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Comments on the validity of the Effective Field Theory approach to physics beyond the Standard Model

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1 Introduction

We consider an EFT where the SM is extended by a set of higher-dimensional operators, and assume that it reproduces the low-energy limit of a more fundamental UV description. The theory has the same field content and the same linearly-realized $SU(3) \times SU(2) \times U(1)$ local symmetry as the SM. The difference is the presence of operators with canonical dimension D larger than 4. These are organized in a systematic expansion in D, where each consecutive term is suppressed by a larger power of a high mass scale Λ. Assuming baryon and lepton number conservation, the Lagrangian takes the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \cdots , \tag{1.1}$$

where each $\mathcal{O}_i^{(D)}$ is a gauge-invariant operator of dimension D and $c_i^{(D)}$ is the corresponding coefficient. Each coefficient scales like a given power of the couplings of the UV theory; in particular, for an operator made of n_i fields one has

$$c_i^{(D)} \sim (\text{coupling})^{n_i - 2} . \tag{1.2}$$

This follows from simple dimensional analysis after restoring $\hbar \neq 1$ in the Lagrangian since couplings, as well as fields, carry $\hbar$ dimensions [1–3] (see also Refs. [4, 5]). An additional suppressing factor $(\text{coupling}/4\pi)^{2L}$ may arise with respect to the naive scaling if the operator is first generated at L loops in a perturbative expansion.

This EFT is intended to parametrize observable effects of a large class of BSM theories where new particles, with mass of order Λ, are much heavier than the SM ones and much heavier than the energy scale at which the experiment is performed. The main motivation to use this framework is that the constraints on the EFT parameters can be later re-interpreted as constraints on masses and couplings of new particles in many BSM theories. In other words, translation of experimental data into a theoretical framework has to be done only once in the EFT context, rather than for each BSM model separately.

The EFT framework is non-renormalizable and therefore it has a limited energy range of validity. In this note we address the question of the validity range at the quantitative level. We will discuss the following points:

- What is the energy range in which the EFT makes sense as a quantum field theory? Under what conditions does it give a faithful description of the low-energy phenomenology of some BSM theory?
- When is it justified to truncate the EFT expansion at the level of dimension-6 operators? To what extent can experimental limits on dimension-6 operators be affected by the presence of dimension-8 operators? Are there physically important examples where dimension-8 operators cannot be neglected?
- When is it justified to calculate the EFT predictions at tree level? In what circumstances may including 1-loop corrections modify the predictions in an important way?

Finally, we will formulate some practical conclusions concerning experimental EFT analyses and presentation of results, such that they can applied to constrain a wider range of UV theories beyond the SM.

It is important to realize that addressing the above questions cannot be done in a completely model-independent way, but requires a number of (broad) assumptions about the new physics. An illustrative example is that of the *Fermi theory*, which is an EFT for the SM degrees of freedom below the weak scale after the W and Z bosons have been integrated out. In this language, the weak interactions of the SM fermions are described by 4-fermion operators of D=6, such as:

$$\mathcal{L}_{\rm eff} \supset \frac{c^{(6)}}{\Lambda^2}\,(\bar{e}\gamma_\rho P_L \nu_e)(\bar{\nu}_\mu \gamma_\rho P_L \mu) + \text{h.c.}\,, \qquad\qquad \frac{c^{(6)}}{\Lambda^2} = -\frac{g^2/2}{m_W^2} = -\frac{2}{v^2}\,. \tag{1.3}$$

This operator captures several aspects of the low-energy phenomenology of the SM, including for example the muon decay, $\mu \to e\nu\nu$, and the inelastic scattering of neutrinos on electrons $\nu e \to \nu\mu$. It can be used to adequately describe these processes as long as the energy scale involved (i.e. the momentum transfer between the electron current and the muon current) is well below $\Lambda = m_W$. However, the information concerning Λ is *not* available to a low-energy observer. Instead, only the scale $\Lambda/\sqrt{|c^{(6)}|} \sim v = 2m_W/g$ is measurable at low energies, which is not sufficient to determine Λ without knowledge of the coupling g. For example, from a bottom-up viewpoint, a precise measurement of the muon lifetime gives indications on the energy at which some new particle (i.e. the W boson) is expected to be produced in a higher-energy process, like the scattering $\nu e \to \nu\mu$, only after making an assumption on the strength of its coupling to electrons and muons. Weaker couplings imply lower scales: for example, the Fermi theory could have ceased to be valid right above the muon mass scale had the SM been very weakly coupled, $g \approx 10^{-3}$. On the other hand, a precise measurement of the muon lifetime sets an upper bound on the mass of the W boson, $m_W \lesssim 1.5$ TeV, corresponding to the limit in which the UV completion is maximally strongly coupled, $g \sim 4\pi$. This example illustrates the necessity of making assumptions (in this case on the value of the coupling g) when assessing the validity range of the EFT, that is, when estimating the mass scale at which new particles appear.

