Crab Cavity: Impedance & Stability

LHC-CC09, 3rd LHC Crab Cavity Workshop
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- longitudinal stability and impedance budget
- transverse impedance budget

Acknowledgments: W. Hofle, T. Linnecar, J. Tuckmantel

Beam and machine parameters

Energy	TeV	0.45	7.0
RF frequency	MHz	400.8 (200.4)	400.8
RF voltage	MV	8.0 (3.0)	16.0
synchrotron frequency f_s	Hz	66.08 (28.64)	23.86
revolution frequency f_0	kHz	11.245	11.245
betatron tune Q_β H/V		59.3/64.28	59.3/64.31
longitudinal emittance	eVs	0.6-1.0	1.0-2.5
rms bunch length	ns	0.4	0.275
nominal (upgrade) bunch current	mA	0.2 (0.7)	0.2 (0.7)
number of bunches (symmetric) M		2808 (3564)	2808 (3564)
nominal (upgrade) beam current (symmetric bunches) I_0	A	0.7 (2.5)	0.7 (2.5)

Longitudinal stability

- Feedback and feedforward systems and longitudinal damper at 400 MHz (~ 1 MHz bandwidth)
- No longitudinal bunch-by-bunch feedback - difficult to do better than natural damping (D. Boussard et al.)
- We rely only on Landau damping due to synchrotron frequency spread inside the bunch:
  - limitation on broad-band impedance budget (ImZ/n)
  - controlled longitudinal emittance blow-up (factor 4) during the ramp
  - proposal for the 2nd harmonic RF system at 800 MHz to be used in bunch-shortening mode (T. Linnecar, E.S., 2007)

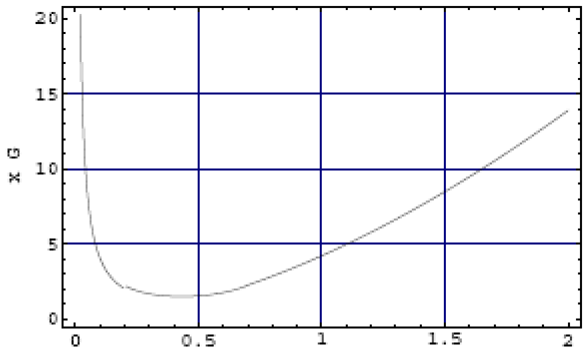
Longitudinal stability

Threshold for coupled-bunch instability (equally spaced bunches) due to resonant impedance with frequency
$f_r = f_0 n_r = pMf_0 + nf_0 + mf_s$

$$R_{sh} < \frac{|\eta| E}{eI_0} (\frac{\Delta p}{p})^2 \frac{\Delta\omega_s}{\omega_s} \frac{F}{f_0 \tau} xG(x),$$

f_0 is the revolution frequency, $\eta = 1/\gamma^2 - 1/\gamma_t^2$, I_0 is the average beam current, $\frac{\Delta p}{p}$ is the relative momentum spread, $\frac{\Delta\omega_s}{\omega_s}$ is the relative synchrotron frequency spread, $F \sim 0.3$ is defined by the particle distribution.
(*V. Balbekov, S. Ivanov, 1984*)

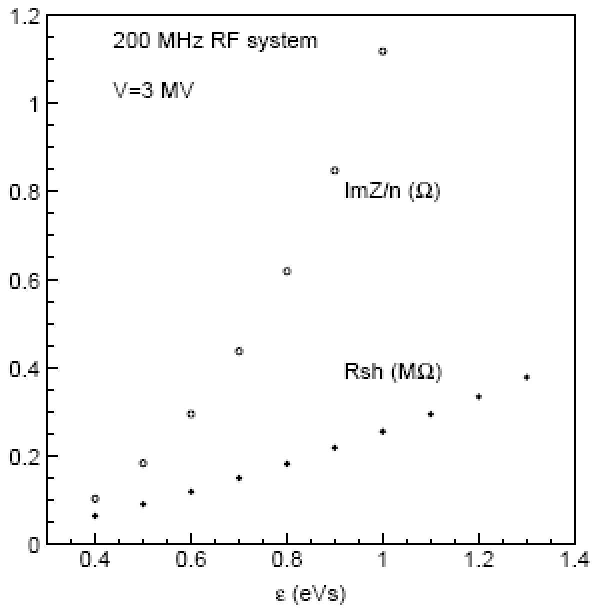
Function
$$xG(x) = x \min\{J_m^{-2}(\pi x)\},$$
$$x = f_r \tau$$



