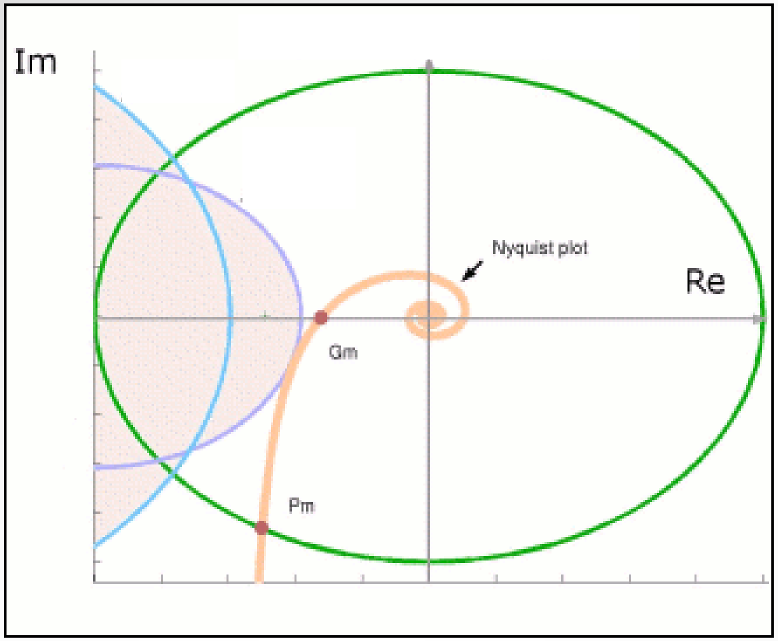
Verification of the Design of the Beam-based Controller

Jürgen Pfingstner

2. June 2009

Content



1. Analysis of the contoller with standard control engineering techniques
2. Uncertanty studies of the response matix and the according control performance

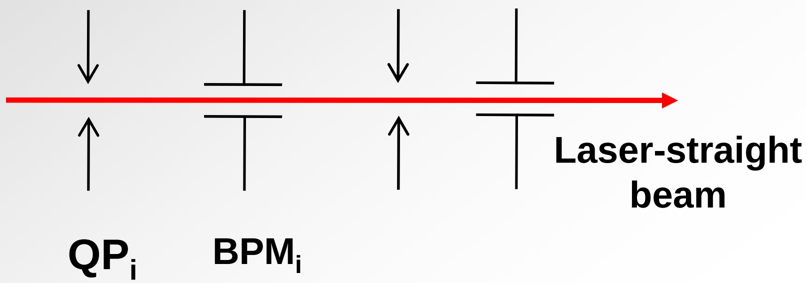
Analysis of the contoller with standard control engineering techniques

- Daniel developed a controller, with common sense and feeling for the system

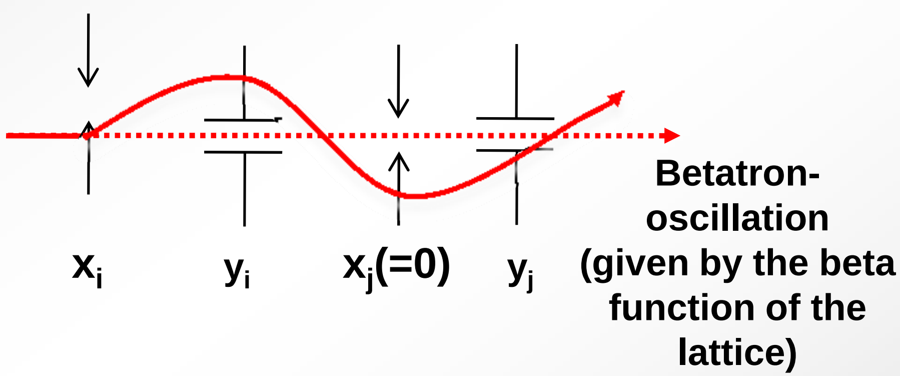
- I tried to verify this intuitive design with, more abstract and standardized methods:
    - Standard nomenclature
    - z – transformation
    - Time-discrete transfer functions
    - Pole-zero plots

