

Strongly versus weakly coupled QGP

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QGP is expected to be strongly coupled around T_c : how does this feature manifest itself in terms of different quantities, how do we observe it on lattice ?

QGP: state of strongly interacting matter for weakly interacting gas of quark and gluons ? $T \gg \Lambda_{QCD}, g \ll 1$

$$2\pi T \gg m_D \sim gT \gg g^2 T$$

EFT approach: EQCD

← Magnetic screening scale:
non-perturbative

Perturbative series is an expansion in g and not α_s
Loop expansion breaks down at some order (weak coupling may still work)

$$\text{Problem : } g(\mu = 10^2 \text{ GeV}) = \sqrt{4\pi\alpha_s(\mu = 10^2 \text{ GeV})} \simeq 1 \quad g(\mu = 10^{16} \text{ GeV}) \simeq 1/2$$

In this talk :

Fluctuations of conserved charges, color screening, topological susceptibility

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k \quad \text{hadronic}$$

$$\frac{p(T, \mu_u, \mu_d, \mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k \quad \text{quark}$$

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T, V, \mu_a, \mu_b, \mu_c) \Big|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$



information about carriers of the conserved charges (hadrons or quarks)



probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3} (\langle B^2 \rangle - \langle B \rangle^2)$$

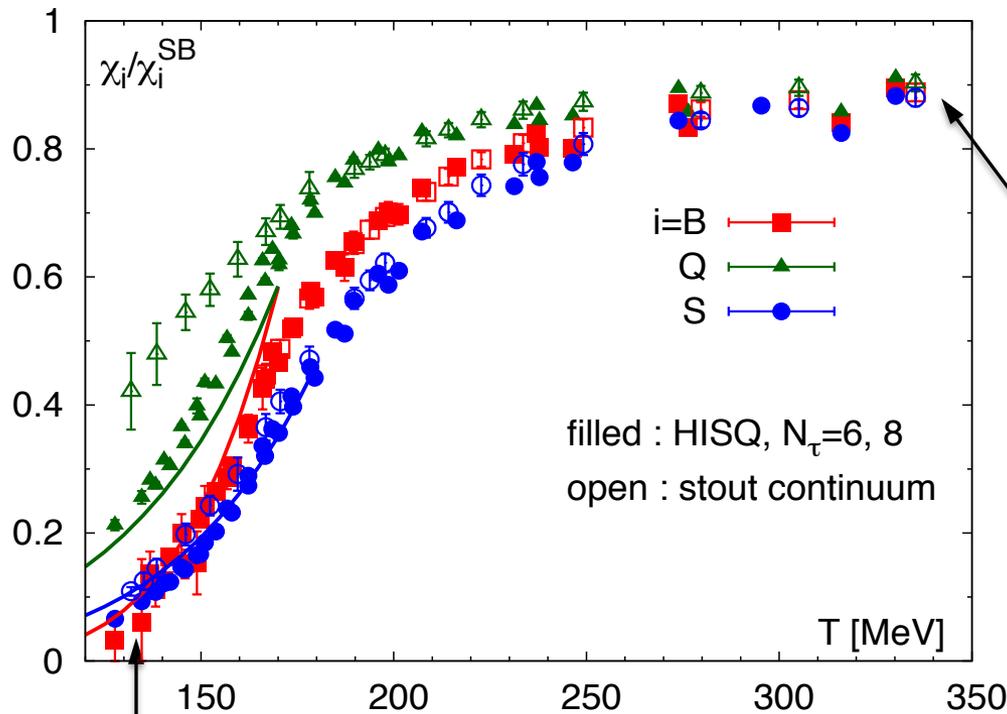
baryon number

$$\chi_Q = \frac{1}{VT^3} (\langle Q^2 \rangle - \langle Q \rangle^2)$$

electric charge

$$\chi_S = \frac{1}{VT^3} (\langle S^2 \rangle - \langle S \rangle^2)$$

strangeness



Ideal gas of massless quarks :

$$\chi_B^{\text{SB}} = \frac{1}{3} \quad \chi_Q^{\text{SB}} = \frac{2}{3}$$

$$\chi_S^{\text{SB}} = 1$$

conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

conserved charges are carried by massive hadrons

Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

$$P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T) \cosh\left(\frac{\mu_S}{T}\right) +$$

$$B_{S=1}(T) \cosh\left(\frac{\mu_B - \mu_S}{T}\right) + B_{S=2}(T) \cosh\left(\frac{\mu_B - 2\mu_S}{T}\right) + B_{S=3}(T) \cosh\left(\frac{\mu_B - 3\mu_S}{T}\right)$$



$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

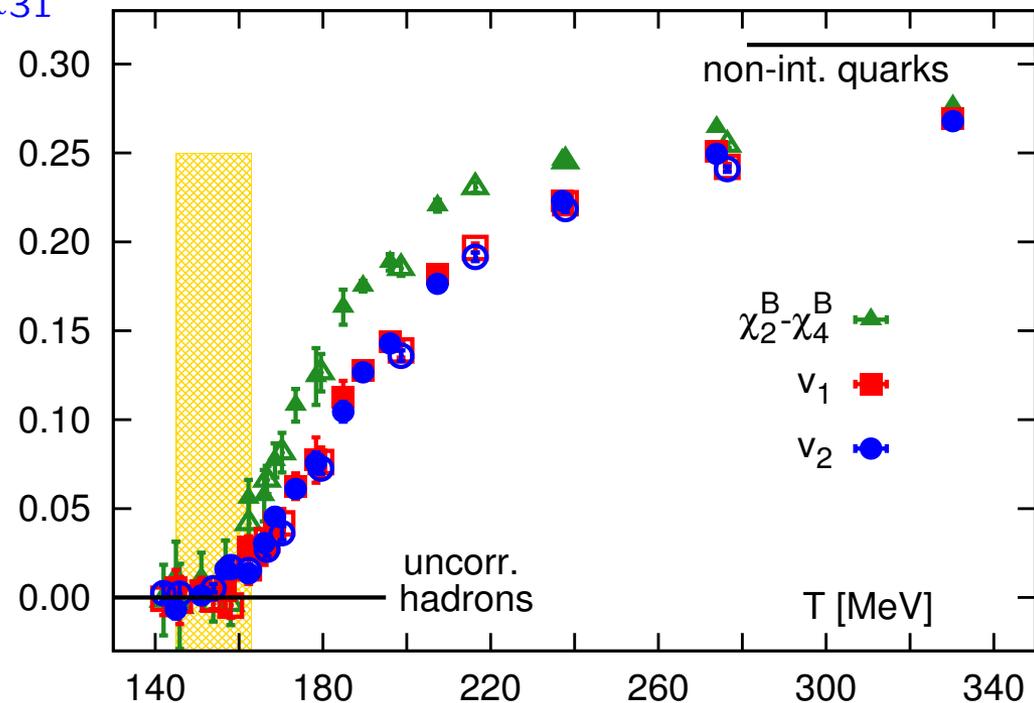
$$v_2 = \frac{1}{3} (\chi_4^S - \chi_2^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