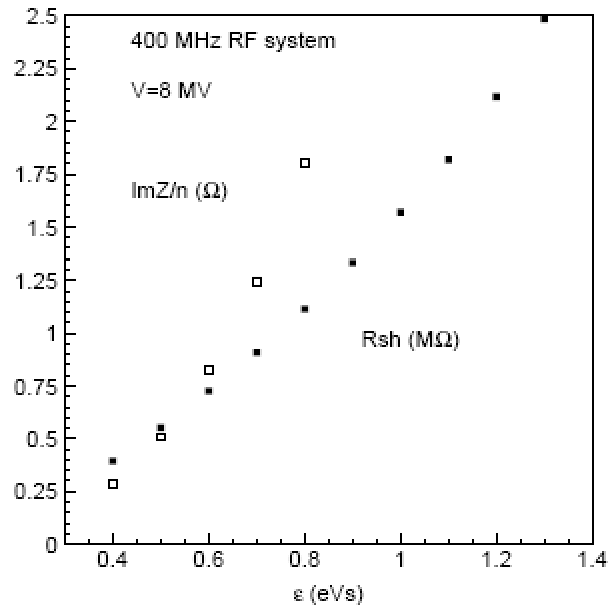
$f_r \tau$

Stability on 450 GeV flat bottom

200 MHz RF system



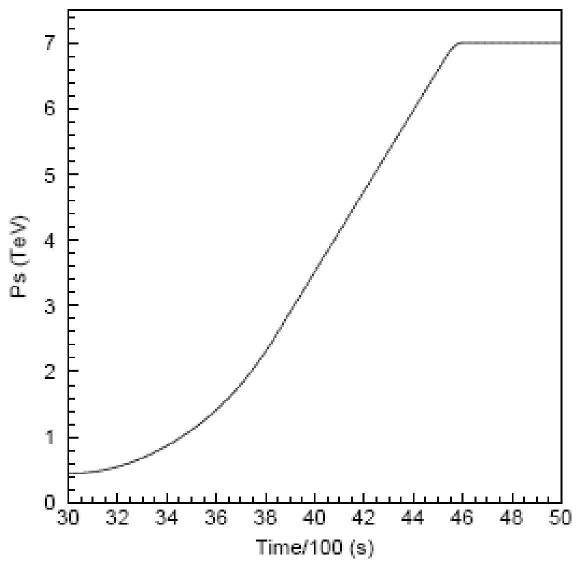
400 MHz RF system



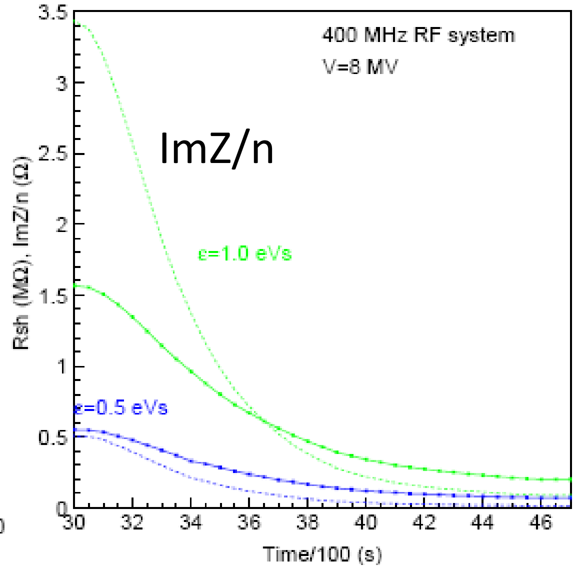
- 200 MHz: 120 kOhm limit for emittance of 0.7 eVs (symmetric bunches) and nominal intensity, 20 kOhm for upgrade intensity and different F
- 400 MHz RF system can be used as Landau cavity (to increase synchrotron frequency spread)

Impedance limit during the LHC cycle for nominal bunch and beam current and different emittances