The lack of observation of any new physics prompts us to adopt the EFT approach to set limits on possible deviations from the SM. In such bottom-up approach, the available experimental data are used to determine the confidence intervals on the coefficients of D=6 operators. The validity of this procedure depends on some aspects of the experimental analyses and on the assumptions made on the BSM theory replacing the EFT at the scale Λ. In the rest of this note we discuss these conditions in more detail.

2 General discussion

On a practical level, once the EFT parametrization is adopted, data from any experiment can be used, without further assumptions, to set limits on, or determine, the value of the effective coefficients. As we will shortly discuss, working at the level of $D = 6$ operators is sufficient in the vast majority of cases. In a situation where no new physics effects are observed, the experimental results can be thus expressed into the limits[1]

$$\frac{c_i^{(6)}}{\Lambda^2} < \delta_i^{\rm exp}(M_{\rm cut})\,. \tag{2.1}$$

The functions $\delta_i^{\rm exp}$ depend on the values, here collectively denoted by $M_{\rm cut}$, of the kinematic variables (such as transverse momenta or invariant masses) which set the typical energy scale characterizing the process and which may be subject to cuts in a collider analysis. For example, when EFT is applied to describe inclusive on-shell Higgs decays one has $M_{\rm cut} \approx m_h$. Another example is e^+e^- collisions at a fixed center-of-mass energy $\sqrt{s}$, in which case $M_{\rm cut} \approx \sqrt{s}$. However, for certain physically important processes these considerations are less trivial, especially in the context of experiments in hadron colliders. For example, for production of two on-shell particles in proton-proton collisions, the relevant scale for $M_{\rm cut}$ is the center-of-mass energy of the partonic collision $\sqrt{\hat{s}}$, or equivalently the invariant mass of the final pair, which may or may not be reconstructed in practice. As the energy scale of the process determines the range of validity of the EFT description, it is extremely important that the experimental limits $\delta_i^{\rm exp}$ are reported by the collaborations for various values of $M_{\rm cut}$. For processes occurring over a wide energy range, unlike for inclusive Higgs decays or e^+e^- collisions, knowledge of $\delta_i^{\rm exp}$ for just one kinematic point severely limits interpretation of the EFT results as constraints on specific models beyond the SM. The maximum value of $M_{\rm cut}$ is

[1] In case a deviation from the SM is observed then Eq. (2.1) should be turned into a confidence interval, $\delta_i^{\rm d,exp}(M_{\rm cut}) < \frac{c_i^{(6)}}{\Lambda^2} < \delta_i^{\rm u,exp}(M_{\rm cut})$.

set by the scale $M_{\rm unitarity}$ where the EFT ceases to be a unitary theory, but the larger range $v \lesssim M_{\rm cut} \lesssim M_{\rm unitarity}$ should be considered.

While extracting the bounds on the effective coefficients can be done in a completely model-independent way, determining whether they constrain a non-vanishing portion of the theoretical parameter space requires some further assumption on the (unknown) UV theory. What one needs is a power counting, i.e. a set of rules to estimate the coefficients of the effective operators in terms of the couplings and mass scales of the UV dynamics. This includes, in particular, specifying the selection rules that operate in the low-energy theory. The simplest situation is when the microscopic dynamics is characterized by a single mass scale Λ and a single new coupling g_* [3]. This particular power counting prescription smoothly interpolates between the naive dimensional analysis ($g_* \sim 4\pi$) [2,6], the simple Λ^2 counting with $g_* = 1$ as discussed e.g. in Ref. [7,8], and the very weak coupling limit $g_* \ll 1$. While this is not a unique prescription, it covers a large selection of popular scenarios beyond the SM. In this class falls the Fermi theory described previously, as well as weakly coupled models where a narrow resonance with universal couplings to matter is integrated out. Moreover, despite the large number of resonances, also some theories with a strongly-interacting BSM sector belong to this category (e.g. the holographic composite Higgs models [9] or, more generally, theories where the strong sector has a large-N description). The scaling of the effective coefficients with g_* is then determined by Eq. (1.2) and by selection rules. For example, if the coupling strength of the Higgs boson to the new dynamics is g_*, then the coefficient of an operator with four Higgs fields and two derivatives will scale like g_*^2. On the other hand, approximate chiral symmetry implies that the coefficient of an operator with a fermion scalar bilinear and three Higgs fields scales as $y_f g_*^2$, where y_f is the corresponding Yukawa coupling. Clearly, the naive estimates of the effective coefficients obtained with such power counting agree with what one would find by integrating out heavy particles in a specific BSM model satisfying the initial assumptions.