should vanish !

- v_1 and v_2 do vanish within errors at low T
- v_1 and v_2 rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Strange hadrons are heavy treat them
As Boltzmann gas

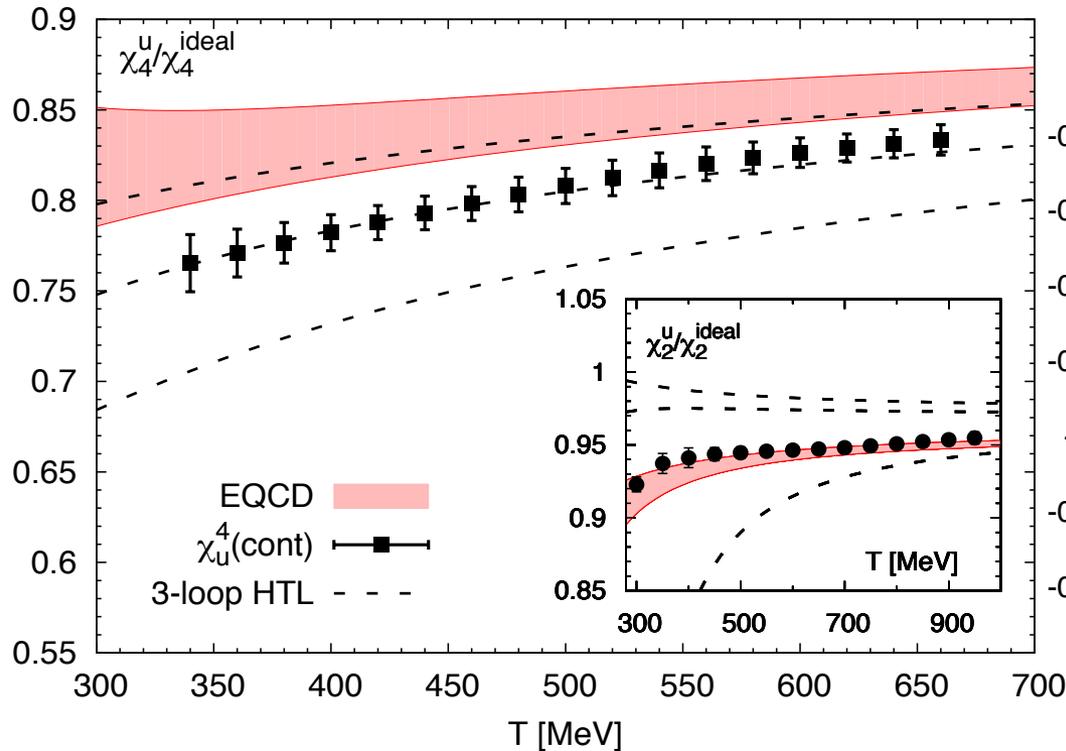
Bazavov et al, PRL 111 (2013) 082301



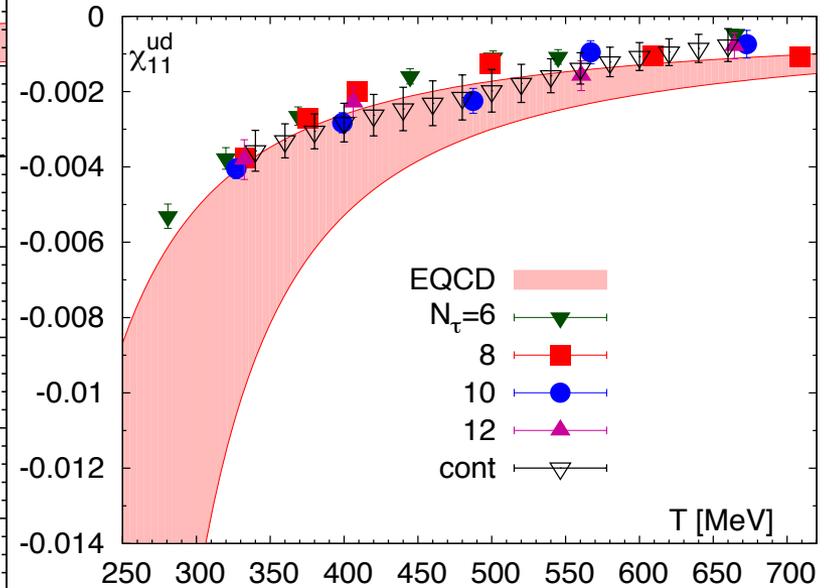
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations



- Good agreement between continuum extrapolated lattice results and the weak coupling approach
- Quark number correlations vanish at any loop order but can be calculated in EQCD and the EQCD calculations agree with the continuum extrapolated lattice results

