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FLUCTUATION PHENOMENA in DENSE QUARK MATTER

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Plan of the Talk

1. Pre-Critical region on the QCD phase diagram
2. Color superconductivity vs. BCS
3. Fluctuation propagator (FP)
4. The uses of FP: electrical conductivity and sound absorption
5. Fluctuation induced color diamagnetism

B.K., M.Andreichikov, M.Lukashov, PRD91, 074010 (2015), Mod.Phys.Lett. A31, 1650179 (2016), EPJ Web of Conf. 125, 04013 (2016).

Inspiration from papers by A.Larkin, L.Aslamazov, A.Varlamov

1 Pre-Critical region on the QCD phase diagram

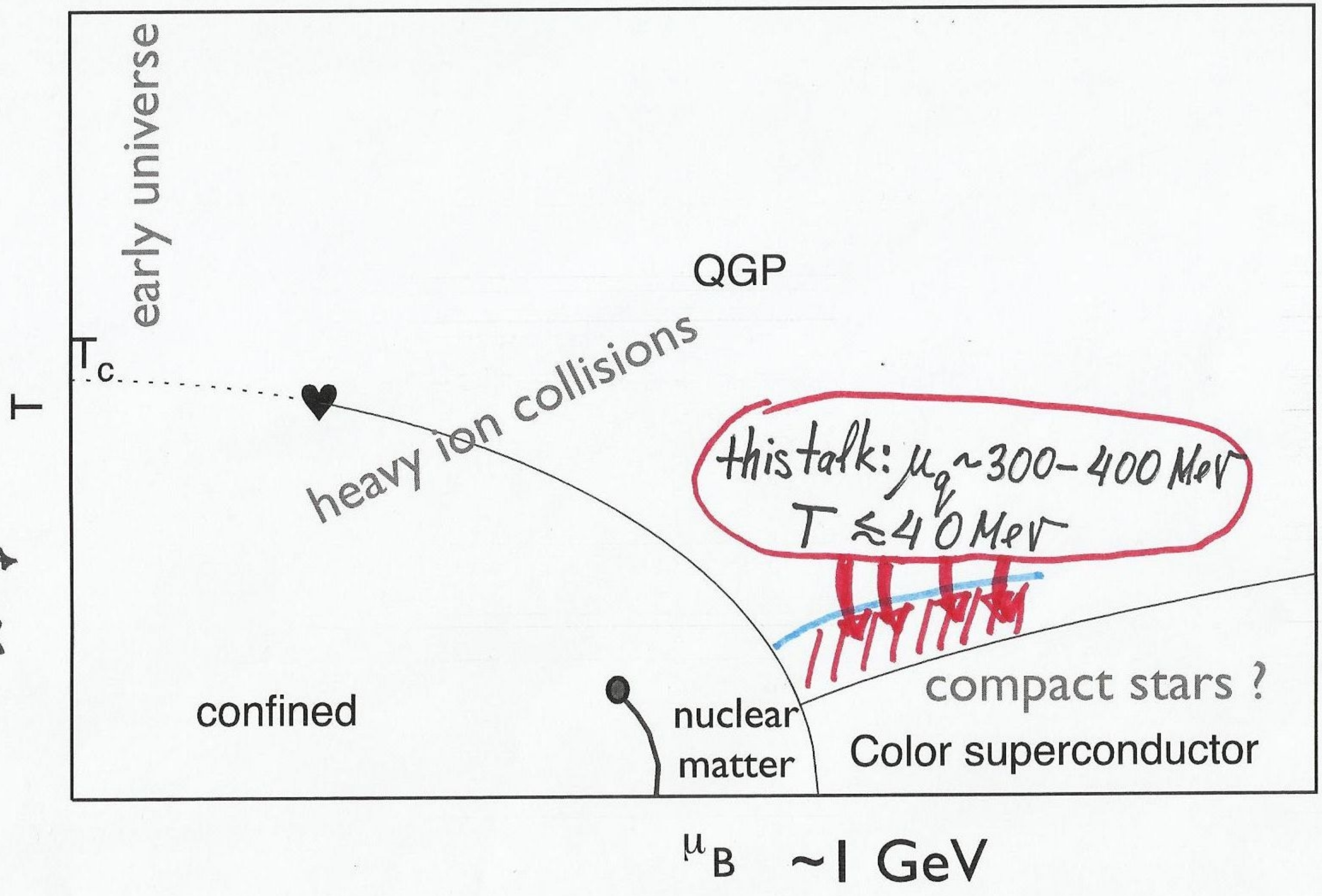
QCD phase diagram – bird's-eye-view,
or "theorist's science fiction" (Owe Philipsen)