For a given power counting, it is relatively simple to derive limits on the theoretical parameter space that are automatically consistent with the EFT expansion, provided the relevant energy of the process is known. Consider again the case of a single scale Λ and a single coupling strength g_*. Then the bounds (2.1) can be recast as limits on these two parameters by using the power counting to estimate $c_i^{(6)} = c_i^{(6)}(g_*)$ and setting the relevant energy scale to $M_{\rm cut} < \Lambda$; one finds

$$\frac{c_i^{(6)}(g_*)}{\Lambda^2} < \delta_i^{\rm exp}(M_{\rm cut})\,. \tag{2.2}$$

These inequalities determine the region of the plane $(M_{\rm cut}, g_*)$ which is excluded consistently with the EFT expansion. This gives a useful indication of how effective are the experimental data in constraining the class of theories under consideration (i.e. those respecting the assumed power counting), though Eq. (2.2) should obviously not be regarded as a strict exclusion limit on a specific BSM model. If the relevant energy of the process cannot be determined, because for example the kinematics cannot be closed, setting consistent bounds requires a more careful procedure, such as the ones proposed in Refs. [10] and [11] (see also Refs. [12, 13] for related discussions).

In the region of validity of the EFT expansion, the effects of $D = 8$ operators are typically negligible. If two operators with $D = 6$ and 8 respectively contribute at the tree-level to the same observable, then they typically have the same field content after electroweak symmetry breaking.[2] In this case the $D = 8$ operator must have two more powers of the Higgs field or two more derivatives compared to the $D = 6$ one. Its contribution is thus suppressed by a relative factor equal to, respectively, $(g_* v/\Lambda)^2$ and $(E/\Lambda)^2$, where E is the relevant energy of the process. The EFT series is thus built in terms of these two expansion parameters, which must be both small for the description to be valid. Even when this holds true, the contribution of $D = 8$ operators might be exceptionally enhanced compared to those of $D = 6$ ones due to some structural or even accidental reason. We will discuss this possibility in detail in Section 3.

It is important to notice that the BSM contribution from $D = 6$ operators to a given process might be larger than the SM one without invalidating the EFT expansion. One important example where this occurs is when the D=6 operators contributes to an observable which vanishes or is very strongly suppressed in the SM, for example to lepton-flavor violating Higgs decays, electric dipole moments, etc.). Nevertheless, even if the SM contribution is not suppressed by a small parameter, D=6 contributions may dominate over the SM ones while D=8 operators remain subleading compared to the D=6 ones. This occurs, in particular, if the underlying UV dynamics is strongly coupled, i.e. for $g_* \gg g_{SM}$. Consider for example a $2 \to 2$ scattering process. The SM contribution to the amplitude will be at most of order g_{SM}^2 at high energy, where g_{SM} is a SM coupling. The correction from $D = 6$ operators involving derivatives will in general grow quadratically with the energy and can be as large as $g_*^2(E^2/\Lambda^2)$. When this occurs, for $\Lambda > E > \Lambda\,(g_{SM}/g_*)$ the BSM contribution dominates over the SM one, while the EFT expansion is still valid (hence $D = 8$ operators are subdominant). In this case the largest contribution to the cross section comes from the square of the $D = 6$ contribution, rather than from its interference with the SM. The best sensitivity to $c_i^{(6)}/\Lambda^2$ is thus expected to come from the highest value of the relevant energy scale accessible in the experiment. Notice that although the contributions to the cross section proportional to $(c_i^{(6)})^2$ and $c_i^{(8)}$ are both of order $1/\Lambda^4$, the latter (generated by the interference of $D = 8$ operators with the SM) has a relative suppression of order $(g_{SM}/g_*)^2$ independently of the energy, and can thus be safely neglected. A well known process where the above situation occurs, i.e. where the energy-growing contribution from $D = 6$ operators can dominate over the SM one, is the scattering of longitudinally-polarized vector bosons. Similarly, depending on the UV dynamics, the same can happen in other $2 \to 2$ scatterings, such as Higgs associated production with a W or Z boson (VH) or dijet searches at the LHC. An illustrative example is discussed in Appendix A.