The model of the accelerator

1.) Perfect aligned beam line



2.) One misaligned QP



a.) 2 times x_i -> 2 times amplitude
-> 2 times y_j
b.) x_i and x_j are independent

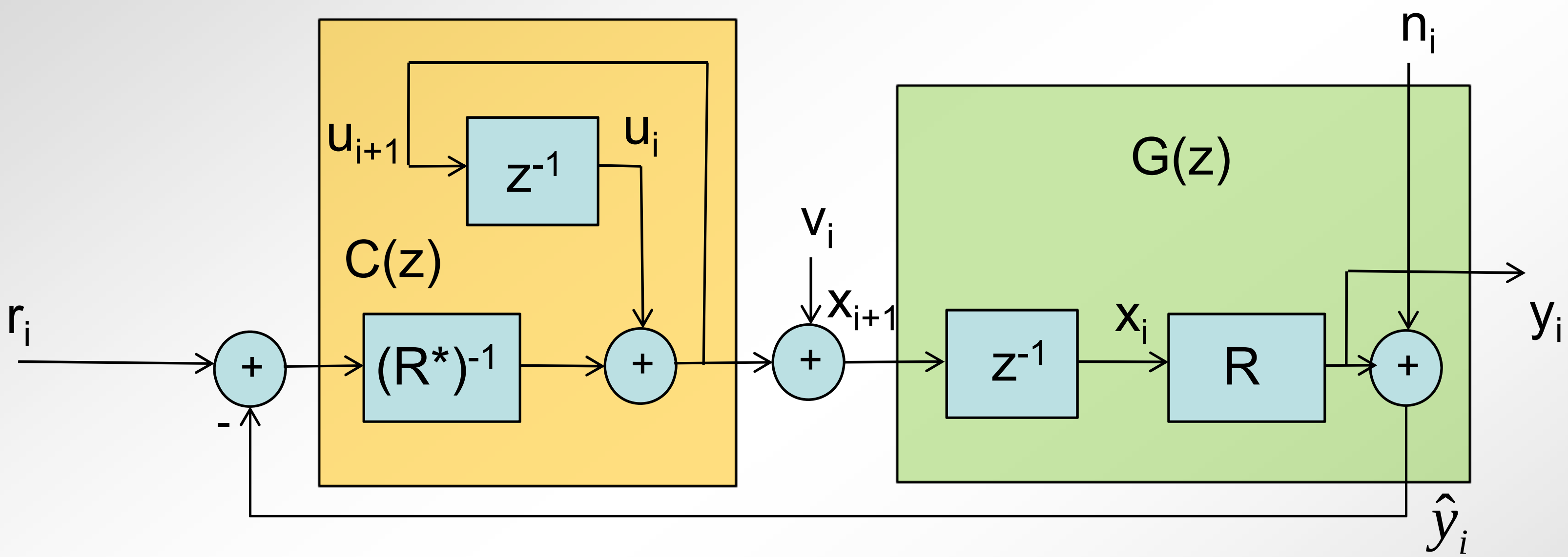
$\Rightarrow$ Linear system without 'memory'

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & \cdots \\ r_{21} & r_{22} & \\ \vdots & & \ddots \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \end{pmatrix}$$

$$\Rightarrow y = Rx$$

y … vector of BPM readings
x … vector of the QP displacements
R … response matrix

Mathematical model of the controlled system



r_i … set value (0)
$\hat{y}_i$ … BPM measurements
y_i … real beam position
v_i … ground motion
n_i … BPM noise

u_i, u_{i+1} … controller state variables
x_i, x_{i+1} … plant state variables (QP position)

C(z) … Controller
G(z) … Plant

System elements (SISO analogon!)