~170 MeV

$T \gtrsim T_c$ RHIC and LHC

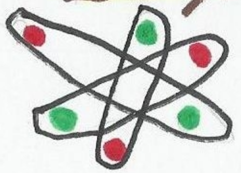
$T \gtrsim T_c$, $\mu = 0$, or small →
→ lattice calculations

$T \lesssim T_c/3$, $\mu \lesssim \frac{GeV}{3}$ →
→ a shaky ground of models

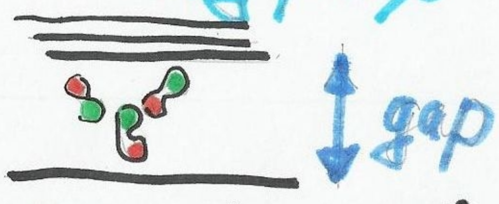


2 Color superconductivity v.s. BCS

CS – attraction in the $\bar{3}$ color channel

BCS	CS
 Cooper pairs	 quarkpairs (Shafrothpairs)

δT – fluctuation region $\quad \delta T / T_c \equiv Gi$ – Ginzburg-Levanyuk number

BCS	CS
$Gi \simeq 10^{-12}$	$Gi \simeq 10^{-2}$
$n^{1/3}\xi \sim 10^4$	$n^{1/3}\xi \sim 1$
$\Delta : \omega_D : E_F \simeq 1 : 10^2 : 10^4$	$\Delta : \Lambda : \mu \simeq 1 : 8 : 4$
Δ ← gap	Λ ← UV cutoff
	Fluctuating pairs
	
	$\tau \propto (T - T_c)^{-1}$

2 more on CS vs. BCS

BCS

$$\frac{d\vec{p}}{(2\pi)^3} \cong \frac{p_F m}{2\pi^2}\,\frac{d\Omega_{\vec{p}}}{4\pi}\,d\xi$$

$$\xi = \frac{\vec{p}^{\,2}}{2m} - \tilde{\mu}$$

Matsubara propagator

$$G(\vec{p},\varepsilon_n) = \frac{1}{i\varepsilon_n - \frac{\vec{p}^{\,2}}{2m} + \tilde{\mu}}$$

$$\varepsilon_n = (2n+1)\pi T,\ \beta \equiv \frac{1}{T}$$

CS

$\frac{p_F \mu}{2\pi^2}$

$$\frac{d\vec{p}}{(2\pi)^3} \cong \left[\rho(\mu) + \left(\frac{\partial\rho}{\partial\xi}\right)_{\mu}\xi\right]\frac{d\Omega_{\vec{p}}}{4\pi}\,d\xi$$

$$\xi = \sqrt{\vec{p}^{\,2}+m^2} - \mu,\quad \mu = m + \tilde{\mu}$$

$\frac{d\rho}{d\xi}$ – density of states energy dependence – important

Matsubara propagator

$$G(\vec{p},\varepsilon_n) = \frac{1}{\gamma_0(i\varepsilon_n+\mu) - \vec{\gamma}\vec{p} - m}$$

$$\varepsilon_n = (2n+1)\pi T$$

3 Fluctuation Propagator (FP)

Derivation $\begin{cases} \text{a) from time dependent Ginzburg-Landau} \\ \text{b) from Dyson equation} \end{cases}$

a) $F[\Psi] = g\left[\varepsilon|\Psi(\vec{r},t)|^2 + \eta^2|\vec{\nabla}\Psi(\vec{r},t)|^2 + \cancel{O(|\Psi|^4)}\right] \rightarrow$

$\rightarrow g\left[\varepsilon|\Psi|^2 + \eta^2\vec{q}^{\,2}|\Psi|^2\right]$, $\varepsilon = \frac{T-T_c}{T_c}$, $\Psi(\vec{r},t)$ – quark pair field, order parameter

