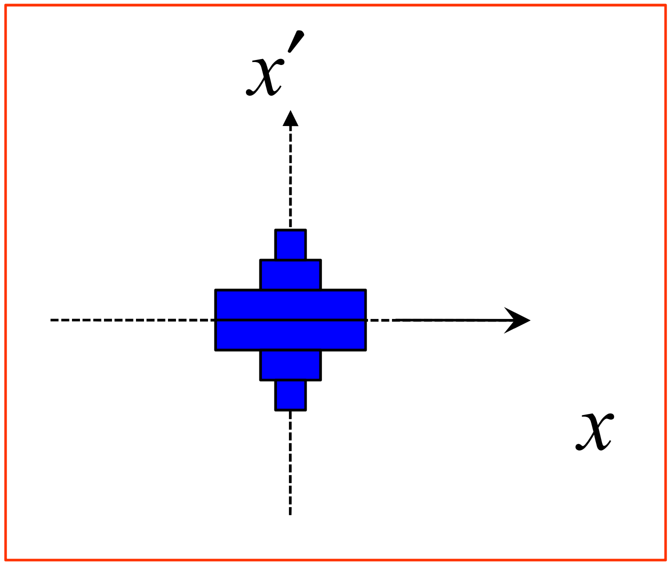
Instantaneous motion is described by a point in “phase space”:



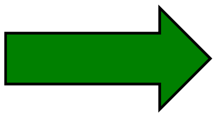
Motion on a horizontal plane

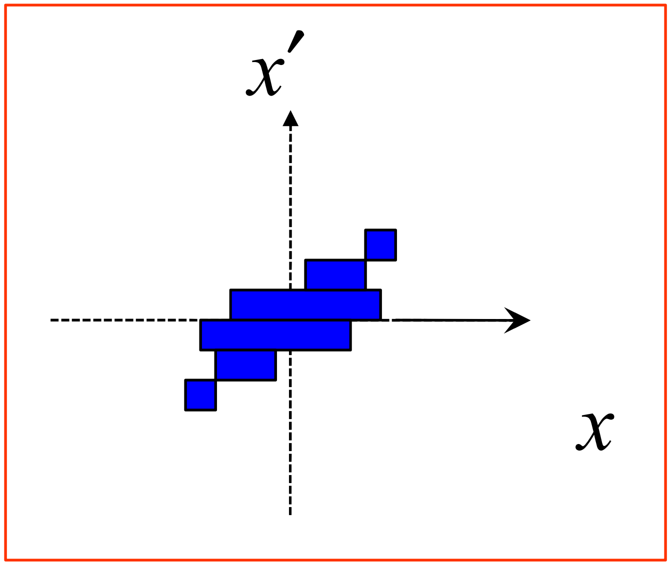






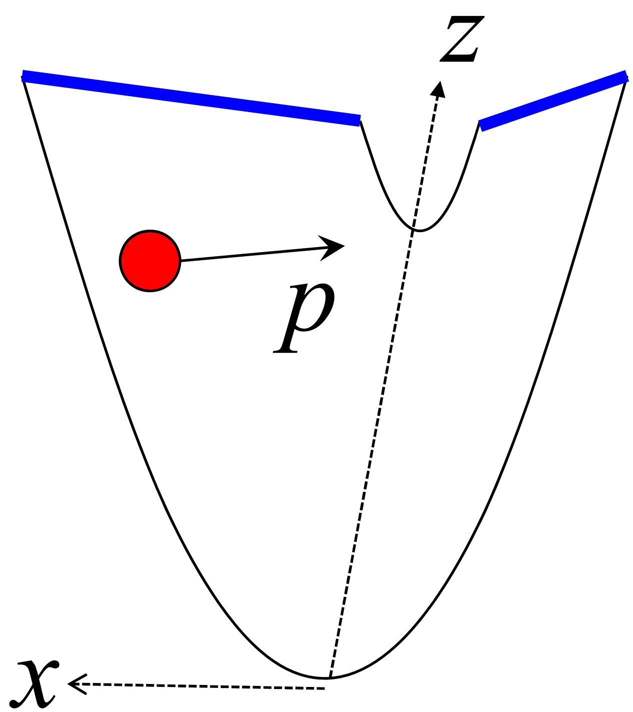
$t = 0$

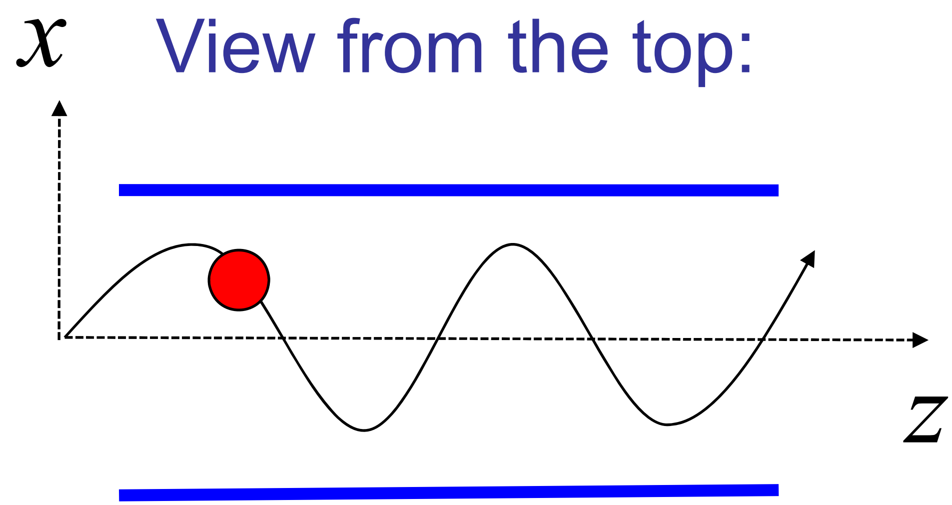


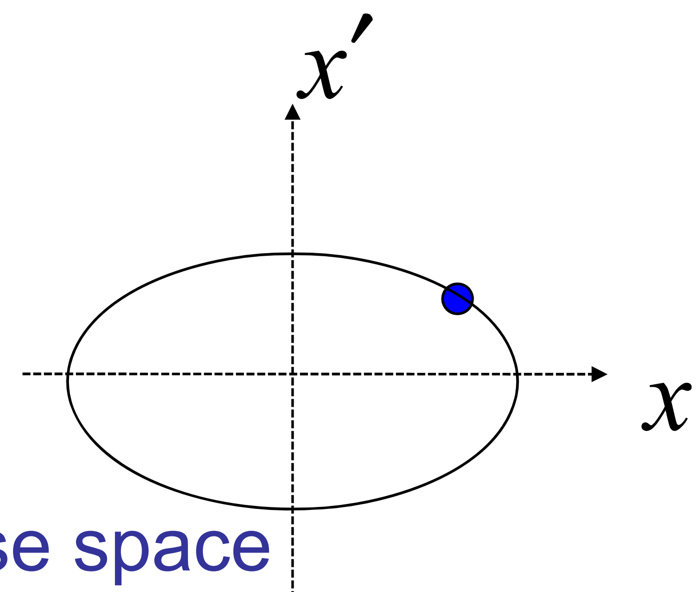


$t > 0$

Consider a parabolic groove:







Individual particles will travel
on elliptical trajectories in phase space

General Solution

$$x(s) = a_x \sqrt{\beta_x(s)} \cos\left[\psi_x(s) + \xi_x\right]$$

where

$$\psi_x(s) = \int_0^s \frac{ds}{\beta_x(s)}$$

with a similar result for *y*

$a_x,\ \xi_x$ constants to be determined from initial conditions

$\beta_x(s)$ determined by the beam-line

Beam Ellipse

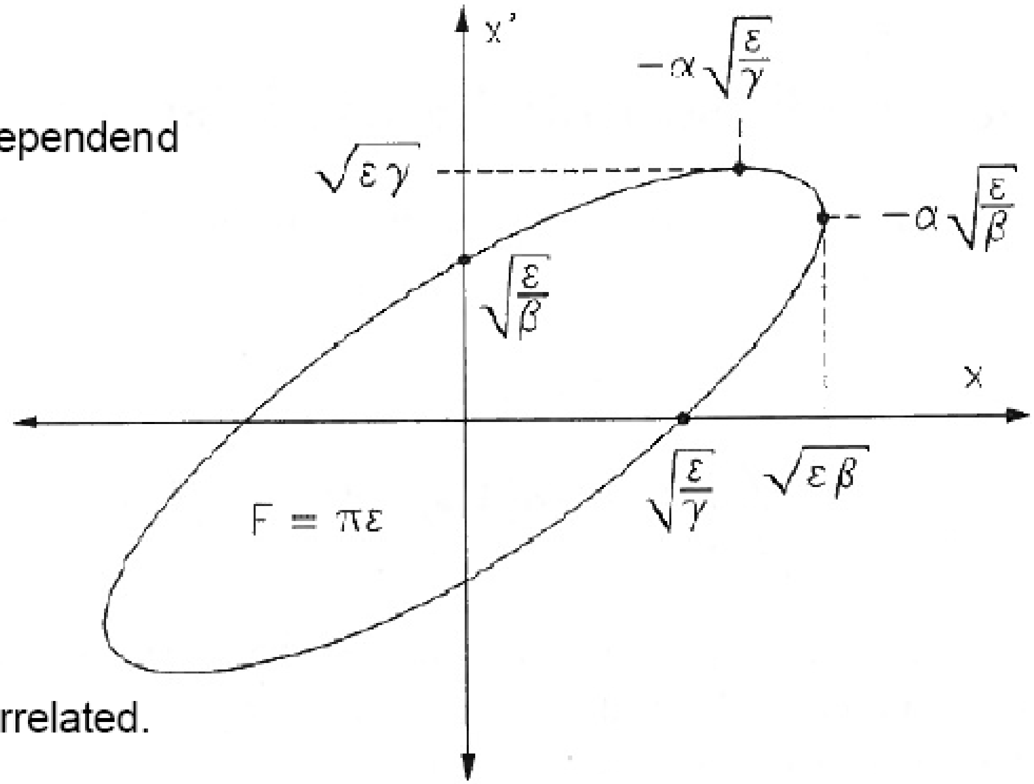
Beam ellipse and its orientation is described by 4 parameters $\varepsilon, \beta, \alpha, \gamma$
called Courant - Snyder or Twiss parameters

$$\varepsilon = \gamma x^2 + 2\,\alpha\, x\, x' + \beta\, x'^2$$

the three ellipse orientation parameters β, α, γ are connected by the relation

$$\gamma = \frac{1+\alpha^2}{\beta}$$

therefore only of these parameters are independend



$\sqrt{\beta\varepsilon}$ is the beam half width

$\sqrt{\gamma\varepsilon}$ is the beam half divergence

α describes how strong x and x' are correlated.