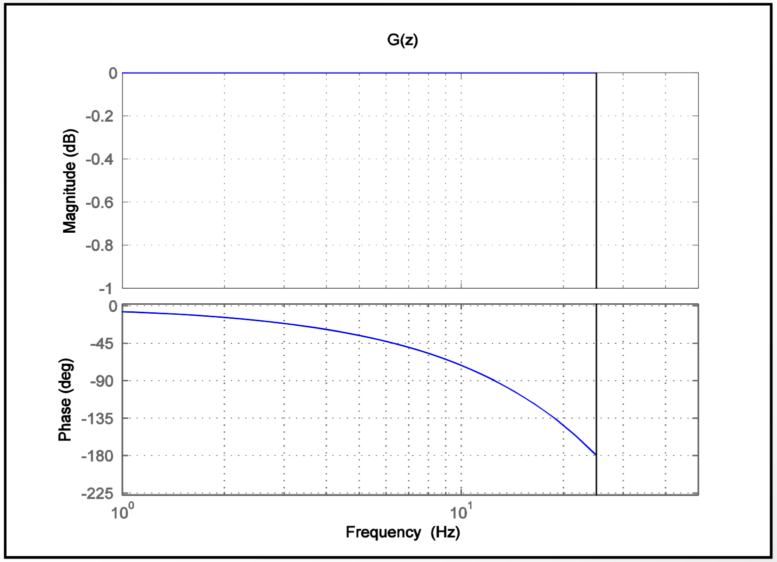
$$G(z) = \frac{R}{z}$$

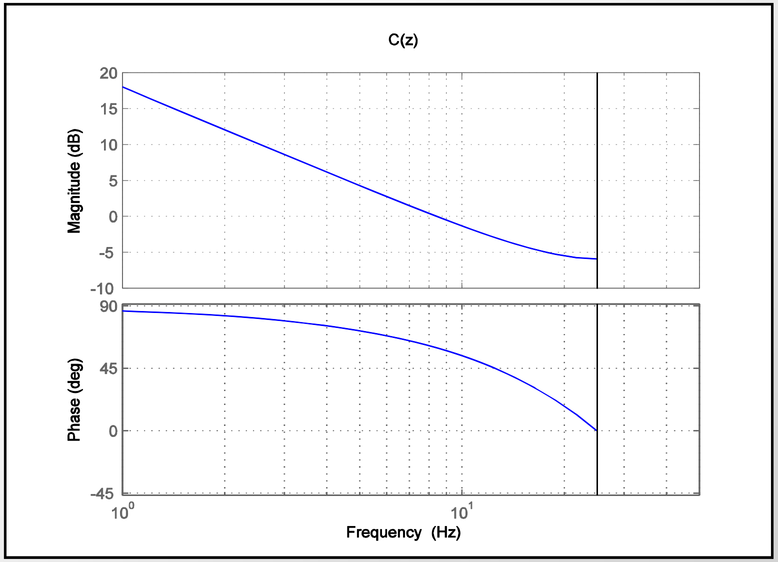
- simple
- allpass
- non minimum phase

$$C(z) = (R^{*})^{-1} \frac{z}{z-1}$$

- I controller

Be aware about the mathematical not correct writing of the TF (matrix instead of scalar)