It is worth discussing at this point one technical issue regarding the contribution of D=6

[2] This is not true if different particles may contribute in an intermediate state, as is the case for example in $h \to 4\ell$ decays, however these exceptions are not relevant for the following discussion.

and D=8 coefficients to the likelihood used to derive the results. We have seen that when the UV theory is strongly coupled and the deviations from the SM are large, the $D = 6$ squared term dominates the cross section, while the $D = 8$ one is suppressed by a ratio of weak to strong couplings. The same holds true in computing the likelihood of course. If instead the deviations from the SM predictions are small, [3] the $D = 6$ quadratic terms can be neglected in the cross section but should be retained in the likelihood. This can be easily seen as follows. A cross section σ (or any other experimentally measured observable) can be schematically written as

$$\sigma \simeq \sigma_{\rm SM}\left(1+2\frac{\delta^{(6)}}{A_{\rm SM}}c^{(6)}+2\frac{\delta^{(8)}}{A_{\rm SM}}c^{(8)}+\left(\frac{\delta^{(6)}}{A_{\rm SM}}c^{(6)}\right)^2+\cdots\right) \tag{2.3}$$

where $A_{\rm SM}$ and $\sigma_{\rm SM}$ denote, respectively, the SM amplitude and SM cross section, while $\delta^{(6)} \sim O(E^2/\Lambda^2)$, and $\delta^{(8)} \sim O(E^4/\Lambda^4)$ parametrize the effect of higher-dimensional operators. We have shown terms up to $O(1/\Lambda^4)$, denoting those further suppressed with the dots. The χ^2 function (again, schematically) has the form:

$$\begin{aligned}\chi^2 \propto (\sigma-\sigma_{\rm exp})^2 &= (\sigma_{\rm SM}-\sigma_{\rm exp})^2+4\sigma_{\rm SM}\left(\sigma_{\rm SM}-\sigma_{\rm exp}\right)\left(\frac{\delta^{(6)}}{A_{\rm SM}}c^{(6)}\right)+4\sigma_{\rm SM}^2\left(\frac{\delta^{(6)}c^{(6)}}{A_{\rm SM}}\right)^2\\ &+4\sigma_{\rm SM}\left(\sigma_{\rm SM}-\sigma_{\rm exp}\right)\left(\frac{\delta^{(8)}}{A_{\rm SM}}c^{(8)}\right)+2\left[\sigma_{\rm SM}\left(\sigma_{\rm SM}-\sigma_{\rm exp}\right)\right]\left(\frac{\delta^{(6)}c^{(6)}}{A_{\rm SM}}\right)^2+\cdots,\end{aligned} \tag{2.4}$$

where $\sigma_{\rm exp}$ is the experimentally measured value of the cross section, and the dots stand for $O(1/\Lambda^6)$ terms. The dimension-8 term in the second line enters formally at the same order $1/\Lambda^4$ as the one proportional to $(c^{(6)})^2$ in the first line, but it can be always neglected within the EFT validity regime where $c^{(6)} \ll c^{(8)}E^2/\Lambda^2$. Indeed, in the strong coupling regime, $g_* \gg 1$, $(c^{(6)})^2$ is dominant, while for a weak coupling, $g_* \lesssim 1$, the multiplicative factor $(\sigma_{\rm SM}-\sigma_{\rm exp})$ is small and effectively scales like $1/\Lambda^2$. Similarly, the $(c^{(6)})^2$ term in the second line is multiplied by $(\sigma_{\rm SM}-\sigma_{\rm exp})$ and can be neglected in this regime. On the contrary, the $(c^{(6)})^2$ term in the first line is not suppressed and in fact it should be retained to ensure that the χ^2 has a local minimum. It is also easy to show that including the term proportional to $c^{(8)}$ affects the best fit value of $c^{(6)}$ only by an amount of $O(E^2/\Lambda^2)$. [4] We thus conclude that while dimension-8 operators can be neglected, square terms from $D = 6$ should be retained.