for $\alpha > 0$ beam is converging,

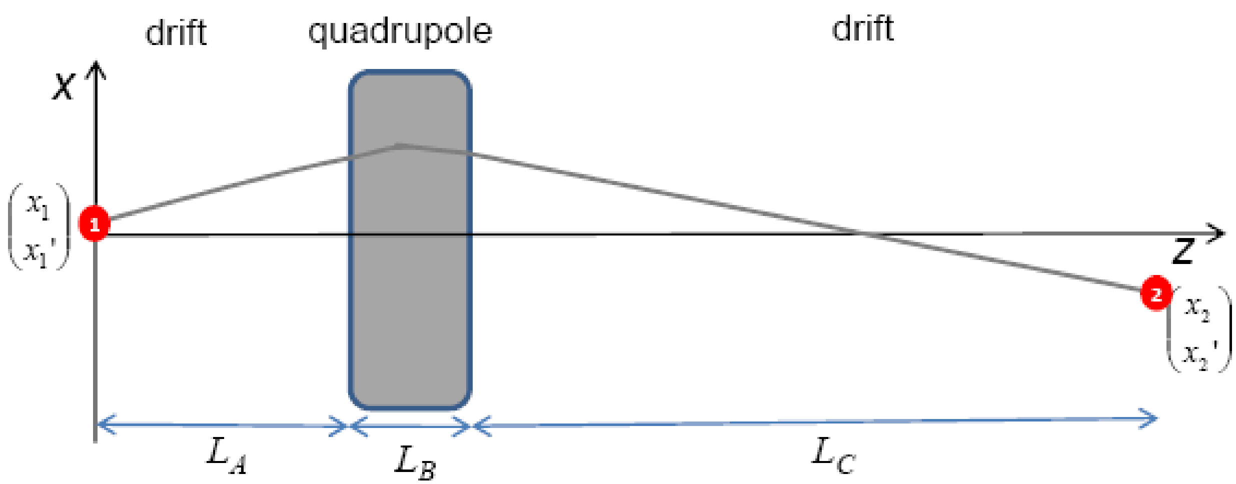
for $\alpha < 0$ beam is diverging.

for $\alpha = 0$ beam size has minimum (waist) or maximum (anti - waist)

H. Braun

Beam Transport

Transport of single particle described with matrix algebra



$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = M_C \cdot M_B \cdot M_A \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\sqrt{k}L_B\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L_B\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L_B\right) & \cos\left(\sqrt{k}L_B\right) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad M_{Quadrupole} = \begin{pmatrix} \cos\left(\sqrt{k}L\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L\right) & \cos\left(\sqrt{k}L\right) \end{pmatrix}$$

generic names of matrix elements $M = \begin{pmatrix} c & s \\ c' & s' \end{pmatrix}$

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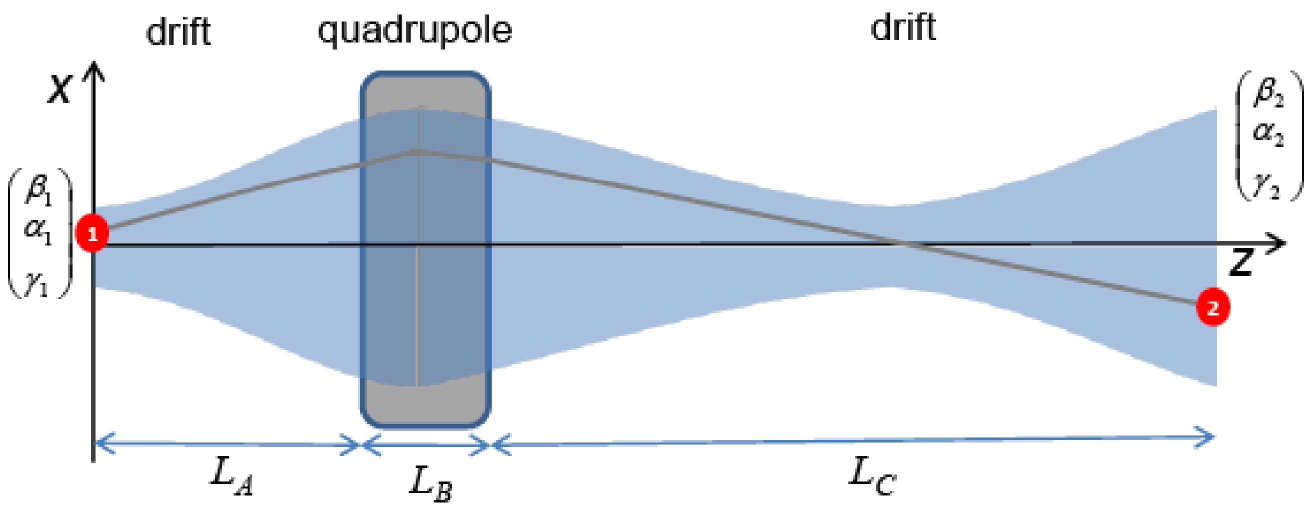
Transport of Twiss Parameters



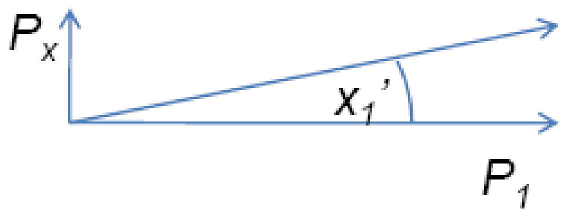
$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} c^2 & -2cs & s^2 \\ -cc' & cs'+c's & -ss' \\ c'^2 & -2c's' & s'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix}$$

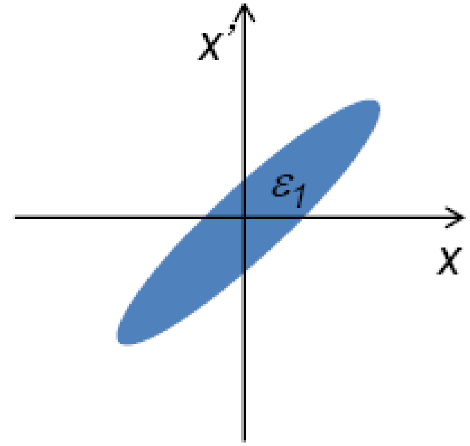
$$\beta_2 = c^2\,\beta_1 - 2cs\,\alpha_1 + s^2\,\gamma_1 \qquad | \cdot \varepsilon$$

$$w_2^2 = c^2\,\beta_1\,\varepsilon - 2cs\,\alpha_1\,\varepsilon + s^2\,\gamma_1\,\varepsilon$$

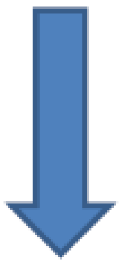
H. Braun

Effect of Acceleration on ε

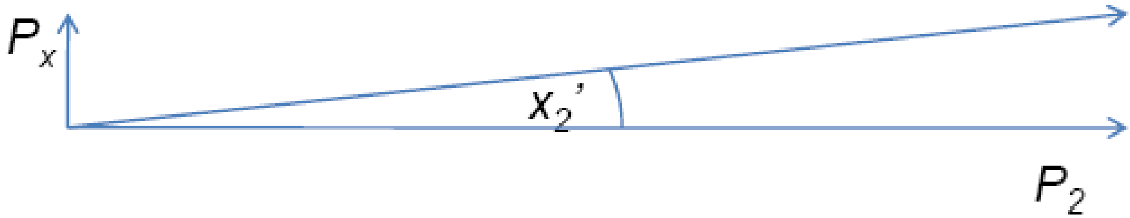


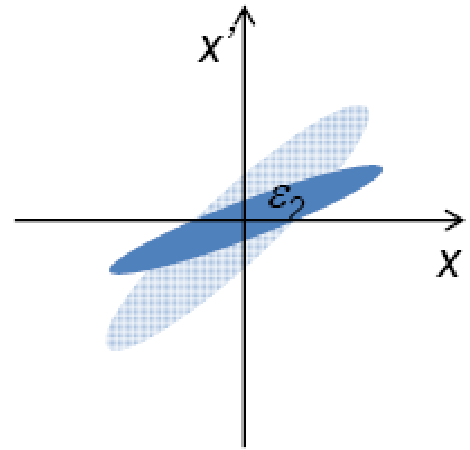


acceleration



$$\varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m\beta_{rel}\gamma_{rel}\, c$$





Normalised emittance: $\varepsilon_N = \beta_{\mathrm{rel}}\gamma_{\mathrm{rel}}\varepsilon$

is preserved during acceleration

"geometric" emittance

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Common Units for ε

mm·mrad, m·rad, μm, m, nm

1 mm·mrad=10^{-6} m·rad=1μm=10^{-6} m=10^{3}nm

Often a π is added to the unit to indicate that the numerical value describes a surface in x, x' space divided by π, i.e. 1 π·mm·mrad

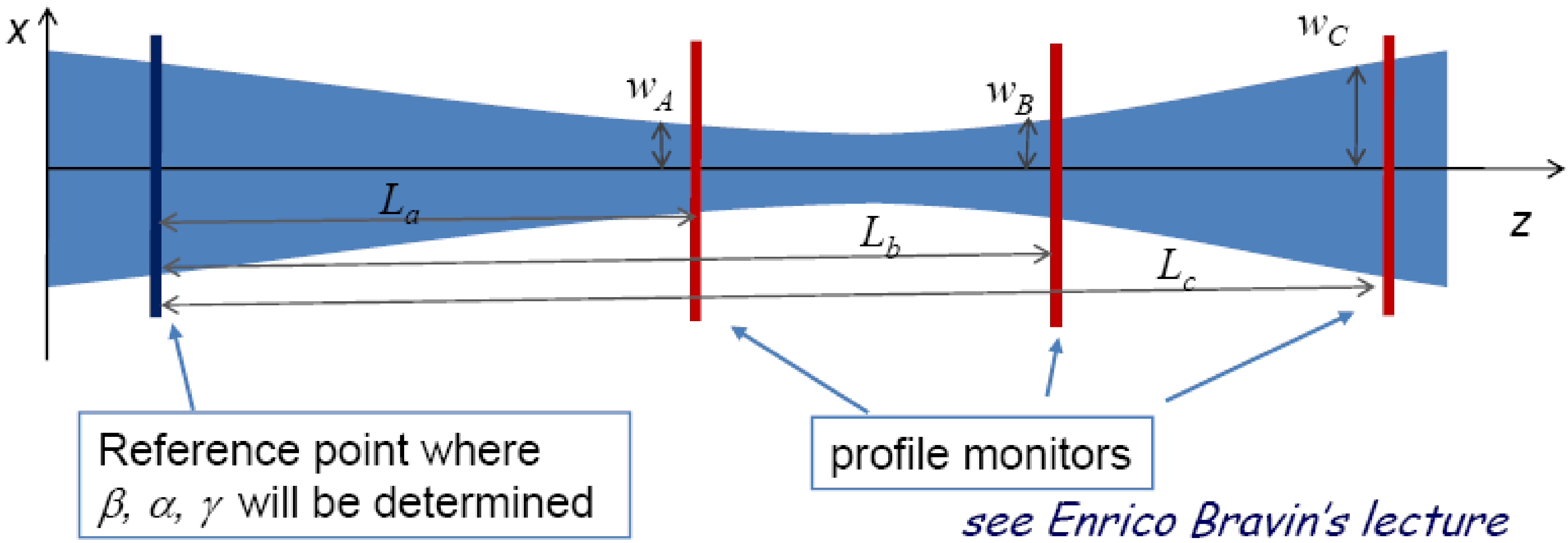
The units for normalised emittance are the same as for geometric emittance

Due to the various definitions it is recommended to always mention the emittance definition used when reporting measurement values !