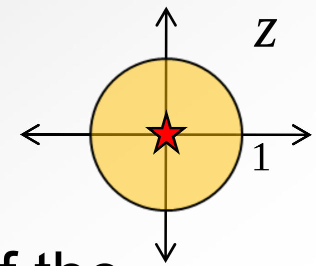


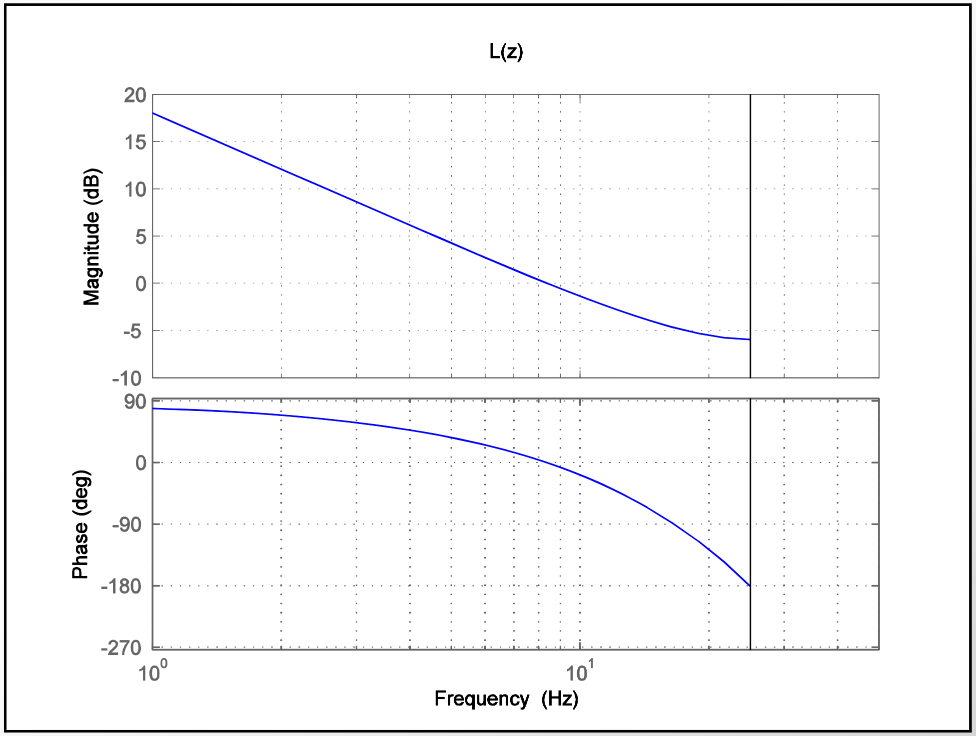


Stability and Performance

$$L(z) := C(z)G(z) = R(R^*)^{-1}\frac{1}{z-1}$$

- Stability
  - necessary attenuation at high frequencies
  - all poles at zero



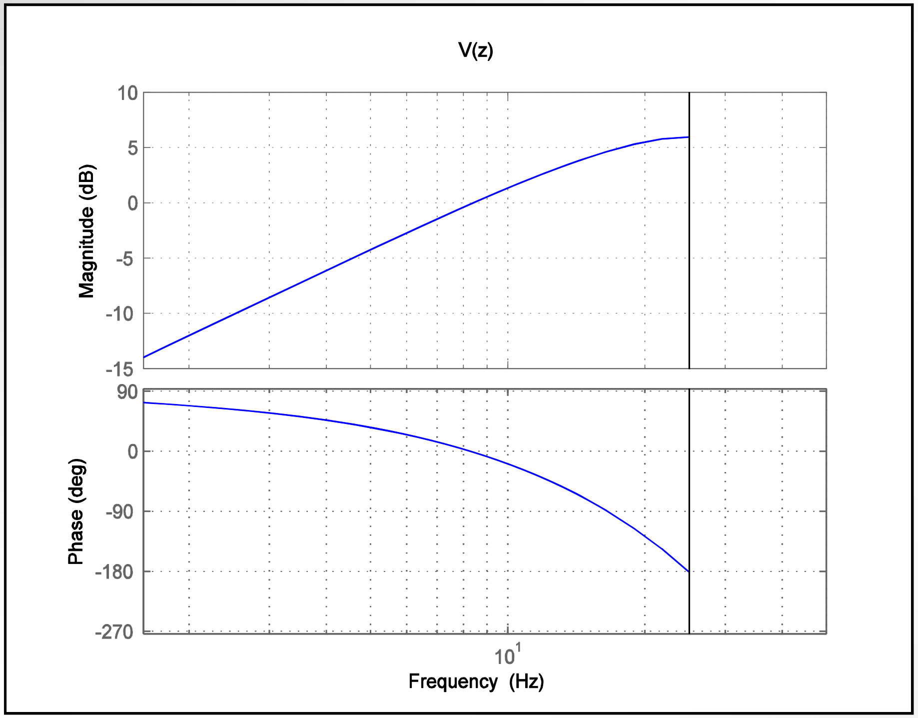


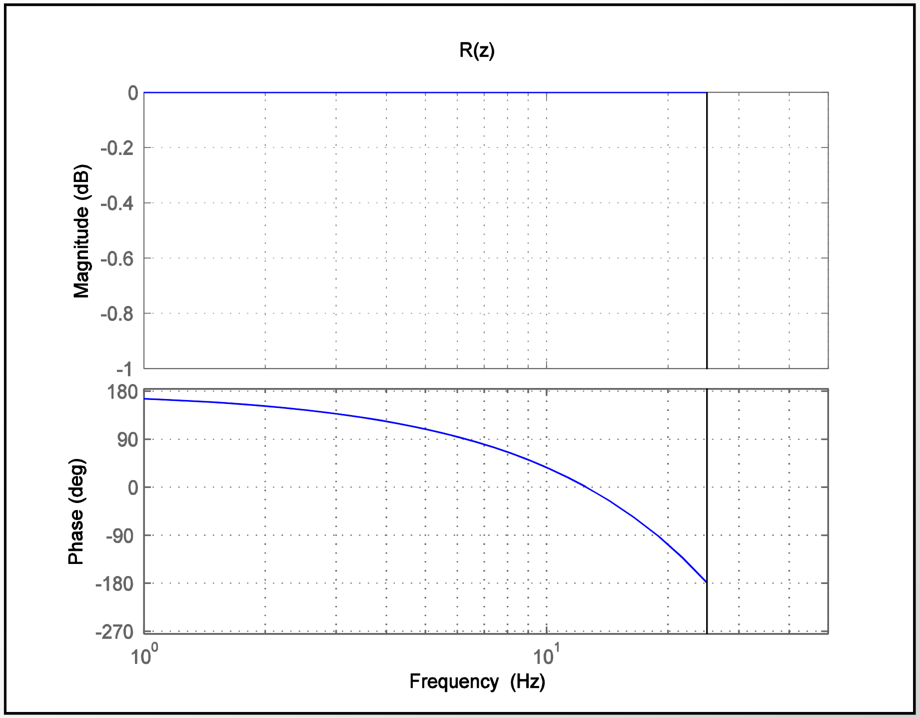
- Performance of the interesting transfer functions
  - $V(z) := \frac{y(z)}{v(z)} = \frac{G(z)}{1+L(z)} = R\frac{z-1}{z}\frac{1}{z-(1-R(R^*)^{-1})}$
  - $N(z) := \frac{y(z)}{n(z)} = \frac{1}{1+L(z)} = \frac{z-1}{z-(1-R(R^*)^{-1})}$
  - $R(z) := \frac{y(z)}{r(z)} = \frac{L(z)}{1+L(z)} = R(R^*)^{-1}\frac{1}{z-(1-R(R^*)^{-1})}$

Important transfer functions

$V(z)$
(ground motion behavior)

$R(z) = N(z)$
(set point following and measurement noise)





Conclusions

- Controller is:
  - very stable and robust (all poles at zero)
  - integrating behavior (errors will die out)
  - good general performance
  - simple (in most cases a good sign for robustness)
  - measurement noise has a strong influence on the output

Further work

- H_∞ optimal control design

Uncertanty studies of the response matix and the according control performance

Controller is robust, but is it robust enough?

yes

no

done

- Answer to that question by simulations in PLACET

- Plan A:
  - Use methods from **robust control** to adjust the controller to the properties of the uncertainties (e.g. pole shift)
- Plan B:
  - Use **adaptive control** techniques to estimate R first and than control accordingly