Bazavov et al, PRD88 (2013) 094021, Ding et al, PRD92 (2015) 074043

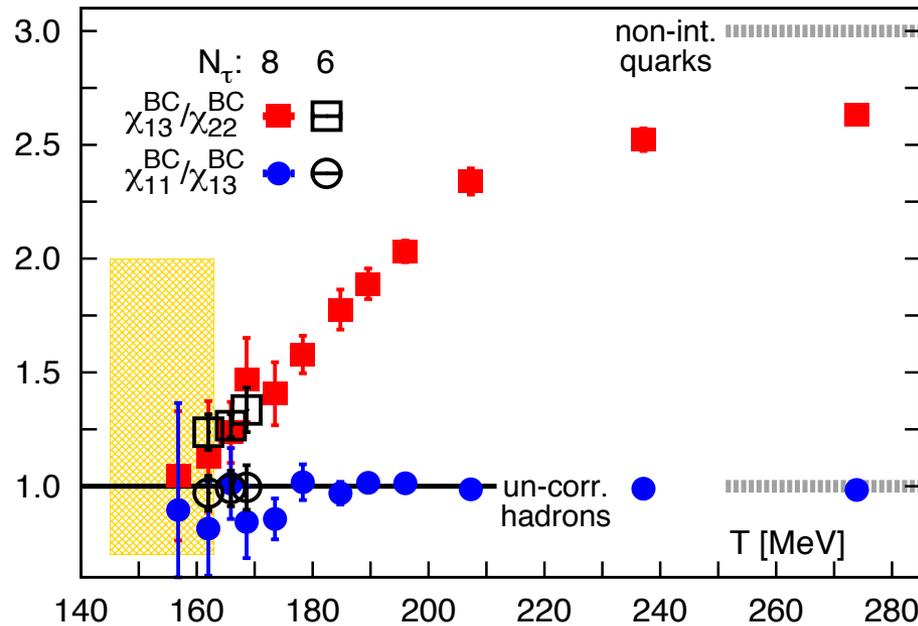
Fluctuation and correlations and deconfinement of charm

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T, \mu_X, \mu_Y, \mu_C) / T^4}{\partial \mu_X^n \partial \mu_Y^m \partial \mu_C^l}$$

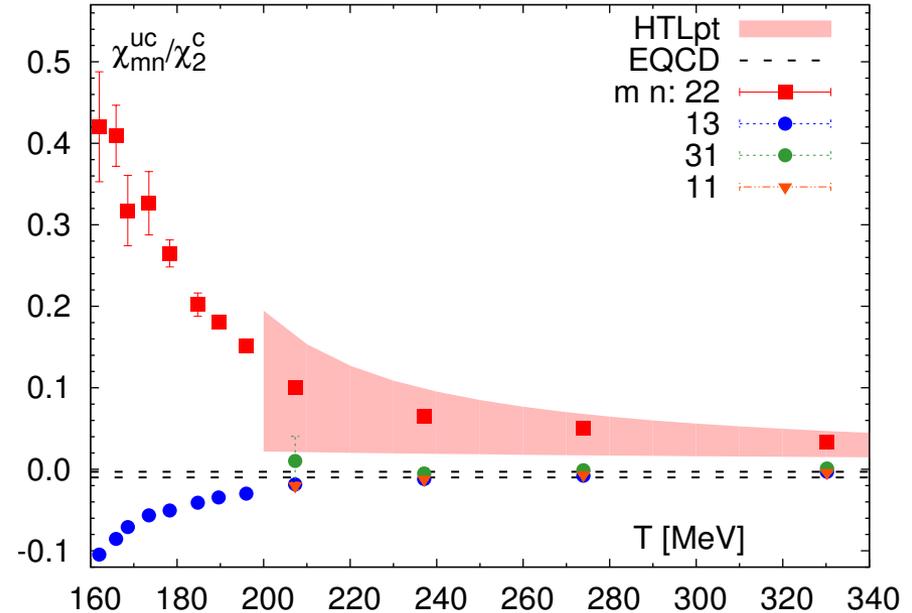
Bazavov et al, PLB 737 (2014) 210

$m_c \gg T \Rightarrow$ only $|C|=1$ sector contributes

In the hadronic phase all BC -correlations are the same !



Hadronic description breaks down just above T_c
 \Rightarrow open charm deconfines above T_c



The charm-light quark correlations can be understood in terms of weak coupling calculations for $T > 250$ MeV but are much larger than the weak coupling result close to T_c