ε Measurement - I

Twiss parameter β, α, γ are a priori not known, they have to be determined together with ε.

Method A



see Enrico Bravin's lecture

$$w^2 = c^2\ \beta\,\varepsilon - 2\,c\,s\ \alpha\ \varepsilon + s^2\ \gamma\,\varepsilon,\ \text{for drift}\ \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad w^2 = \beta\,\varepsilon - 2\ L\ \alpha\ \varepsilon + L^2\ \gamma\,\varepsilon$$

H. Braun

Derivation of Twiss params:

$$w_A^2 = \beta\,\varepsilon - 2\,L_A\,\alpha\,\varepsilon + L_A^2\,\gamma\,\varepsilon$$
$$w_B^2 = \beta\,\varepsilon - 2\,L_B\,\alpha\,\varepsilon + L_B^2\,\gamma\,\varepsilon$$
$$w_C^2 = \beta\,\varepsilon - 2\,L_C\,\alpha\,\varepsilon + L_C^2\,\gamma\,\varepsilon$$

⇓ can be rewritten in Matrix notation

$$\begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} 1 & -2L_A & L_A^2 \\ 1 & -2L_B & L_B^2 \\ 1 & -2L_C & L_C^2 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2L_A & L_A^2 \\ 1 & -2L_B & L_B^2 \\ 1 & -2L_C & L_C^2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}$$

$$\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta \cdot \gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

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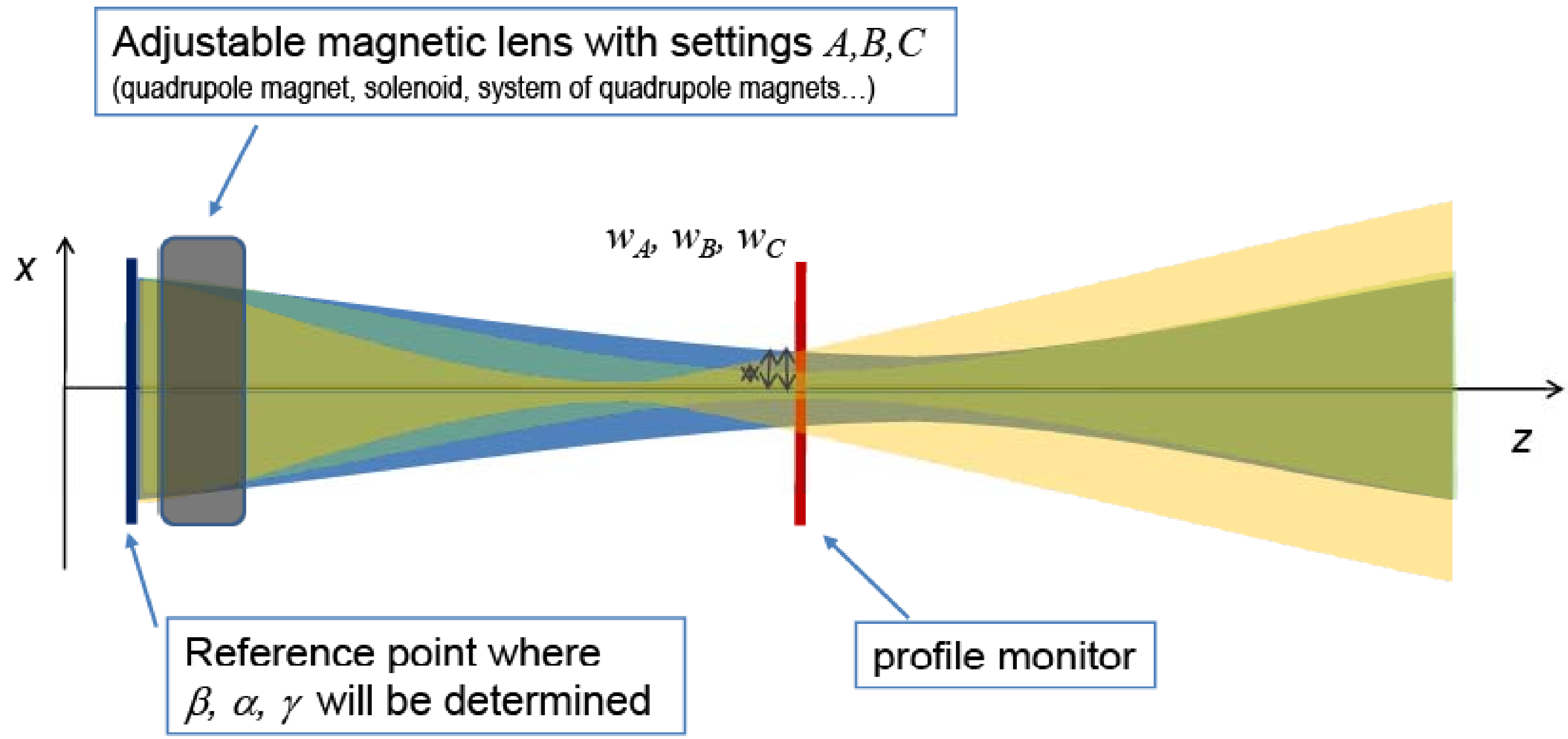
ε Measurement - II



$$w^2 = c^2\,\beta\varepsilon - 2cs\,\alpha\,\varepsilon + s^2\,\gamma\,\varepsilon, \qquad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

H. Braun

Change quad strength:

$$w_A^2 = c_A^2 \beta \varepsilon - 2 c_A s_A \alpha \varepsilon + s_A^2 \gamma \varepsilon$$
$$w_B^2 = c_B^2 \beta \varepsilon - 2 c_B s_B \alpha \varepsilon + s_B^2 \gamma \varepsilon$$
$$w_C^2 = c_C^2 \beta \varepsilon - 2 c_C s_C \alpha \varepsilon + s_C^2 \gamma \varepsilon$$

$\Downarrow$ can be rewritten in Matrix notation

$$\begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} c_A^2 & -2c_A s_A & s_A^2 \\ c_B^2 & -2c_B s_B & s_B^2 \\ c_C^2 & -2c_C s_C & s_C^2 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} \Rightarrow \begin{pmatrix} c_A^2 & -2c_A s_A & s_A^2 \\ c_B^2 & -2c_B s_B & s_B^2 \\ c_C^2 & -2c_C s_C & s_C^2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} w_A^2 \\ w_B^2 \\ w_C^2 \end{pmatrix} = \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix}$$

$$\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2 \left(\beta \cdot \gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

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Need 3 or more measurements:

To determine ε, β, α at a reference point in a beamline one needs at least three w measurements with different transfer matrices between the reference point and the w measurements location.

Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.

Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.

Three w measurements are in principle enough to determine ε, β, α
In practice better results are obtained with more measurements.
However, with more than three measurements the problem is over-determined.
χ^2 formalism gives the best estimate of ε, β, α
for a set of n measurements w_i, $i=1$-n with transfer matrix elements c_i, s_i.

H. Braun

χ^2 Formalism

Measured half beam width w_i

Predicted half beam width $\sqrt{c_i^2\beta\varepsilon - 2c_i s_i \alpha\varepsilon + s_i^2\gamma\varepsilon}$,
but $\beta\varepsilon, \alpha\varepsilon, \gamma\varepsilon$ a priori not known

Find set of $\beta\varepsilon, \alpha\varepsilon, \gamma\varepsilon$ values, which minimises

$$\chi^2 = \sum_{i=1}^{n}\left(c_i^2\beta\varepsilon - 2c_i s_i\alpha\varepsilon + s_i^2\gamma\varepsilon - w_i^2\right)^2$$

Conditions for χ^2 minimum

$$\frac{\partial\chi^2}{\partial\beta\varepsilon} = 0 \quad\Rightarrow\quad \beta\varepsilon\sum_{i=1}^{n}c_i^4 - 2\alpha\varepsilon\sum_{i=1}^{n}c_i^3 s_i + \gamma\varepsilon\sum_{i=1}^{n}c_i^2 s_i^2 = \sum_{i=1}^{n}c_i^2 w_i^2$$

$$\frac{\partial\chi^2}{\partial\alpha\varepsilon} = 0 \quad\Rightarrow\quad \beta\varepsilon\sum_{i=1}^{n}c_i^3 s_i - 2\alpha\varepsilon\sum_{i=1}^{n}c_i^2 s_i^2 + \gamma\varepsilon\sum_{i=1}^{n}c_i s_i^3 = \sum_{i=1}^{n}c_i s_i w_i^2$$

$$\frac{\partial\chi^2}{\partial\gamma\varepsilon} = 0 \quad\Rightarrow\quad \beta\varepsilon\sum_{i=1}^{n}c_i^2 s_i^2 - 2\alpha\varepsilon\sum_{i=1}^{n}c_i s_i^3 + \gamma\varepsilon\sum_{i=1}^{n}s_i^4 = \sum_{i=1}^{n}c_i^2 w_i^2$$