η – fluctuation correlation length

Relaxation of the order parameter:

$$-\gamma\frac{\partial\Psi(\vec{r},t)}{\partial t} = \frac{\delta F}{\delta\Psi^*} + \zeta(\vec{r},t),$$ ζ – Langevin forces, thermodynamic fluctuations

A formal solution $\Psi(\vec{r},t) = \hat{L}\zeta(\vec{r},t)$,

$$\hat{L} = -\left[\gamma\frac{\partial}{\partial t} + g(\varepsilon + \eta^2\vec{q}^{\,2})\right]^{-1} \rightarrow \hat{L}(\vec{q},\omega) = -\frac{1}{g}\frac{1}{\varepsilon - i\frac{\gamma}{g}\omega + \eta^2\vec{q}^{\,2}}$$

i) $\hat{L}$ is singular at $T \rightarrow T_c$ and small ω, q

ii) to identify $\hat{L}$ with the **FP** fluctuation-dissipation theorem must hold

③ FP-fluctuation-dissipation (FD) theorem:

Equal-time correlator $\langle \psi^*(\vec{r}',t)\psi(\vec{r},t)\rangle = \langle |\psi_{\vec{p}}|^2\rangle$

averaged over fluctuations in thermal equilibrium

$$\hat{L}^{-1} = -\left(\gamma\frac{\partial}{\partial t} + \hat{H}\right),\quad \hat{H} = \varrho(\varepsilon + \eta^2\vec{q}^{\,2})$$

$$\langle |\psi_{\vec{p}}|^2\rangle = \frac{\int \mathcal{D}\psi^*\mathcal{D}\psi\, |\psi_{\vec{p}}|^2 \exp(-\hat{H}/T)}{\int \mathcal{D}\psi^*\mathcal{D}\psi \exp(-\hat{H}/T)} = \frac{1}{\varrho(\varepsilon+\eta^2\vec{q}^{\,2})}$$

On the other hand, the equal-time correlator reads

$$\langle \psi^*(\vec{r},t)\psi(\vec{r}',t)\rangle = \langle \zeta^*(\vec{r},t)\hat{L}^*\hat{L}\zeta(\vec{r}',t)\rangle = 2T\gamma\int\frac{d\vec{p}}{(2\pi)^3} e^{i\vec{p}(\vec{r}-\vec{r}')} \times$$

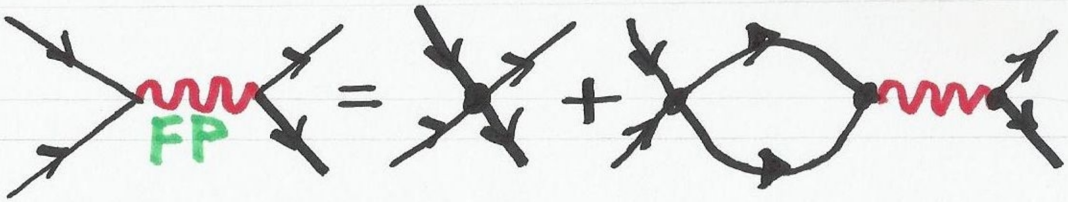
$$\times \int\frac{d\omega}{2\pi} L^*(\vec{p},\omega)L(\vec{p},\omega) = \frac{1}{\varrho(\varepsilon+\eta^2\vec{q}^{\,2})}.$$

$\zeta(\vec{r},t)$ is a Gaussian Langevin force

$\langle \zeta^*\zeta\rangle = 2T\gamma\,\delta^{(3)}(\vec{r}-\vec{r}')\,\delta(t-t')$

Conclusion: the FD theorem holds

3 FP - Microscopic (Dyson equation) derivation



$$L(\vec{q},\omega_k)=\frac{1}{-\frac{1}{g}+\Pi(\vec{q},\omega_k)}$$, Π - polarization operator, hard density loop, $q \ll p$, $T \ll \mu$ (C. Manuel, B. K.)