Quasi-particle model for charm degrees of freedom

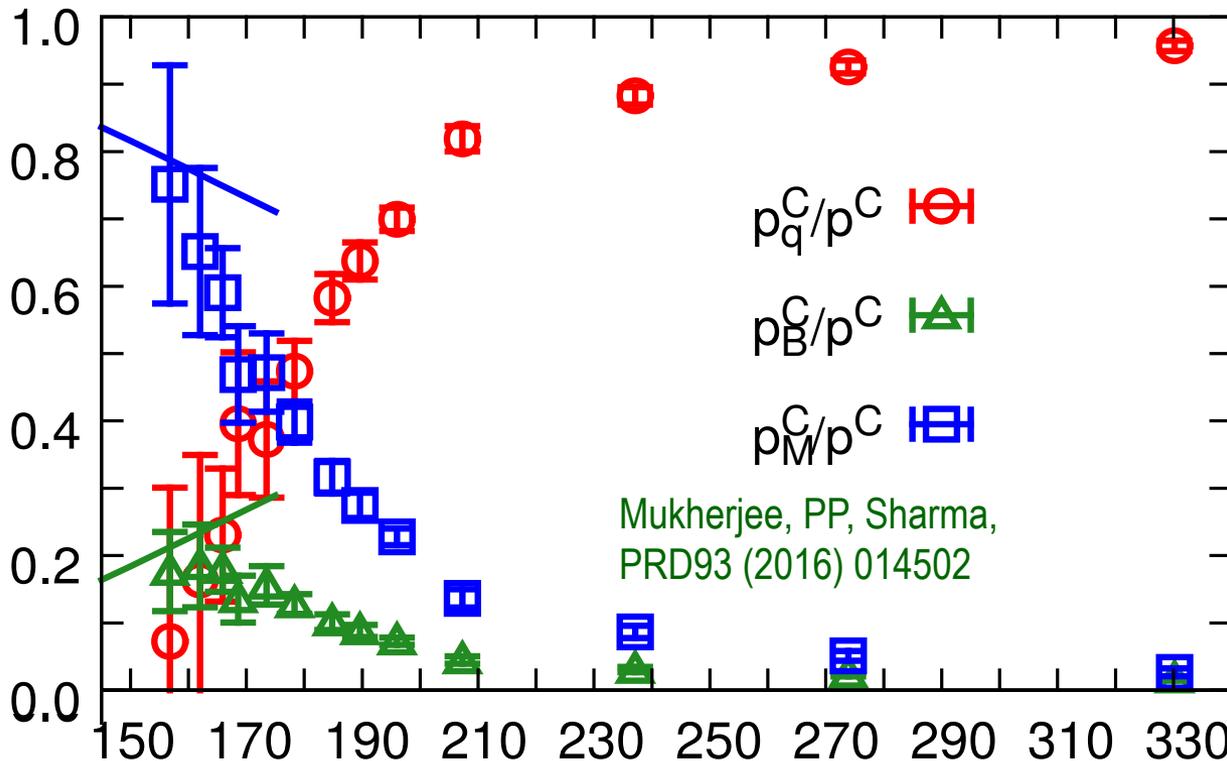
Charm dof are good quasi-particles at all T because $M_c \gg T$ and Boltzmann approximation holds

$$p^C(T, \mu_B, \mu_c) = p_q^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T) \cosh(\hat{\mu}_C)$$

$$\chi_2^C, \chi_{13}^{BC}, \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T)$$

$$\hat{\mu}_X = \mu_X/T$$

Partial meson and baryon pressures described by HRG at T_c and dominate the charm pressure then drop gradually, charm quark only dominant dof at $T > 200$ MeV



Partial pressures drop because hadronic excitations become broad at high temperatures (bound state peaks merge with the continuum)

See
 Jakovác, PRD88 (2013), 065012
 Biró, Jakovác, PRD(2014)065012

Vice versa for quarks

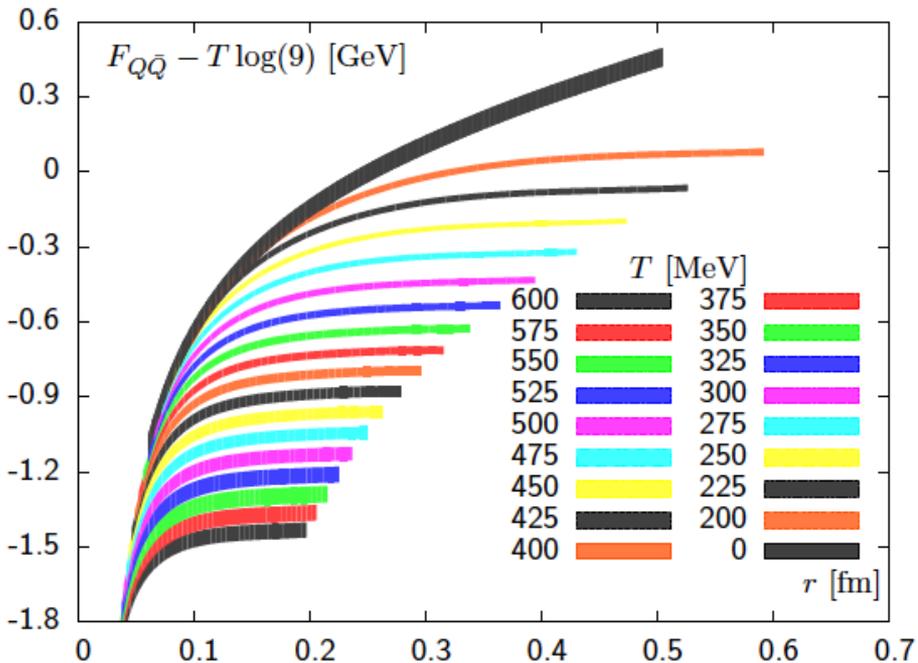
Deconfinement and color screening

Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

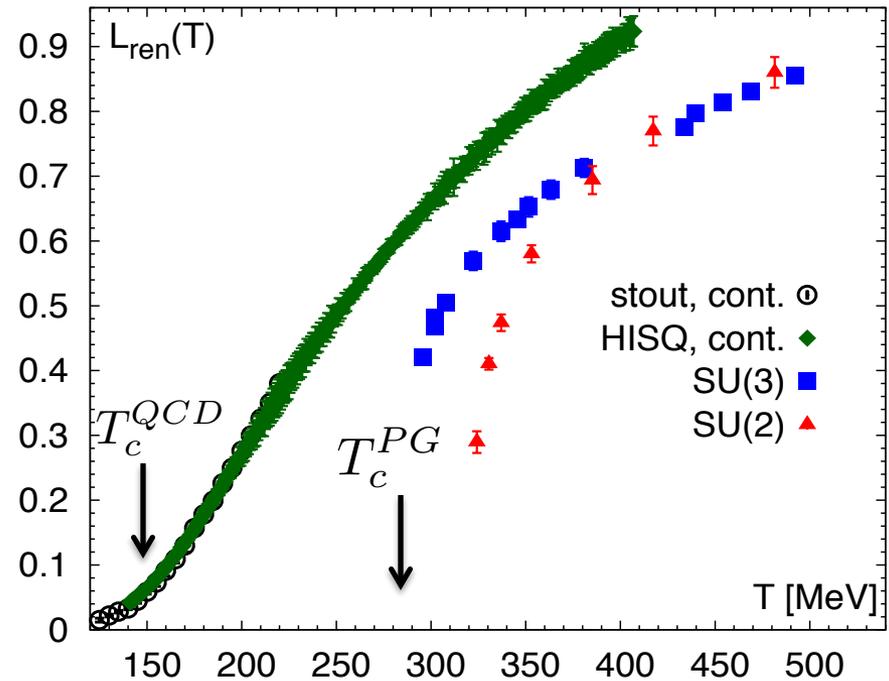
$$L = \mathcal{P} \exp \left(ig \int_0^{1/T} d\tau A_0(\vec{x}, \tau) \right) \quad \exp(-F_{Q\bar{Q}}(r, T)/T) = \frac{1}{9} \langle \text{tr} L(r) \text{tr} L^\dagger(0) \rangle$$