Tests in PLACET

- Script in PLACET where the following disturbances can be switched on and off:
  - Initial energy E_{init}
  - Energy spread ΔE
  - QP gradient jitter and systematic errors
  - Acceleration gradient and phase jitter
  - BPM noise and failures
  - Corrector errors
  - Ground motion
- Additional PLACET function *PhaseAdvance*
- 2 Test series (Robustness according to machine drift):
  - Robustness regarding to machine imperfection with perfect controller model
  - Robustness regarding to controller model imperfections with perfect machine
- Analysis of the controller performance and the resulting R

Test procedure

1.) Misalign the QP at the begin of the simulation to create an emittance growth at the end of the CLIC main linac

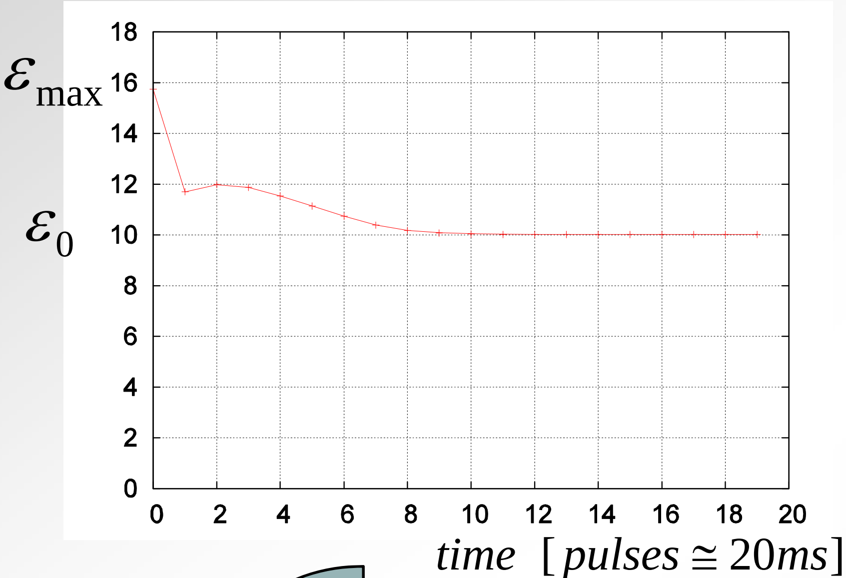
2.) Observe the feedback action in respect to the resulting emittance over time

3.) Change certain accelerator parameter and repeat 1 and 2.

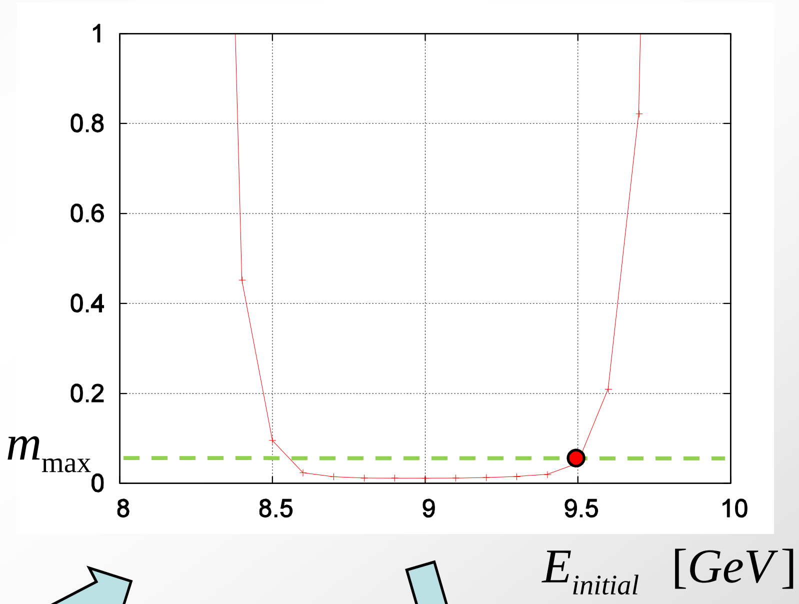
=> The test focus on the stability and convergence speed more than on the steady-state emittance growth and growth rate. (see metric)