(ω_k – Matsubara)

$$\Pi(\vec{q},\omega_k)=T\sum_{\varepsilon_n}\int\frac{d\vec{p}}{(2\pi)^3}G(-\vec{p},-\varepsilon_n)G(\vec{p}+\vec{q},\varepsilon_n+\omega_k)\simeq T\sum_{\varepsilon_n}\left[A(\vec{q}=0,\omega_n,\varepsilon_k)+B(\omega_k,\varepsilon_n)\vec{q}^{\,2}\right].$$

In this way we fix (express) γ and η^2: $\gamma=\frac{\pi\varrho}{8T_c}$, $\eta^2=\frac{8\pi}{T_c}\mathcal{D}$,

$\mathcal{D}$ - diffusion coefficient

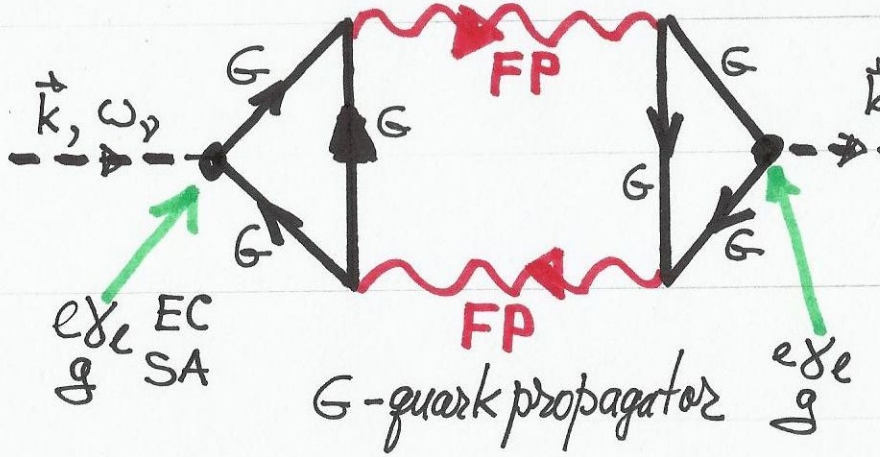
Finally $$L(\vec{q},\omega)=-\frac{1}{\varrho}\,\frac{1}{\varepsilon+\frac{\pi}{8T_c}\left(-i\omega+\mathcal{D}\vec{q}^{\,2}\right)}\,.$$

4 The uses of FP: a) Electrical conductivity (EC)

b) sound absorption (SA)

A single diagram containing two singular FP-s dominates in all transport processes

Aslamazov- Larkin diagram



$\propto (T-T_c)^{-\alpha}$

$\alpha = 1/2$ EC

$\alpha = 3/2$ SA

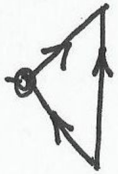
G - quark propagator

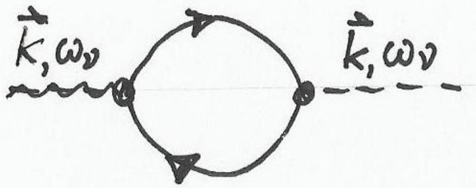
Three loops - simplifying assumptions are needed

4 The uses of FP-electrical conductivity

Drude vs. AL → classical vs. quantum

$$AL(\vec{k},\omega_\nu) = -4T\sum_{\Omega_j}\int\frac{d\vec{p}}{(2\pi)^3}B^2(\vec{k},\vec{p},\omega_\nu,\Omega_j)\times$$
$$\times L(\vec{k}+\vec{p},\Omega_j+\omega_\nu)L(\vec{p},\Omega_j)$$

$B =$  $= eT\sum_{\varepsilon_n}\int\frac{d\vec{p}\,'}{(2\pi)^3}\mathrm{Tr}\{\vec{\gamma}\,G(\vec{p}\,',\varepsilon_n)G(\vec{p}\,'+\vec{k},\varepsilon_n+\omega_\nu)\times G(\vec{p}-\vec{p}\,',\Omega_j-\varepsilon_n)\}$

Drude 

$$\sigma_D = \frac{1}{3}e^2 v_F^2\rho\frac{\tau}{1+\omega\tau+D\vec{k}^2\tau}\rightarrow\Big|_{\substack{\omega\to 0\\ \vec{k}\to 0}}$$
$$\rightarrow\frac{1}{3}e^2v_F^2\tau\rho$$

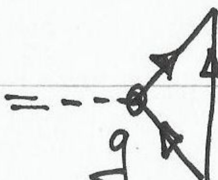
$$\sigma_{AL} = \frac{\pi B^2}{12\rho^2}\int\frac{d\vec{q}}{(2\pi)^3}\frac{\vec{q}^{\,2}}{\left(\varepsilon+\frac{\pi}{8T_c}D\vec{q}^{\,2}\right)^3}\propto\left(\frac{T_c}{T-T_c}\right)^{1/2}.$$