$$F_{Q\bar{Q}}(r \rightarrow \infty, T) = 2F_Q(T) \quad \Rightarrow \quad L_{ren} = \exp(-F_Q(T)/T)$$

2+1 flavor QCD, continuum extrapolated (TUMQCD, to be published)



free energy of static quark anti-quark pair shows Debye screening at high temperatures



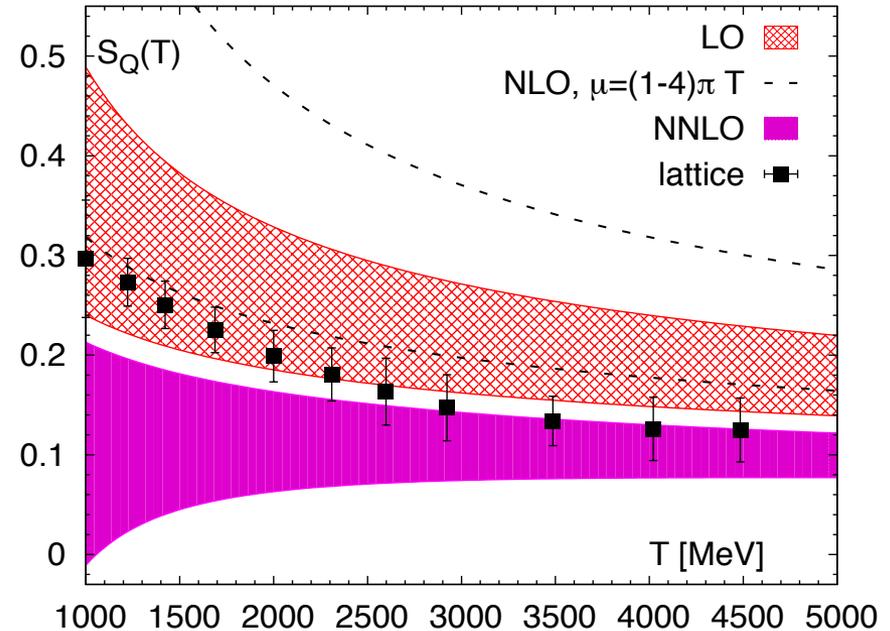
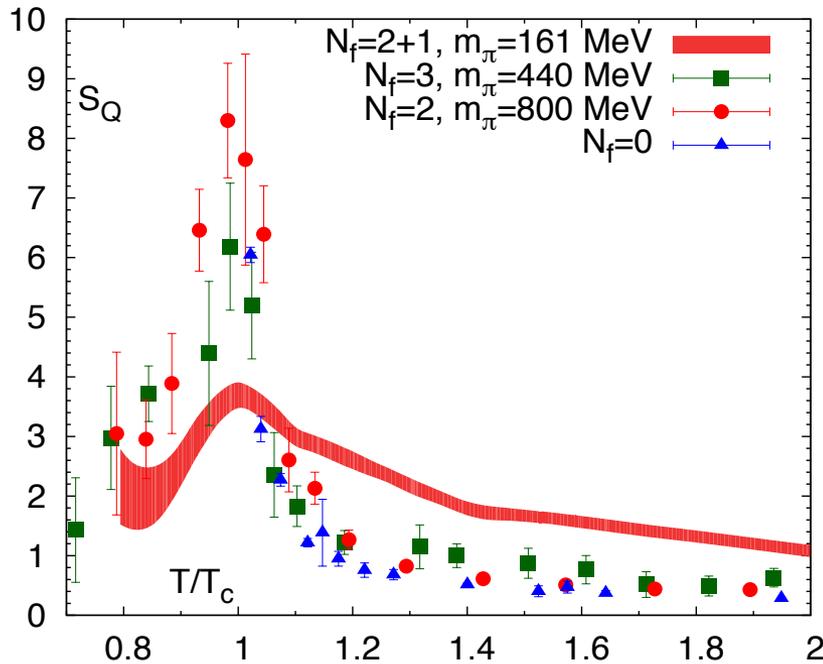
SU(N) gauge theory \neq QCD !