Our discussion so far was limited to tree-level effects of $D = 6$ operators. The EFT can be consistently extended to an arbitrary loop order by computing observables perturbatively in the SM couplings. The corresponding series is controlled by the expansion parameter

[3] This can occur either because the UV theory is weakly coupled or because, despite strong coupling, the new physics scale is much higher than the energy probed by the experiment.

[4] Indeed, minimizing Eq. (2.4) with respect to $c^{(6)}$, one finds, schematically,

$$c^{(6)} \simeq \frac{\sigma_{\rm exp}-\sigma_{\rm SM}}{\sigma_{\rm SM}}\frac{A_{SM}}{\delta^{(6)}}-c^{(8)}\frac{\delta^{(8)}}{\delta^{(6)}}\,. \tag{2.5}$$

$g_{SM}^2/16\pi^2$, which adds to the two EFT parameters $(g_*^2 v^2/\Lambda^2)$ and E^2/Λ^2 already discussed. One-loop effects of D=6 operators are formally suppressed by $O(g_{SM}^2/16\pi^2)$, and are thus subleading compared to the tree-level contributions (some exceptions are discussed in the next subsection). Note that including loop corrections in the EFT context is less crucial than for a pure SM calculation. This is because the experimental precision is typically better than the magnitude of the SM loop corrections, therefore including loop corrections is essential to obtain a correct description of physical processes. In the case of the EFT, we are yet to observe any *leading-order* effect of higher-dimensional operators. There do exist situations, however, where including NLO corrections may be important for obtaining an adequate description of physical processes in the EFT. For example, it is well known that NLO QCD corrections to the SM predictions of certain processes at the LHC can be of order 1, and then large k-factors are expected to apply to the EFT corrections as well. Another example is the one-loop Higgs corrections to well-measured electroweak precision observables [14, 15]. Since deviations of the Higgs couplings due to D=6 operators can be relatively large (up to $O(10\%)$) without conflicting with current experimental data, the 1-loop effects, in spite of the suppression factor, can be numerically important for observables measured with a per-mille precision. Next, NLO effects are expected to be more important for observables that arise at one loop in the SM, such as $h \to \gamma\gamma$. Finally, for LHC observables such as Higgs p_T that scan a large energy range the running of Wilson coefficients may lead to sensitivity to several different combinations of Wilson coefficients, which may reduce parameter degeneracies [16]. The calculation of NLO effects in the context of EFT is currently an active field of study, see e.g. [8, 17–26]. It is very important to identify all cases where 1-loop effects of D=6 operators can be relevant.

To summarize, we have argued that, generically, the use of a tree-level EFT truncated at the level of D=6 operators provides an adequate description of a large class of BSM models. We recommend using this framework as one of the approaches to interpret the Higgs data at the LHC. One important conclusion from this discussion is that the cut-off scale Λ is an integral part of EFT's formulation, but its value cannot be directly determined from low-energy experiments. Therefore, results should be presented by the experimental collaborations as a function of the kinematic variables, here collectively denoted with $M_{\rm cut}$, which set the relevant energy of the process. For the purpose of estimating the validity of the EFT approach, it is useful to compare the EFT constraints obtained with and without including the quadratic contributions of D=6 operators in the theoretical calculations of observables. Significant differences between these two procedures will indicate that the results apply only in the case of strongly-coupled UV theories, where quadratic terms can give the dominant effect at large energies. Finally, we argued that quadratic terms from D=6 operators should always be retained in the calculation of the likelihood function.

3 Limitations of D=6 EFT

The SM Lagrangian extended by D=6 operators is an effective theory that captures the low-energy regime of a large class of models with new heavy particles. However, not every such model can be adequately approximated by truncating the EFT expansion at the D=6 level. In this section we discuss these special cases where a more complicated approach is in order, or where the EFT approach fails completely.

As argued in Section 2, generically we expect that the effect of D=8 operators is subleading compared to that of D=6 ones at energies $E \ll \Lambda$, with $\Lambda \gg m_W$. On the other hand, if $E \sim \Lambda$, the entire tower of operators (D=8, D=10, etc.) contributes, and the EFT expansion is not useful. Nevertheless, there are physical situations when D=8 operators can be relevant, despite the whole EFT expansion being convergent. We identify the following cases:

- **Symmetries**
  A suppression of the D=6 operators can arise as the result of some selection rules. One can envisage situations in which symmetries of the UV theory forbid or suppress certain D=6 operators but not D=8 ones. An example occurs in models with a pseudo Nambu–Goldstone boson Higgs, where dim-6 and dim-8 operators contributing to Higgs pair production via gluon fusion are generated by different mechanisms [30]. In particular, the dim-6 operator $|H|^2 G^a_{\mu\nu} G^{a,\mu\nu}$ is suppressed because it violates the shift symmetry $H \to H + \alpha$ (which is part of the Goldstone symmetry). On the other hand, two D=8 operators with extra derivatives can be constructed (one of them being $D_\lambda H^\dagger D^\lambda H G^a_{\mu\nu} G^{a,\mu\nu}$) which respect the shift-symmetry and whose coefficients are therefore not suppressed. As a consequence, in the energy range $\Lambda\sqrt{c^{(6)}/c^{(8)}} < E < \Lambda$ the contribution from D=8 operators dominates over that from D=6 ones but the EFT expansion is still valid. A similar situation can occur for composite gauge bosons if their dipole interactions involving, e.g., the abelianized $SU(2)_L$ field strength $\hat{W}^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ are characterized by a strong coupling, while the monopole interaction associated with the covariant derivative is associated with the SM weak coupling g [31]. This is technically natural, because the symmetry group which accompanies the field strength is $SU(2)^{global}_L \times U(1)^3_{local}$ and differs from the one associated with the covariant derivative, $SU(2)^{local}_L$. Then, since $\hat{W}^a_{\mu\nu}$ has dimension two, this structure implies, for instance, that the first strong correction to $VV \to VV$ scattering comes from a dimension-8 operator of the form $\hat{W}^4_{\mu\nu}$. Finally, the same can also happen for fermions if they are identified with the goldstino of N spontaneously broken symmetries [32]. In this case the first interactions respecting supersymmetry arise at dimension 8 and include self interaction of the form $\bar{\psi}^2\partial^2\psi^2$ [39].

- **Zero at leading order**
  For certain processes the contribution to the scattering amplitude from D=6 operators, as well as perhaps the SM one, vanishes and the first non-trivial correction appears only at the D=8 level. One well known example is the s-channel production of neutral gauge

boson pairs. Such a process does not occur in the SM nor in the D=6 EFT because triple gauge couplings of neutral gauge bosons arise only from $D \geq 8$ operators. This category includes also processes, such as all scatterings with transverse gauge bosons, where, because of the helicity structure of the amplitudes, the dim-6 operators do not interfere with the SM but the dim-8 ones do [33]. Another example is the triple Higgs production by vector boson fusion whose energy-growing piece originates from $D \geq 8$ operators only [34].

- **Hierarchy of sensitivity**
  The present experimental constraints on physics beyond the SM display a hierarchical structure. For example, electroweak precision observables were measured by LEP-1 with a per-mille accuracy, while for the LHC Higgs observables are currently measured with an $O(10\%)$ accuracy at best. There exist UV theories where integrating out new particles generates D=6 operators affecting Higgs physics, but only D=8 operators affecting electroweak precision observables. One example is a theory with an $SU(2)_L$ triplet of vector resonances coupled only to the Higgs current and not coupled to SM fermions. In such a case, D=8 operators may be phenomenologically as important as the D=6 ones, and both should be retained.

- **Fine-tuning**
  One can imagine a fine-tuned situation where integrating out the heavy states in the UV theory generates D=6 operators with coefficients that are accidentally much smaller than their naive estimate and much smaller than those of the D=8 operators, $c_i^{(6)} \ll c_i^{(8)}$. In such a case, the EFT with only D=6 operators will not correctly approximate the dynamics of the UV theory, and some D=8 operators need to be included for an adequate description. By nature, naive dimensional analysis and simple power counting are just not suited when some parameters are accidentally small. Notice that, contrary to the structural hierarchies described in the previous points, finely tuned or accidental hierarchies are not stable under RGE evolution.

In summary, there do exist physical situations where the inclusion of dimension-8 operators on top of dimension-6 ones is well motivated and where, nonetheless, the EFT expansion remains well defined. This does not mean, however, that introducing a complete set of D=8 operators into EFT analyses is preferable in practice. Indeed, such a framework would be utterly complicated, and, moreover, the existing experimental data do not contain enough information to lift the degeneracy between D=6 and D=8 operators. Instead, it is recommended to focus on the (already challenging) EFT with D=6 operators and address case-by-case the special situations discussed above.