For $\tau = 0.5\,fm$, $\varepsilon = 10^{-2}$ → $\sigma_D = 0.002\,fm^{-1}$, $\sigma_{AL} = 0.08\,fm^{-1}$

4 The uses of FP – sound absorption

Im AL diagram with scalar vertices

Nonzero due to $\frac{\partial \rho}{\partial \xi}$ – not only the Fermi surface crust is involved

$B =$ 

$\propto GGG \propto \left(\frac{\partial \rho}{\partial \xi}\right)_\mu \to g \frac{m}{2\pi^2}\left(\frac{v_F^2+1}{v_F}\right) \ln \frac{\Lambda}{2\pi T_c}$

no γ-matrices

$\Lambda \sim 800$ MeV, $T_c \sim 40$ MeV $\to \ln \sim 1$

$$\mathrm{Im}\, AL = -\omega g^2 (v_F^2+1) \frac{m^2}{2^7 p_F^4 A^{3/2}} \ln^2 \frac{\Lambda}{2\pi T_c} \left(\frac{T}{T-T_c}\right)^{3/2}$$

$$A = \frac{\pi}{24} v_F^2 \frac{T}{T}$$

Im AL $\propto \omega$ – pairing falls behind the T_c changes caused by the sound wave – Mandelshtam-Leontovich slow relaxation theory

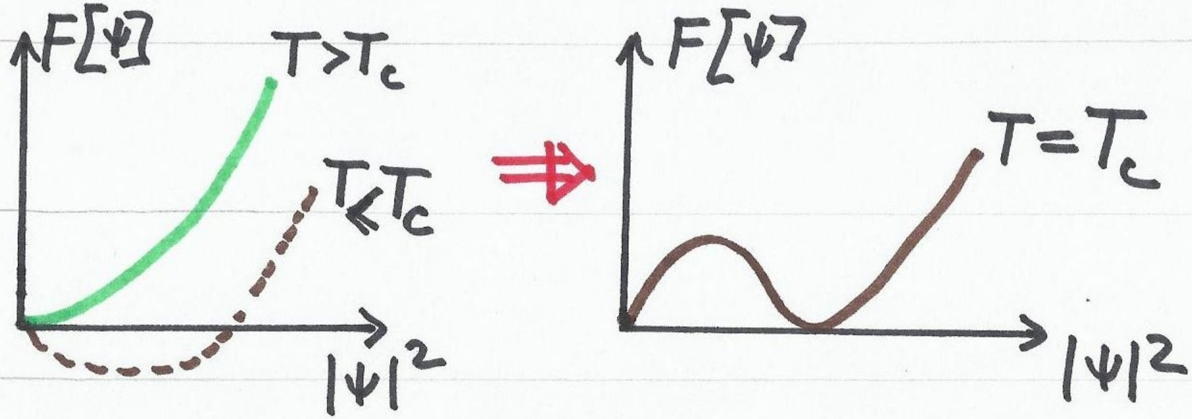
Similar phenomena: singularity of bulk viscosity at T_c (Kharzeev-Tuchin)

singularity of sound velocity (Karsch-Petreczky)

5 Fluctuation induced color diamagnetism – – a short note

Fluctuations of the gauge field $\langle \delta A \delta A \rangle$

Two effects $\begin{cases} \text{(i) lowering of } T_c : T_c' = T_c(1 - g^2 \eta^2 \langle \vec{A}^2 \rangle_\psi) \\ \text{(ii) cubic term in GL – weak 1st order} \end{cases}$



$$F[\psi] = \rho[\varepsilon|\psi|^2 + \eta^2 \vec{q}^{\,2} |\psi|^2] - \frac{1}{2} A_k^l \vec{\nabla}^2 A_k^l$$

Stodolsky, Zakharov: $\langle \int d^3x\, \vec{A}^2 \rangle_{min}$ in $\vec{\nabla}\vec{A} = 0$.

$\langle \vec{A}^2 \rangle \lesssim T_c \Lambda / g^2$, $\langle \vec{A}^2 \rangle \lesssim 1/g^2 \eta^2$, $\langle \vec{A}^2 \rangle \lesssim (50\,\text{MeV})^2$