Similar results with stout action Borsanyi et al, JHEP04(2015) 138

The entropy of static quark

TUMQCD, PRD 93 (2016) 114502

$$S_Q = -\frac{\partial F_Q}{\partial T}$$



At high temperature the static quark only “sees” the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

The onset of screening corresponds to peak in S_Q and its position coincides with T_c

The entropy of the static quark has been calculated at NNLO accuracy

Berwein et al, PRD93 (2016) 034010

Weak coupling (EQCD) calculations work only for $T > 1500$ MeV

Casimir scaling of the Polyakov loop

Instead of fundamental representations consider Polyakov loop P_n in arbitrary representation n

PP, Schadler, PRD92 (2015) 094517

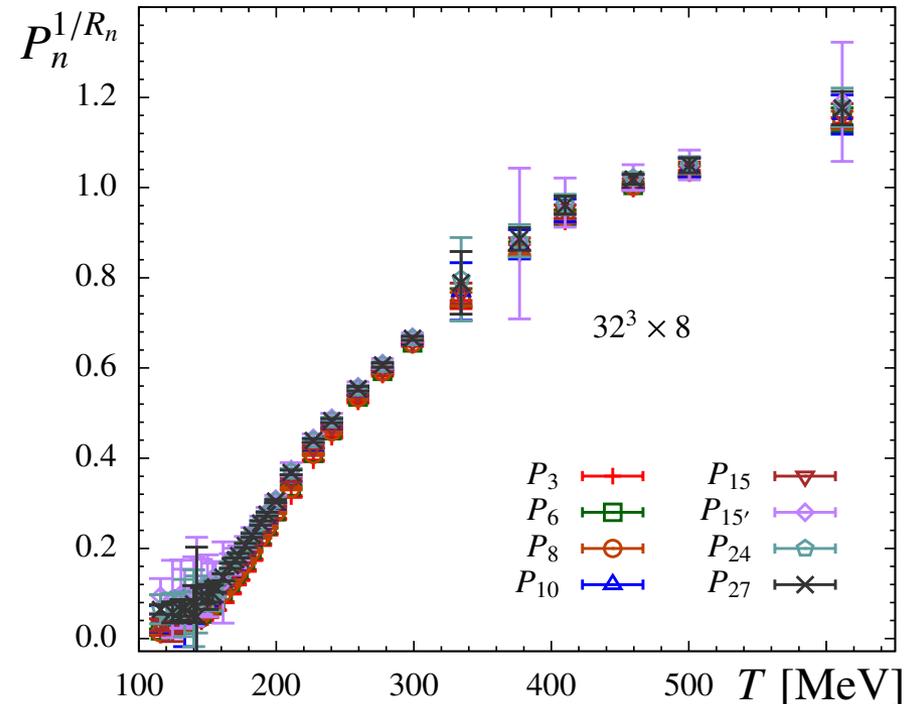
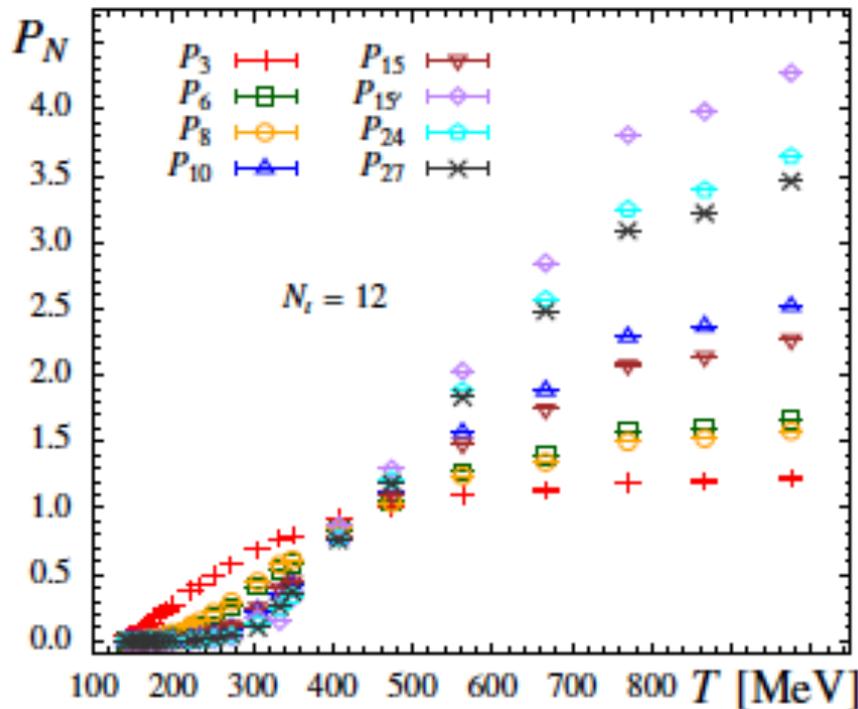
$$P_3 = L_{ren}$$

Use symanzik flow to renormalized the Polyakov loop and reduce the noise

Fodor et al, JHEP 1409 (2014) 018

Casimir scaling: free energy is proportional to quadratic Casimir operator C_n of rep n