These conditions can be rewritten in matrix notation

$$\begin{pmatrix} \sum_{i=1}^{n}c_i^4 & -2\sum_{i=1}^{n}c_i^3 s_i & \sum_{i=1}^{n}c_i^2 s_i^2 \\ \sum_{i=1}^{n}c_i^3 s_i & -2\sum_{i=1}^{n}c_i^2 s_i^2 & \sum_{i=1}^{n}c_i s_i^3 \\ \sum_{i=1}^{n}c_i^2 s_i^2 & -2\sum_{i=1}^{n}c_i s_i^3 & \sum_{i=1}^{n}s_i^4 \end{pmatrix} \cdot \begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n}c_i^2 w_i^2 \\ \sum_{i=1}^{n}c_i s_i w_i^2 \\ \sum_{i=1}^{n}c_i^2 w_i^2 \end{pmatrix}$$

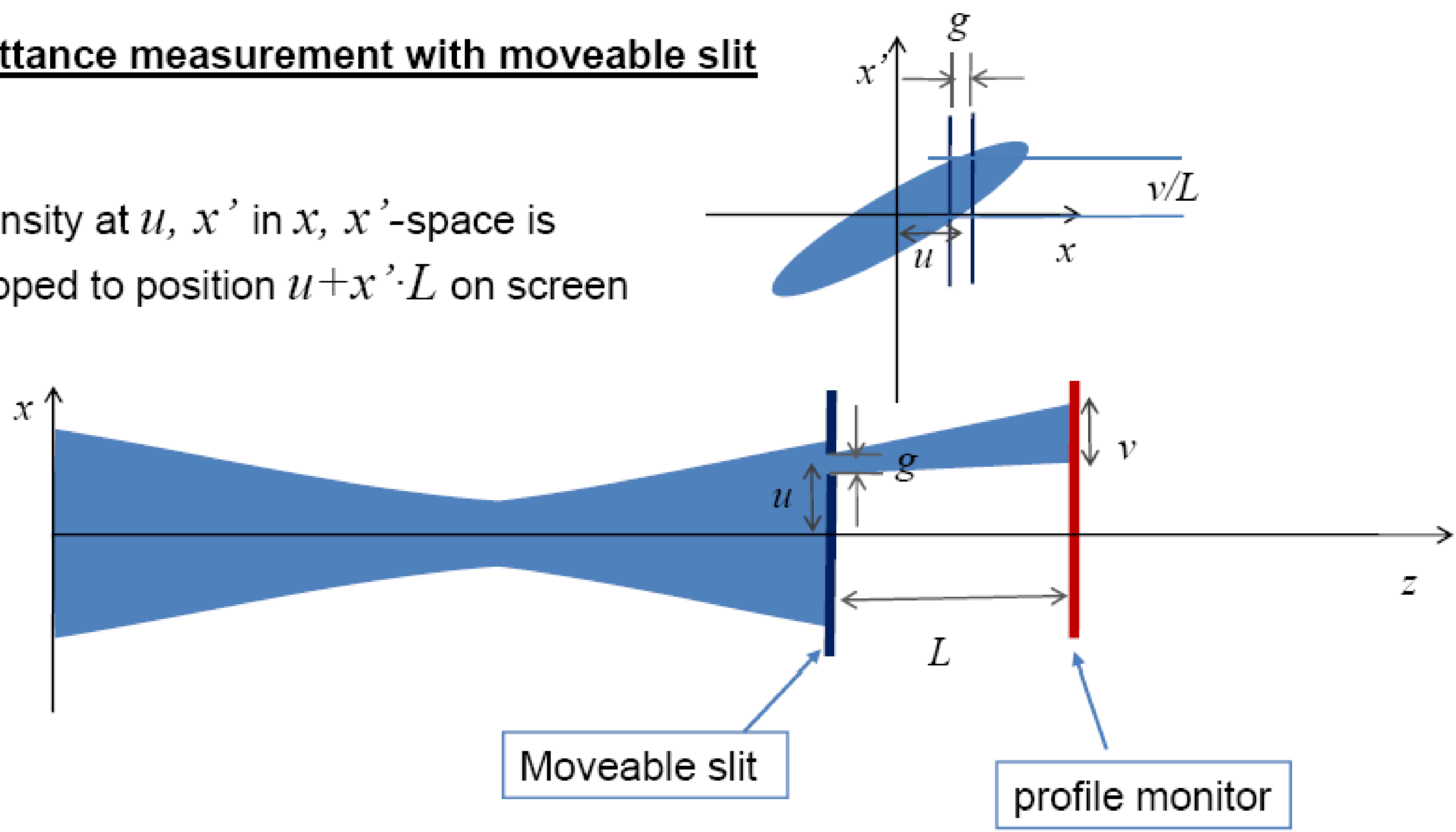
$$\begin{pmatrix} \beta\varepsilon \\ \alpha\varepsilon \\ \gamma\varepsilon \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n}c_i^4 & -2\sum_{i=1}^{n}c_i^3 s_i & \sum_{i=1}^{n}c_i^2 s_i^2 \\ \sum_{i=1}^{n}c_i^3 s_i & -2\sum_{i=1}^{n}c_i^2 s_i^2 & \sum_{i=1}^{n}c_i s_i^3 \\ \sum_{i=1}^{n}c_i^2 s_i^2 & -2\sum_{i=1}^{n}c_i s_i^3 & \sum_{i=1}^{n}s_i^4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \sum_{i=1}^{n}c_i^2 w_i^2 \\ \sum_{i=1}^{n}c_i s_i w_i^2 \\ \sum_{i=1}^{n}c_i^2 w_i^2 \end{pmatrix}$$

$$\sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

H. Braun

Emittance measurement with moveable slit

Intensity at u, x' in x, x'-space is mapped to position $u+x'\cdot L$ on screen



From width and position of slit image mean beam angle and divergence of slice at position u is readily computed.

By moving slit across the beam complete distribution in x, x' space is reconstructed.

Conditions for good resolution: $v >> g$

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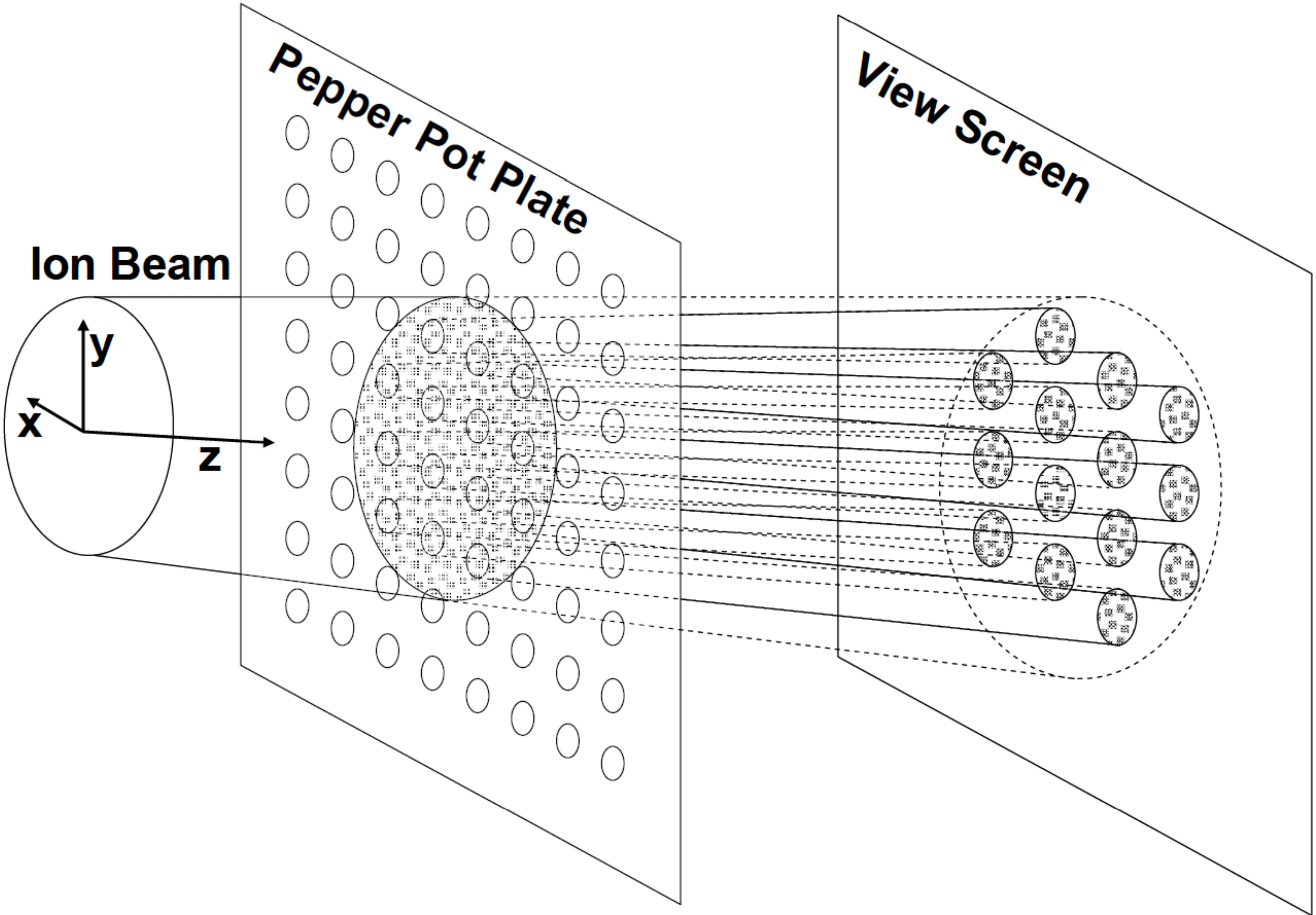
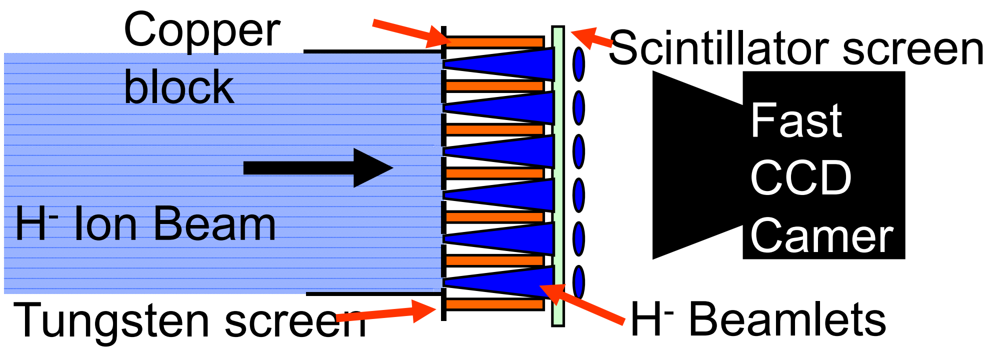
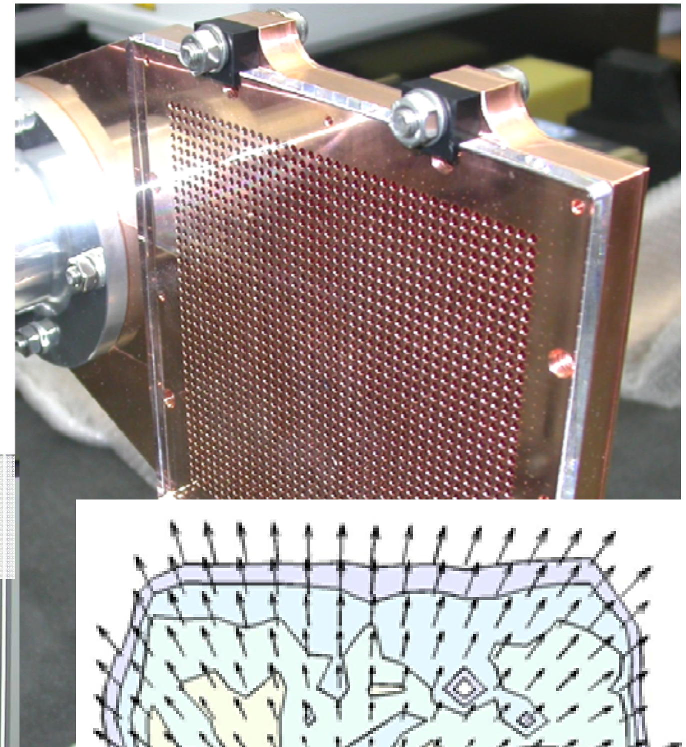
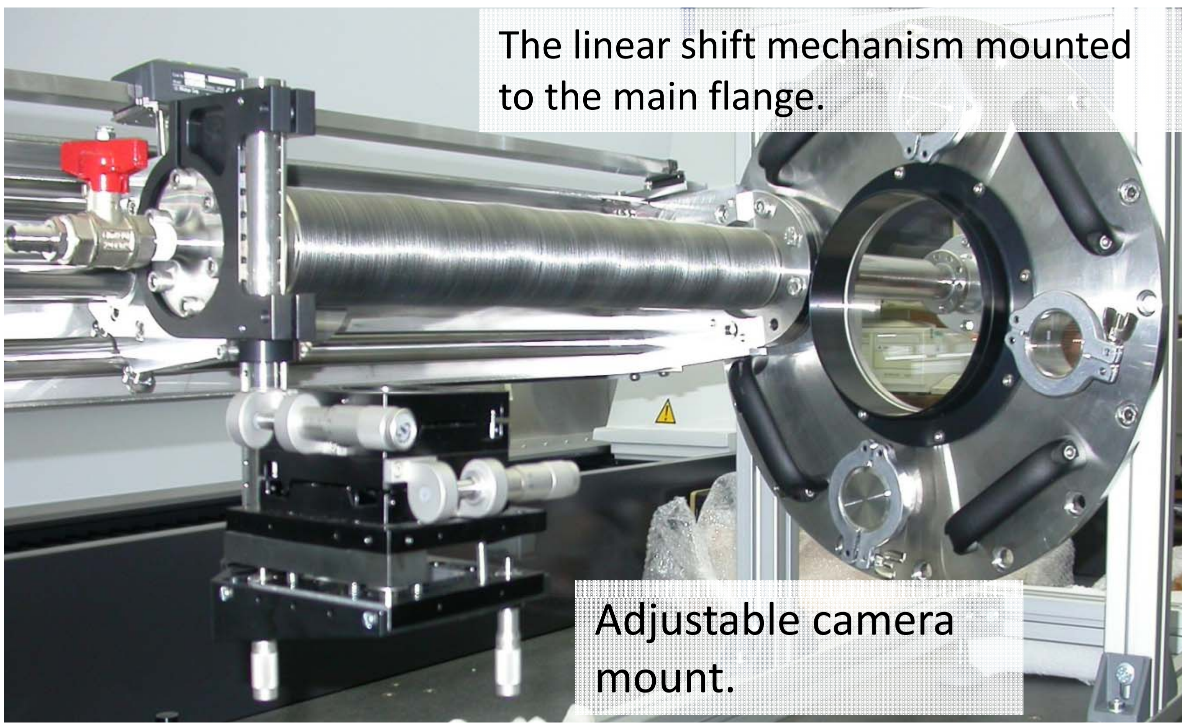
Pepperpot