4 Summary

In this short note we discussed the validity of an EFT where the SM is extended by D=6 operators. One important message is that the validity range cannot be determined using only low-energy information. The reason is that while the EFT is valid up to energies of order of the mass Λ of the new particles, low-energy observables depend on the combinations $c^{(6)}/\Lambda^2$, where the effective coefficient $c^{(6)}$ is a function of the couplings of the UV theory. Furthermore, relative contributions of the D=8 operators, $\frac{c^{(8)}}{c^{(6)}}\frac{E_{\rm exp}^2}{\Lambda^2}$, depend on the assumptions about the UV theory. Only when a particular power counting is adopted, such as the g_*-counting discussed in this note, can these relative contributions be estimated in the bottom-up approach. Note that similar issues regarding validity of the expansion may arise also in the context of LHC pseudo-observables [35] and are addressed in a similar fashion, although details of the power-counting may be different in that case.

The practical conclusion is that the experimental constraints on the Wilson coefficients of D=6 operators be reported as functions $M_{\rm cut} < \Lambda$, where $M_{\rm cut}$ is the limiting value of relevant kinematic variables such as transverse momenta or invariant masses. This is especially important for hadron collider experiments such as those performed at the LHC, where collisions probe a wide range of energy scales. The maximum possible value of $M_{\rm cut}$ is set by the scale $M_{\rm unitarity}$ where the unitarity is lost within the EFT ceases, but the larger range $v \lesssim M_{\rm cut} \lesssim M_{\rm unitarity}$ should be explored. Furthermore, the results should be presented both with and without taking account the quadratic contributions in the Wilson coefficients to the measured cross sections and decay widths. With this way of presentation, the experimental results can be applied to constrain a larger class of theories beyond the SM in a larger range of their parameter space. Other frameworks to present results, for example template cross-sections discussed elsewhere, should also be pursued in parallel, as they may address some of the issues discussed in note.

The energy at which the EFT breaks down typically coincides with the scale where the contribution of D=8 and higher-dimensional operators is of the same order as that of D=6 operators. Conversely, when the EFT expansion is well convergent at the LHC energies, the effects of D=8 operators can be normally neglected. Exceptions from this rule may arise as a consequence of selection rules or for certain well-defined classes of processes. The inclusion of D=8 operators in experimental analyses is justified only when dealing with these special cases, and it is unnecessary and would represent an inefficient strategy in a generic situation. If no large deviations from the SM are observed at the LHC Run-2, stronger constraints on D=6 operators can be set. This will extend the EFT validity range to a larger class of UV theories and, for a fixed $E_{\rm exp}$, leave less room for contributions of D=8 operators. As a consequence, the internal consistency and the validity range of the LO D=6 EFT will only increase. [5] On the other hand, if a deviation from the SM is observed, efforts to include EFT

[5] The validity range can also be improved by means of a global analysis combining different measurements, which often lifts flat directions in the parameter space and leads to stronger constraints on D=6 Wilson coefficients, see e.g. [36].

loop corrections and to estimate the effects of $D > 6$ operators will be crucial to improve characterization of the underlying UV theory.

A Appendix: Example

For our example, we consider the SM extended by a triplet of vector bosons V_μ^i with mass Λ transforming in the adjoint representation of the SM $SU(2)_L$ symmetry. Its coupling to the SM fields is described by [37,38]

$$\mathcal{L} \supset \kappa_H \frac{ig}{2} V_\mu^i H^\dagger \sigma^i \overleftrightarrow{D_\mu} H + \kappa_q \frac{g}{2} V_\mu^i \bar{q}_L \gamma_\mu \sigma^i q_L, \tag{1.1}$$

where $q_L = (u_L, d_L)$ is a doublet of the 1st generation left-handed quarks. In this model V_μ^i couples to light quarks, the Higgs boson, and electroweak gauge bosons, and it contributes to the $q\bar{q} \to Vh$ process at the LHC. Below the scale Λ, the vector resonances can be integrated out, giving rise to an EFT where the SM is extended by D=6 and higher-dimensional operators. Using the language of the Higgs basis, the EFT at the D=6 level is described by the parameter δc_z (correction to the SM Higgs couplings to WW and ZZ) and δg_L^{Zq} (corrections to the Z and W boson couplings to left-handed quarks), plus other parameters that do not affect the $q\bar{q} \to Vh$ process at tree level. The relevant EFT parameters are matched to those in the UV model as

$$\delta c_z = -\frac{3m_W^2}{2\Lambda^2}\kappa_H^2, \qquad [\delta g_L^{Zu}]_{11} = -[\delta g_L^{Zd}]_{11} = -\frac{m_W^2}{2\Lambda^2}\kappa_H \kappa_q \,. \tag{1.2}$$

When these parameters are non-zero, certain EFT amplitudes grow as the square of the center-of-mass energy s of the analysed process, $\mathcal{M} \sim s/\Lambda^2$. Then, for a given value of the parameters, the observable effects of the parameters become larger at higher energies. However, above certain energy scale, the EFT may no longer approximate correctly the UV theory defined by Eq. (1.1), and then experimental constraints on the EFT parameters do not provide any information about the UV theory.