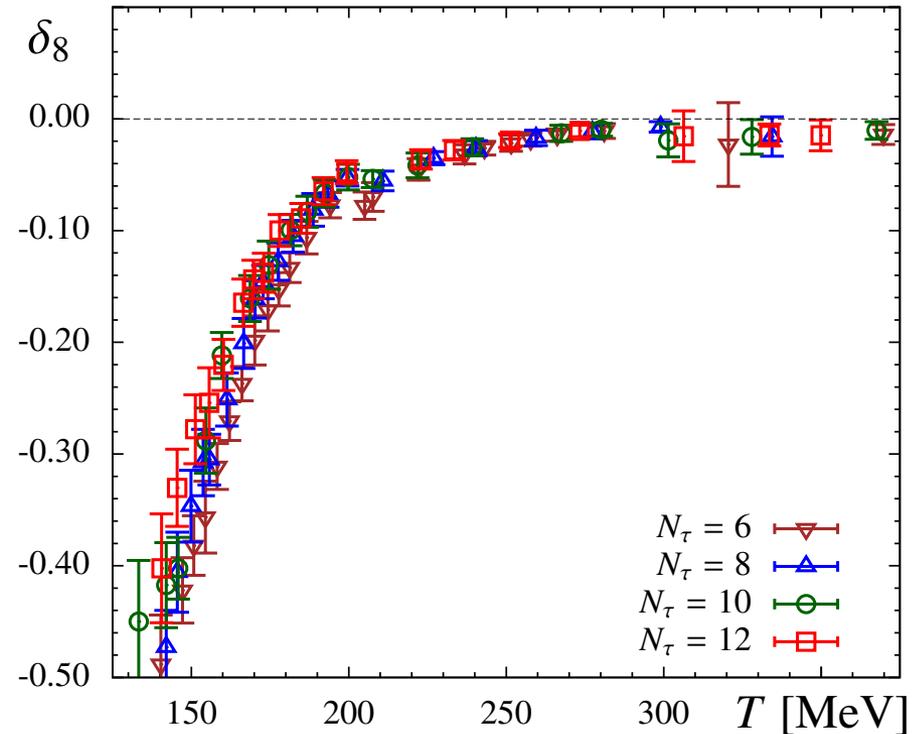
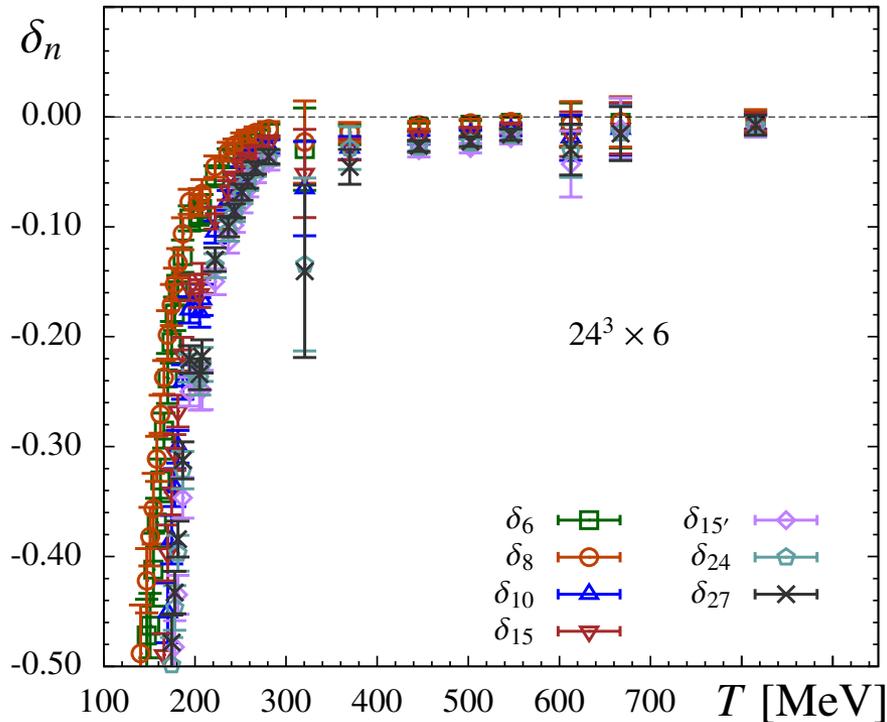
$$R_n = C_n/C_3$$



Expected in weak coupling expansion: e.g. at LO $F_Q^n = -C_n \alpha_s m_D$

Casimir scaling of the Polyakov loop (con't)

$$\delta_n = 1 - P_n^{1/R_n} / P_3$$



Casimir scaling holds for $T > 300$ MeV color screening like in weakly coupled QGP ?

Breaking of Casimir scaling first appear at order α_s^4 in the weak coupling expansion

Berwein et al, PRD93 (2016) 034010

Free energy and singlet free energy of static quark-antiquark

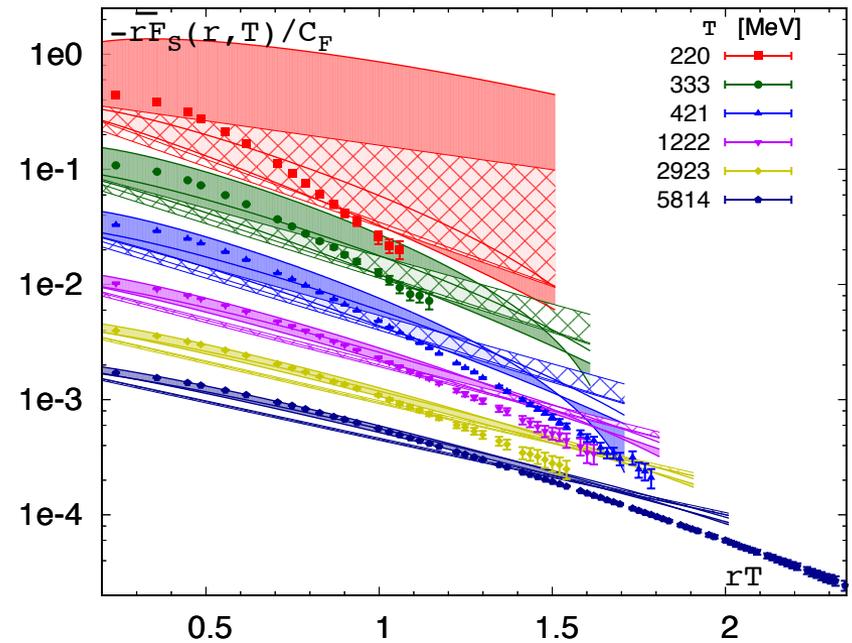
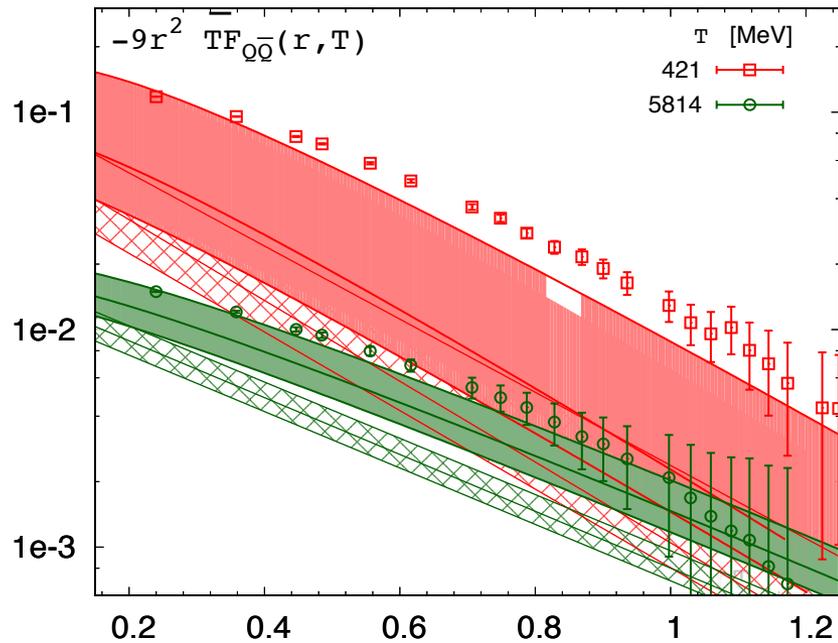
$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9} \langle \text{tr} L(r) \text{tr} L^\dagger(0) \rangle$$