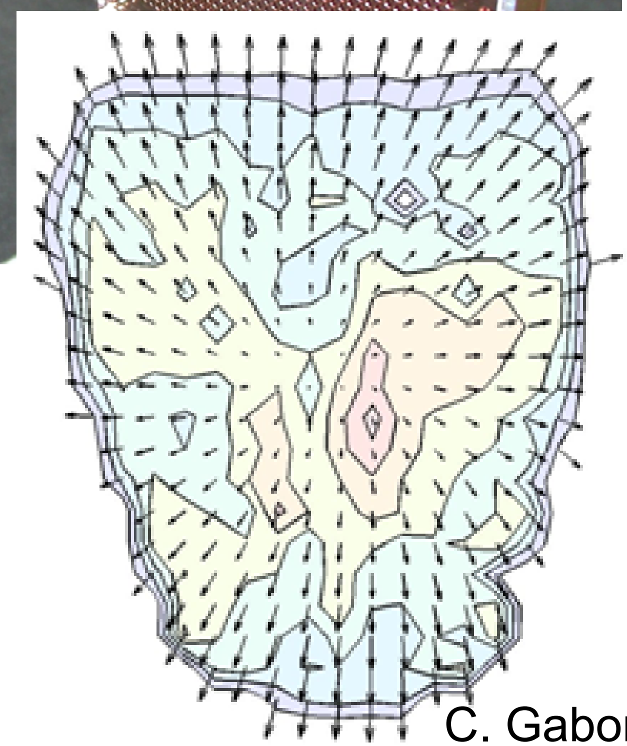


Principle and technical set up of the pepper pot emittance instrument.









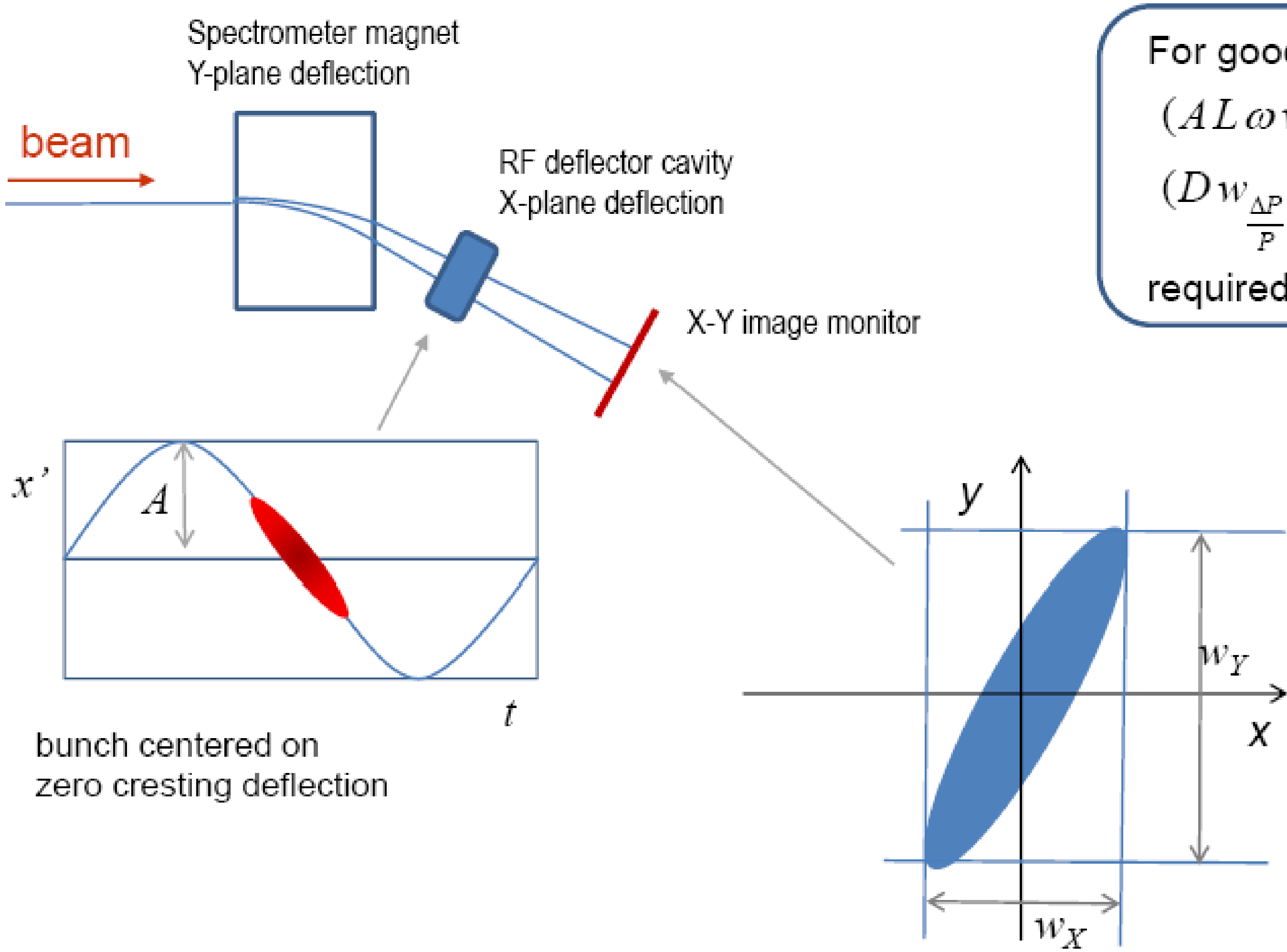
C. Gabor

Longitudinal Emittance

Conjugate variables E (→p), z

$$\varepsilon_L = \sqrt{\left\langle z^2 \right\rangle \left\langle \left(\frac{\Delta p}{p} \right)^2 \right\rangle - \left\langle z \cdot \left(\frac{\Delta p}{p} \right) \right\rangle}$$