Performance metric

Normalized emittance $\varepsilon(t)\ [nm \cdot rad]$



$m(E_{initial})$



$$m := \int_{t=0}^{t=0.4s} \frac{\varepsilon(t) - \varepsilon_0}{\varepsilon_{max} - \varepsilon_0} dt$$

Allowed range for $E_{initial}$ is where $m(E_{initial}) < m_{max} = 0.05$

ε_0 … Emittance produced by the DR and RTML
$\varepsilon(t)$ … Emittance at the end of the main linac
Theoretical minimum for $m_{min} = 0.5 \cdot 0.02 = 0.01$

Results

Quantity	Acceptable values	Nominal values
E_{init}	8.6 – 9.4 GeV	9.0 GeV (± 1 %)
σ_E	0.0 – 9.0 %	2.0 % (± <1%)
QP gradient jitter	0.0 - 0.25 %	< 0.1%
QP gradient error (syst.)	0.0 – 0.25 %	
Acceleration gradient variation	0.0 – 0.3 ‰	< 0.3 ‰
Corrector jitter	0.0 – 8.0 nm (std)	< 10 nm (std)
BPM noise	0.0 – 500 nm (std)	< 50 nm (std)
BPM error distributed *2	110 of 2010 (5%)	
BPM block errors *2	6 of 2010	
Ground motion	0.68 x 10^{-3}nm/min (≈5d)	

*1 … Values are according to the multi-pulse projected emittance

*2 … Failed BPMs that deliver zeros are worse then the one giving random values.

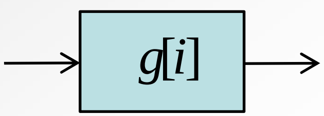
Conclusions

- Controller is very robust in respect to stability
- It is sufficient in respect to disturbances

Thank you for your attention!

z – Transformation

- Method to solve recursive equations
- Equivalent to the Laplace – transformation for time-discrete, linear systems
- Allows frequency domain analysis



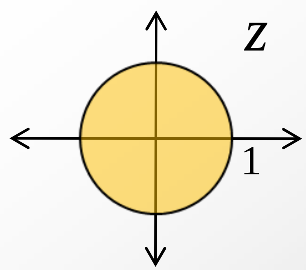
$$G(z) = \sum_{i=0}^{\infty} g[i] z^{i}$$

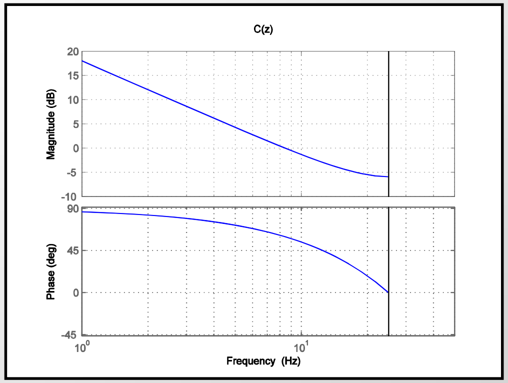
z – transfer function G(z)

$$z = e^{i\omega T_d}$$

Discrete Bode plot

Discrete System





Measures for the performance

- Goal: Find properties of R_{dist} that correspond with the controller performance