$$e^{-F_S(r,T)/T} = \frac{1}{3} \langle \text{tr} L(r) L^\dagger(0) \rangle$$

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9} e^{-F_S(r,T)/T} + \frac{8}{9} e^{-F_O(r,T)/T}$$

$$\bar{F}_i(r,T) = F_i(r,T) - 2F_Q(T)$$

Coulomb gauge



TUMQCD, to be published

Lattice results are in reasonable agreement with NLO weak coupling result for $rT < 0.6$, at larger distances, non-perturbative effects (due to chromo-magnetic sector) become important

Instanton gas at work ?

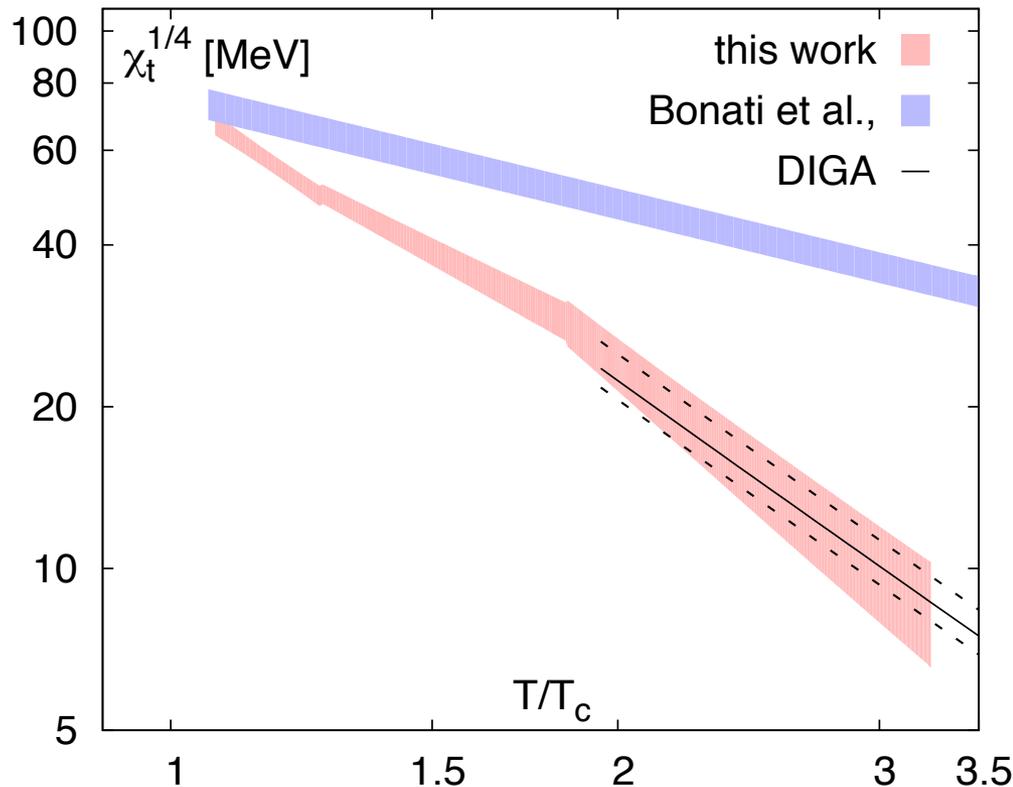
The amount of $U_A(1)$ breaking at high T is reduced because of the reduced instanton density => dilute instanton gas approximation (DIGA), Gross et al, RMP 53 (1981) 43

Topological susceptibility with HISQ action using Symanzik flow

$$\chi_{top} = \frac{1}{V} (\langle Q^2 \rangle - \langle Q \rangle^2)$$

$$\chi_{top}/m_l^2 = \chi_{disc,5} \simeq \chi_{disc}$$

Schadler, Sharma, PP, PLB 762 (2016) 498



DIGA is compatible with the lattice results if a K factor ~ 1.79 is included

Similar K factor was found for SU(3) gauge theory,
Borsányi et al, PLB 752 (2016) 175

Summary

- The deconfinement transition temperature defined in terms of the free energy of static quark agrees with the chiral transition temperature for physical quark mass
- Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, strongly coupled QGP manifest itself as incomplete dominance of quark dof close to T_c
- Charm hadrons can exist above T_c and are the dominant dof for $T < 180$ MeV
- For $T > 300$ MeV weak coupling expansion works well for quark number susceptibilities
- For $T > 300$ MeV Casimir scaling for Polyakov loop for higher representations predicted by weak coupling calculations holds.
- The NLO weak coupling expansion for the r-dependence of the free energy of static quark anti-quark pair agrees with lattice results for $T > 400$ MeV, while the T-dependence of $F_Q(T)$ is described by the weak coupling calculations only for $T > 1500$ MeV
- Dilute instanton gas works for $T > 300$ MeV