Measurement in linac



For good accuracy

$$(A\,L\,\omega\,w_t)^2 >> \beta\varepsilon_X$$

$$(D\,w_{\frac{\Delta P}{P}})^2 >> \beta\varepsilon_Y$$

required

$$y = D_Y \frac{\Delta P}{P}$$

$$w_{\frac{\Delta P}{P}} = \frac{1}{D_Y} w_Y$$

$$x = A\,L\,\sin(\omega t)$$

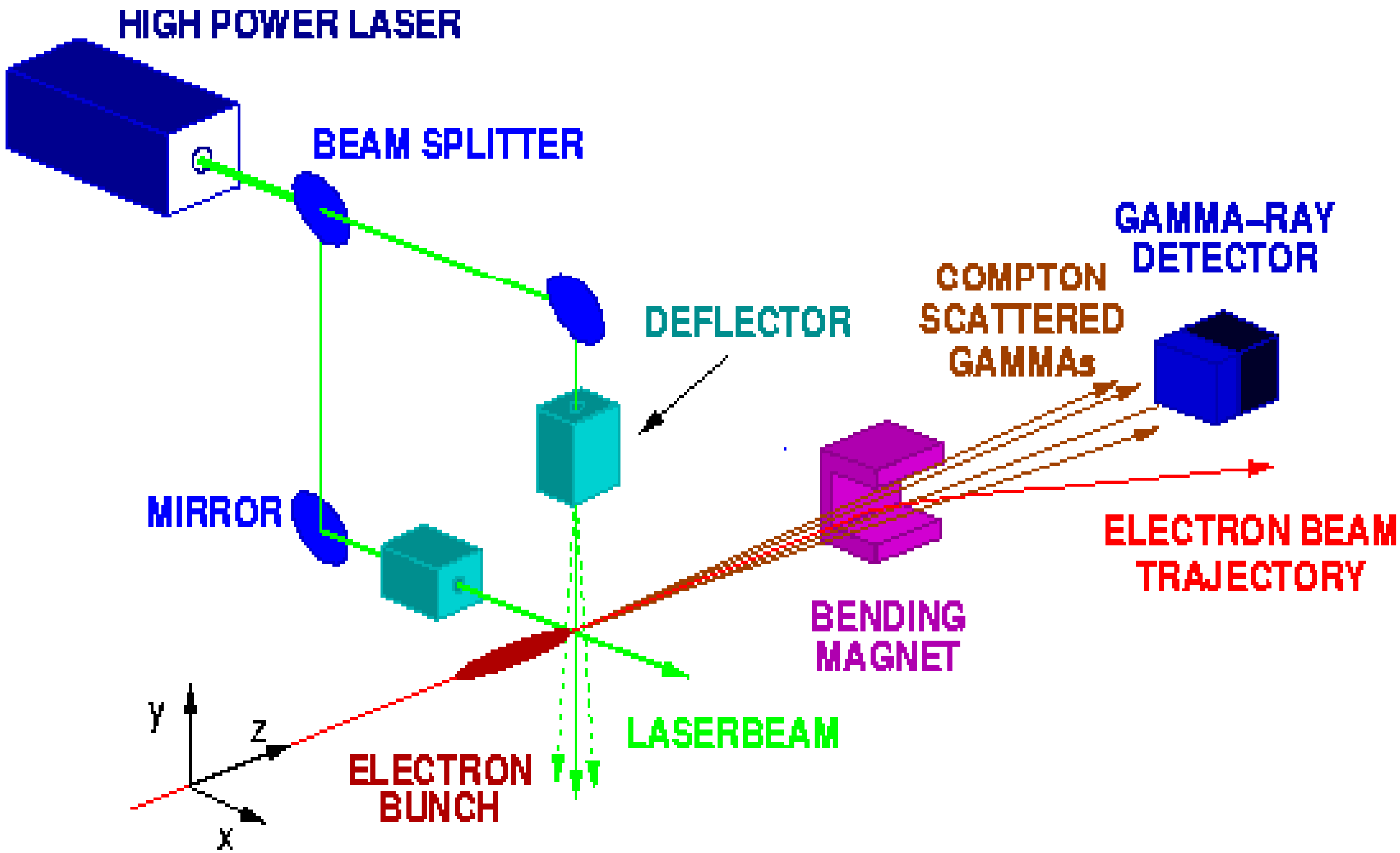
$$w_t \approx \frac{1}{A\,L\,\omega} w_x$$

H. Braun

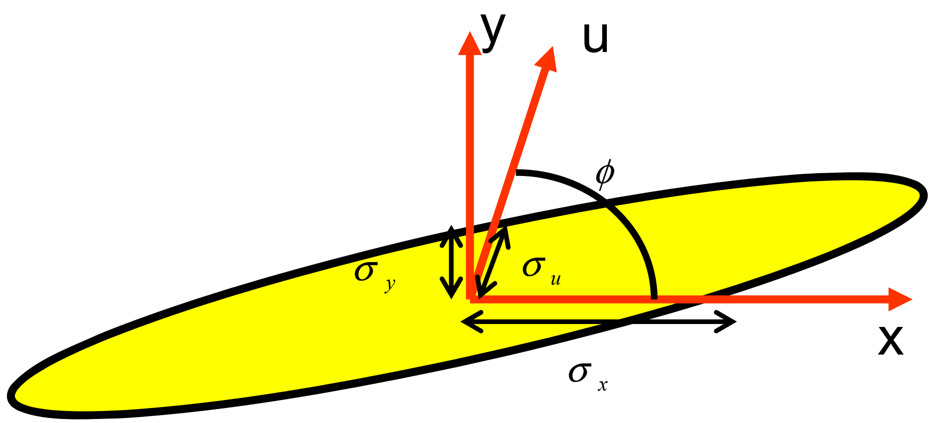
Measuring the Transverse Beam Profile

- Traditional method is to sweep a solid wire across the beam.
- Measure background vs relative position of wire and beam.
- Micron-scale precision required for LC
- Solid wires would not stand the intense beams of the LC
- Solid wires could ablate, harming SC surfaces nearby.
- So: replace wire with a laser beam.
- Count Comptons downstream.

Laserwire


HIGH POWER LASER
BEAM SPLITTER
GAMMA-RAY
DETECTOR
COMPTON
SCATTERED
GAMMAs
DEFLECTOR
MIRROR
ELECTRON BEAM
TRAJECTORY
BENDING
MAGNET
LASERBEAM
ELECTRON
BUNCH
y
z
x

Skew Correction: x-y coupling



$$\phi_{\text{optimal}} = \tan^{-1}\left(\frac{\sigma_x}{\sigma_y}\right)$$

$$\approx 68° - 88° \text{ at ILC}$$

Error on coupling term:

$$\delta\langle xy\rangle = \sigma_x\sigma_y\left[4\left(\frac{\delta\sigma_u}{\sigma_u}\right)^2 + \left(\frac{\delta\sigma_x}{\sigma_x}\right)^2 + \left(\frac{\delta\sigma_y}{\sigma_y}\right)^2\right]^{\frac{1}{2}}$$

ILC LW Locations E_b = 250 GeV

σ_x(μm)	σ_y (μm)	ϕ_{opt}(°)	σ_u (μm)
39.9	2.83	86	3.99
17.0	1.66	84	2.34
17.0	2.83	81	3.95
39.2	1.69	88	2.39
7.90	3.14	68	4.13
44.7	2.87	86	4.05