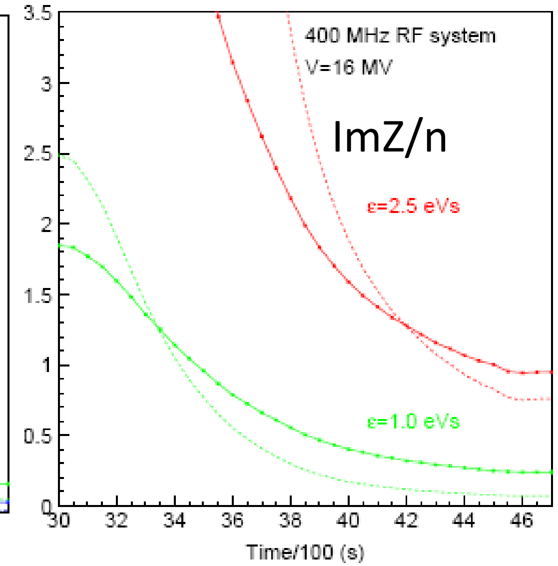
Momentum



V=8 MV@400 MHz



V=16 MV@400 MHz



→ Threshold narrow- and broad-band impedances decrease with beam energy: $R_{sh} \sim (\varepsilon^2/E)^{3/4}$, $ImZ/n \sim (\varepsilon^2/E)^{5/4}$ → controlled emittance blow-up $\varepsilon \sim E^{1/2}$

Beam stability in 400 MHz RF system

- Limit of 150 kOhm for 1 eVs at 7 TeV, nominal intensity (symmetric bunches)
- Factor 1/2 for different bunch distribution (formfactor F)
- Factor 1/3.5 for upgrade intensity
- Factor 4 for 2.5 eVs (nominal) emittance

➢ 85 kOhm in frequency range (100-600) MHz, relaxed $\sim (2\tau f_r)^{5/3}$ outside minimum at $f_r \approx 0.4/\tau$, (τ is 4 σ bunch length)

Summary: longitudinal impedance budget

- Requirement for HOM damping in LHC given so far is 60 kOhm (defined by 200 MHz RF at 450 GeV)
- For nominal intensity
  - in 400 MHz RF system we have 80 kOhm for small emittance beam (1 eVs) at 7 TeV, 300 kOhm for 2.5 eVs
  - in 200 MHz RF system it is 70 kOhm , but the 400 MHz RF system can be used as Landau system
- Assumption: no loss of Landau damping due to broad-band impedance (ImZ/n > 0.1 Ohm, budget estimation in LHC DR - 0.07 Ohm), possible for small emittances (<0.7 eVs) at injection into 200 MHz RF system or at 7 TeV in the 400 MHz RF system (< 1 eVs)

➢ 10 kOhm for upgrade intensity and two identical cavities

Transverse coupled- bunch instability (narrow-band impedances)

- Instability thresholds are determined by
  - betatron frequency spread:
    - system nonlinearities
    - octupoles
    - space charge
    - long range beam-beam
  - synchrotron frequency spread (m>0)
  - chromaticity

- In LHC there is a bunch-by-bunch transverse feedback system (20 MHz bandwidth) to damp injection oscillations and unstable rigid bunch motion

➢ Find which growth rates can be damped without significant transverse emittance blow-up with present transverse damper HW

Instability growth time (1/2)

- A resonant transverse impedance with resistive part Z_T [Ohm/m] at resonant frequency f_r will drive coupled bunch mode (n, m) when $f_r = (n+pM+Q_\beta)f_0 + mf_s$ with the growth rate

$$\frac{1}{\tau_m} = \frac{1}{m+1}\frac{1}{4\pi Q_\beta}\frac{cI_0 Z_T}{E/e}F(\omega_r\tau - \omega_\xi\tau)$$

f_0 and f_s are revolution and synchrotron frequency,

M is number of (symmetric) bunches,

Q_β is betatron tune, $\omega_\xi = Q_\beta\,\omega_0\xi/\eta$, ξ is chromaticity,

Formfactor F(x) for water-bag bunch: F(0)=1 (the worst mode m=0),

$F(x > 0.5) \approx 0.5$

Instability growth time (2/2)