To illustrate this point, we compare the UV and EFT descriptions of $q\bar{q} \to Vh$ for three benchmark points:

- **Strongly coupled:** $\Lambda = 6$ TeV, $\kappa_H = \kappa_q = 3$;
- **Moderately coupled:** $\Lambda = 2$ TeV, $\kappa_H = \kappa_q = 1$;
- **Weakly coupled:** $\Lambda = 1$ TeV, $\kappa_H = \kappa_q = 1/2$;

Clearly, all three benchmarks lead to the same EFT parameters at the D=6 level. However, because Λ varies, these cases imply different validity ranges in the EFT. This is illustrated in Fig. 1. In all cases, the EFT is valid near the production threshold, but above a certain energy $E_{\max}$ the EFT is no longer a good approximation of the UV theory. Clearly, the value of $E_{\max}$ is different in each case. For the moderately coupled case, it coincides with the energy

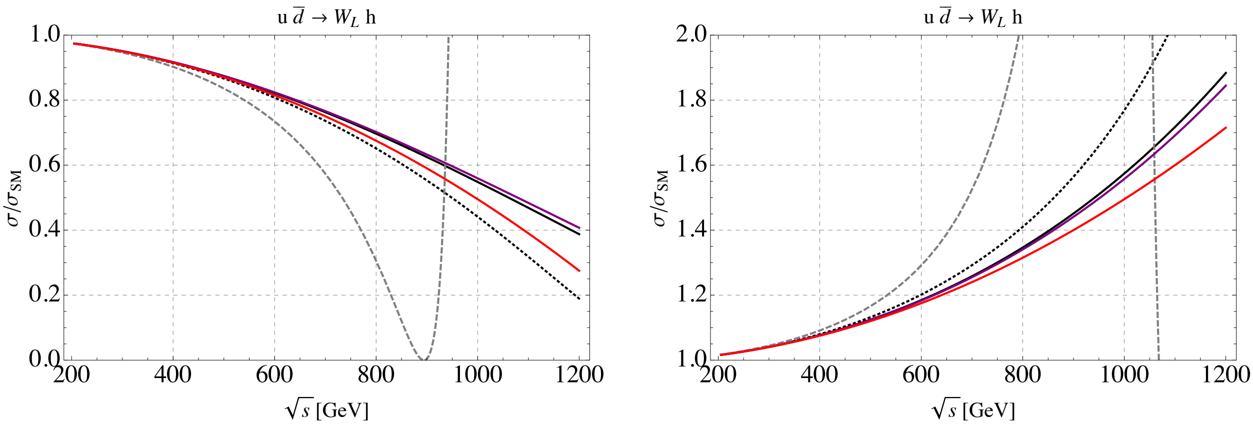

Figure 1: *Left:* The partonic $u\bar{d} \to W^+h$ cross section as a function of the center-of-mass energy of the parton collision. The black lines correspond to the $SU(2)_L$ triplet model with $m_V = 1$ TeV, $\kappa_H = \kappa_q = 1/2$ (dashed), $m_V = 2$ TeV, $\kappa_H = \kappa_q = 1$ (dotted), and $m_V = 6$ TeV, and $\kappa_H = \kappa_q = 3$ (solid). The corresponding EFT predictions are shown in the linear approximation (red), and when quadratic terms in D=6 parameters are included in the calculation of the cross section (purple). *Right:* The same, for the sign of κ_q flipped, so that the production rate is enhanced rather than suppressed compared to the SM.

at which the linear and quadratic EFT approximations diverge. From the EFT perspective, this happens because D=8 operators can no longer be neglected. However, for the strongly coupled case, the validity range extends beyond that point. In this case, it is the quadratic approximation that provides a better approximation of the UV theory. As discussed in the main text, that is because, for strongly-coupled UV completions, the quadratic contribution from D=6 operators dominate over that of D=8 operators in a larger energy range.

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