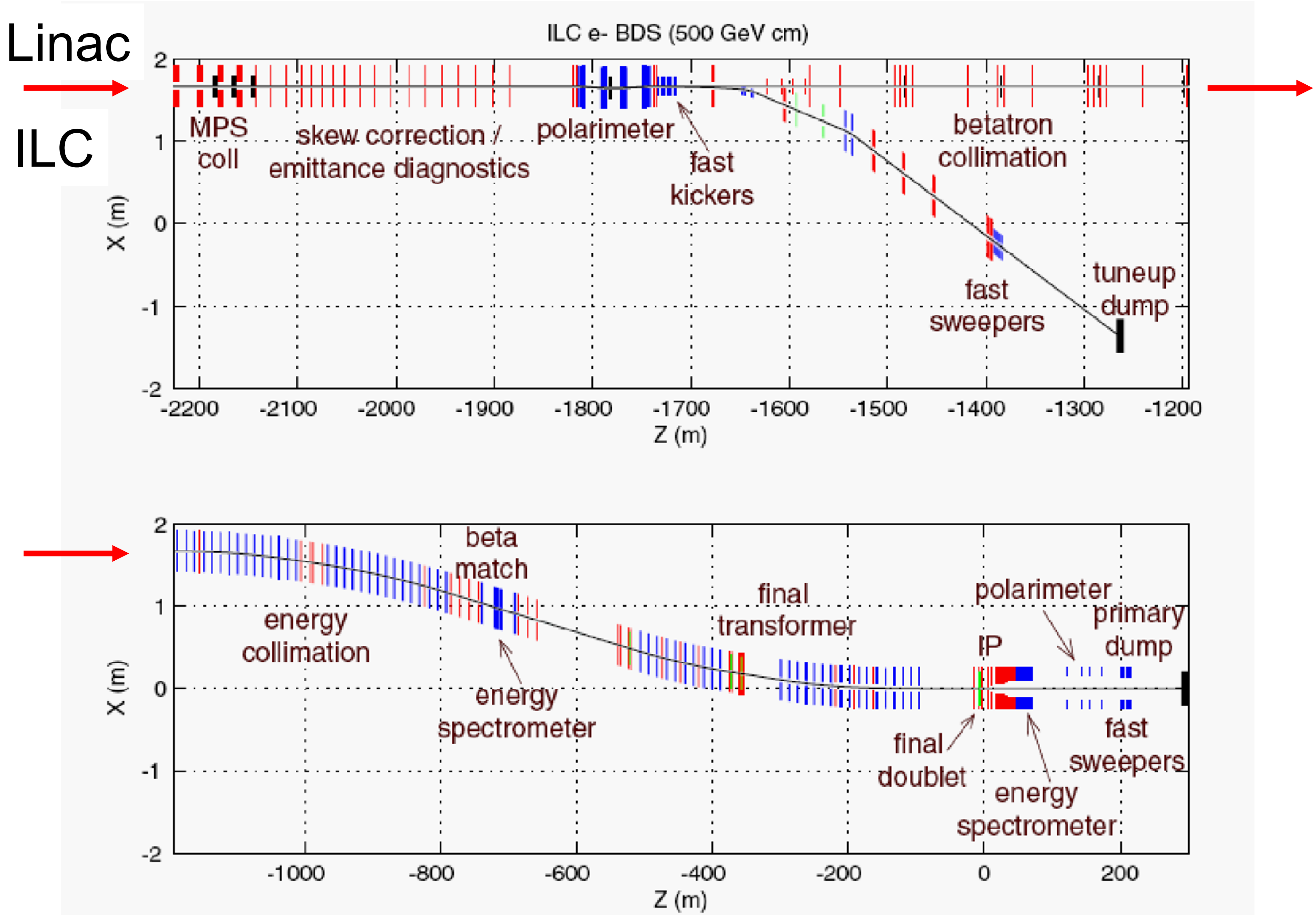
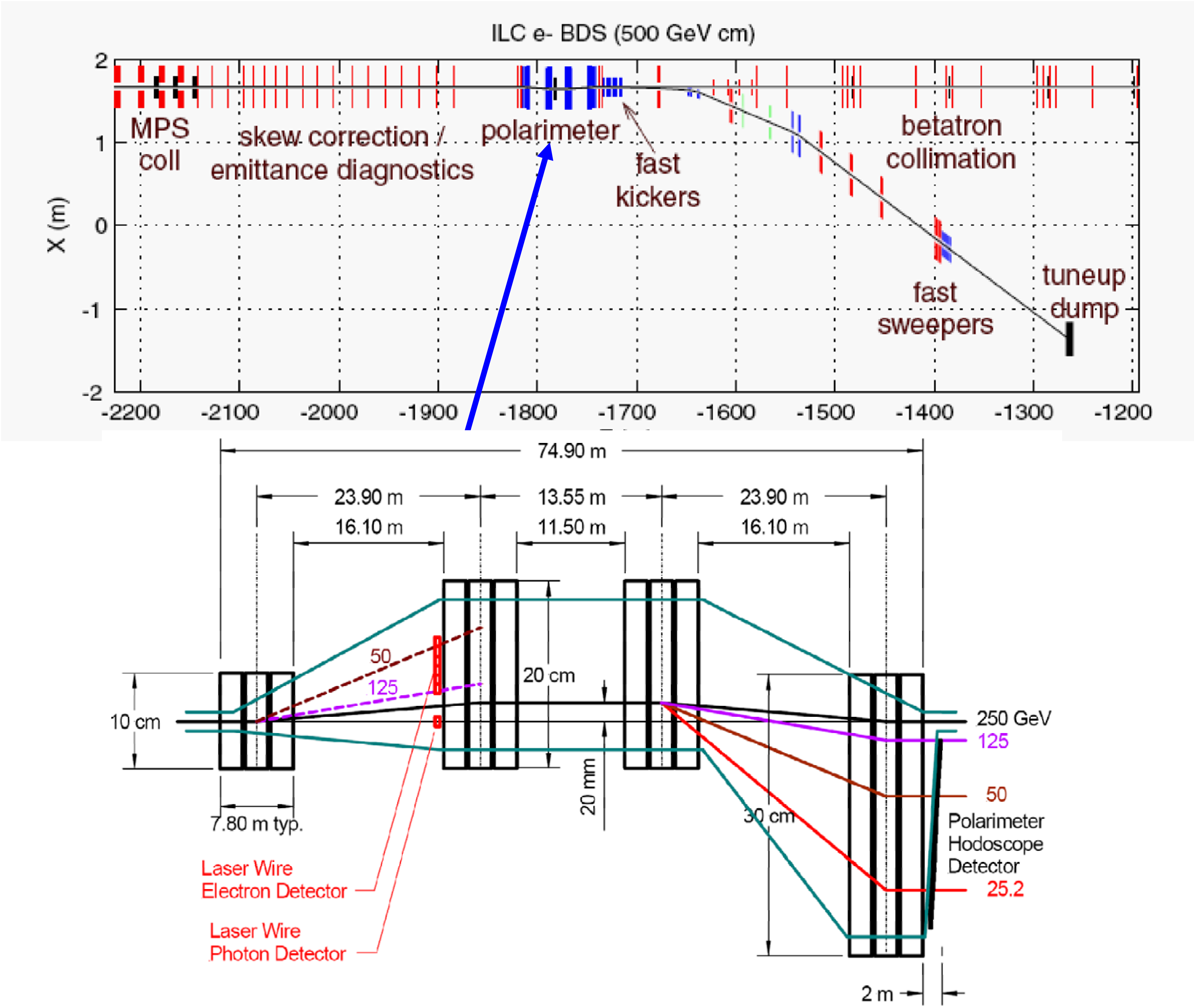


FIGURE 2.7-2. BDS layout showing functional subsystems, starting from the linac exit; X – horizontal position of elements, Z – distance measured from the IP.



ILC e- BDS (500 GeV cm)
X (m)
2
1
0
-1
-2
-2200
-2100
-2000
-1900
-1800
-1700
-1600
-1500
-1400
-1300
-1200
MPS
coll
skew correction /
emittance diagnostics
polarimeter
fast
kickers
betatron
collimation
fast
sweepers
tuneup
dump
74.90 m
23.90 m
13.55 m
23.90 m
16.10 m
11.50 m
16.10 m
50
125
20 cm
10 cm
250 GeV
125
50
20 mm
30 cm
7.80 m typ.
Polarimeter
Hodoscope
Detector
25.2
Laser Wire
Electron Detector
Laser Wire
Photon Detector
2 m

Laser wire : Measurement precision

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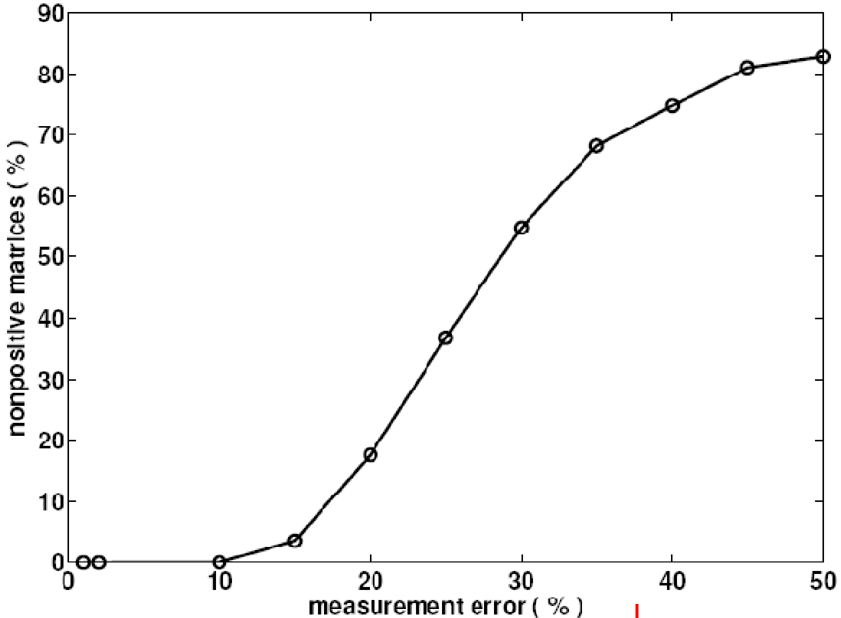
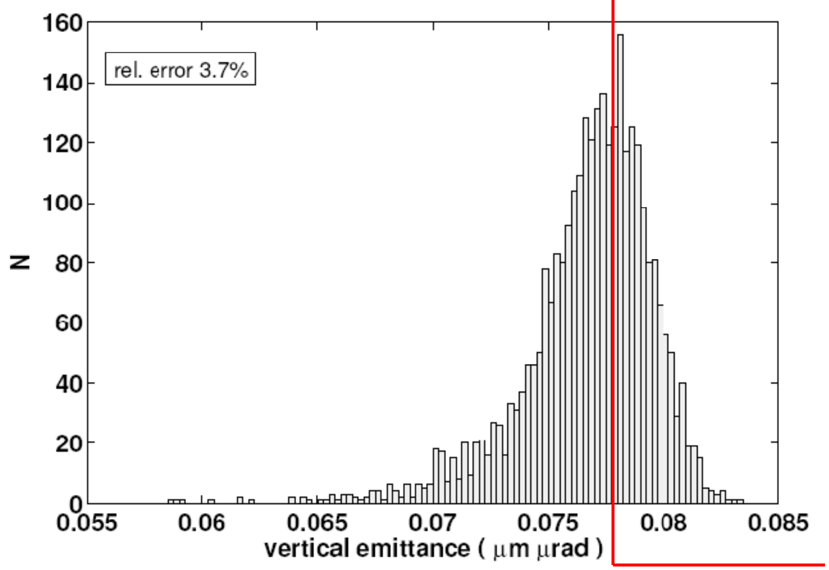
I. Agapov, G. B., M. Woodley



The Goal: Beam Matrix Reconstruction

NOTE: Rapid improvement with better σ_y resolution



Reconstructed emittance of <u>one ILC train</u> using 5% error on σ_y

Assumes a 4d diagnostics section
With 50% random mismatch of initial optical functions