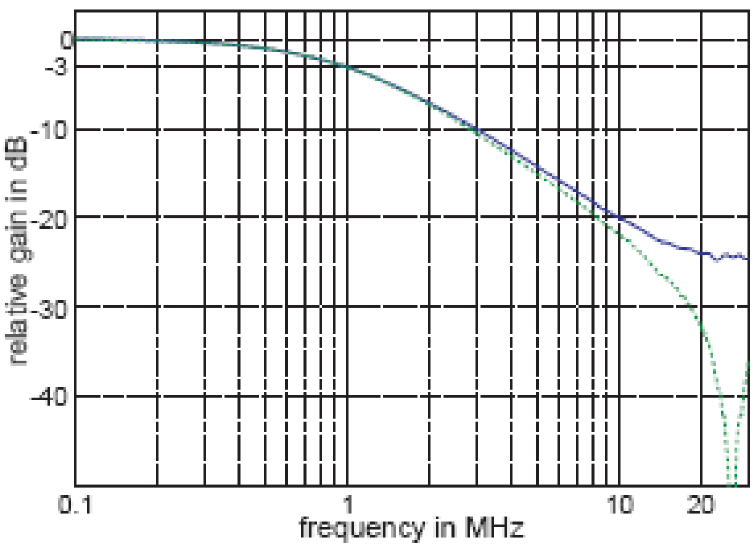
- Growth rate ~ 1/E → maximum at low energy
- At 450 GeV, nominal intensity and one cavity
  minimum τ_{inst} = 0.15 [s] for Z_T =1 MOhm/m (ξ=0):
  Z_T [MOhm/m] < 0.15 /(1-x)/τ_d , $x=(f_r - f_\xi)\tau < 0.8$
  Z_T [MOhm/m] < 0.3 (1+2x)/τ_d , $x > 0.8$
  τ_d [s] is the damping time by transverse damper,
  τ is the bunch length, typically 1.0 ns < τ < 1.5 ns
- For upgrade intensity factor 3.5 down

Damping time (1/2)

- Specifications of the transverse feedback:
  damping time τ_d =3.6 ms, but (W. Hofle et al.)
  - simultaneous injection oscillation damping
  - resistive wall instability growth time ~17 ms at injection for ultimate intensity (E. Metral, 2008)
  - decoherence time 68 ms (for tune spread 1.3 x10^{-3} and chromaticity $\xi \neq 0$)
  - strict budget for transverse emittance blow-up (2.5%), 2.2% blow-up at ultimate intensity
  - gain roll-off for high frequencies

Damping time (2/2)

Power amplifier
frequency characteristics



→ roll-off of gain for kicker and tetrode anode voltage (W. Hofle et al., EPAC'08)

- Frequency roll-off: -1 dB at 1 MHz and -24 dB at 20 MHz

  → $\tau_{d\ max}$ = 60 ms

- ➢ Crab cavity impedance (for x=0)

  Z_T < 2.5 [MOhm/m]

  (The same threshold from betatron frequency spread)

- Formfactor F_m for different longitudinal particle distribution (not water-bag) – up to factor 1/4
- n_c identical cavities: factor $1/n_c$
- Weight function $\beta/\langle\beta\rangle$ if beta-function at Crab cavity location is different from average

Warning: definition of transverse shunt impedance R_s

- Resonant impedance in [Ohm/m]

  $Z_T(\omega)=D(\omega)\,R_s/[1+jQ(w-w^{-1})]$,

  where $w=\omega/\omega_r$

  $\rightarrow Z_T(\omega_r)=D(\omega_r)\,R_s$

- A. Chao, K.Y. Ng

  $D(\omega)=c/\omega$, $D(\omega_r)=c/\omega_r$
- A. Zotter, S. A. Kheifets

  $D(\omega)=\omega_r/(j\omega)$, $D(\omega_r)=1/j$
- S. Y. Lee

  $D(\omega)=2c/(b^2\omega)$, $D(\omega_r)=2c/(b^2\omega_r)$,

  b – beam pipe radius
- G. Dome

  $D(\omega)=\omega_r^2/(c\omega)$, $D(\omega_r)=\omega_r/c$

- At the resonant frequency

  $Rs=|V_T|^2/(2\,P)$,

  where P is the power loss in the cavity walls and HOM damper for a given level of deflecting voltage V_T on the cavity axis (``circuit" definition, = ½ ``linac" Rs)

- Measured R_s [Ohm] = $Z_T\ c/\omega_r$

- Calculated - …

Summary: transverse impedance budget

- Threshold for the nominal intensity and one cavity at 450 GeV determined by the damping time of 60 ms is 2.5 MOhm/m
- With margin for particle distribution:
  - $0.6/(1-f_r)$ MOhm/m f_r [GHz] < 0.8
  - $1.2(1+2f_r)$ MOhm/m f_r [GHz] > 0.8
  - 3 MOhm/m at 800 MHz $\rightarrow$ 0.4 MOhm/m for upgrade intensity and 2 cavities
- Additional factor proportional to local beta-function $\beta/\langle \beta \rangle$