$$abs_{norm}$$

$$M = R_{dist} - R_{nom}$$

$$m_{norm} = \frac{\sum_i \sum_j |m_{ij}|}{\sum_i \sum_j |r_{norm,ij}|}$$

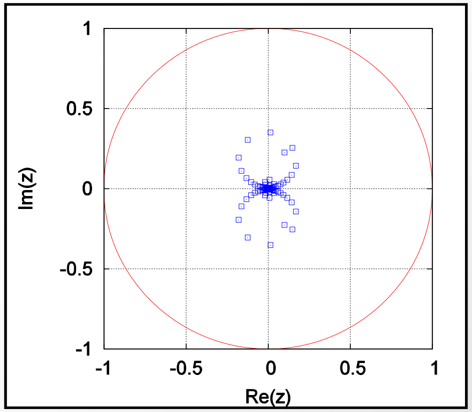
R_{dist} … disturbed matrix
R_{nom} … nominal matrix
abs_{norm} … absolut matrix norm

$$ev_{norm}$$

$$L = E - R(R^*)^{-1}$$

$$EV = eig(L)$$

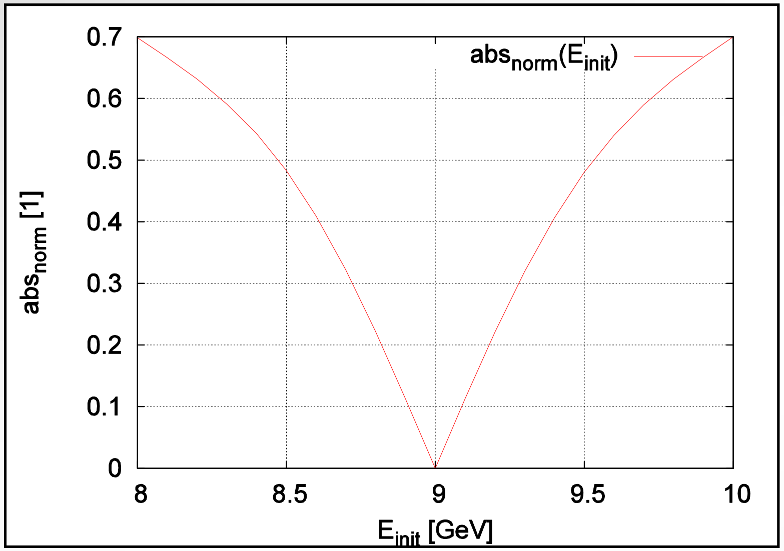
$$ev_{norm} = 1 - \max_i(|ev_i|)$$



ev_{norm} … eigenvalue norm
L … matrix that determines the poles of the control loop

Information from abs_{norm} and ev_{norm}

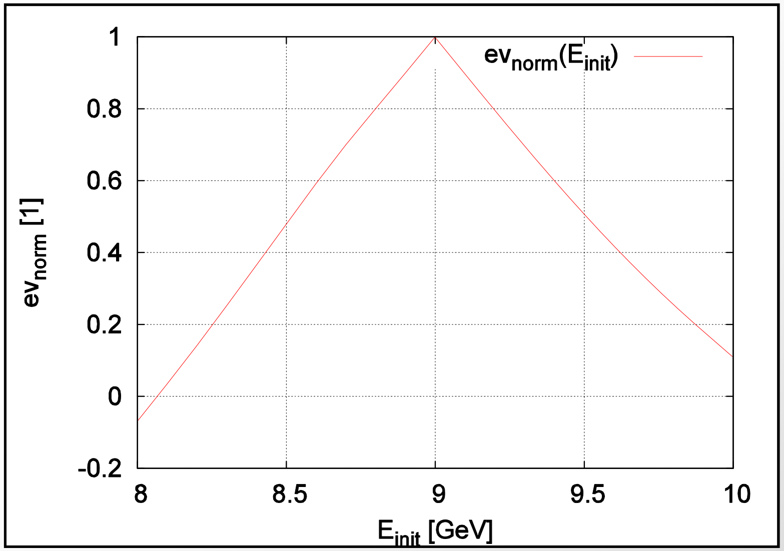
abs_{norm}



Controller works well for:

- $abs_{norm} = 0.0 – 0.4$

ev_{norm}



Controller works well for:

- $ev_{norm} = 0.5 – 1.0$