The true emittance is 0.079 μm μrad

H^- Neutralisation

The process $\mathrm{H}^- + \gamma \rightarrow \mathrm{H}^0 + e^-$

has threshold energy ~0.75 eV

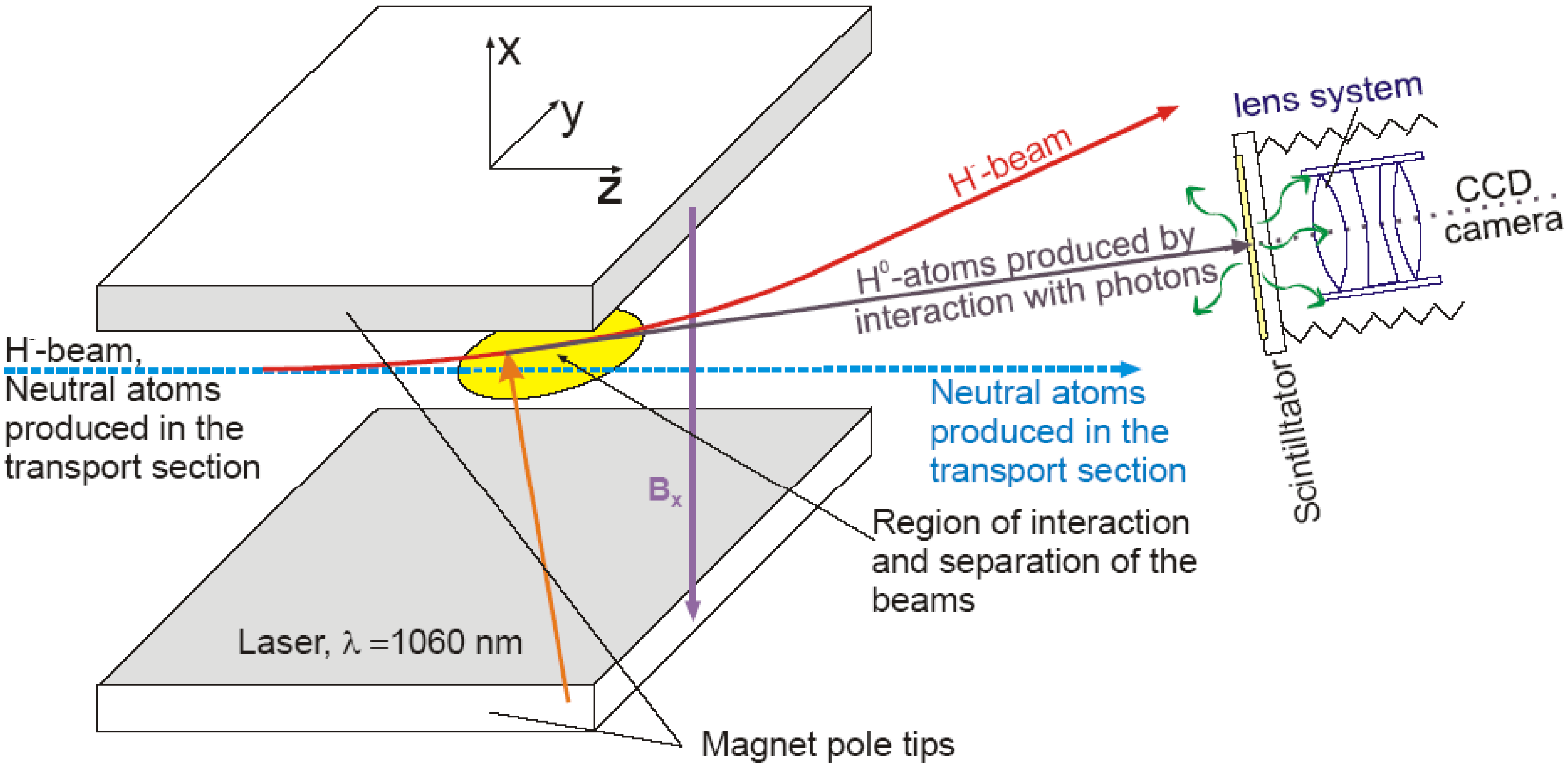
so it can be driven by a Nd:YAG laser operating at 1060 nm.

A focussed laser beam can thus be used to

- Measure emittance of H- beam
- Enable proton production by laser-induced stripping.

All the previous technical issues apply…

Schematic Operation

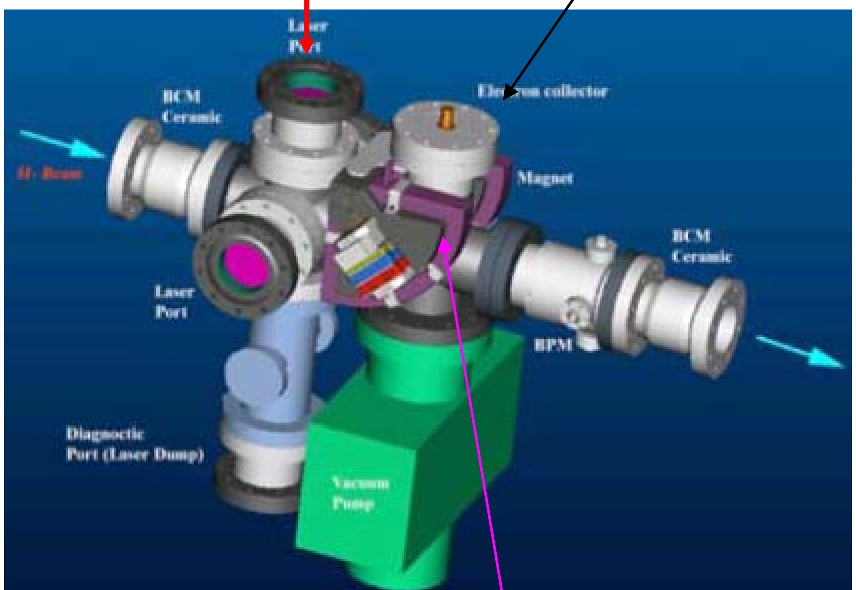


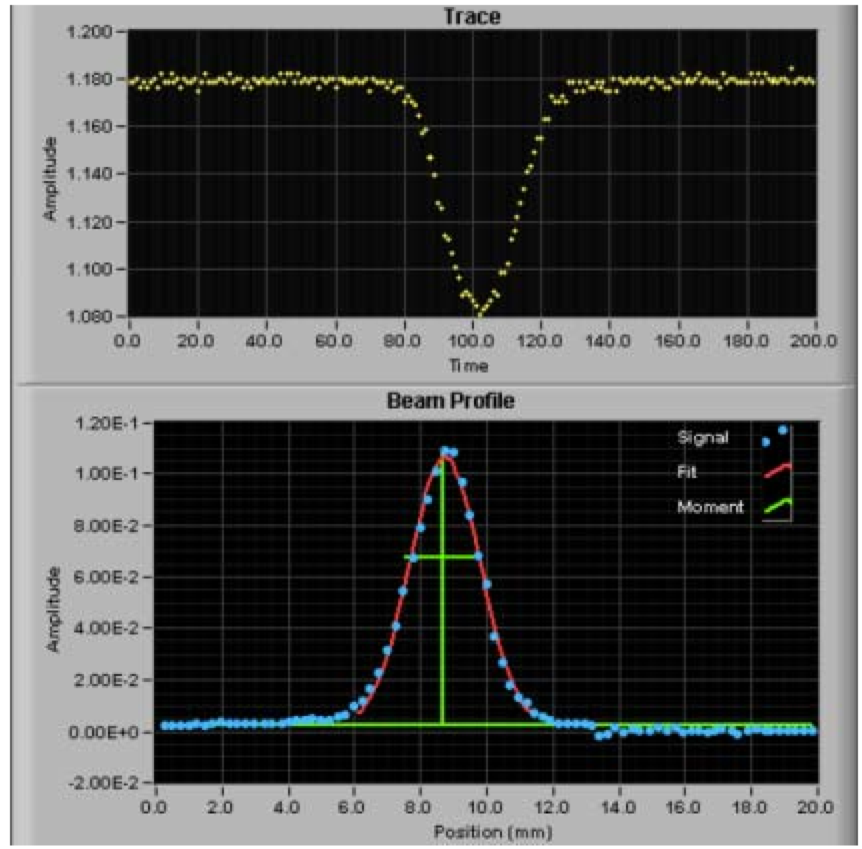
Front End Test Stand (RAL) – electrons + neutrals
SNS (detect electrons)

SNS laser-wire system

Laser

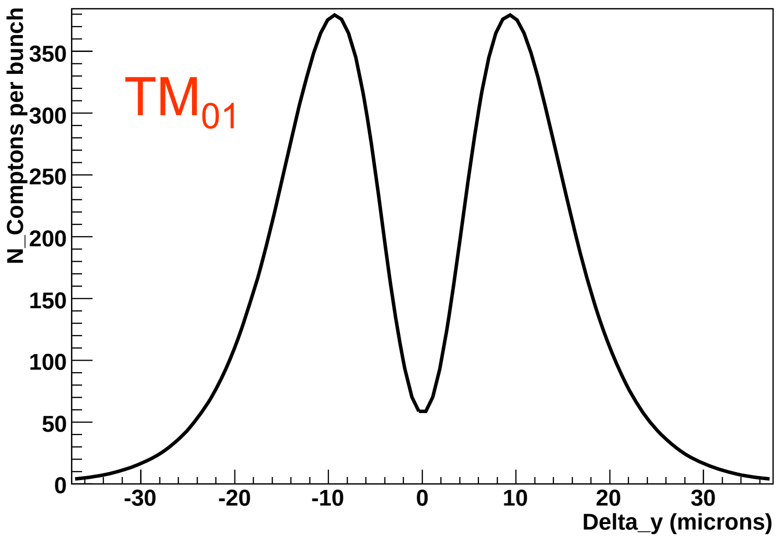
e⁻ detector

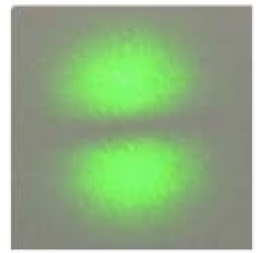


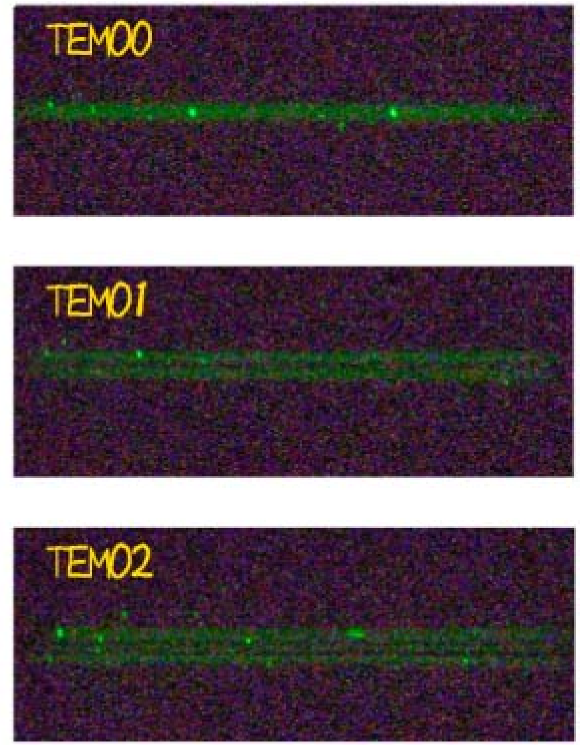


dipole to extract e^-

Higher Order Modes







Their presence increases the effective "emittance" of the laser

$$\sigma_x \sigma_{x'} = \frac{\lambda}{4\pi} \rightarrow M^2 \frac{\lambda}{4\pi} \qquad (M^2 > 1)$$

pure TM_{00}

property of a realistic laser

Summary

- Emittance is an important parameter for accelerators
  - Determines the final luminosity of a collider
  - Determines the quality of a beam in a light source
  - Determines the aperture of a beam at any location, given a known set of optics.
- Measurement:
  - Pepperpot for low energy protons
  - Transverse beam profile plus knowledge of optics: e.g. quad scans
  - Laser-wires for electron/positron and H^-
  - Shintake monitor for 10s nm scale beams

Thanks to:

- A. Assadi
- H. Braun (CAS 2008)
- P. Forck
- K. Wittenburg
- C. Gabor

Whose ideas I have used and whose slides I have borrowed!

Enjoy the problem set !